University admissions in Germany: empirical and experimental evidence

Dorothea Kübler

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Introduction

- Quotas for special groups of students play an important role in practice. How should they be implemented?
- We analyze an existing mechanism with quotas and propose an alternative mechanism for the problem.
- Theory, data, simulations, and experiments are used to understand the existing market and to propose a re-design.

Introduction

- Places for medicine and related subjects in Germany allocated by centralized procedure
(Winter term 2010/2011: 56 000 applicants for 13 000 places)
- Admissions procedure *sequential* and consists of
  (1) a *priority-based part*, where fraction of total capacity is allocated among “special applicants” on basis of their preferences and exogenous admission criteria, and
  (2) a *two-sided part*, where remaining seats are allocated among remaining applicants on basis of applicants’ and universities’ preferences.
- Procedure is sequential: First the seats in the priority-based part are filled. Then all remaining applicants are considered in the two-sided part.

Current procedure

(Westkamp, 2012, and Braun, Dwenger, Kübler, 2010)

Current procedure: Overview

- **Part 1: Priority-based part**
  - 20 % of places reserved for applicants with very good grades; 20 % for those with the longest waiting time
  - Allocation of places on basis of applicants' preferences and *exogenous admission priorities*.
- **Part 2: Two-sided part**
  - Conducted about one month after priority-based part
  - All remaining places allocated among remaining applicants on basis of applicants' preferences and criteria chosen by universities.
- Only difference between two types of places is time of allocation.
- Applicants can submit separate preference lists for each part, but at the same point in time.
- For the remainder, abstract from waiting-time quota.

Current procedure: Priority-based part

- **Selection:** Order applicants w.r.t. average grades and select as many as places are available in priority-based part, that is $q^1 = \sum_u q_{(u,1)}$.
  - **Terminology:** Selected applicant $\hat{=}$ top-grade applicant
- **Assignment:**
  1. Each applicant "applies" to top choice (wrt ranking submitted for first part). Each university accepts applicants in the order of average grades until capacity $q_{(u,1)}$ is filled, or there are no more applicants who ranked it first.
  2. Each applicant "applies" to second choice. Each university accepts applicants in order of average grades until remaining capacity is filled, or there are no more applicants who ranked it second.

     $\vdots$
- *Boston mechanism* (Abdulkadiroglu and Sönmez, 2003)

Current procedure: Two-sided part

- **Available capacity at u** = initial capacity for two-sided part + remainder from priority-based part ($=: q_{(u,2)}$)
- **Evaluation:** Universities evaluate *remaining* applicants and submit strict rankings
- **Assignment:**
  1. Each university offers admission to $q_{(u,2)}$ most preferred individual applicants. Each applicant *temporarily* accepts best offer (wrt list submitted for second part).
  2. Each university offers admission to $q_{(u,2)}$ most preferred applicants among those who have not rejected it in first round. Each applicant *temporarily* accepts best offer.

  ⋮
- *University-proposing deferred acceptance algorithm* (Gale and Shapley, 1962)

Current procedure: Example

- Eight students ($s_1, \ldots, s_8$) indexed in increasing order of average grades (s_1 is best)
- Four universities (W, X, Y, Z), each with (initially) one place in priority based and one place in two-sided part
- Students' and universities' preferences

$$P_{s_i}: \quad W \succ X \succ Y \succ Z, \qquad \forall i = 1, 2, ..., 8$$
$$P_u: \quad s_1 \succ s_2 \succ s_3 \succ s_4 \succ s_5 \succ s_6 \succ s_7 \succ s_8, \qquad \forall u = W, X, Y, Z.$$

Current procedure: Example

- Suppose all applicants report truthfully in both parts of the procedure
- Outcome is

$$\mu = \begin{pmatrix} W & X & Y & Z \\ s_1|s_5 & s_2|s_6 & s_3|s_7 & s_4|s_8 \end{pmatrix}.$$

- The matching μ is not an equilibrium of the revelation game induced by the mechanism.

Current procedure: Discussion

- Distinctive feature of German system: Sequential procedure with capacity redistribution
- Strong incentives for manipulating the procedure, which are known to matchmaker. Top-grade applicants are advised that
  (a) chance of being assigned to a university in priority-based part decreases significantly if it is not ranked first,
  (b) it may be beneficial to truncate preference lists for the first part, and
  (c) they lose guaranteed priority over others in the two-sided part.
- Clearinghouse provides information about grades (and ranks in part 1) necessary to get into universities in previous years.

Current procedure: Empirical findings

- Data set contains all applications (around 60,000) in biology, medicine, pharmacy, psychology, animal health and dentistry for the winter term 2006/2007.
- Focus on two aspects of strategic behavior
  - (a) manipulation of ROLs in Boston mechanism
  - (b) truncation of ROLs due to sequential structure
- Findings:
  - (a) significant difference between drop between number of applications ranking an over-demanded university first and second compared to drop for non-overdemanded universities (not for second and third, third and fourth...)

Current procedure: Empirical findings

(b) Fraction of applicants submitting truncated preference lists in top-grade procedure

Grade	Truncations
1.0 – 1.2	.602
1.3 – 1.5	.496
1.6 1.8	.406
1.9 2.1	.319
≥ 2.2	.390

# of univ. ranked	All applicants	Top-grade applicants
1	.111	.261
2	.061	.097
3	.075	.118
4	.055	.080
5	.096	.081
6	.602	.364
N	61,317	3,274

Current procedure: Simulation

- Unique property of German mechanism that three different rank-order lists can be submitted.
- Interpret ROLs submitted in second part as true preferences (Why? University preferences highly correlated; truncations are exhaustive)
- Assume that universities rank applicants according to their grades.

Current procedure: Simulation results

- Preferences received by applicants selected in top-grade procedure, by mechanism and preferences submitted:

	Stated pref.	True pref.	True pref. & truncate in first part
1st preference	.795	.803	.968
2nd preference	.096	.098	.022
3rd preference	.038	.036	.004
4th preference	.021	.025	.000
5th preference	.021	.027	.000
6th preference	.010	.010	.000
Unassigned	.019	.002	.006

Current procedure: Simulation results

- Simulations show that around 20 percent of selected students are better off when we truncate their ROL after first choice.
- But: simulations are based on assumptions about the applicants' strategies and about preferences of universities.

Experiment

Implementing quotas in university admissions:
An experimental analysis
(Braun, Dwenger, Kübler, Westkamp, 2013)

Experiment

Experiment has two three goals:

- Check whether too few top-grade students truncate their ROLs in first part of current procedure
- Compare current mechanism to a mechanism based on SDA with quotas
- Compare implementation in weakly dominant strategies when the strategies are simple (truth telling) or more complex (strategic misrepresentation).

Related literature

- **Optimal stable matching mechanism in applications**
  Roth (1984), Kamada, Kojima (2010)
- **Boston mechanism**
  Abdulkadiroglu, Sönmez (2003), Ergin, Sönmez (2006)
- **Controlled choice in matching problems**
  Abdulkadiroglu, Sönmez (2003), Kamada, Kojima (2011), Ehlers, Hafalir, Yenmez, Yildirim (2011), Kojima (2010) Hafalir, Yenmez, Yildirim (2013), Echenique, Yenmez (2012), Kominers, Sönmez (2012)
- **Two-sided matching experiments**
  Chen, Sönmez (2006), Pais, Pinter (2008), Featherstone, Niederle (2009), Echenique, Wilson, Yariv (2009), Calsamigla, Haeringer, Klijn (2010)
- **Sequential mechanisms**
  Dur, Kesten (2013)

Treatments

Two treatments

1. Sequential allocation (**MSEQ**)
   - First part: Boston mechanism for top-grade applicants
   - Second part: SDA (student proposing deferred acceptance) to allocate all remaining places
2. Simultaneous allocation (**MSIM**)
   - SDA with capacity redistribution (Westkamp, 2012)

Alternative mechanism

- Simultaneous allocation of all seats using SDA with capacity redistribution
- In a typical round of the SDA
  - applicants apply to most preferred university among those that have not rejected them yet
  - each university *u*,
    first *temporarily* admits the $q_{(u,1)}$ top-grade applicants with the best average grades (all top-grade applicants if fewer than $q_{(u,1)}$)
    and then *temporarily* admits its most preferred *remaining* applicants up to its *remaining capacity*

- Westkamp (2012) develops general approach to *matching problems with complex constraints*
- Desire to achieve some target distribution of student types but should not waste capacity to achieve this, i.e., accept violations of affirmative action policy
- Allow for universities to prefer some violations over others, i.e., complex preferences for two-sided part (e.g. 50 % of *remaining* places allocated on basis of interviews, 50 % on basis of objective criteria)
- Neither a special case of, nor more general than other matching problems with constraints
- Controlled choice constraints of Abdulkadiroglu and Sönmez (2003): each student has one characteristic and schools have fixed upper bounds on the number of students with a certain characteristic they are willing to admit (majority quota)

Alternative mechanism: Properties

- Algorithm produces the *applicant optimal "stable" matching* (Roth, 1984)
- Strategic properties:

Theorem

(i) *For MSEQ, it is a weakly dominant strategy for each student to submit her preferences truthfully for the second part of the mechanism.*

(ii) *For MSIM, it is a weakly dominant strategy for each student to submit her preferences truthfully.*

Equilibrium outcomes

Theorem

Consider any complete information Nash equilibrium of MSEQ such that

(i) *all students submit preferences truthfully for the second part of the mechanism, and*

(ii) *no top-grade student who is matched to a university u in the second part of MSEQ could have been matched to u in the first part of MSEQ by unilaterally deviating to a strategy which ranks u as her top choice for that part.*

Then this equilibrium coincides with the outcome of MSIM under truth-telling.

Example revisited

- The outcome of MSIM is given by

$$\nu = \begin{pmatrix} W & X & Y & Z \\ s_1 | s_2 & s_3 | s_4 & s_5 | s_6 & s_7 | s_8 \end{pmatrix}.$$

- This outcome is also the only equilibrium outcome of MSEQ with truth-telling at the second stage and where top-grade students are matched as early as possible.
- In all of our experimental markets, all Nash-equilibria of MSEQ in which no applicant employs a weakly dominated strategy yield the same matching of applicants as MSIM under truth-telling.
- For this and all other markets studied in the experiment, the only arbitrariness in equilibrium outcomes of MSEQ is in which type of place students get at their assigned universities.

Experimental Design I

- Experimental markets with 8 applicants $(1, \ldots, 8)$ and 4 universities (W, X, Y, Z) with two seats each
- Applicants indexed in increasing order of average grades
- One seat at each university initially reserved for top-grade applicants $(1, \ldots, 4)$
- Participants in the experiment always took the role of applicants
- Monetary payoffs (irrespective of which type of place received): 22 EUR for obtaining a place at first choice, 16 for second, 10 for third, and 4 for fourth.

Overview of market characteristics

Four markets were implemented that differ with respect to applicants' and universities' preferences:

	Preferences of		Incentive	With weakly
	students	universities	to misrepresent[1]	dominant strategies
Market 1	aligned	aligned	s_2, s_3, s_4	s_2, s_3, s_4
Market 2	aligned	split aligned	s_2, s_3, s_4	
Market 3	split aligned	aligned	s_2, s_4	s_2, s_4
Market 4	split aligned	split aligned	s_3, s_4	

Notes: [1] Top-grade students who can improve their payoffs by misrepresenting their true preferences in the first stage of MSEQ.

▸ Markets

Experimental design II

- Each participant randomly assigned to one of the two treatments and then played all 4 markets three times (in randomly changing roles), leading to 12 rounds per subject
- Participants in treatment MSIM submit one, those in MSEQ submit two preference lists
- At the end, one round randomly chosen to determine payoffs
- Full information about all relevant market characteristics (preferences of students and universities, capacities, quotas)
- 10 sessions for each treatment (160 participants)

Strategic coaching

- Simultaneous mechanism: Participants were told that truth-telling would always be optimal for them
- Sequential mechanism: Participants were told that
  - truth-telling always optimal for second part
  - truth-telling not always optimal in first part and truncations or skipping sometimes profitable
- Motivation: Matchmakers often try to nudge participants towards optimal application strategies
  - Examples: German university admissions, school choice in NYC and Boston,...

Results: Truth-telling

Significant differences between the mechanisms in the way applicants misrepresent their preferences:

- First stage of MSEQ: Most top-grade students either truncate or truncate and re-order their preferences.
- MSIM and second stage of MSEQ: More than 90% of applicants submit a full preference list, and more than 75% of truth-telling.

Mechanism	Truth-telling		Misrepresentation of preferences		
	All pref.	1st pref.	Trunc. (T)	Re-ord. (R)	R+T
MSIM	81.02%	87.82%	2.35%	16.00%	0.63%
MSEQ, 1st	13.68%	60.83%	50.76%	12.15%	23.40%
MSEQ, 2nd	75.35%	85.42%	4.90%	16.88%	2.88%

Results: Performance compared to equilibrium, share of rounds

- Share of rounds in which the realized matching coincides with the equilibrium matching

	MSIM	MSEQ	MSIM − MSEQ
Market 1: Fully aligned	0.9111	0.2778	0.6333***
	(0.2862)	(0.4504)	(0.0563)
Market 2: Student aligned	0.7701	0.4000	0.3701***
	(0.4232)	(0.4926)	(0.0691)
Market 3: University aligned	0.8333	0.1667	0.6667***
	(0.3748)	(0.3748)	(0.1667)
Market 4: Split aligned	0.5778	0.1333	0.4444***
	(0.4967)	(0.3418)	(0.0636)
Markets 1–4	0.7731	0.2444	0.5287***
	(0.4194)	(0.4304)	(0.0317)

Results: Performance by market

- Compare preference ranks achieved under both mechanisms relative to the equilibrium

$$M_j^{agg} = \frac{\sum_i (y_{ij}^e - y_{ij}^r)}{8}$$

	MSIM	MSEQ	MSIM − MSEQ
Market 1: Fully aligned	0.0000	-0.0111	0.0111***
	(0.0000)	(0.0358)	(0.0038)
Market 2: Student aligned	-0.0029	-0.0125	0.0096*
	(0.0188)	(0.0421)	(0.0049)
Market 3: University aligned	-0.0319	-0.1639	0.1319***
	(0.0984)	(0.1662)	(0.0204)
Market 4: Split aligned	-0.0764	-0.0639	0.0125
	(0.1453)	(0.1198)	(0.0199)
Markets 1–4	-0.0249	-0.0660	0.0411***
	(0.0824)	(0.1296)	(0.0081)

Results: Performance by applicant

	MSIM	MSEQ	MSIM − MSEQ
Student 1	-0.0084	-0.0639	0.0555***
	(0.0126)	(0.0125)	(0.0178)
Student 2	-0.0701	-0.3278	0.2577***
	(0.0302)	(0.0301)	(0.0426)
Student 3	-0.0644	-0.3833	0.3189***
	(0.0324)	(0.0323)	(0.0458)
Student 4	-0.0812	-0.2694	0.1882***
	(0.0384)	(0.0382)	(0.0541)
Student 5	-0.0308	0.2833	-0.3141***
	(0.0283)	(0.0281)	(0.0399)
Student 6	-0.0112	0.0306	-0.0418
	(0.0278)	(0.0276)	(0.0392)
Student 7	0.0588	0.2250	-0.1662***
	(0.0311)	(0.0309)	(0.0439)
Student 8	0.0084	-0.0222	0.0306**
	(0.0099)	(0.0098)	(0.0139)

Results: Weakly dominant strategies

- Strategies supporting the equilibrium in MSEQ are not unique. Thus, coordination problems can arise.
- However, in Market 1 all top-grade applicants have weakly dominant strategies.
  - s_2 truncates after first choice in 38 % of cases.
  - s_3 truncates after first or second choice in 26 % of cases.
  - s_4 truncates after first or second choice in 14 % of cases.
- Similarly in Market 3 where s_2 and s_4 have a weakly dominant strategy
  - s_2 truncates after first choice in 44.4 % of cases.
  - s_4 truncates after first or second choice in 22.2 % of cases.
- Failure of majority of top-grade participants to play weakly dominant strategy

Results: Learning

- There is some learning by top-grade applicants in MSEQ, but even in the final rounds 9-12, they are significantly better off in MSIM than in MSEQ.

▸ Learning

Conclusion

Conclusion

- Experimental results support empirical findings that top-grade students fail to use truncation strategies optimally.
- Implementation in weakly dominant strategies is less successful when students have to misrepresent compared to strategy proof mechanism.
- German clearinghouse is aware of the problem. Procedure will be changed in the near future to a variant of university-proposing GS where quotas and capacity redistribution are implemented by the universities (similar to MSIM).

Backup slides

Market 1

- Applicants' preferences

  $A1, \ldots, A8$: $W > X > Y > Z$

- Universities' preferences

  W, X, Y, Z: $A1 > A2 > \ldots > A8$

- Under truth-telling, MSEQ yields

$$\mu = \begin{array}{cccc} W & X & Y & Z \\ A1, A5 & A2, A6 & A3, A7 & A4, A8 \end{array}$$

- Unique equilibrium outcome

$$\nu = \begin{array}{cccc} W & X & Y & Z \\ A1, A2 & A3, A4 & A5, A6 & A7, A8 \end{array}$$

Market 2

- Applicants' preferences

  $A1, \ldots, A8$: $W > X > Y > Z$

- Universities' preferences

  $W : A1 > A3 > A2 > A4 > A5 > A6 > A7 > A8$

  $X : A1 > A5 > A2 > A3 > A4 > A6 > A7 > A8$

  $Y, Z : A1 > A2 > A3 > A4 > A5 > A6 > A7 > A8$

- Under truth-telling, MSEQ yields

$$\mu = \begin{matrix} W & X & Y & Z \\ A1, A5 & A2, A6 & A3, A7 & A4, A8 \end{matrix}$$

- Unique equilibrium outcome

$$\nu = \begin{matrix} W & X & Y & Z \\ A1, A3 & A2, A5 & A4, A6 & A7, A8 \end{matrix}$$

Market 3

- Applicants' preferences

  $A1, A2, A5, A6 : W > Y > X > Z$

  $A3, A4, A7, A8 : X > Y > W > Z$

- Universities' preferences

  $W, X, Y, Z\text{:}\ A1 > A2 > \ldots > A8$

- Under truth-telling, MSEQ yields

$$\mu = \begin{array}{cccc} W & X & Y & Z \\ A1, A5 & A3, A7 & A2, A6 & A4, A8 \end{array}$$

- Unique equilibrium outcome

$$\nu = \begin{array}{cccc} W & X & Y & Z \\ A1, A2 & A3, A4 & A5, A6 & A7, A8 \end{array}$$

Market 4

- Applicants' preferences

  $A1, A3, A5, A7 : W > Y > X > Z$

  $A2, A4, A6, A8 : X > Y > W > Z$

- Universities' preferences

  $W, Y, Z : A1 > A2 > A3 > A4 > A5 > A6 > A7 > A8$

  $X : A1 > A5 > A2 > A3 > A4 > A6 > A7 > A8$

- Under truth-telling, MSEQ yields

$$\mu = \begin{array}{cccc} W & X & Y & Z \\ A1, A5 & A2, A6 & A3, A7 & A4, A8 \end{array}$$

- Unique equilibrium outcome

$$\nu = \begin{array}{cccc} W & X & Y & Z \\ A1, A3 & A2, A4 & A5, A6 & A7, A8 \end{array}$$

Truthtelling in first stage of MSEQ

Applicant	First pref.	Second pref.	Third pref.	Fourth pref.
$A1$	92.50%	40.56%	32.50%	33.89%
$A2$	58.89%	27.78%	25.28%	22.50%
$A3$	54.44%	26.67%	22.22%	19.72%
$A4$	37.50%	27.78%	20.00%	17.50%

Back

Learning by top-grade applicants in MSEQ

Table: Difference in individual performance measure between MSIM and MSEQ, by student and round

	Rounds 1–4	Rounds 5–8	Rounds 9–12	All rounds
Student 1	0.1167**	0.0250	0.0250	0.0555***
	(0.0457)	(0.0184)	(0.0188)	(0.0178)
Student 2	0.4417***	0.2583***	0.0720	0.2577***
	(0.0826)	(0.0655)	(0.0693)	(0.0426)
Student 3	0.5167***	0.2083***	0.2316***	0.3189***
	(0.0893)	(0.0707)	(0.0742)	(0.0458)
Student 4	0.2167**	0.1833**	0.1639*	0.1882***
	(0.1071)	(0.0888)	(0.0837)	(0.0541)
Students 1–4	0.3229***	0.1688***	0.1231***	0.2051***
	(0.0427)	(0.0333)	(0.0336)	(0.0214)

Back

Matching with complex constraints

- Matching problems with complex constraints:

  Each college c characterized by a sequence of strict rank order lists $(\succ_{(c,t)})_{t=1}^{T}$ and sequence $(q_{(c,t)})_{t=1}^{T}$ of *capacity redistribution functions*, where

  $$q_{(c,t)} : \{0, \ldots, |I|\}^{t-1} \to \{0, \ldots, |I|\}$$

- Interpretation:
  - Student type t = set of acceptable students w.r.t. $\succ_{(c,t)}$
  - Target distribution $\overline{q}_{(c,t)} = q_{(c,t)}(0, \ldots, 0)$
  - Capacity redistribution functions express preferences over deviations from target distribution

- Two simple assumptions on capacity redistribution guarantee existence of student optimal stable (w.r.t. to capacity redistribution) matching and group strategy-proofness of

Current procedure: Incentives for manipulation

- **Within part 1**
  - Relative to truthful revelation, top-grade applicants can often benefit from *skipping* some universities in the first part.
- **Across parts:**
  - Relative to truthful revelation, top-grade applicants can often benefit from *truncating* preference list for first part in order to guarantee participation in two-sided part.

Stability

Definition

A matching $\mu = (\mu^1, \mu^2)$ is *stable* with respect to $P = (P_s)_{s \in S}$ if

(i) no student is matched to an unacceptable university,

(ii) no university assigns a seat in its regular quota to an unacceptable student,

(iii) no top-grade student could be matched to a better university in the top-grade quota,

(iv) no student-university pair blocks the matching in the regular quota.

A matching $\mu = (\mu^1, \mu^2)$ is *strongly stable*, if it is stable and matches students as early as possible.

Matching outcomes

Theorem

Let $P = (P_s)_{s \in S}$ be an arbitrary profile of student preferences.

(i) *The outcome of MSIM under truth-telling is the unique student optimal strongly stable matching with respect to P.*

(ii) *Let (Q^1, Q^2) be a Nash-equilibrium of the game induced by MSEQ such that $Q^2_s = P_s$ for all students s.*

   (1) *The outcome of MSEQ under (Q^1, Q^2) is stable with respect to P.*

   (2) *If $f^{SEQ}(Q^1, Q^2)$ matches students as early as possible, then $f^{SEQ}(Q^1, Q^2) = f^{SIM}(P)$.*

Experimental design

- Differences between sequential mechanism and current German admissions procedure
  - No payoff difference between receiving assignment in first or second part
  - Students have full information about preferences of all market participants
  - Student- instead of university-proposing DA in second stage
- Reasons for student-proposing DA
  1. In "large" markets no significant difference between SDA and UDA (Kojima and Pathak, 2009; Azevedo and Leshno, 2011)
  2. Make mechanisms as similar as possible to focus on effects of sequential versus simultaneous allocation

Results: Performance compared to equilibrium, share of students

- Share of students who realize their equilibrium matching

	MSIM	MSEQ	MSIM − MSEQ
Market 1: Fully aligned	0.9750	0.7153	0.2597^{***}
	(0.1562)	(0.4516)	(0.0178)
Market 2: Student aligned	0.9353	0.7653	0.1701^{***}
	(0.2461)	(0.4241)	(0.0185)
Market 3: University aligned	0.9556	0.7346	0.2208^{***}
	(0.2062)	(0.4418)	(0.0182)
Market 4: Split aligned	0.8736	0.7083	0.1653^{***}
	(0.3324)	(0.4548)	(0.0210)
Markets 1–4	0.9349	0.7309	0.2040^{***}
	(0.2468)	(0.4436)	(0.0095)