On the negation of indicative conditionals*

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Abstract

A debated aspect of the analysis of indicative conditionals of the form "if A then C" concerns whether they have as their negation the conjunction "A and not C" or the conditional negation "if A then not C". We argue that neither theory is adequate, but that both forms of negation can be pragmatically retrieved from a Kratzer-style analysis of conditionals in which the negation of "if A then C" is equivalent to the weak negation "if A possibly not C". This paper lays out the relevant pragmatic hypotheses and presents the results of one experimental study intended to test those predictions.

1 Negating conditionals

By an indicative conditional sentence, we mean an *if-then* sentence in which both the antecedent and the consequent are in the indicative mood, as in the following examples:

(1) If John was in Paris, then Mary was in New York.

(2) If John visits Paris tomorrow, then Mary will be pleased.

(3) If this figure is a rectangle, then it is a square.

The question we are investigating is how the insertion of a sentential negation operator (such as "it is not the case that") is understood, and more specifically how the denial of such sentences is expressed (as in a response starting by "No,..."), depending on the sentence and on the context.

A large part of the recent literature on the negation of conditionals has been focused on the opposition between two families of theories: on the one hand accounts based on the material conditional analysis, predicting the negation of "if A then C" to be the conjunction "A and not C" (see [6], [8], [10], [9]), and on the other suppositional theories predicting the negation to be the conditional negation "if A then not C" (including possible-world theories [20], trivalent theories [15], [1], and probabilistic theories [2], [4], [16], [5]).[1] Several experiments have been conducted in recent years by psychologists of reasoning to advance this debate (viz. [7], [3], [11]), indicating a preference for conditional negation, but with systematic exceptions (for example, [7] report differences depending on whether the conditional sentence is in the past tense, or the future tense).

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[1] Suppositional theories are all inspired from Ramsey's remarks in[18], who already concluded, from what has since been called the Ramsey Test: "in a sense If p, q and If $p, \neg q$ are contradictories".

Our claim is that the debate has been unduly restricted to that opposition. In particular, most accounts of the psychology of conditionals have ignored a third family of accounts, predicting the negation of a conditional sentence to be equivalent to "if A, possibly not C' (viz. [13], [12], [14] and other analyses in the family of strict conditionals, without the uniqueness assumption). In our view, this modal negation is the baseline negation for all indicative conditionals, but the other two kinds of negations are pragmatically recoverable from it based on additional features of the context and on constraints regarding the information shared between the participants in a conversation. In what follows we first give a more precise operationalized version of those hypotheses, and then present the result of one experimental study in which they were tested.

2 Basic framework

Our approach assumes a version of Krater's modal account of conditionals, which provides a compositional version of Lewis's theory. Specifically, Lewis predicts the following relations between the negation of a conditional, the conditional negation of the consequent, and the conjunction of the antecedent with the negation of the consequent (">" stands for Lewis's conditional connective):

(4) $A \wedge \neg C \models (A > \neg C) \models \neg(A > C)$

The first sentence is true provided A is true and C false, the second provided some $A\neg C$-world is closer than any AC-world (or if there are no possible A-world), and the third if for every AC-world there is an $A\neg C$-world at least as close. Moreover, $\neg(A > C)$ is in Lewis's theory exactly a way of expressing "if A then it might be that not C" ([13]:21), namely a conditional negation in which only the possibility of the consequent is asserted conditional on the antecedent.[2]

Assuming this pattern of asymmetric entailments, standard pragmatic considerations suggest that the semantic negation of a conditional would by default be the weakest negation, but that in some contexts, it may be more informative to pick a stronger negation provided some additional information is at hand. Specifically, we are making the following two hypotheses:

(1) the conjunctive negation "A and not C" of a conditional of the form "if A then C" should be favored over a conditional negation ("if A then not C", or "if A, it might be that not C") only if the speaker is sufficiently informed or sufficiently confident about both the truth status of the antecedent and the consequent. In particular, in case the speaker is uncertain about the antecedent, she should preferably use a conditional negation.

(2) among conditional negations, the strong negation "if A then not C" should be favored over the weak negation "if A then it might be that not C" only if the speaker has sufficient information or sufficient reason to believe that the possibilities described by the antecedent exclude the possibilities described by the consequent.

For example, assuming geometric competence, and for a context in which one is reasoning about a figure about whose shape the only information one has is that it is a polygon, one can predict from (1) the fact that it would be too strong to deny "if this figure is a rectangle, it is a triangle" by "this figure is a rectangle and it is not a triangle". From (2) one can predict that it would be too weak to deny the same conditional sentence by "if this figure is a rectangle,

[2] Within Kratzer's framework, which we detail in the more extended version of this paper, the occurrence of "might" in negated conditionals is directly predicted, instead of stipulated.

it might not be a triangle", but also that it would be too strong to deny "if this figure is a rectangle, then it is a square" by "if this figure is a rectangle, then it is not a square".

3 Experimental hypotheses

Our proposal relies on a more specific typology of the denials of conditional sentences of the form "if A then C". First, we distinguish between *conjunctive* and *conditional* denials, depending on whether the antecedent is unconditionally asserted or not; and secondly, we distinguish between *weak* and *strong* denials, depending on whether the negation of the consequent is under the scope of a possibility operator (*possibly/need/may not*), or equivalently, scoping above a necessity operator (*not necessarily*). One substantial assumption we are making is that, when no overt modality is used to mark a weak negation, we are dealing with a strong denial. The typology is illustrated in Table 1.

	Weak	**Strong**
Conjunctive	'A and possibly not C'	'A and not C'
Conditional	'if A then possibly not C'	'if A then not C'

Table 1: Four types of denials

In the experiment we present in the next section, we each time presented our participants with a dialogue between two interlocutors, one of whom is asserting a conditional, and the other denying it, starting a sentence with 'No...' (following a methodology first used by [17] from an unpublished pilot study reported therein). The purpose of the experiment was to elicit negations of conditional sentences in order to get a comparative measure of the occurrence of each negation depending on the environment.

The main relational predictions we tested for were the following:

(1) The higher the unconditional probability of the antecedent, the more we expect to see conjunctive negations. This corresponds to the idea that a conjunctive negation will be favored if the contradictor is in a position to know the truth of the antecedent.

(2) The lower the conditional probability of the consequent given the antecedent, the more we expect to see strong negations. This corresponds to the idea that a strong negation will be favored if the contradictor is in a position to assume that the antecedent excludes the consequent.

In the experiment, we manipulated two main variables. One variable concerns the probability of occurrence of the antecedent; the other concerns the conditional probability of the consequent given the antecedent. Both probabilities are presented as transparent to the contradictor of the conditional, but not necessarily to the assertor of the conditional, who is always presumed uncertain about the realization of the antecedent.

An important caveat is that we never presented the antecedent as having probability zero (it is never an impossible event). Because of that, we did not expect to see a fifth form of denial of the conditional in terms of the plain negation of the antecedent. Suppose for example that I am talking about a game that happened yesterday and say: "if the Reds won, then it must have been a terrific party". If the hearer knows that the Reds did not win, then the most natural way of rejecting the sentence is to update my information by saying: "actually, the Reds did not win". Such cases would be easy to incorporate as controls, but were deliberately set aside.

	P(if square then black)			
P(square)	0	1/6	3/6	5/6
1/4	□□□□□□ ●●●○○○ ●●●○○○ ●●●○○○	■□□□□□ ●●●○○○ ●●●○○○ ●●●○○○	■■■□□□ ●●●○○○ ●●●○○○ ●●●○○○	■■■■■□ ●●●○○○ ●●●○○○ ●●●○○○
3/4	□□□□□□ ●○	■□□□□□ ●○	■■■□□□ ●○	■■■■■□ ●○
1	□□□□□□	■□□□□□	■■■□□□	■■■■■□

Table 2: The 12 conditions defined by the combination of the probability of the conditional sentence and the probability of its antecedent.

4 Experimental Study

4.1 Method

4.1.1 Material and design

We compared denials for two kinds of sentences, a conditional sentence of the form "if A then C", and a control consisting of the categorical assertion of the consequent "C". Each sentence was administered to a different group of subjects.

The conditional sentence referred to a chip drawn at random from a display of chips of different colors and shapes. The target sentence was "if the chip is square, it will be black" (here abbreviated as: 'if A, C'). The probability of the conditional $P(C|A)$ and the probability of its antecedent $P(A)$ were determined by choosing specific numbers of chips in each color and shape. There was always a first row of six square chips with 0, 1, 3, or 5 black chips allowing the values of 0, 1/6, 3/6, and 5/6 for $P(C|A)$. To determine the values of $P(A)$ an appropriate number of round chips (half of them black in all the cases) was added to the first row. $P(A)$ was set to 1/4, 3/4, or 1 by adding 18 chips, two chips, or none, respectively. In brief, there were four values for $P(C|A)$ crossed with three values for $P(A)$, hence twelve conditions (see Table 2).

The categorical sentence was "the chip will be black". Only one row of six chips was presented, identical with the $P(A) = 1$ conditions of the conditional group, that is, zero, one, three, or five black chips out of 6, defining four levels for the probability of the target sentence.

In summary, the design was mixed. In the conditional group there were two factors: the probability for the sentence to be true (four levels) crossed with the probability of the antecedent (three levels). In the categorical group only the first factor with the same four levels was considered.

To get the expression of the denial we used the following scenario reproduced here in full (translated from French):

> Pierre and Marie own a set of chips that consists of black chips and white chips that can be square or round. They have displayed the following chips :
>
> [there follows one specific display for each condition]

Someone is going to draw a chip at random.
Pierre says : [conditional group] "if it is a square chip, it will be black".
[categorical group] "the chip will be black".

Marie disagrees. She wants to contradict Pierre.
But the place is noisy. Here are the only words that can be heard, following the order in which Marie has uttered them: [conditional group] *No; square; chip; black.*
[categorical group] *No; chip; black.*

Your task : Observe the chips carefully, then restore the missing words to make a full sentence. It is likely that Marie said: "No !.." Please fill in the dots.

4.1.2 Participants

Participants were recruited from the same pool of students as in the first experiment (in which none of them had participated). They were randomly allocated to one of the two groups, and within the conditional group, to one condition. They received the scenario on a sheet of paper. Nonnative speakers of French were discarded, leaving 264 answers (conditional group N = 198; categorical group N = 66).

4.2 Results

4.2.1 Classification of denials

A substantial proportion of participants took an argumentative point of view (29%), giving reasons to indirectly predict the outcome instead of giving direct predictions. The great majority of these justifications were quantified sentences such as *all/some/ square chips are not black*; *not all square chips are black; no square chip is black* that expressed generalizations; a minority referred to some specific evidence (e. g. *there is one square chip that is not black*). These answers have in common that they did not refer to the singular chip to be drawn but provided reasons to make a prediction about it. We did not include such answers in the analysis, in order to focus on denials making direct reference to "the chip", or "it". After they were discarded, there were 187 answers to classify, out of which nine were either incomprehensible or showed incomprehension of the task (e.g., *this is not a chip*), hence a total of 178 answers (conditional group N = 123 ; categorical group N = 55).

Conditional group. Altogether, 93% of all the denials belonged to one of the four types predicted. The following percentages are relative to these predicted responses (N = 115), leaving aside the 7% that were unpredicted. There were more conditional (61%) than conjunctive responses (39%); and more weak (63%) than strong responses (37%). The corresponding cross-classification was: weak conditional 40%; strong conditional 21%; weak conjunctive 23%; strong conjunctive 16%.

- *Conditional responses.* The most frequent response was the weak form $<$ if A, neg NEC $C >$ = *if it is a square chip, it will not necessarily be black* (24% of the predicted responses). The remaining weak forms (16%) were various expressions of probability for C to occur, of possibility for not-C, or for C or not-C to occur. The strong conditional, $<$ if A, neg $C >$ = *if it is a square chip, it will not be black* was the second most frequent response (21%).

- *Conjunctive responses.* The majority of the weak forms was $<$ A and neg NEC $C >$ = *the square chip will not necessarily be black* (13%). The remaining weak forms (10%) were various expressions of possibility for not-C, or for C or not-C to occur, or of likelihood for C to occur, e. g. *there is /little/more/ chance for the square chip to be black*. The majority of the strong forms was $< A$ and not-$C >$ = *the square chip will not be black* (13%) and there were a few $< A$ and neg POSS $C >$ = *the square chip cannot be black* (3%).

Figure 2 shows the distribution of the answers based on all the observations (N = 123). The unpredicted responses constituted 7% of the responses and were of the type "not-A and C", or "if not-A then not-C", or "if C then A", with roughly equal frequency.

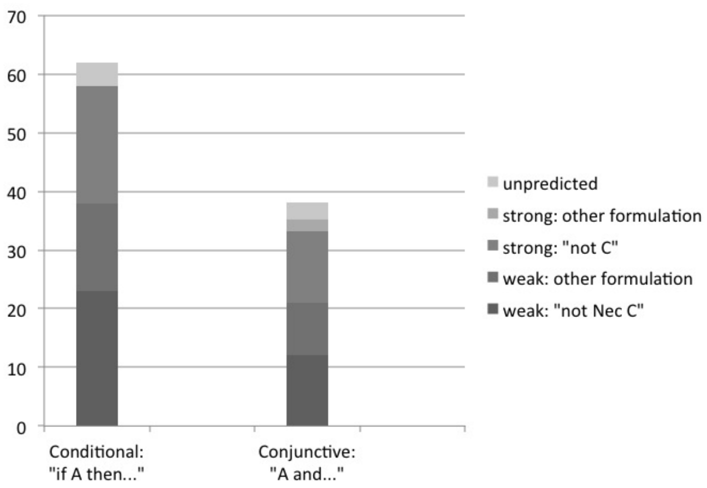

Figure 1: Percentage of conditional and conjunctive responses showing the weak and strong formulations.

Categorical group. The strong responses $<$ neg $C >$ = *the chip will not be black* constituted the majority (64% of the observations). The majority of the weak responses (22%) were various expressions of probability or possibility for not-C, or for C to occur similar to the categorical responses above, and the remaining (14%) were $<$ neg NEC $C >$ = *the chip will not necessarily be black*.

4.2.2 Test of the predictions

We first compare the frequency of weak responses in the two groups, which differ by the sentence to be denied (conditional versus categorical). The data are: conditional sentence = 73/123=59%, categorical sentence = 20/55=36%. The difference is significant (chi-square = $9.46, p < .01$). In the context that we have chosen, the denial of a conditional sentence is more frequently expressed by a weak negation, as opposed to a strong negation in the case in which the consequent is categorically asserted.

We now turn to the test of the relational predictions in the case of the conditional group. The rate of responses in which A is categorically affirmed, and conjunctions uttered, is predicted

$P(A)$	conditional	conjunctive	Total
1/4	27	12	39
3/4	26	10	36
1	19	23	42
Total	72	45	117

Table 3: Frequency distribution of conditional and conjunctive denials as a function of the probability of the antecedent of the conditional sentence to be denied.

$P(C\|A)$	weak	strong	Total
5/6	21	9	30
3/6	27	8	35
1/6	18	12	30
0	7	15	22
Total	73	44	117

Table 4: Frequency distribution of weak and strong denials as a function of the probability of the conditional sentence to be denied.

to be an increasing function of the probability of the antecedent of the conditional sentence. Kendall's S test (with Jonckheere's correction for continuity) applied to the data in Table 3 confirmed the general trend ($z = 2.14, p < .02$). However, the figures in Table 3 suggest that the increase is not uniform. A decomposition of the total chi-square ($= 7.43, df = 2$) after partitioning the table between the first two rows and the bottom row yielded a chi-square of 7.36 ($df = 1$), while the top two rows contributed only for .07 ($df = 1$), showing that the rate for the two lower values of $P(A)$ do not differ and that the level $P(A) = 1$ is responsible for the general trend.

The second relational prediction is that the rate of strong negations should be a decreasing function of the probability of the conditional sentence to be denied. Table 4 displays the relevant data.

There was indeed a significant trend for strong denials to be more frequent as $P(C|A)$ decreases (Kendall's S test with Jonckheere's correction, $z = 2.77, p < .005$). The trend was marked essentially for the null value, as shown by the partition of the total chi-square ($12.82, df = 3, p < .005$) in which the opposition of the first three rows against the last one ($df = 1$) yielded a value (10.79) close to the total chi-square.

4.3 Discussion

The results of this experiment support both the prediction that the four types of denials would occur, and the relational predictions we made. In particular, we see that conjunctive negations are favored as the antecedent of the conditional becomes more certain, and secondly, that strong negations are favored as the consequent becomes less probable relative to the antecedent. Furthermore, the comparison between denials of conditional sentences and denials of categorical sentences, which shows a significant preference for weak over strong negations in the case of conditional sentences, supports the main hypothesis whereby weak negations are semantically fundamental for indicative conditionals (assuming the semantic negation of the future tense sentence "the chip will be black" to be "the chip will not be black").

5 Further work

One aspect we have not investigated concerns the interaction of negation with tense in conditionals. Handley et al. in [7] point out in their first study that "participants were more likely to indicate that p and not-q followed in the past compared to the future tense condition". We surmise that the contrast, assuming it is robust, can be pragmatically explained along the lines we have outlined here (specifically, for past tense clauses favoring the implicature that the event has happened). Secondly, in this paper we have investigated how the negation of conditional sentences is expressed, but a different task would be to investigate experimentally how negated conditionals of the form "if A then not C" and "it's not the case that if A then C" are interpreted. In other words, the task would be to directly compare the interpretations of the outer and inner negation of a conditional. In the framework we have adopted, "if A then not C" and "not (if A then C)" have different truth-conditions, which is what justified our distinction between strong and weak conditional negation. Other analyses, notably Schlenker's analysis in terms of plural definite descriptions [19], also make the distinction at the semantic level, but for the latter it has been suggested that a presupposition of homogeneity could pragmatically make "not (if A then C)" equivalent to "if A then not C" (see [21]). We conjecture that "not (if A then C)" will generally be interpreted as weaker than "if A then not C", but we leave an empirical investigation of this question for further work.

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