Preference Formation in School Choice

COMSOC Summer School on Matching Problems, Markets and Mechanisms

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The school choice problem

School choice procedures refer to explicit procedures used to assign children / students to schools taking into account their preferences

- In systems without tradition of choice, motivation comes from willingness to **take parents' preferences into account** and idea that competition will induce schools to **respond to demand**
- In systems with tradition of unregulated choice, motivations comes from willingness to address **congestion** and equity concerns that unregulated markets raise
  - Congestion arises from saturation or in urban contexts as soon as there is some (even slight) preference polarization

The school choice problem (cont'd)

Seminal article by Abdulkadiroğlu and Sömnez (AER, 2003) introduces **mechanism design approach** to analysis of school choice procedures

- Students have **exogenous preferences** over schools
- Students benefit from **priorities** at these schools
- Schools have capacities
- A school choice procedure is a procedure that matches students to schools taking as inputs students' preference reports, school priorities and capacities

$\rightarrow$ Goal is to design best procedure according to some criteria

Specificities of the school choice problem

— Relative to standard two-sided matching problem
  – Exogenous priorities: preferences only on one side of the market (! Still differs from assignment as schools can be strategic)
  – Coarse priorities are common: will require tie-breaking

Has spurred distinct and specific "school choice" lit

— Applications tend to involve many students

Much recent interest in large market properties

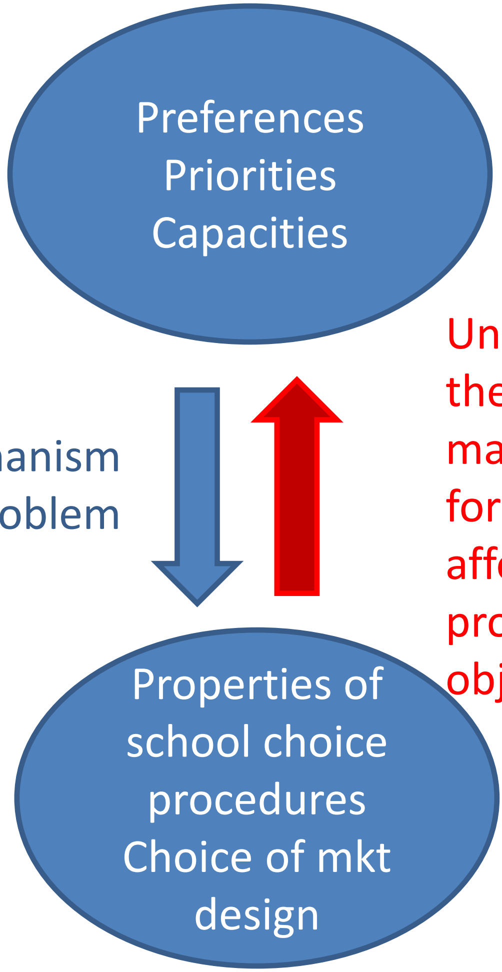
— Nature of good
  – School attendance obligation in most countries
  – Public policy interest

Influences priorities and objectives, little research

  – Key input to community and individual socialization
  – Multi-attribute nature of good (and partial observability)

Nature of preferences: **focus of this talk**

Objectives



Mechanism design problem

Understand interactions between the school choice procedure (i.e. the market design) and preference formation since this will eventually affect ability of school choice procedures to meet original policy objectives

Rmk 1: School choice will be main motivation but some of the issues relevant to preference formation in other matching contexts

Objectives for today: give you a taste for wide open research area !

1. Set-up and typology of channels for preference formation

2. Appl’n 1: Interdependent preferences

3. Appl’n 2: Preferences over peers

1. Set-up and channels for preference formation

Canonical model of school choice

C schools, with **capacities** q_c, c = 1, …, C.

S students, with strict **preferences** P_s over schools

It will be useful to assume a cardinal representation for preferences:

$cP_sc' \Leftrightarrow u_{sc} > u_{sc'}$

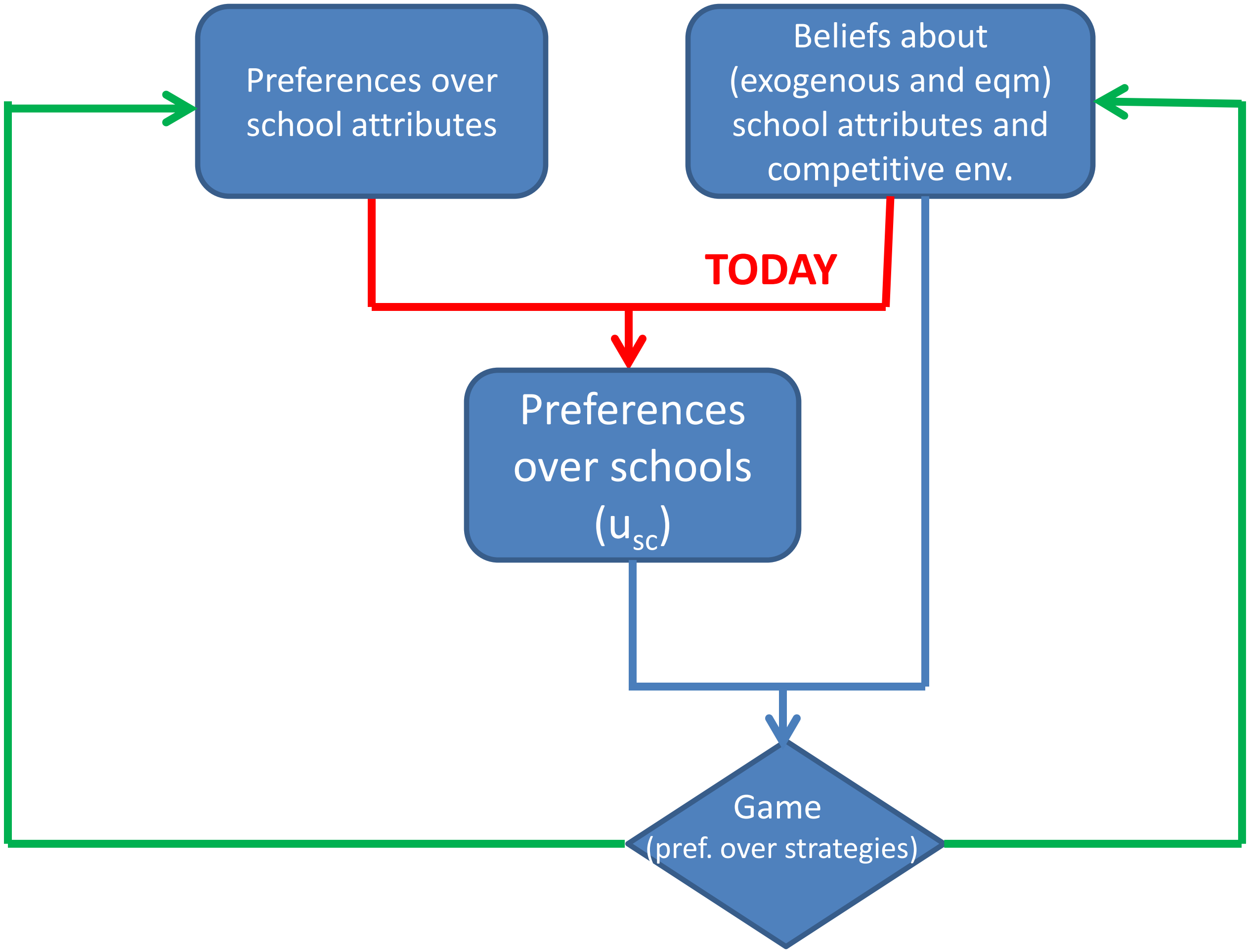
School *c* is acceptable if $u_{sc} \geq 0$

Students benefit from **priorities** at schools

- Let e_{sc} be the priority from which student s benefits at school s.
  $e_{sc} > e_{s'c}$ means that student s has priority over s' at school c
- **Coarse priorities**: ∃s,s' such that $e_{sc} = e_{s'c}$
- **Strict priorities**: ∀ s ≠ s', $e_{sc} > e_{s'c}$ or $e_{s'c} > e_{sc}$

A typology for how the chosen procedure can affect preferences

Pure preference channel: Procedure and rest of environment influence framing, saliency, …



TODAY

Information-channel: Procedure and rest of environment influence information and beliefs

The information channel - examples

1. Interdependent preferences
   - In practice, students may be imperfectly informed about the quality of the school they apply to
   - Observing the outcome of the match can be informative about this quality if students' preferences are sufficiently congruent and students observe different signals

2. Preferences over peers
   - Students (parents) care the quality of theirs peers in school
   - Students may want to want to make sure to be in the same school as their friends
   - Coordination znd beliefs about who will be matched where becomes important

The information channel – examples (contd)

3. Costly preference acquisition (not covered today)
   - Idea is that students do not have full information about schools. Discovering characteristics of these schools takes time, requires on-site visit, talking to current and past students, …. i.e. it is costly
   - The issue now will be: how many schools should you investigate? (Lee and Schwartz, 2011)
     - Expected benefits from an additional investigation decline
     - Cost constant

2. Appl'n 1: Interdependent preferences

Motivation

— In practice, students may be imperfectly informed about the quality of the school or college they apply to

— Observing the outcome of the match can be informative about this quality if students' preferences are sufficiently congruent and students observe different signals

→ Implications for stability and other properties of school choice procedures?

Model (adapted* from Chakraborty et al. 2010)

(unobserved) school qualities δ_c $\in \Delta$ (finite)

Students receive **signals x_{sc}** $\in X$

} Pr(δ,x) joint probability distribution

Priorities e_{sc} are common knowledge

Preferences:

$w_{sc}(\delta,x)$ = student s' util from school c, *given* vector of qualities and signal realizations (δ,x) (intrinsic preference)

<u>Special cases</u>: $w_{sc}(\delta,x) = \delta_c$ (pure common value)

$w_{sc}(\delta,x) = x_{sc}$ (private value) – back to std case

Given information I, **student s' expected utility from c** is given by

$$u_{sc}(I) = \sum_{\delta,x} w_{sc}(\delta, x) \Pr(\delta, x | I)$$

* Chakraborty et al. study interdependent preferences over students

Notion of stability

- Because procedure used to reach matching will influence students' beliefs about qualities, stability cannot be defined solely on the basis of the resulting matching, *but also with respect to procedure used to reach it.*

- Define generic direct mechanism Γ: X x [0,1] → set of matchings M

  Randomization device ω

- Information structure: each student receives signal based on $(\hat{x},\omega)$
  - Special cases: (1) students only observe their own match; (2) students observe the entire match; (3) students observe cutoffs, …
  - Notion of coarser or less coarse information structure

Notion of stability (cont'd)

Consider following extensive form game:

1. Nature selects δ, x according to $\Pr(\delta, x)$; each student receives signal vector x_s
2. Students report $\hat{x}_s$
3. Matching $\mu(\hat{x}, \omega)$ is generated
4. Each student receives message z_s
5. Student s either accepts $\mu(\hat{x}, \omega)(s)$, rejects it and/or offers to rematch with (other) school c
6. Any school that received a rematching offer accepts or rejects (possibly also dropping one of the students it was matched to)

A **mechanism is stable** under information structure z if there exists a Perfect Bayesian eqm of this game in which all students report their signals truthfully and accept their assignment on the equilibrium path

Stability is difficult to achieve

Thm 1 (Chakraborty et al. 2010): If a mechanism is stable under some information structure, then it is also stable for a coarser information structure

Intuition: the more info you give, the more likely students will learn something new

Result (Chakraborty et al. 2010): There **may not** exist a stable mechanism even if the only information students receive concerns their own match

Intuition: Suppose a student does not have priority at a school, and he receives a signal that this school is of high quality. By reporting a low signal, he could mislead the other students that this school is not worth it and so secure a place (example of Chakraborty et al pretty knife-edge: one student is perfectly informed but that information is not useful to him, only to affect allocation – example 2)

But some good news in empirically relevant environments

Result (Chakraborty et al., 2010): If students benefit from the same priorities at all schools, then there exists a stable mechanism (serial dictatorship) when the only information that gets revealed is individual matches.

Appl'n: purely score-based university admission procedures

Lots of open questions remaining

- Theory says existence of a stable mechanism will depend on the structure of priorities, degree of interdependence / congruence in preferences, ….
  - Becomes an empirical question!
- Other properties of mechanisms unexplored (strategyproofness, efficiency, …)
- Example of practices and information structure, where information is generated during the procedure
  - In pure score-based university admission systems, cutoffs are often made public (China, Hungary, Germany, Ukraine)
  - In Antwerp (first-come, first-served), schools are asked to open building for queues « so that they are not visible ».
  - Increasing practice of online / phone implementation of first-come, first-served procedures.

Appl'n 2: Preferences over peers

Motivation

In practice, students (parents) care about who else goes to the school

- Friends
- Racial or socio-economic composition
- Academic caliber

$\rightarrow$ Implications for stability and other properties of school choice procedures?

Existing work on preferences over peers in matching models

- There is some work on « preferences over colleagues » in the two-sided matching literature
  - couples are also a special case (see David's lectures)
- When students have preferences over peers as well as over schools, the core may be empty (in other words, a stable matching may not exist)
- Literature has focused on identifying restrictions on preferences to restablish existence (e.g. Dutta-Massó, 1997) or seek maximally stable allocations (Echenique-Yenmez, 2007, Pycia, 2012)
  - Some interesting insights such as the fact that existence breakdown will depend on relative importance of peer effects versus other drivers of preferences

Less studied consequences of peer effects

- Multiple equilibria
  - Though Brock and Durlauf (2006) suggest (in another setting) that multiple equilibria less likely when the number of alternatives goes up, a relevant case in the school choice context
- Discrepancy between NE outcome and welfare maximizing outcome
- Congestion, even in the absence of an overall capacity constraint

Exception: Calsamiglia, Miralles, Mora (2013)

A toy model with endogenous preferences

- There are 2 schools, $c \in \{c_1, c_2\}$, each with capacity ½.
- There is a continuum of students of mass 1, indexed by s, and characterized by:
  - Their socio-economic status β_s
  - Their preferences for attending each school u_{sc}

— Students' preferences over schools take the following form:

$$u_{sc} = \alpha_s \delta_c + \varepsilon_{sc}$$

Relative importance of peers in utility function

school c' s endogenous quality

idiosyncratic preference for school c

δ_c = mean socio-economic status of pupils in school c

Further assumptions on $u_{sc} = \alpha_s \delta_c + \varepsilon_{sc}$

- $\boldsymbol{\beta_s} \in \{\boldsymbol{\beta_L}, \boldsymbol{\beta_H}\}$ with $\beta_L < \beta_H$ and $\boldsymbol{\alpha_s} \in \{\boldsymbol{\alpha_L}, \boldsymbol{\alpha_H}\}$ with $\alpha_L < \alpha_H$
  - Captures idea that how much parents care about the quality peers is correlated with socio-economic status (Burgess et al. 2009, Coldron et al, 2009)
  - Does not account for homophily that might also be at play
- Mass of H-type students is **λ**
- $\boldsymbol{\varepsilon_{sc}}$ are i.i.d. across students and schools, with mean zero (let F be the cdf of ε_{s1} - ε_{s2})
  - Except for quality of student intake, there is no difference in the aggregate perceived quality of the two schools, and there is no intrinsic preference for one school over the other across socio-economic status (can be relaxed)
- Assume that $\boldsymbol{\alpha_H(\beta_H - \beta_L) \leq (\bar{\varepsilon} - \underline{\varepsilon})}$ (will ensure that whatever the social mix of a school, there is always some students from both types who like it best)

Allocation under the student-proposing DA (Azevedo-Leshno, 2012)

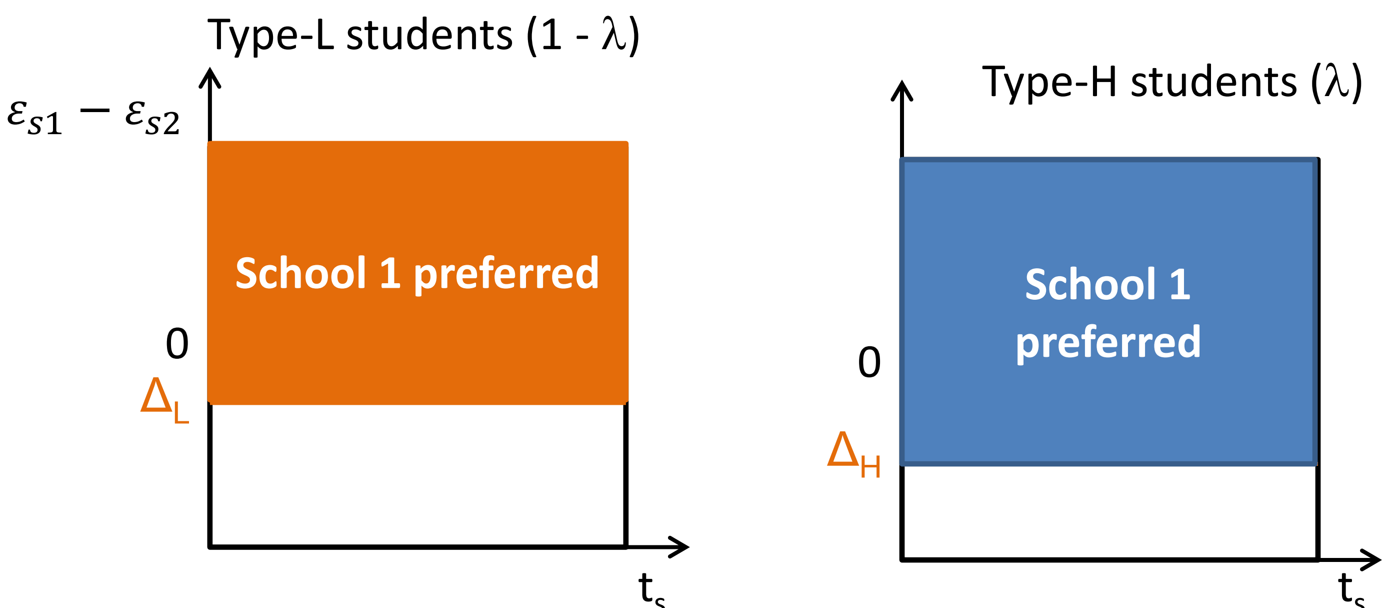
Assume there is common random tie-breaking rule ($t_s \in [0,1]$).

Student s prefers school 1 to school 2 iff

$$u_{s1} = \alpha_s \delta_1 + \varepsilon_{s1} > u_{s2} = \alpha_s \delta_2 + \varepsilon_{s2}, \text{ i.e.}$$

$$\varepsilon_{s1} - \varepsilon_{s2} > -\alpha_s(\delta_1 - \delta_2)$$

Suppose that $\delta_1 > \delta_2$



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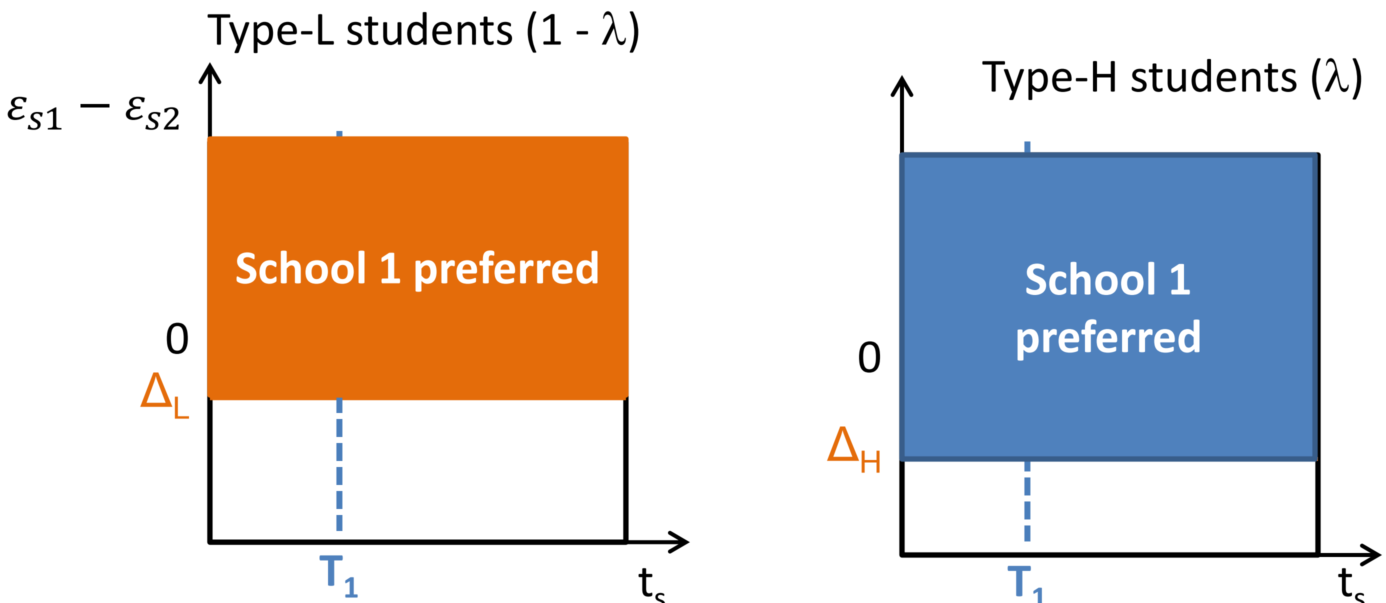

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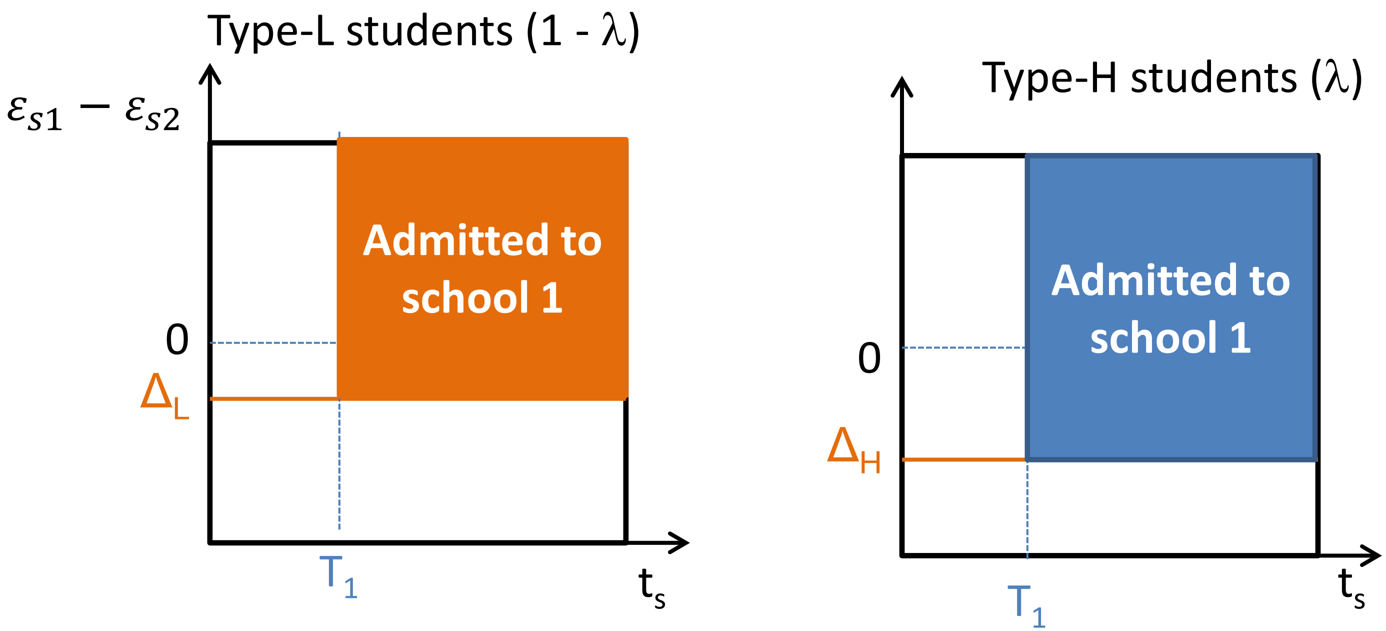


T_1 is such that:

$$(1 - T_1)\big(1 - F(\Delta_L)\big)(1 - \lambda) + (1 - T_1)(1 - F(\Delta_H))\lambda = \frac{1}{2}$$

An equilibrium is characterized by

- School-specific cutoffs, T_c (min. priority draw for admission in school c)

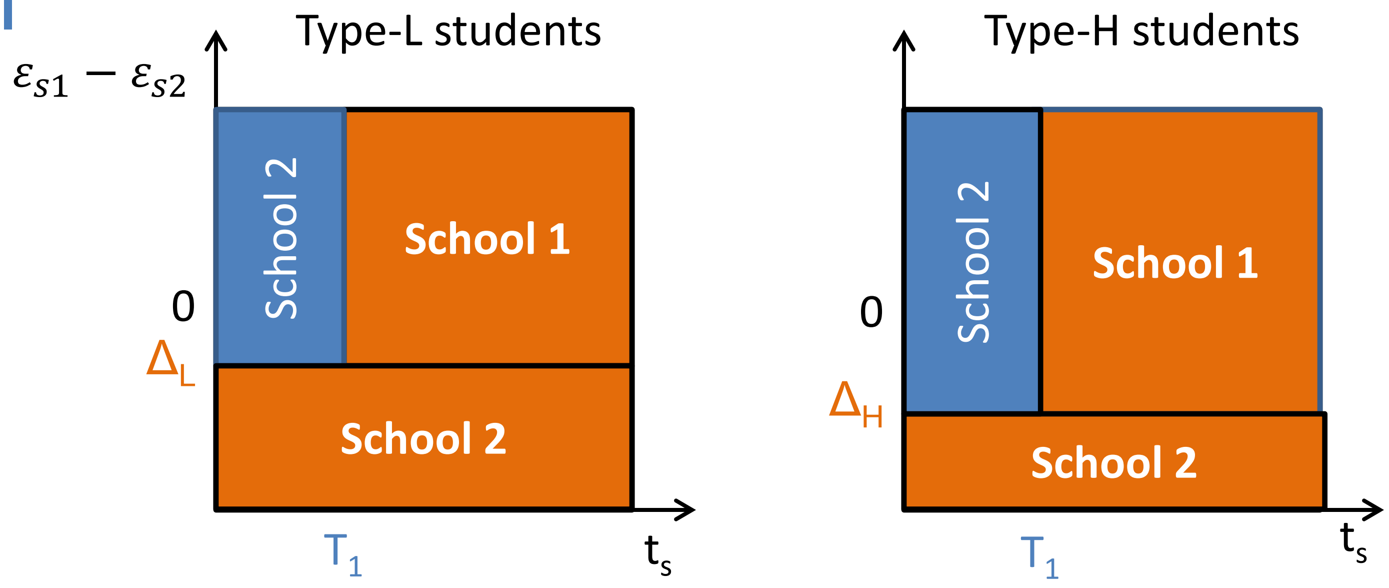
- Two indifference thresholds, Δ_L and Δ_H, that determine the value of $\varepsilon_{s1} - \varepsilon_{s2}$ such that student s is indifferent between school 1 and school 2:

$$u_{s1} = u_{s2} \Leftrightarrow \Delta_s \equiv \varepsilon_{s1} - \varepsilon_{s2} = -\alpha_s(\delta_1 - \delta_2)$$

- Equilibrium values for δ_1 and δ_2:

$$\delta_c = E[\beta_s | \mu(s) = c]$$

Final allocation



- Students in the orange areas go to their first choice school and students in the blue areas get their second choice
- Greater fraction of H-type students in school 1 (→ consistent with $\delta_1 > \delta_2$)

Equilibrium properties

- [When F has increasing hazard rate], there are 3 eq: 2 asymmetric eq + one symmetric eqm with $\delta_1 = \delta_2$ and $T_1 = T_2 = 0$
  - In asymmetric eq, the H-types are those who are most censored (= “unhappy and vocal parents”)
  - In the symmetric eqm, everybody gets his/her first choice

- Comparative statics for the eqm with $\delta_1 > \delta_2$:
  - $\alpha_H \nearrow \Longrightarrow T_1 \nearrow$, segregation ↗ and less people get their first choice (polarisation)
  - $\alpha_L \nearrow \Longrightarrow T_1 \searrow$, segregation ↘ and more people get their first choice
  - When $\alpha_H = \alpha_L$, there is a unique eqm

  Polarization and congestion comes from the fact that people from different SES **care differently** about peers, <u>not</u> that people care about peers (≠ homophily)

What changes with peer effects

- **Given profile of ROLs by all other students**, it is a **BR** to submit truthful (**given resulting school composition**) ROLs
  - NE concept, no longer dominant strategies as in standard model with exogenous preferences
  - Greater informational requirement for data to be generated from equilibrium

- Multiple equilibria are possible
- As with any other externality, NE outcome may not maximize welfare

Social diversity quotas

- Suppose now that the policy is to promote social diversity in schools and it is implemented using a double quota. At each school,
  - H-type students have priority over fraction $\lambda/_2$ of the seats
  - L-type students have priority over fraction $^{(1-\lambda)}/_2$ of the seats

  (these quotas ensure that student composition is as close as possible to population composition, conditional on demand)

- Eqm has now type-specific cutoffs, T_{1L}, T_{1H}, T_{2L}, T_{2H}

Equilibrium with social diversity quotas

In the unique eqm, $\delta_1 = \delta_2$
Every student gets his/her first choice
The social mix of both schools is the same (no social segregation)

Model can be extended (with some changes in results) to settings where some school exogenously better and more than 2 schools.

Model helps explain observed correlation in the UK between the level of school segregation, admission policies fostering or not social diversity, the fraction of parents getting their first choices and the level of appeals (Coldron report, 2008)

What do we learn from this simple model with peer effects?

- In our setting, given fixed preferences, outcome is pareto efficient … but different rules will lead to different preferences
- One way to compare welfare across preference profiles is rank maximality
  - Preference polarization induces congestion and poor performance based on rank maximality partial order
- Lesson: Preference polarization may be exacerbated, or to the contrary, reduced by the mechanism
  - Because of the way participants adjust to the **strategic incentives**
    - Boston mechanism: congestion reduced
    - HBS course allocation: congestion increased, resulting in ex-post and ex-ante inefficiency
    - Early-labor job market (decentralized): congestion results in applicants and departments being unmatched
  - Because of the way it influences preferences – **this lecture**

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