Centre for Data Analytics

Insight

Preference Inference Based on Pareto Models

Anne-Marie George, Nic Wilson

Motivation

Preference Inference is relevant in many fields like recommender systems and multi-objective optimization where one wants to reason over user preferences. Preference Inference based on Pareto models can also be seen as a prediction of voting/decision making outcomes based on prior experience. Here, we assume that each individual has a (known) total order on the alternatives that is realised by a function to the rational numbers, called evaluation function.

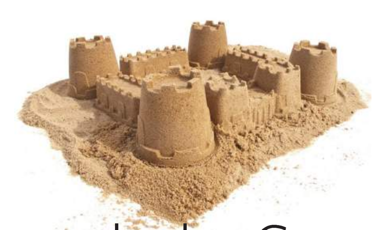
Participants form (unknown) groups, within which they come to decisions by combining their evaluation functions with an operator $\oplus$, e.g., addition, multiplication, etc. Then, one alternative is chosen over another, if it is preferred in all groups.

Preference Structure

Alternatives: Set $\mathcal{A}$ of items the user can choose from.

Evaluations: Set $\mathcal{C}$ of functions $\mathcal{A} \to \mathbb{Q}^{\geq 0}$ from the alternatives to the non-negative rational numbers to rate the alternatives (the optimum is 0).

Operator $\oplus$: Associative, commutative and strictly monotonic operation on $\mathbb{Q}^{\geq}$ to combine evaluations.

		 Playa de la Concha	 Parque Cristina Enea	 Aquarium
Evaluations	Ana	1	3	2
	Bruno	3	3	1
	Clara	3	1	3
$\oplus$	Ana+Clara	4	4	5

Preference Statements

A set of non-strict and strict preference statements Γ is provided by the user.

Non-Strict Statements: "Park is preferred to beach." 

Strict Statements: "Beach is strictly preferred to aquarium." 

Preference Models

Pareto Models

- A Pareto model $P = \{C_1, \ldots, C_k\}$ is a (possibly empty) set of disjoint subsets of evaluations $C_i \subseteq \mathcal{C}$, e.g., groups of participants that come to decisions together.
- $\mathcal{P}$ is the set of Pareto models.
  $\mathcal{P}(1)$ is the set of models $\{C_1, \ldots, C_k\} \in \mathcal{P}$ with singleton sets $|C_i| = 1$, i.e., every individual votes for itself.
  $\mathcal{P}^s$ is the set of Pareto models $\{C\}$ that consist of a single set $C \subseteq \mathcal{C}$, i.e., a single group that makes the decisions.
- A Pareto model $P = \{C_1, \ldots, C_k\}$ induces an order relation on $\mathcal{A}$ by comparing $\oplus$-combinations of the sets in a Pareto manner, i.e., one alternative is preferred to another if all groups of participants prefer it.

Example: Let $\oplus$ be the addition on $\mathbb{Q}$ and $P = \{\{\text{Bruno, Clara}\}\}$.

		$>^{\oplus}_P$		$\equiv^{\oplus}_P$	
Bruno + Clara	3+3=6	>	3+1=4	=	1+3=4

Decision Problems

Preference Consistency Problem (PCP)

Given: Set of preference models $\mathcal{M}$ over a preference structure $\langle \mathcal{A}, \mathcal{C}, \oplus \rangle$, set of preference statements Γ on alternatives $\mathcal{A}$.

Question: Does there exist a model in $\mathcal{M}$ that satisfies Γ?

all preference models $\mathcal{M}$ / possible candidates for user model $\emptyset$?

Example:

$\Gamma =$ 

Pareto model $\{\{\text{Ana, Clara}\}\}$ satisfies Γ. Hence, Γ is $\mathcal{P}$-consistent.

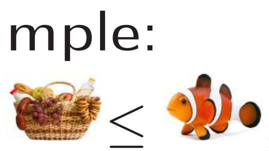
Preference Deduction Problem (PDP)

Given: Set of preference models $\mathcal{M}$ over a preference structure $\langle \mathcal{A}, \mathcal{C}, \oplus \rangle$, set of preference statements Γ and statement φ on alternatives $\mathcal{A}$.

Question: Do all model in $\mathcal{M}$ that satisfy Γ also satisfy φ? ($\Gamma \models_{\mathcal{M}} \varphi$?)

all preference models $\mathcal{M}$ / possible candidates for user model satisfy φ?

Example:

$\Gamma =$  , $\varphi =$  , $\varphi' =$ 

The Γ-satisfying Pareto model, $\{\{\text{Ana, Clara}\}\}$, $\{\{\text{Bruno, Clara}\}\}$ and $\{\{\text{Clara}\}\}$, satisfy φ', i.e., $\Gamma \models_{\mathcal{P}} \varphi'$. However, the Γ-satisfying Pareto model $\{\{\text{Ana, Clara}\}\}$ does not satisfy φ, i.e., $\Gamma \not\models_{\mathcal{P}} \varphi$.

Results

$\mathcal{P}$

Let $\mathcal{C}^{\leq\Gamma} := \{C \in \mathcal{C} \mid \bigoplus_{c \in C} c(\alpha_\varphi) \leq \bigoplus_{c \in C} c(\beta_\varphi) \text{ for all } \varphi \in \Gamma\}$ be the sets of evaluations (i.e., groups of participants) that do not oppose Γ. Define $\mathcal{C}^{<\Gamma}$ analogously.

- If $\Gamma \models_{\mathcal{P}} \varphi$, then $\Gamma \cup \{\overline{\varphi}\}$ is $\mathcal{P}$-inconsistent. The reverse is not necessarily true.
- Γ is $\mathcal{P}$-consistent if and only if Γ is $\mathcal{P}^s$-consistent, i.e., $\bigcap_{\alpha<\beta\in\Gamma} \mathcal{C}^{<\{\alpha<\beta\}} \cap \bigcap_{\alpha\leq\beta\in\Gamma} \mathcal{C}^{\leq\{\alpha\leq\beta\}} \neq \emptyset$.

	Pareto Models $\mathcal{P}$	Pareto Models $\mathcal{P}(1)$
PCP	NP-complete (reduction from SAT)	solvable in $O(\lvert\Gamma\rvert\lvert\mathcal{C}\rvert)$ by constructing $C^{\leq\Gamma}$
PDP	coNP-complete (reduction from SAT)	

$\mathcal{P}(1)$

Let $C^{\leq\Gamma} := \{c \in \mathcal{C} \mid c(\alpha_\varphi) \leq c(\beta_\varphi) \text{ for all } \varphi \in \Gamma\}$ be the evaluations (i.e., participants) that do not oppose Γ.

- If $\Gamma \models_{\mathcal{P}(1)} \varphi$, then $\Gamma \cup \{\overline{\varphi}\}$ is $\mathcal{P}(1)$-inconsistent. The reverse is not necessarily true.
- Let $\Gamma \subseteq \mathcal{L}^{\mathcal{A}}$ be $\mathcal{P}(1)$-consistent. $C^{\leq\Gamma}$ is the set of evaluations that are contained in Γ-satisfying Pareto models.
- Γ is $\mathcal{P}(1)$-consistent if and only if $\forall\, \alpha < \beta \in \Gamma$ there exists $c \in C^{\leq\Gamma}$ with $c(\alpha) < c(\beta)$.

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