# Minisum and Minimax Committee Election Rules for General Preference Types: Winner Determination and Axiomatic Properties

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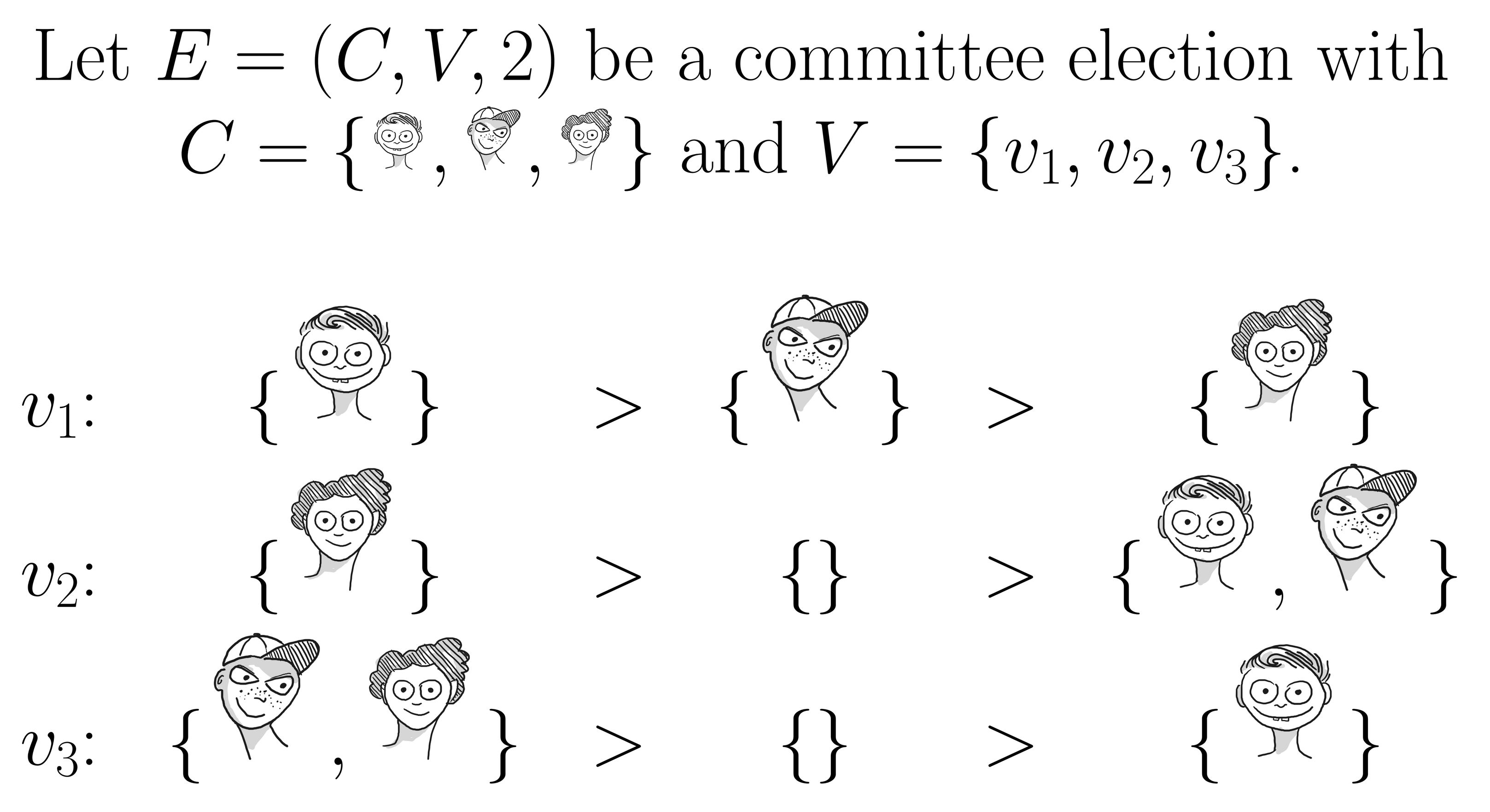
## Preference Types: $\ell$-Ballots

- Given a large candidate set, it might be impossible to rank all candidates.
- Dividing the candidates into two groups might be insufficient if the voter wants to express different intensities.
- The intermediate approach: We allow the voters to group the candidates into a fixed number of groups, possibly empty (cf. the papers by Obraztsova et al. [3] and Baumeister et al. [2]).
- We call such a ballot an **$\ell$-ballot**.
- $E = (C, V, k)$ is a committee election, where $C = \{c_1, \dots, c_m\}$ denotes the set of candidates, $V = (v_1, \dots, v_n)$ the list of $\ell$-ballots, and $k \in \mathbb{N}$ the committee size.
- $F_k(C)$ denotes the set of all committees with size $k$ over the candidates in $C$.
- Our committee election rules **minimize the dissatisfaction** of the voters with the elected committees.

## Minisum/Minimax $\ell$-Group Rules

- We define the **distance** $\delta_\ell$ between an $\ell$-ballot $v$ and a committee $W \in F_k(C)$ by
$$\delta_\ell(v, W) = \sum_{c \in C} |v(c) - W(c)|,$$
where $W(c) = 1$ if $c \in W$, and otherwise $W(c) = \ell$, and where $v(c)$ denotes the group number of $c$ in $v$.
- The **minisum $\ell$-group rule** minimizes the sum of the voters' distances to the winning committees.
$$f^\ell_{sum}(C, V, k) = \operatorname*{argmin}_{W \in F_k(C)} \sum_{v \in V} \delta_\ell(v, W)$$
- The **minimax $\ell$-group rule** minimizes a voter's maximal distance to the winning committees.
$$f^\ell_{max}(C, V, k) = \operatorname*{argmin}_{W \in F_k(C)} \max_{v \in V} \delta_\ell(v, W)$$

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|---|---|---|---|
| $v_1$: | 1 | 3 | 5 |
| $v_2$: | 6 | 2 | 2 |
| $v_3$: | 4 | 4 | 0 |
| Minisum | 11 | 9 | **7** |
| Minimax | 6 | **4** | 5 |

## Axiomatic Properties

| Properties | $\ell$-group rules minisum | minimax |
|---|---|---|
| Non-imposition, Homogeneity | ✓ | ✓ |
| Consistency | ✓ | × |
| Independence of clones | ✓ | × |
| Committee monotonicity | ✓ | × |
| (Candidate) monotonicity | ✓ | ✓ |
| Positive responsiveness | ✓ | × |
| Pareto criterion | ✓ | ✓ |
| (Committee) Condorcet consistency | × | × |
| Solid coalitions, Consensus committee | × | × |
| Unanimity | strong | strong |

## Winner Determination

- Computing a winning committee under the minisum $\ell$-group rule is easy.
- Deciding whether there exists a committee so that a voter's maximal distance is lower than a given distance $d$ is NP-complete.
- However, this problem is in FPT when parameterized by $d$.

**Minimax $\ell$-Score**

**Given:** A committee election $E = (C, V, k)$, and a nonnegative integer $d$.
**Question:** Is there a committee $W \in F_k(C)$ such that $\max_{v \in V} \delta_\ell(v, W) \leq d$?

**Theorem.** *There is an algorithm solving* Minimax $\ell$-Score *whose running time is in* $\mathcal{O}\left((mn + m \log m)\left(\frac{\sqrt{33}}{2}d\right)^d\right)$. *In particular, the* Minimax $\ell$-Score *problem is fixed-parameter tractable when parameterized by* $d$.

## Future Work

- consider different rules for $\ell$-ballots and identify which properties are satisfied
- identify rules that fulfill Condorcet consistency and committee Condorcet consistency
- adapt the systems of proportional representation to our setting
- redefine the axiom of justified representation [1] to handle our types of votes

## References

[1] H. Aziz, M. Brill, V. Conitzer, E. Elkind, R. Freeman, and T. Walsh. Justified representation in approval-based committee voting. In *Proceedings of the 29th AAAI Conference on Artificial Intelligence*, 2015.

[2] D. Baumeister, T. Böhnlein, L. Rey, O. Schaudt, and A. Selker. Minisum and minimax committee election rules for general preference types. In *Proceedings of the 22nd European Conference on Artificial Intelligence (ECAI16)*, 2016. Short Paper. To appear.

[3] S. Obraztsova, E. Elkind, M. Polukarov, and Z. Rabinovich. Doodle poll games. In *Proceedings of the First IJCAI-Workshop on Algorithmic Game Theory*, 2015.