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CHAPTER 19

School Placement of Trainee Teachers: Theory and Practice

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19.1 Introduction

The traditional study programs of teachers-to-be for upper elementary and secondary education in Slovakia and the Czech Republic involve the specialization of each student in two subjects, e.g., Mathematics and Physics, Chemistry and Biology, Slovak language and English, etc. In addition to the study of various topics of these school subjects and principles of Pedagogics and Psychology, each curriculum contains a practical placement in a real school several times during the studies. School placement enables the students to find connections between the theoretical knowledge and its practical use in a class. It can be organized in several modes, each providing a different level and type of support to the trainees and putting different levels of requirements on their competences.

The aim of this work is to describe mathematical models, computational complexity results and proposed algorithms for the tasks connected with the school placement of trainee teachers. They depend on the concrete type of the placements, so now we describe briefly their two types as used at the home university of the author.

During the *group placement (observational)* a group of students (ideally of 4-6 students) visit real classes, they observe the teacher at her work, her didactic methods, reactions to the behavior of pupils, etc, and after the class they engage in common discussion to analyze everything that has happened. The computational difficulty of this placement problem comes from the requirement to assign each student to two different groups (one for each of her specialization subjects) in such a way that a non-conflicting schedule should be possible.

Individual placement means that a student teaches pupils herself, but under a supervision of a qualified teacher. Students of our university teach both their specialization subjects during one placement period, and for practical reasons, the whole placement should take place at the same school. There is a requirement to teach *at least* a certain number of classes for each subject and have a supervising teacher for both specialization subjects, which leads to certain capacity restrictions on the side of schools.

To ensure the quality of practical placements, the university approves of both

Year	1967	1974	1989	2003	2006	2011	2012	2014
# of combinations	3	4	6	9	13	19	26	31

Table 19.1: Two-subject combinations of P.J. Šafárik University graduates

schools and supervising teachers. The university staff also construct the assignment for the whole cohort of students. We shall provide some figures to give an indication of how complicated and time-consuming this task is.

At the time of writing this text (academic year 2016/2017) there are 10 universities in Slovakia providing the teachers' studies. Previously, there was a relatively clear separation of various subjects: Science faculties offered subjects of natural sciences, like Mathematics, Physics, Biology, Chemistry, etc., Philosophical faculties offered languages and humanities, whilst Pedagogical faculties (in addition to primary and special education) offered sports and arts education. In the 1990s many new universities and faculties were established in Slovakia and also the teaching profession became less popular. As a result, universities started to compete for students in several different ways. One approach was to offer a greater degree of flexibility for their studies, namely, to allow a much larger number of different two-subject combinations. To be able to achieve this, most universities joined forces of their faculties. So, for example, at the University of Prešov, a student can choose *any* two subjects offered by 6 faculties (Faculty of Arts, Faculty of Humanities and Natural Sciences, Faculty of Sports, Faculty of Education and two theological faculties) plus three from the Institute of Minority Languages and Culture - Ruthenian, Hungarian and Roma. This means a choice from almost any pair of more than 21 different subjects; the explicit list of study programs for applicants in 2017/2018 contains 135 different two-subject combinations. For Pavol Jozef Šafárik University in Košice, where we conducted our research, we can give more detailed data, based on the official graduates' lists of the University. The first students of the Science faculty graduated in 1967 and originally, the only possible combinations were Mathematics-Physics, Chemistry-Physics and Chemistry-Biology. Year 1974 saw the first graduates of the combination Mathematics-Chemistry; in 1989 Informatics appeared among the specialization subjects and after the Institute of Geography was established, combinations with Geography emerged in 2003. A further important catalyst for the rise in the number of combinations was the foundation of the Faculty of Arts; the first graduates of common study programs were in 2011. Currently, the Faculty of Science offers 6 subjects and the Faculty of Arts 8 subjects; in 2014 the number of different two-subject combinations of graduates was 31.

There are many objections against so many different subject combinations. First, it is very complicated to coordinate so many different study groups (even with the help of the most recent computers and information systems) and to create the timetable for them, but more important are scientific and economic objections. Namely, it is difficult for a future teacher to cope with very distant subject areas at the level required for successful teaching in the school-leaving years. Moreover, complementary subjects, for example Mathematics and Physics, clearly help the teacher to master each of them to a greater depth. Further, for some specific subject combinations the position of the graduates in the labor market is difficult, as most schools do not have enough teaching hours for these subjects. This was demonstrated also during our study, where the most common combination found among the trainees without a place was Geography-Psychology.

Our research responded to the current situation. We wanted to provide a tool for replacing the most time-consuming part of the students' placement management by a user-friendly computer program. The administrative staff can use the list of students with their specializations and the list of schools with their affiliated supervising teachers (the data they always had to deal with) and as the output they receive a list of groups, a list of students with their assigned school, a list of schools with their assigned students and other supporting documents helping them to find an ad-hoc solution to various unexpected cases that can emerge. The data we provided could even be used for a deeper analysis of the current situation of the teachers' studies.

Related Work. Frieze and Yadegar (1981) model the assignment of students of colleges of education to teaching practice in the following way. Each student has to be assigned to one of a set of schools and there she is supervised by a tutor who comes from the college. Each school and tutor have a capacity (i.e., the maximum number of trainee-teachers that they can accept or supervise, respectively). Moreover, there is a cardinal *satisfaction value* for each student-school-tutor triple. Frieze and Yadegar propose an integer linear program to find a schedule with maximum total satisfaction.

A related task is that of assigning students to projects. Each project is supervised by a lecturer, and projects as well as lecturers have their capacities (enabling a lecturer to have a lower total capacity than the sum of capacities of the projects she is supervising). In a series of papers Abraham et al. (2007) and Manlove and O'Malley (2008) suppose that students have preferences over projects, and lecturers have preferences over students, or over projects. The optimality criterion is *stability*, and the authors study the computational complexity of various problems connected with algorithms for finding stable assignments. By contrast, Kwanashie et al. (2015) want to optimize the *profile* of a matching, i.e., a vector whose rth component indicates how many students have their rth-choice project in the assignment.

Recall that in our trainee teacher assignment problem each student needs two places in a specific structure. In the group placement the student practices one of her subjects in the first part of the placement period and the other subject in its second period. This is similar to the problem studied by Irving (1998) in the context of Scottish medical education. Each medical student must take on two positions in two half-years, namely a medical post and a surgical post, respecting the capacities of medical and surgical units in the two half-years. Irving seeks a stable matching maximizing the number of fulfilled seasonal preferences of students. However, while the types of posts are the same for each medical student, in our case, the pairs of subjects may vary for different students, plus in the group assignment problem we need to combine several students with the same specialization subject into groups.

Similarities with the structure of the trainee teacher assignment problem connected with individual placement can be found in the problems associated with the hospitals/residents matching problem. Motivated by the requirement of the couples (pairs of residents) not to be separated, McDermid and Manlove (2010) study the hospitals/residents problem with sizes of residents (a couple is a 'resident' of size 2) and capacities of hospitals. Let us remark that in the trainee teachers assignment problem each student also needs two places at one school, but the school capacities are more structured, related to individual subjects.

Similar 'multidimensional' constraints appear in a recent work of Delacrètaz et al. (2016) modeling the refugee resettlement problem. A refugee family needs a certain number of units of different public services, such as school seats, hospital beds, slots in language classes or employment training programs and each country or municipality has only restricted numbers of those. Delacrètaz et al. (2016) propose several mechanisms for this allocation problem and study their properties.

Let us also mention here further theoretical work connected with teacher placement. While the trainee teachers and the schools do not have (or, at least they are not asked about them) preferences over the other side of the market, when teachers are looking for a job, their preferences and preferences of schools really matter. Cechlárová et al. (2016) use stability as the criterion for the matching and study the computational complexity of various modifications of the basic problem.

19.2 Basic Notation

We shall denote by A the set of n applicants (students, trainee teachers) and by P the set of k subjects. For ease of exposition, elements of the set P will sometimes be referred to using letters such as M, F, I or B, to serve as a reminder of real subjects taught at schools, such as Mathematics, Physics, Informatics or Biology.

Each applicant $a \in A$ is characterized by a pair $\boldsymbol{p}(a) = \{p_1(a), p_2(a)\} \subseteq P$ of different subjects. The subset of applicants whose specialization involves a subject $p \in P$ (while the other subject may be arbitrary) will be denoted by A_p, the subset of applicants with specialization $\{p, q\}$ by $A_{p,q}$. Further notation differs for the two kinds of placement and so it is postponed to the respective sections.

19.3 Group Placement: Observational

During this type of placement, groups of students of size between 4 and 6 visit classes that are led by a teacher. Their task is to observe everything that happens during a class, take notes about how the teacher presents the material, how she reacts to the pupils' behavior, etc. After the class, students together with the teacher analyze her methods, approaches, ask questions, etc. The aim of this type of placement is to see how to apply in practice the principles that students learned so far only theoretically.

Students	Specialization
Anna, Boris	MF
Cyril, Daniel	MI
Eva, František	IF

Table 19.2: Students for group placement

Group	Students
M	Anna, Boris, Cyril, Daniel
I	Cyril, Daniel, Eva, František
F	Anna, Boris, Eva, František

Table 19.3: Minimum number of groups

Although the basic pedagogical and psychological principles are common for all subjects, many methods and approaches are content-dependent, so the requirement is that students should visit classes where their specialization subjects are taught. Therefore this type of placement is divided into two periods. In the first period the student visits classes of one specialization subject of hers and in the second period she visits classes of her other specialization subject. So the task is to create one-subject groups of size 4-6 students in such a way that each student is in exactly two groups in two different periods. The groups are then scheduled in such a way that each group, if possible, visits a class in an elementary school, a general education high school and also in a specialized high school. To save human and financial resources, it is desirable to keep the number of groups as small as possible.

Example 19.1. *To get an idea of the essence of the problem, let us consider six students whose data are given in Table 19.2. If we create exactly one group of students for each subject, we will not be able to schedule them. For, suppose that group M will be scheduled for the first placement period. Then, since Anna's specialization is MF and Daniel's specialization is MI, both I and F groups have to be scheduled for the second placement period. However, in this case, Eva and František cannot visit classes for both their specialization subjects. If, instead, we split M into two 'mathematical' groups M1={Anna, Boris} and M2={Cyril, Daniel}, then a valid schedule could be M1 and I in the first period and M2 and F in the second period.*

Formally, an instance of the MIN-TAP-G problem is $I = (A, P)$ where P is the set of subjects and A is the set of students, where each $a \in A$ is characterized by a pair $\boldsymbol{p}(a) \subset P$. We want to find a family $\mathscr{B} = \mathscr{B}^1 \cup \mathscr{B}^2$ with $\mathscr{B}^1 = \{B_1^1, B_2^1, \ldots, B_k^1\}$ and $\mathscr{B}^2 = \{B_1^2, B_2^2, \ldots, B_k^2\}$ in such a way that

$$B_p^1 \cup B_p^2 = A_p \text{ and } B_p^1 \cap B_p^2 = \emptyset \text{ for each subject } p \in P \tag{19.1}$$

$$\text{for each } a \in A_{p,q} \text{ there exists } i \in \{1, 2\} \text{ such that } a \in B_p^i \text{ and } a \in B_q^{3-i}. \tag{19.2}$$

Sets $B_1^1, B_2^1, \dots, B_k^1$ are the groups scheduled for the first period, sets $B_1^2, B_2^2, \dots, B_k^2$ are the groups scheduled for the second period. Requirements (19.1) and (19.2) mean that each student is a member of exactly one group for her first subject and of exactly one group for her second subject and that the family enables a valid schedule, i.e., the two groups containing a given student are never scheduled for the same period. The problem MIN-TAP-G is to find a valid family with the minimum number of groups. The proof of the intractability of this problem uses a reduction that is closely related to the construction of an auxiliary graph presented when designing an algorithm for finding a minimum size odd cycle transversal by Reed et al. (2004).

Theorem 19.1. MIN-TAP-G *is* NP*-complete even when* $|A_{p,q}| \leq 1$ *for each pair* $\{p, q\} \subset P$.

Proof. We provide a polynomial transformation from the NP-complete problem BIPARTITE GRAPH (Garey and Johnson, 1979). Here, a graph $G = (V, E)$ and an integer t are given; the question is whether it is possible to delete a set W (an odd cycle transversal) of at most t vertices from G in such a way that the subgraph $G = (V \backslash W, E')$ induced on the set of vertices $V \backslash W$ is bipartite.

Let an instance I of MIN-TAP-G for $G = (V, E)$ be defined as follows. The sets of subjects and applicants are $P = \{p(v); v \in V\}$ and $A = \{a(e); e \in E\}$, where the pair of subjects of a student $a(e)$ corresponding to edge $e = \{u, v\}$ is $\{p(u), p(v)\}$.

First suppose that $G = (V \backslash W, E')$ is bipartite with the bipartition $U \cup U'$. We create $|V| + |W|$ groups as follows. For each $v \in U$ we have $B^1_{p(v)} = \{a(e); v \in e\}$ in $\mathscr{B}^1$ and for each $v \in U'$ the group $B^2_{p(v)} = \{a(e); v \in e\}$ in $\mathscr{B}^2$. For each vertex $v \in W$ we create two groups: group $B^1_{p(v)} = \{a(e); e = \{v, w\}; w \in U'\} \in \mathscr{B}^1$ and group $B^2_{p(v)} = \{a(e); e = \{v, w\}; w \in U\} \in \mathscr{B}^2$. It is easy to see that this a valid partition.

Conversely, suppose that we have a valid partition. It is easy to see that to obtain a bipartite subgraph of G it is enough to delete the vertices that correspond to the subjects p with both sets B_p^1 and B_p^2 nonempty. □

The computer program that is currently in use for group placement was developed by Silvia Bodnárová (2015) for her bachelor thesis. Although an $O(log\ n)$ approximation algorithm exists (Garg et al., 1994) for finding a minimum odd cycle transversal, she used a very simple heuristic to find a valid partition that consists of dividing each set $A_{p,q}$ into two halves A' and A'': in the first period the first half A' practices subject p and the second half A'' subject q; for the second period they simply switch the subjects. Although the number of sets in the obtained partition may be very far from the optimum (i.e., for the instance of Example 19.1 it outputs two sets for each of the subjects M,I,F), the fact that the sets still have to be divided into smaller groups implies that the final solution is in practice not so bad. Recall that the size of the final groups should be between 4 and 6 students (smaller groups are tolerated, for example if the total number of students with the given subject is too small). To get the final groups, observe that each integer $g \geq 8$ can be expressed in the form $g = 4x_6 + 5x_5 + 4x_4$ where x_6, x_5, x_4 are nonnegative integers. Table 19.4 lists the values of these coefficients minimizing the sum $x_6 + x_5 + x_4$ according to the division remainder z of g expressed in the form $g = 6y + z$.

Silvia's program applied to the lists of students from years the 2010 to 2016 was able to deal with each cohort of students in milliseconds, whilst for the single merged list of all students from these seven years containing more than 750 students it still needed less than 0.1 seconds. Perhaps surprisingly, the more students were in the list, the 'better' results were obtained. With smaller samples, sometimes groups containing only 2 or 3 students appeared (this only happened if for some p,the number of students in A_p was less than 16), but the members of the administrative staff were happy to find suitable mergers by hand, given that they were dealing with at most 20 students instead of around 100.

Division remainder	0	1	2	3	4	5
x_6	y	$y-2$	$y-1$	$y-1$	y	y
x_5	0	1	0	1	0	1
x_4	0	2	2	1	1	0

Table 19.4: The numbers of groups of sizes 6,5 and 4 for sets in a valid family

19.4 Individual Placement: Teaching

Individual placement means that the trainee teacher is alone with a group of pupils, observed by a supervising teacher. Students of our university teach both their specialization subjects during one placement period and for practical reasons and also to model everyday professional practice, the whole placement should take place at the same school. The requirement to teach *at least* a certain number of classes for each subject and have a supervising teacher for *both* specialization subjects leads to certain capacity restrictions on the side of schools, derived from the number of available supervising teachers, but also from the number of classes scheduled at the given school.

19.4.1 Mathematical Model and Complexity Results

There is a set S of m schools. Each school $s \in S$ has a certain capacity for each subject: the vector of capacities of school s is $\boldsymbol{c}(s) = (c_1(s), \dots, c_k(s)) \in \mathbb{N}^k$. An entry $c_p(s)$ of $\boldsymbol{c}(s)$ will be referred to as a *partial capacity* of school s for subject p, meaning that it is the maximum number of applicants from A_p that school s is able to accept. Again, we shall sometimes write $c_M(s), c_I(s)$, etc.

We also suppose that an applicant a can provide a subset $S(a) \subseteq S$ of *acceptable* schools, i.e. schools to which he/she is willing to go. This can be justified as follows: a student has already done a placement at an elementary school, so now she would like to visit a different type of school. Or, a school is neither within the town of the student's residence nor in the locality of the university, so the need to commute may be prohibitive.

An instance of TAP is a triple $I = (P, A, S)$ of the sets of subjects, applicants (characterized by their specializations and sets $S(a)$ of acceptable schools) and schools (along with their partial capacities). Given an instance of TAP, an *assignment* $\mathcal{M}$ is a subset of $A \times S$ such that each applicant $a \in A$ is a member of at most one pair in $\mathcal{M}$. We shall write $\mathcal{M}(a) = s$ if $(a, s) \in \mathcal{M}$ and say that applicant a is *assigned* (to school s); if there is no such school, applicant a is *unassigned*. The set of applicants assigned to a school s will be denoted by $\mathcal{M}(s) = \{a \in A; (a, s) \in \mathcal{M}\}$. We shall also denote by $\mathcal{M}_p(s)$ the set of applicants assigned to school s whose specialization includes subject p and by $\mathcal{M}_{p,r}(s)$ the set of applicants assigned to s whose specialization is exactly the pair $\{p, r\}$. More precisely, $\mathcal{M}_p(s) = \{a \in A : (a, s) \in \mathcal{M} \;\&\; p \in \boldsymbol{p}(a)\}$ and $\mathcal{M}_{p,r}(s) = \{a \in A : (a, s) \in \mathcal{M} \;\&\; \{p, r\} = \boldsymbol{p}(a)\}$. An assignment $\mathcal{M}$ is *feasible* if $\mathcal{M}(a) \in S(a)$ for each $a \in A$ and $|\mathcal{M}_p(s)| \leq c_p(s)$ for each $s \in S$ and each $p \in P$.

MAX-TAP denotes the problem of deciding, given an instance J of TAP and an integer ℓ, whether a feasible assignment exists that assigns at least ℓ applicants. A special case of MAX-TAP asking for an assignment that leaves no student unassigned will be denoted by FULL-TAP.

Example 19.2. *Again, to get an idea of the difficulty of the problem, let us have a look at the following example. Trying to assign students one by one may lead to a maximal matching (no other student can be assigned). Clearly, not every maximal matching is maximum. In the* TAP *instance with the set of subjects* $P = \{1, 2, 3\}$, *given in Figure 19.5, the maximum matching is* $\mathcal{M}_1 = \{(a_1, s_1), (a_2, s_1), (a_3, s_2)\}$ *of size 3. Matching* $\mathcal{M}_2 = \{(a_3, s_1)\}$ *is maximal and its size is 1. This shows that the size of a maximal matching can be only one third of the size of a maximum matching. Cechlárová et al. (2015a) prove that it cannot be less.*

school	capacities for			applicant	type	acceptable schools
	1	2	3			
s_1	2	1	1	a_1	$\{1, 2\}$	s_1
s_2	0	1	1	a_2	$\{1, 3\}$	s_1
				a_3	$\{2, 3\}$	s_1, s_2

Table 19.5: An instance of TAP for Example 19.2

If there are only two subjects in a given instance then all applicants are in a sense equivalent and MAX-TAP reduces to the famous maximum cardinality matching problem in a bipartite graph. In what follows we list several results concerning the intractability of FULL-TAP and MAX-TAP as well as the identified polynomially solvable cases (Cechlárová et al., 2015b).

Theorem 19.2. FULL-TAP *is* NP-*complete even when* $|S(a)| \leq 3$ *for each* $a \in A$ *and*

(i) $|P| = 3$ *and no partial capacity of a school exceeds* 2*; or*

(ii) $|P| = 4$ *and no partial capacity of a school exceeds* 1,

but it is polynomially solvable if either $|P| = 2$, *or* $|P| = 3$ *and no partial capacity of a school exceeds 1; or if each applicant is allowed to list at most 2 acceptable schools and all partial capacities are at most 1.*

Theorem 19.3. MAX-TAP *is* NP*-complete, even in the following restricted cases:*

(i) $S(a) = S$ *for each applicant and no partial capacity exceeds 2;*

(ii) *each applicant is allowed to list at most 2 acceptable schools and all partial capacities are at most 1.*

The identified efficiently solvable special cases are of little practical significance. Therefore, the approximability of the optimization version of FULL-TAP was explored by Cechlárová et al. (2015a). Two greedy-like approximation algorithms were proposed. The theoretical approximation guarantees were $1/2$ and $2/k$ respectively, and although computational experiments showed a much better performance on the instances generated, we still preferred an exact algorithm based on integer programming.

19.4.2 Integer Linear Program, its Implementation and Application to Real Data

Taking into account the intractability results for MAX-TAP, we decided to use integer linear programming. This approach allows also for some special features that were encountered in the real data. First, we allow the possibility that some applicants, usually only a small proportion, need a placement for one subject only. This situation occurs if an applicant has received recognition for one of their subjects as a result of some other activity (e.g., teaching the subject in question in a specialized summer camp, working in a counseling center, etc.) or if the applicant has failed an exam that is a prerequisite for a particular subject and cannot therefore study that subject before resiting and passing the exam at a later date. Some placement schools also ask not to be sent more than a certain number of trainees at once, regardless of their specialization. We shall also describe the special treatment used in cases where students specializing in one subject are allowed to do their practical placement in another (related) subject. This is the case for students of Psychology, as the number of supervising teachers for them is not sufficient, moreover, elementary schools and most secondary schools do not teach Psychology as a separate subject. The common practice therefore is to allocate these students to either Ethics or Civics courses instead.

Let J be an instance of TAP with applicants $A = \{a_1, \ldots, a_n\}$, schools $S = \{s_1 \ldots, s_m\}$ and subjects $P = \{p_1, \ldots, p_k\}$. Let us associate with each applicant $a_i \in A$ a vector v^i of length k such that $v^i_p = 1$ if the specialization of a_i involves subject p and $v^i_p = 0$ otherwise. In our context, $v^i_p = 1$ for at most two values of p, for a given i, $(1 \leq i \leq n)$. Further, each applicant a_i has an ordered list of length $\ell(a_i)$ consisting of acceptable schools in $S(a_i)$. Let $pos(i, r, j) = 1$ if position r in the ordered list of applicant a_i contains school s_j, where $i = 1, 2, \ldots, n$, $r = 1, 2, \ldots, \ell(a_i)$, $j = 1, 2, \ldots, m$ and $pos(i, r, j) = 0$ otherwise.

The set of variables will be $X = \{x^i_r \ := 1, 2, \ldots, n, r = 1, 2, \ldots, \ell(a_i) + 1\}$ with the following interpretation:

$$x^i_r = \begin{cases} 1 & \text{if } a_i \text{ is assigned to the school in position } r \text{ in her list} \\ 0 & \text{otherwise} \end{cases}$$

for $r = 1, 2, \ldots, \ell(a_i)$, and

$$x^i_{\ell(a_i)+1} = \begin{cases} 1 & \text{if } a_i \text{ is unmatched} \\ 0 & \text{otherwise.} \end{cases}$$

We consider the following integer linear program:

$$\begin{aligned} \sum_{i=1}^{n} \sum_{r=1}^{\ell(a_i)} x^i_r &\rightarrow max \\ \sum_{r=1}^{\ell(a_i)+1} x^i_r &= 1 \text{ for } i = 1, \ldots, n \\ \sum_{i=1}^{n} \sum_{r=1}^{\ell(a_i)} pos(i,r,j) v^i_p x^i_r &\leq c_p(s_j) \text{ for } j = 1, \ldots, m,\ p = 1, \ldots, k \\ x^i_r &\in \{0,1\} \text{ for } j = 1, \ldots, n,\ r = 1, \ldots, \ell(a_i) + 1 \end{aligned}$$

The first set of constraints ensure that each applicant is assigned to exactly one school or is unassigned. The second set of constraints (notice that they are linear, since $pos(i, r, j)$ and v^i_p are constants) ensure that the number of applicants from A_p that are assigned to school s_j does not exceed the partial capacity $c_p(s_j)$ of subject p at school s_j. It is obvious that an optimal solution of this linear integer program corresponds to a solution of MAX-TAP.

Let us now describe how we handled the possibility that it may be acceptable to assign applicants whose specialization involves a certain subject (for simplicity, let us suppose that the index of this subject is 1), to places of some related subjects (here, again for ease of exposition, let us suppose that these related subjects are indexed by 2 and 3). First, let us denote the set of applicants a_i in $A_1 \backslash (A_2 \cup A_3)$, i.e., those with $v^i_1 = 1$ and $v^i_2 = v^i_3 = 0$, by A'. For each $a_i \in A'$ we created two clones a_{i+n} and a_{i+2n}, such that

$$\begin{aligned} &v^{i+n}_1 = 0; \quad v^{i+n}_2 = 1; \quad v^{i+n}_3 = 0; \\ &v^{i+2n}_1 = 0; \ v^{i+2n}_2 = 0; \ v^{i+2n}_3 = 1; \\ &v^{i+n}_j = v^{i+2n}_j = v^i_j \text{ for each } j > 3. \end{aligned}$$

The lists of acceptable schools for both clones of a_i are the same as that of a_i. The constraints applied to a_i are applied in similar fashion to a_{i+n} and a_{i+2n}.

Since we require that at most one of the three clones be matched, the unmatched position $\ell(a_i) + 1$ may be 0 for at most one of the three clones. Thus the sum across the 3 unmatched positions must be greater than or equal to 2, i.e. for each $a_i \in A'$ we add a constraint

$$x^i_{\ell(a_i)+1} + x^{i+n}_{\ell(a_i)+1} + x^{i+2n}_{\ell(a_i)+1} \geq 2.$$

The results of the application of an implementation with IP solver CPLEX 12.4 to the list of 175 schools used before 2015 were reported by Cechlárová et al. (2015b). The software that is currently in use at our university was implemented

Year	# of students	without replacement for Psychology		with replacement for Psychology	
		time	# of assigned	time	# of assigned
2010	103	0.66	95	0.87	101
2011	118	3.42	90	16.18	98
2012	100	0.49	77	8.11	93
2013	138	1.10	114	34.17	116
2014	82	0.38	73	0.60	78
2015	107	0.61	93	0.91	100
2016	102	0.59	94	0.90	100

Table 19.6: Experiments with real data

by Michal Barančík (2015) in Java using the open source library *lpsolve.jar*. Experiments were carried out on a desktop PC with an Intel i5-2500 3.3Ghz processor, with 4GB of memory running Windows 7 Enterprise.

We tested this implementation using the real data of students from the years 2010 to 2016. Their numbers varied from a minimum of 82 to a maximum of 138. The list of schools was in all cases the same, containing 53 schools, all in Košice. The numbers of assigned students and run-times in seconds for individual years are contained in Table 19.6.

However, the free Java LP-solver got close to its limits. When we submitted combined two-year lists, in several cases we did not obtain a solution within 10 minutes, so decided to stop the computation.

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