Computational challenges in fair division

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The general problem

- Input:
  - A collection of items
  - Users (or agents) that have utilities for bundles of items
- Goal:
  - Allocate the items to the agents so that the allocation is fair according to specific fairness criteria

- Variations:
  - Divisible vs indivisible items, restricted utility functions, different notions of fairness

- Many applications: e.g., ICT, multi-agent systems, negotiations, peace treaties, etc.
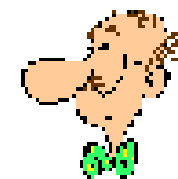
Example: Alice and Bob get divorced ☹

items

	car	house	dog	bicycle	
Alice	1	5	1	3	
	4	3	1	2	Bob

Utilities

Example: Alice and Bob get divorced ☹

items

Alice

| 1 | (5) | 1 | 3 |

Bob

| (4) | 3 | (1) | (2) |

An envy-free allocation

Example: Alice and Bob get divorced ☹

items

Alice

| 1 | (5) | (1) | 3 |

| (4) | 3 | 1 | (2) |

Bob

An equitable allocation

Fairness criteria

- Proportionality: each user feels she got a fair share
- Envy-freeness: no user envies the bundle of any other user
- Equitability: all users are equally happy
- Max-min fairness: the least happy user is as happy as possible

Other criteria

- Efficiency of allocations
  - Pareto efficiency
  - Social welfare
- Efficiency of computation
  - Fast computation (e.g., polynomial-time)
  - Preferably in a distributed way
- Resistance to manipulability
  - Strategy-proofness

Example: envy-free cake cutting

- Input:
  - A divisible item (cake)
  - Two agents, each having private utilities over parts of the cake
- Goal:
  - Allocate pieces of the cake to the agents so that nobody envies the part allocated to the other player

- Good news:
  - We know how to solve the problem for 2 and 3 agents
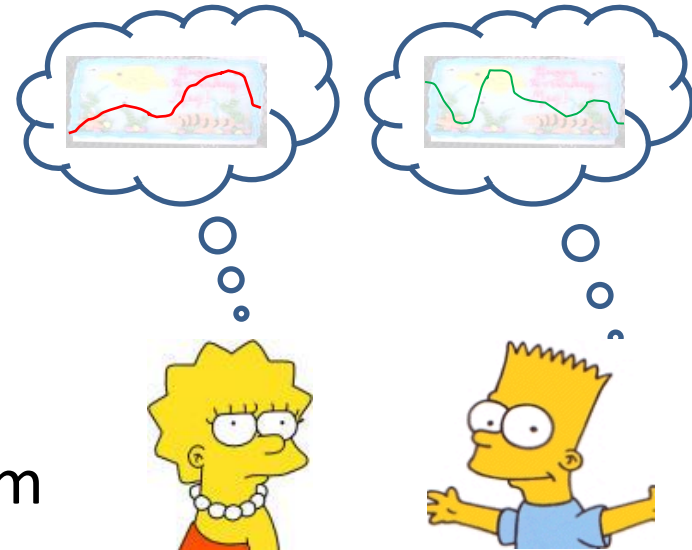
Many related issues

- What is the computational complexity of the problem?
  - Looks like searching for a needle in a haystack
- Provable lower bounds?
  - All we know is that envy-freeness is slightly more difficult to achieve than proportionality
- Restricted utilities
- More expressive models
  - E.g., restriction for contiguous pieces
  - E.g., no utility for trimmings
- What about other fairness criteria?
  - E.g., approximate equitability

Another example: the Santa Claus problem

- Input:
  - Santa Claus has a bag full with toys
  - Several kids, each having a utility for each toy
- Goal:
  - Compute an allocation so that the utility of the least happy kid is maximized

- Good news:
  - We know how to compute $O(\log n)$-approximate allocations

Related challenges

- Improved approximation algorithms
- Inapproximability results
- Restricted utilities
- Other fairness objectives with indivisible items
- More expressive models

Many more issues

- Tradeoffs between fairness and efficiency
  - E.g., fairness and Pareto-efficiency
- What is the price of fairness?
  - How suboptimal can the social welfare be in a fair (proportional, envy-free, equitable, max-min fair) allocation?
- Strategy-proofness
  - Incompatible with fairness (in general)
  - Monetary incentives, transferable utilities