# Matching Schemes in Practice

Péter Biró
Institute of Economics
Hungarian Academy of Sciences
birop@econ.core.hu

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# Computational Social Choice via matching schemes

We have an economic/social choice problem
with

- participants
- possible outcomes

+Some objective facts
+true preferences of the players over the possible outcomes.

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We have an economic/social choice problem: e.g., school choice with

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- possible outcomes: matchings

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We shall design such rules, or mechanisms, that lead to a 'good' solution given the objective facts and true preferences.

The mechanism may be

- decentralised (e.g., college admissions in the US)
- coordinated (e.g., college admissions in the UK)
- centralised (e.g., college admissions in Hungary and Spain)

# Computational Social Choice via matching schemes

We have an economic/social choice problem : e.g., market with

- participants : buyers and sellers
- possible outcomes : matchings with prices

+Some objective facts (e.g., age of buyer)
+true preferences of the players over the possible outcomes.

We shall design such rules (or mechanism) that lead to a 'good' solution given the objective facts and true preferences.

The mechanism may be

- decentralised (e.g. usual market)
- coordinated (e.g. eBay)
- centralised (e.g. Google's auction for TV ads in the US)

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  answers by social scientists, economists, game theorists
- **Is there any decentralised / coordinated / centralised mechanism that leads to a 'good' solution?**
  answers by game theorists, mechanism designers
- **Can we compute a 'good' solution efficiently via a centralised mechanism?**
  answers by computer scientists, algorithm theorists

# College Admissions as Gale and Shapley (1962) imagined

The solution by the Gale-Shapley mechanism is

- fair: an application is rejected by a college only if its quota is filled with better applicants
- student-optimal: no student could be admitted to a better college in any other fair solution

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An economic benefit of having a centralised system:

- Popular colleges always obtain their targeted numbers

# The Gale–Shapley algorithm in practice

Allocating residents to positions:

- National Resident Matching Program since 1952!
- and many other professions in the US and other countries...
  (e.g., Scottish Foundation Allocation Scheme – SFAS)

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Admission systems in education:

- New York high schools since 2004,
  Boston high schools since 2005
- Higher education admissions in Spain (1998)
- Higher education admissions in Hungary since 1985
- Secondary school admissions in Hungary since 2000
  (Original Gale–Shapley model and algorithm!)

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# Game Theory

Unit Director: László Á. KÓCZY

Researchers:

Péter BIRÓ Péter Csóka Helga HABIS László Á. KÓCZY András SIMONOVITS Balázs SZIKLAI

| Research focus | Projects | Research fields | Publications | Matching schemes

## Matching schemes
collection of applications

### Matching schemes in Europe

| Country | Student admissions | Job market | Kidney exchange | Other applications |
|---|---|---|---|---|
| France | | professeur allocation | | |
| Germany | higher education | | | |
| Hungary | secondary schools, higher education | | | |
| Israel | | | | dormitories |
| Netherlands | | | The Dutch program | |
| Spain | higher education | | The Spanish Program | |
| Turkey | higher education | | | |
| UK | | SPA-SFAS, TIS | NHS Blood and Transplant | |

## SPA and SFAS - stable matching of medical graduates to posts in Scotland

**Webpage:** Scottish Foundation Allocation Scheme
**Short description of the program:**
The four main medical schools in Scotland (Glasgow, Edinburgh, Aberdeen and Dundee) produce around 800 graduates per year. The allocation of these graduates to their first positions has been carried out by a centralized matching scheme since 2000. Initially, when the allocation was to Pre-Registration House Officer posts, or PRHOs, the scheme was known as SPA (Scottish PRHO Allocations). SPA ran for six years, but since 2006 the allocation has been to so-called Foundation Programmes, and the scheme has been changed and renamed SFAS (Scottish Foundation Allocation Scheme).
**SPA (2000 -- 2005)**
In the SPA scheme, the requirement was that each applicant be allocated to two six-month positions, a medical post and a surgical post (except for a small number of applicants who required only one of these.) Applicants were required to rank their preferred n medical posts and preferred n surgical posts separately, in strict order of preference, and these rankings were submitted to the central body. (The value of n varied from year to year, in the range from 3 to 6.) Applicants were also invited to indicate whether they wished their medical post or their surgical post to come first chronologically (their so-called seasonal preference). Consultants in charge of each medical and each surgical unit were then sent the names of those applicants who had listed them, and were in turn required to submit strictly ranked preferences over their applicants, together with the number of posts on offer. The matching program was based primarily on the classical Gale-Shapley applicant-oriented algorithm. This was used to find the applicant-optimal stable matchings to medical and surgical posts separately. Following this, the two matchings were combined , using a network-flow based approach, to ensure that each candidate's allocation was to two different six-month periods, and that no unit's capacity was exceeded during any six-month period, at the same time maximizing the number of applicants whose seasonal preference was satisfied. Technical details of the algorithm appear in the webpage. Typically some 90-95% of positions were allocated by this first round of matching. Subsequently, details of unfilled positions were circulated to unmatched or partially matched applicants, and these applicants were matched in a second, somewhat ad hoc, round.
**SFAS (2006 -- )**
The current arrangements for medical training involve the allocation of each graduate to an integrated two-year Foundation Programme, which involves a range of medical disciplines. From the point of view of the matching algorithm, all that is required is an allocation of each applicant to (at most) one programme, respecting the capacities of the programmes. So, on the surface this appears to be an instance of the classical Hospitals-Residents problem. However, two factors combined to add extra interest to this process.
Firstly, as a matter of policy, programme directors are no longer asked to rank their applicants in order of preference. Instead, applicants are ranked in a so-called master list, in order of score – each applicant has a numerical score allocated partly on the basis of academic performance and partly as a result of the assessment of their application form. (Application forms are complex, and require detailed responses to challenging questions.) The range of possible scores (approximately 40 – 100) is much smaller than the number of applicants (around 750 each year), so there are inevitably ties of substantial length in the master list. The individual programme preference lists are constructed from the master list, so these also typically contain ties. It turns out that, even in the presence of a master list, random or arbitrary tie-breaking can lead to variations in the size of the

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Secondly, pairs of applicants to SFAS may declare themselves to be a couple, with a view to being assigned to geographically compatible positions. Such applicants are required to submit individual preference lists, just like single applicants. These are then combined in a pre-specified way to form a joint preference list for the couple, omitting any incompatible allocations. (A complete compatibility matrix for all pairs of programmes is available to applicants.) Under a natural extension of the stability concept, it is well known that a stable matching may not exist in the presence of couples, and that it is an NP-complete problem to determine whether such a matching exists. These results continue to hold in the presence of a master list [4]. The current SFAS program uses a heuristic that attempts to find a stable matching that allocates as many applicants as possible. Simulations reported in [4] suggest that, as long as the proportion of couples is relatively low, it is highly likely that a stable matching can be found, and this has so far been borne out in practice.

Description was given by Robert W. Irving.

**References:**


- [1] Robert W. Irving. Matching medical students to pairs of hospitals: a new variation on a well-known theme, in Proceedings of ESA'98, the Sixth Annual European Symposium on Algorithms, Venice Italy, 1998, Lecture Notes in Computer Science, vol. 1461 (Springer 1998), pp. 381-392.
- [2] R.W. Irving, D.F. Manlove and S. Scott. The stable marriage problem with master preference lists, Discrete Applied Mathematics vol. 156 (2008), pp. 2959-2977. Postprint.
- [3] R. W. Irving and D. F. Manlove, Finding large stable matchings. ACM Journal of Experimental Algorithmics vol. 14 (2009), section 1 article no. 2, 30 pages. Postprint.
- P. Biro, R.W. Irving and I. Schlotter, Stable matching with couples: Theory and Practice, Technical Report. Dept of Computing Science, University of Glasgow, TR-2011-324. To appear in ACM Journal of Experimental Algorithmics.

# Scottish Foundation Allocation Scheme

Special features:

1. ties
2. couples

Theory: Each of these two features makes the problem of finding a 'good' solution NP-hard, so heuristics are used...

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- R. W. Irving and D. F. Manlove, Finding large stable matchings. ACM Journal of Experimental Algorithmics vol. 14 (2009), section 1 article no. 2, 30 pages.
- P. Biró, R.W. Irving and I. Schlotter, Stable matching with couples – an empirical study. To appear in ACM Journal of Experimental Algorithmics.


The National Resident Matching Program (NRMP) is a private, not-for-profit corporation established in 1952 to provide a uniform date of appointment to positions in graduate medical education (GME) in the United States.

## News from the NRMP!

**New>SLEEP MEDICINE JOINS THE MATCH!**

The NRMP is pleased to welcome Sleep Medicine (SLPM) as a new fellowship match for the 2012 appointment year. Sponsored by the American Academy of Sleep Medicine, the Sleep Medicine fellowship match will open for registration on July 27, 2011 with Match Day on November 16, 2011. For more information about the Sleep Medicine Fellowship Match, including the Schedule of Dates, click on the Fellowship Matches at the top of this page or contact our Helpdesk Specialists toll free at 1-866-617-5834.

**New>NRMP SEEKING COMMENTS ON "ALL-IN" POLICY**



The National Resident Matching Program (NRMP) Board

## Main Residency Match

The 2011 Main Residency Match was the largest in NRMP history, with more than 26,000 positions and almost 38,000 applicants. Read the Match Day press release; view the Advance Data Tables; and listen to the podcast from NRMP Executive Director Mona M. Signer.

### Communications

Visit the Communications page for more information about and access to recent NRMP web conferences and webcasts.

## Data and Reports

Visit the Data and Reports section for recent reports and historic NRMP match data.

**Results of the 2010 NRMP Program Director Survey** *(PDF, 164 pages)* This report presents the results of selected items from the 2010 NRMP Program Director

| Step 1 | |
|---|---|
| Each partner should first arrange an individual preference list on separate sheets of paper. In the example, the letters refer to a specific program in a particular hospital in that city. | |
| Partner I | Partner II |
| 1) New York City - A<br>2) Chicago - A<br>3) Evanston - B<br>4) Los Angeles - A<br>5) New York City - B | 1) Chicago - X<br>2) Chicago - Y<br>3) Boston - X<br>4) Chicago - Z<br>5) New York City - X<br>6) New York City - Y |

| Step 2 | |
|---|---|
| Next, both partners must decide together how to prepare their lists as pairs of programs. For example, they could consider all the possible pairings where the hospital programs are in the same general location, as indicated in the list below. In some cases one rank in the pair may be designated "No Match" to indicate that one partner is willing to go unmatched if the other is matched to a position. Note that the list below is not necessarily in the order that will eventually be submitted. | |
| Partner I | Partner II |
| New York City - A<br>New York City - A<br>Chicago - A<br>Chicago -A<br>Chicago -A<br>Evanston -B<br>Evanston -B<br>Evanston -B<br>New York City -B<br>New York City -B<br>New York City -A<br>Chicago -A | New York City -X<br>New York City -Y<br>Chicago -X<br>Chicago -Y<br>Chicago -Z<br>Chicago -X<br>Chicago -Y<br>Chicago -Z<br>New York City -X<br>New York City -Y<br>No Match<br>No Match |

# Hospitals / Residents problem with couples

National Resident Matching Program (since 2009 in SFAS too)

Couples can submit joint preference lists...

| **Applicants:** | **Bill** | **Adam and Eve** |
|---|---|---|
| 1st choice:<br>2nd choice: | Queens<br>Memorial | (Memorial, Queens) |

**the ranking of NY Queens Hospital:** Eve, Bill

**the ranking of NY Memorial Hospital:** Bill, Adam

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**the ranking of NY Queens Hospital:** Eve, Bill
**the ranking of NY Memorial Hospital:** Bill, Adam

Roth (1984): 'Fair' solution may not exist.

Ronn (1990): The related decision problem is NP-complete.

B.-Irving-Schlotter (2011-JEA):
NP-complete even for master lists (relevance in SFAS)

Heuristics are used in the applications...

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# Game Theory

Unit Director: László Á. KÓCZY

Researchers:

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| Research focus | Projects | Research fields | Publications | Matching schemes

## Matching schemes
collection of applications

### Matching schemes in Europe

| Country | Student admissions | Job market | Kidney exchange | Other applications |
|---|---|---|---|---|
| France | | professeur allocation | | |
| Germany | higher education | | | |
| Hungary | secondary schools, higher education | | | |
| Israel | | | | dormitories |
| Netherlands | | | The Dutch program | |
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| Turkey | higher education | | | |
| UK | | SPA-SFAS, TIS | NHS Blood and Transplant | |

## Higher education admissions in Hungary

**Webpage:** National Higher Education Information Centre
**Short description of the program:** The program was established in 1985, the number of applicants is between 100000 and 170000 every year. The students apply for studies rather than to institutions (but here we refer to the studies as colleges for simplicity). The program uses a special Gale-Shapley algorithm, the college-oriented version was replaced with the applicant-oriented version in 2007. The main special features of the program are the following:
1. Ties can occur, since students are ranked according to their scores, and two students may have the same score at a particular college. The attempted solution is a so-called *stable score-limit*, which satisfies the condition that no overdemanded college can decrease its score-limit without violating its quota. See related results in [1].
2. Beside the upper quotas some colleges may have lower quotas as well. This feature is studied in [2].
3. Some sets of colleges may have common upper quotas. This feature is studied in [2].
4. Teachers can apply for pair of studies, which is like having couples in the Hospitals Residents problem.
Description was given by Péter Biró.
**Another description of the program in Hungarian and further links:** Felsőoktatási MŰHELY

**References:**


- [1] P. Biró. Student Admissions in Hungary as Gale and Shapley Envisaged. Technical Report. Dept of Computing Science, University of Glasgow, TR-2008-291.

- [2] P. Biró, T.Fleiner, R.W. Irving and D.F. Manlove. The College Admissions problem with lower and common quotas. Theoretical Computer Science 411, 3136-3153 (2010).
The full version of this paper is available as Technical Report no. TR-2009-303 of the Computing Science Department of Glasgow University, 2009.

- [3] L.Á. Kóczy, A magyarországi felvételi rendszerek sajátosságai. Közgazdasági Szemle 57:(2) pp. 142-164. (2010), in Hungarian.

# Hungarian higher education matching scheme

Special features:

1. ties
2. lower quotas
3. common quotas
4. paired applications

Theory: Each of the latter three features makes the problem of finding a 'good' solution NP-hard, so heuristics are used...

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- P. Biró. Student Admissions in Hungary as Gale and Shapley Envisaged. Technical Report. Dept of Computing Science, University of Glasgow, TR-2008-291.
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**Matching schemes**
collection of applications

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## Matching schemes in America

| Country | Student admissions | Job market | Kidney exchange | Other applications |
|---|---|---|---|---|
| U.S. | high schools | NRMP and others | NEPKE and others | U.S. Navy |
| Canada | | CaRMS | | |

For further information, see the webpage of Al Roth

## Matching schemes in Asia

| Country | Student admissions | Job market | Kidney exchange | Other applications |
|---|---|---|---|---|
| Australia | | | AKX | |
| Japan | | JRMP | | |
| Singapore | high schools | | | |

## Worldwide applications

# Kidney exchanges

In case of a kidney failure a patient go for

- dialysis (-)
- transplantation $(+)$
  this can be
  - cadaveric (from dead body), but there is a shortage
  - living donation

# Kidney exchanges

In case of a kidney failure a patient go for

- dialysis (-)
- transplantation (+)
  this can be
  - cadaveric (from dead body), but there is a shortage
  - living donation

But what if you have a willing donor who is incompatible with you?
Perhaps you can exchange kidney with others!
Kidney exchange programs (US, the Nederlands, UK...)

New England Program For Kidney Exchange

A transplant option for patients with an incompatible living donor

Home | The Program | FAQ | Stories | Living Donors | Links | About Us | Contact | Login

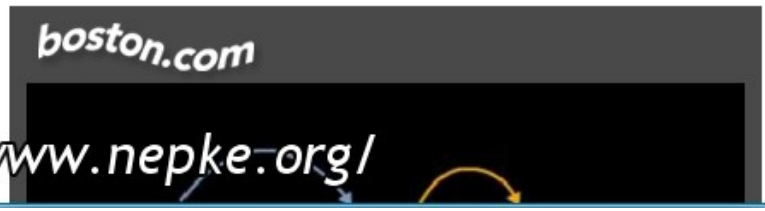
## Welcome

### A Life-Saving Option

The New England Program for Kidney Exchange offers new life-saving options to those seeking a kidney transplant, but whose potential living donor is not a good biological "match" due to either blood type incompatibility or cross-match incompatibility. This option is known as kidney exchange, kidney paired donation, or kidney swap.

NEPKE uses a computer program to find cases where the donor in an incompatible pair can be matched to a recipient in another pair. By exchanging donors, a compatible match for both recipients may be found. You can learn more about the program HERE and read our newsletter here.

NEPKE can also find potential kidney recipients for those generous people who seek to become non-directed living donors (otherwise known as Good Samaritan Donors or Altruistic Donors). Information about that process is available HERE .



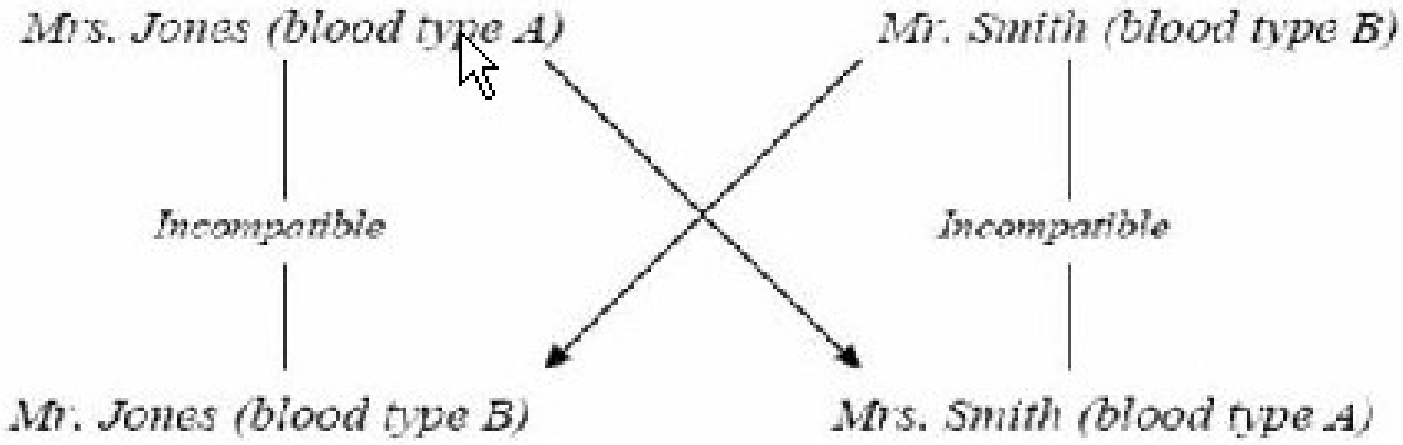

http://www.nepke.org/

**NEWS:**
Transplant centers are being provided with brochures to provide information about this program to their kidney patients.
More News

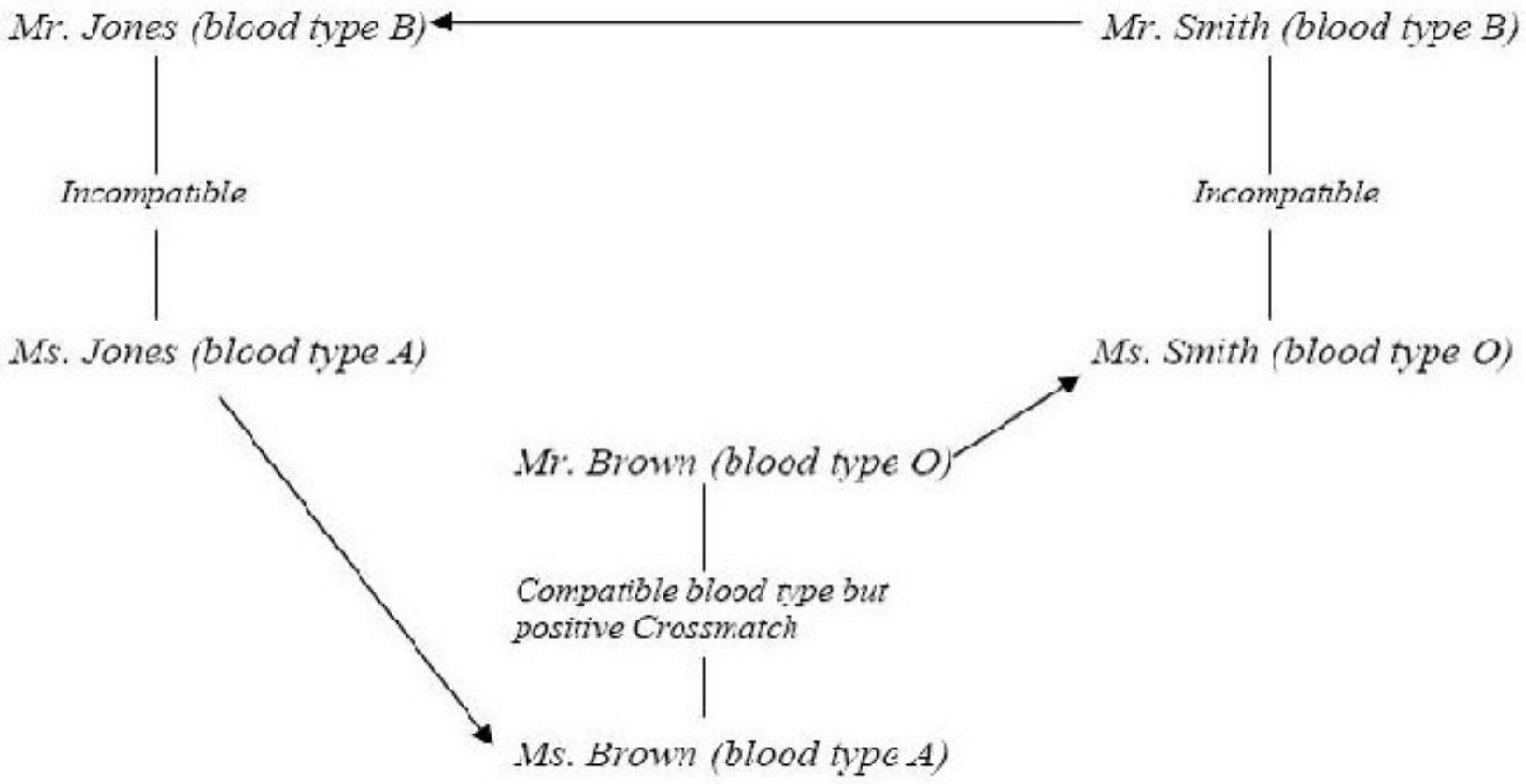
**NEPKE**
Transplants to Date
**83**
!

**Notes:**
There are many good websites on the Internet that help kidney patients learn more about transplant options.
Links

An example of NEPKE matching is shown by the following: Mr. Smith has blood type B and wants to donate to Mrs. Smith, who is blood type A. Mrs. Jones has blood type A and wants to donate to Mr. Jones, who is blood type B. If each of the donors are medically suitable to undergo the operation and each agree to an exchange, then Mr. Smith could donate to Mr. Jones and Mrs. Jones would donate to Mrs. Smith.



In this instance, Mr. Smith would donate to Mr. Jones, Ms. Jones would donate to Ms. Brown, and Mr. Brown would donate to Ms. Smith.



# Kidney exchange scheme in the UK

- matching runs in every three month with around 150 pairs in the pool
- 2-way and 3-way exchanges allowed in the solution
- 'optimality' means that the number of donations is maximal

Theory: The related problem is NP-complete. We implemented an exact algorithm and an IP heuristics and both worked fine!
Led to at least 68 successful transplants since 2007, including six 3-way exchanges...

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- R.J. Johnson, J.E. Allen, S.V. Fuggle, J.A. Bradley and C. Rudge; on behalf of the Kidney Advisory Group. Early Experience of Paired Living Kidney Donation in the United Kingdom. Transplantation, 86(12) : 1672-1677, 2008.
- P. Biró, D.F. Manlove and Romeo Rizzi. Maximum weight cycle packing in directed graphs, with application to kidney exchange programs. Discrete Mathematics, Algorithms and Applications, 1 (4) : 499-517, 2009.

## Kidney exchange program in the UK

**Webpage:** National Matching Scheme for Paired and Pooled (kidney) Donation
**Short description of the program:** NHS Blood and Transplant run the National Matching Scheme for Paired and Pooled (kidney) Donation in the UK. They maintain a database of incompatible (donor,patient) pairs who would be willing to participate in a live-donor kidney exchange with one or more other (donor,patient) pairs. Every three months a matching run is carried out in an attempt to construct an optimal set of exchanges. At present paired (2-way) and pooled (3-way) exchanges are sought, though an exchange involving 3 or more couples has yet to be carried out in the UK. The term "optimal" refers to the fact that the overriding constraint is to maximise the number of transplants that can be carried out, and subject to this, to maximise the overall score of the exchange. The score of an exchange is based on the points system that NHS Blood and Transplant employs for couples involved in the process - see the above webpage for more details. A number of paired kidney donations have been carried out in the UK as a result of this matching scheme, and these have generated much interest in the media - see an article from The Guardian, for example. Description was given by David F. Manlove.
**Recent news:** A BBC article and the corresponding video on the first successful 3-way exchange in the UK. A report by the Human Tissue Authority, and a summary about the involvement of the Glasgow algorithm and complexity research group.

**References:**


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Daily Mail, Thursday, December 6, 2007

# The transplant pact

## Two saved as families exchange kidneys

By Luke Salkeld

THEY were both in desperate need of a kidney donor, and both had relatives who were willing to sacrifice an organ.

But without a family match, strangers Donald Planner and Margaret Wearn instead entered into an extraordinary pact.

Mr Planner's daughter donated her kidney to Mrs Wearn, whose husband gave his kidney to Mr Planner.

The operations took place 170 miles apart in synchronised procedures with the organs transported by ambulances travelling in opposite directions between the two hospitals.


'Completely amazing': Donald Planner with his daughter Suzanne

Margaret and Roger Wearn: 'No different to a direct donation'

organ or he would die. His ... lly reliant on the dialysis

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# Three-way kidney transplant success

By Graham Satchell
BBC News Breakfast reporter

**Step back to nine in the morning on 4 December 2009.**

Six patients are ready for surgery at three different hospitals across the UK.

It is the culmination of months of preparation and a remarkable event in the history of live organ donation in this country.

This is a three-way kidney swap between couples who've never met.

Chris Brent with his sister Lisa Burton

In Aberdeen, 54-year-old Andrea Mullen suffered sudden kidney failure three years ago.

It had a devastating impact on her life. She had to have dialysis three

> **"It's a threefold thing really so it's a real good feelgood factor all round"**
>
> Lisa Burton, who donated a kidney

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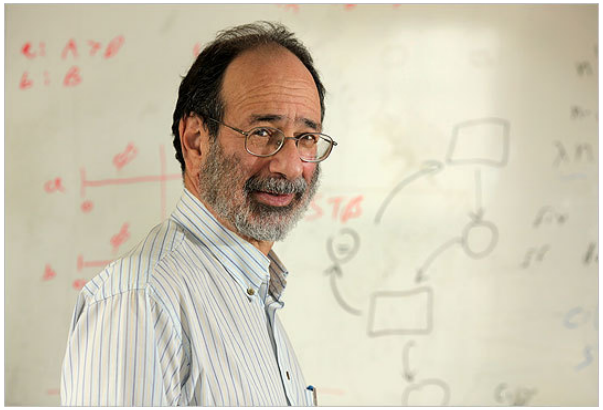
# Centralised mechanism vs central planning



(re-)designed many programs, e.g.

- US resident allocations
- New England kidney exchanges
- NY and Boston school choice

**Al Roth (Science, 1990):** "Note that the centralized markets studied here do not involve central planning as it is most usually understood, since these markets have been designed to be sensitive to the preferences expressed by the participants, rather than to achieve the independent objectives of a planner. What is centralized is not the objective, but the market mechanism itself.

# Future challenges

Theoretical research:

- Studying computational and strategic issues (computer science, game theory)
- What are the benefits of having a centralised scheme? e.g., compare student admissions practices in Poland and Hungary (economics, social sciences)

# Future challenges

Theoretical research:

- Studying computational and strategic issues (computer science, game theory)
- What are the benefits of having a centralised scheme? e.g., compare student admissions practices in Poland and Hungary (economics, social sciences)

New applications?

- **Hungary:** school choice (nursery, kindergarten, primary schools), resident allocation, kidney exchanges?
- **Europe:** united schemes for higher education admissions and kidney exchanges?