An Asymptotic Study of the Axiomatic Properties of Social Decision Schemes

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Randomized Social Choice

A **social decision scheme (SDS)** maps a preference profile of a set of voters N to a **lottery**, i.e., a probability distribution over a set of alternatives A.

Some desirable properties of social decision schemes:

- **Pareto-efficiency**: making a voter better off will make another voter worse off,
- **strategyproofness**: no voter can benefit by misrepresenting his true preferences,
- **anonymity/neutrality**: all voters/alternatives are treated equally.

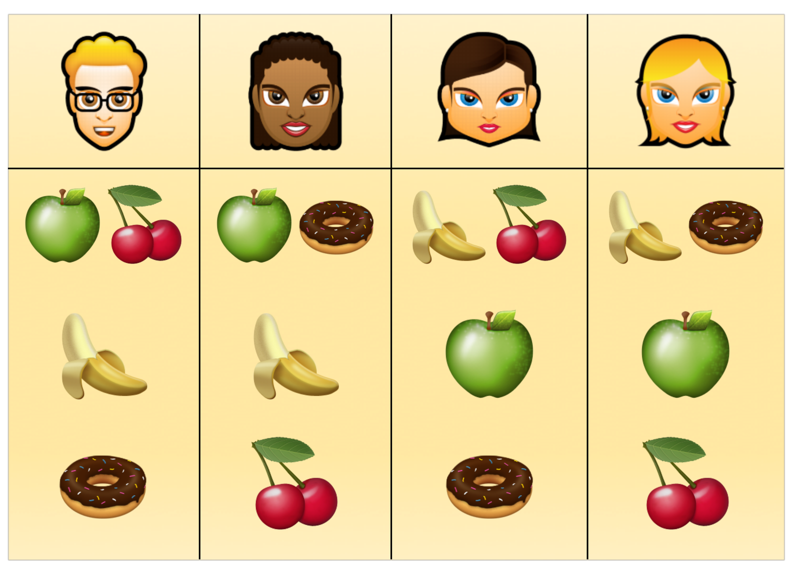
Comparing Lotteries

Stochastic dominance (SD): $p \succcurlyeq^{SD} q$ if and only if:

- for each alternative x, p is at least as likely to return an alternative at least as good as x as is q,
- $\sum_{y \succcurlyeq x} p(y) \geq \sum_{y \succcurlyeq x} q(y) \ \forall x \in A$, or
- the expected utility for p is at least as large as for q for every consistent von-Neumann-Morgenstern utility function.

Theorem (Brandl et al. 2016): There is no *anonymous* and *neutral* SDS that satisfies Pareto-efficiency and strategyproofness, if $|N| \geq 4$ and $|A| \geq 4$.

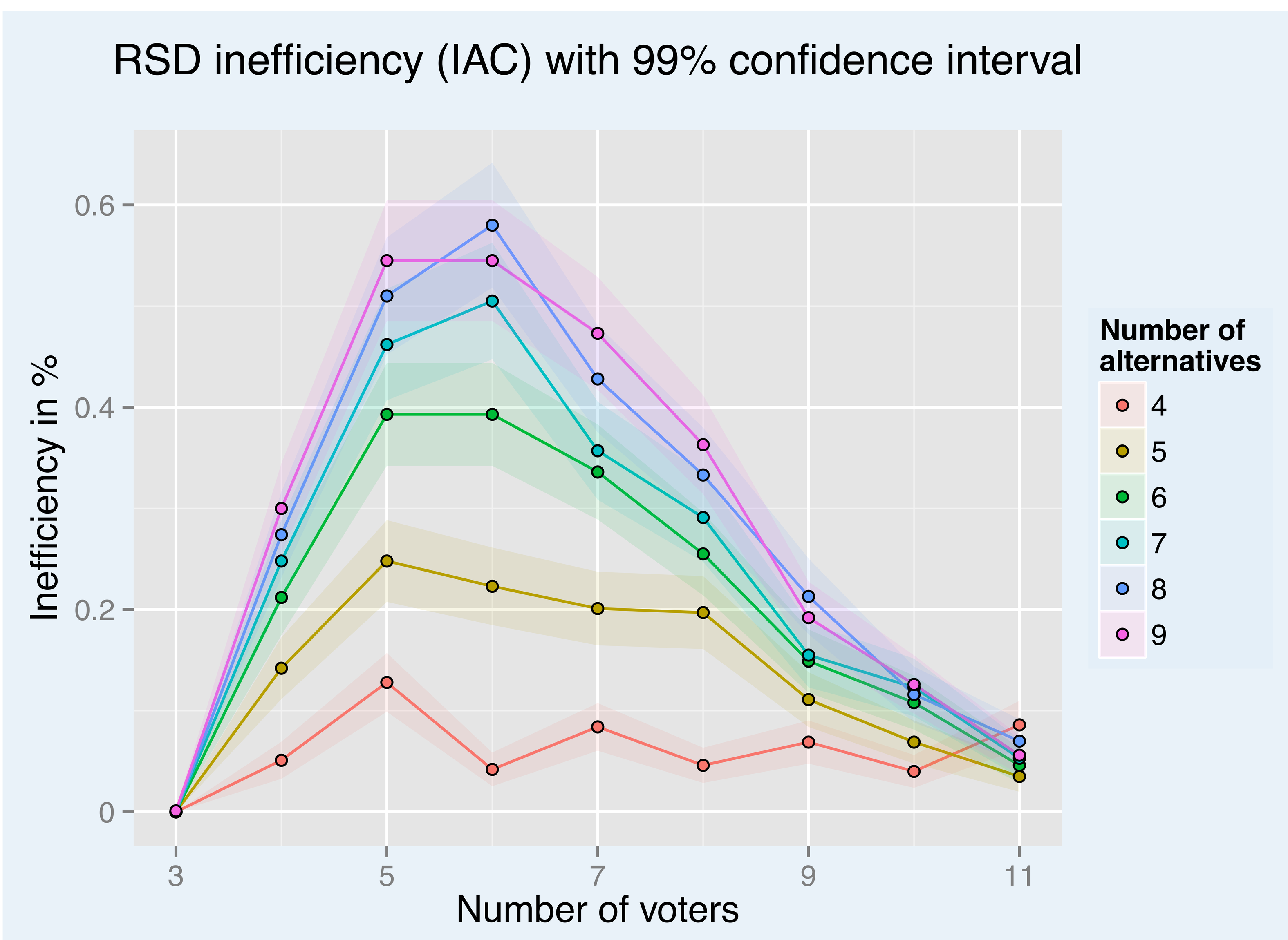
Random Serial Dictatorship (RSD)



Step 1: Choose a dictator uniformly at random.

Step 2: Break ties in top rank by invoking RSD with all other voters.

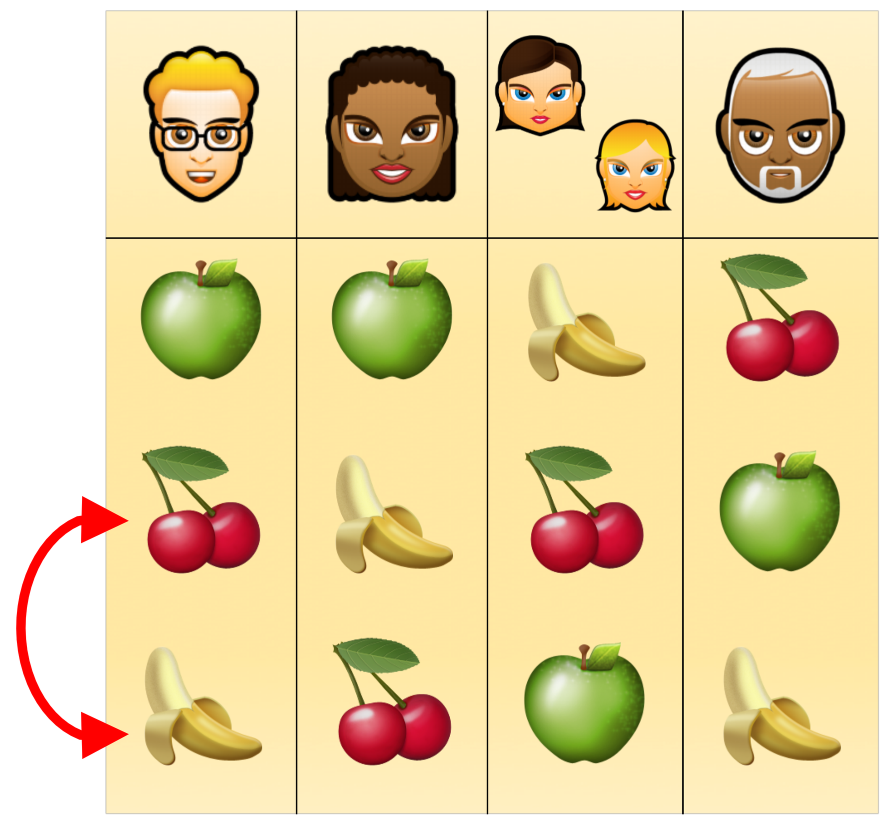
RSD returns 1/3 + 1/3 + 1/6 + 1/6, but all voters strictly prefer 1/2 + 1/2, thus violating Pareto-efficiency.



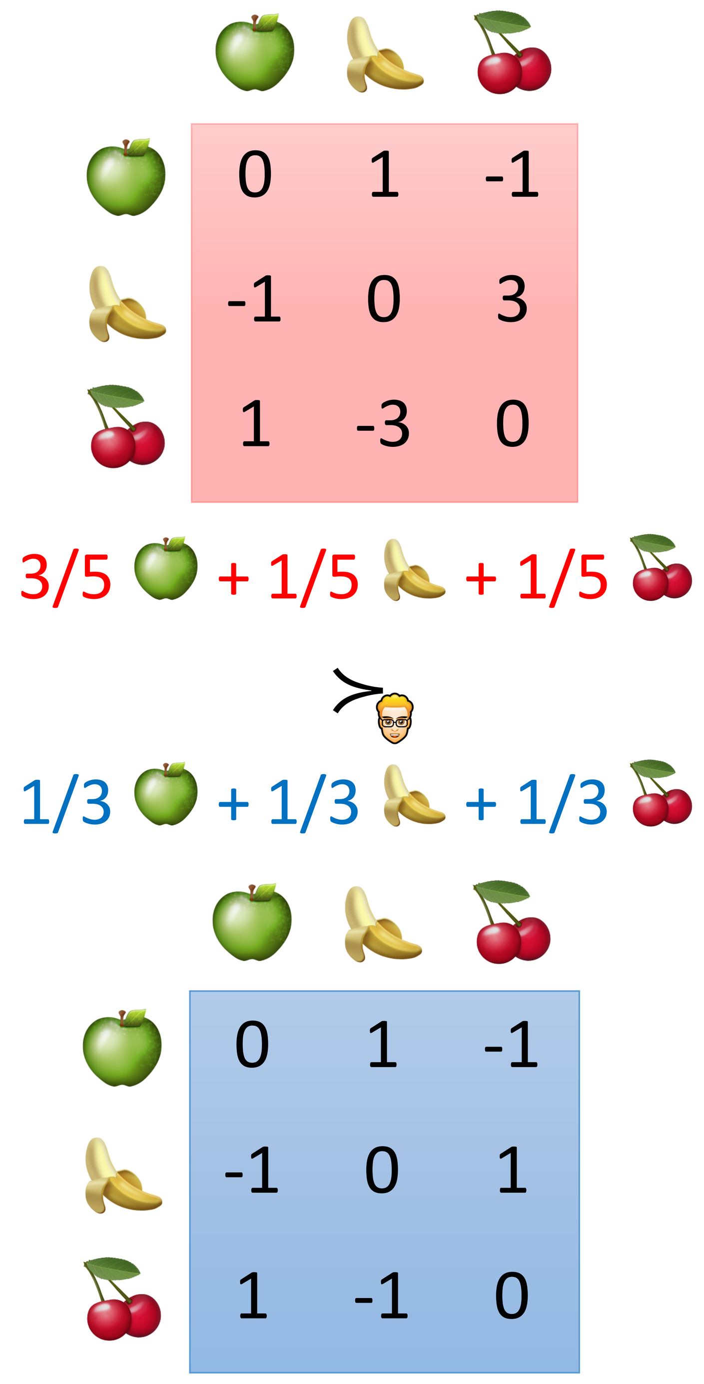
Maximal Lotteries (ML)

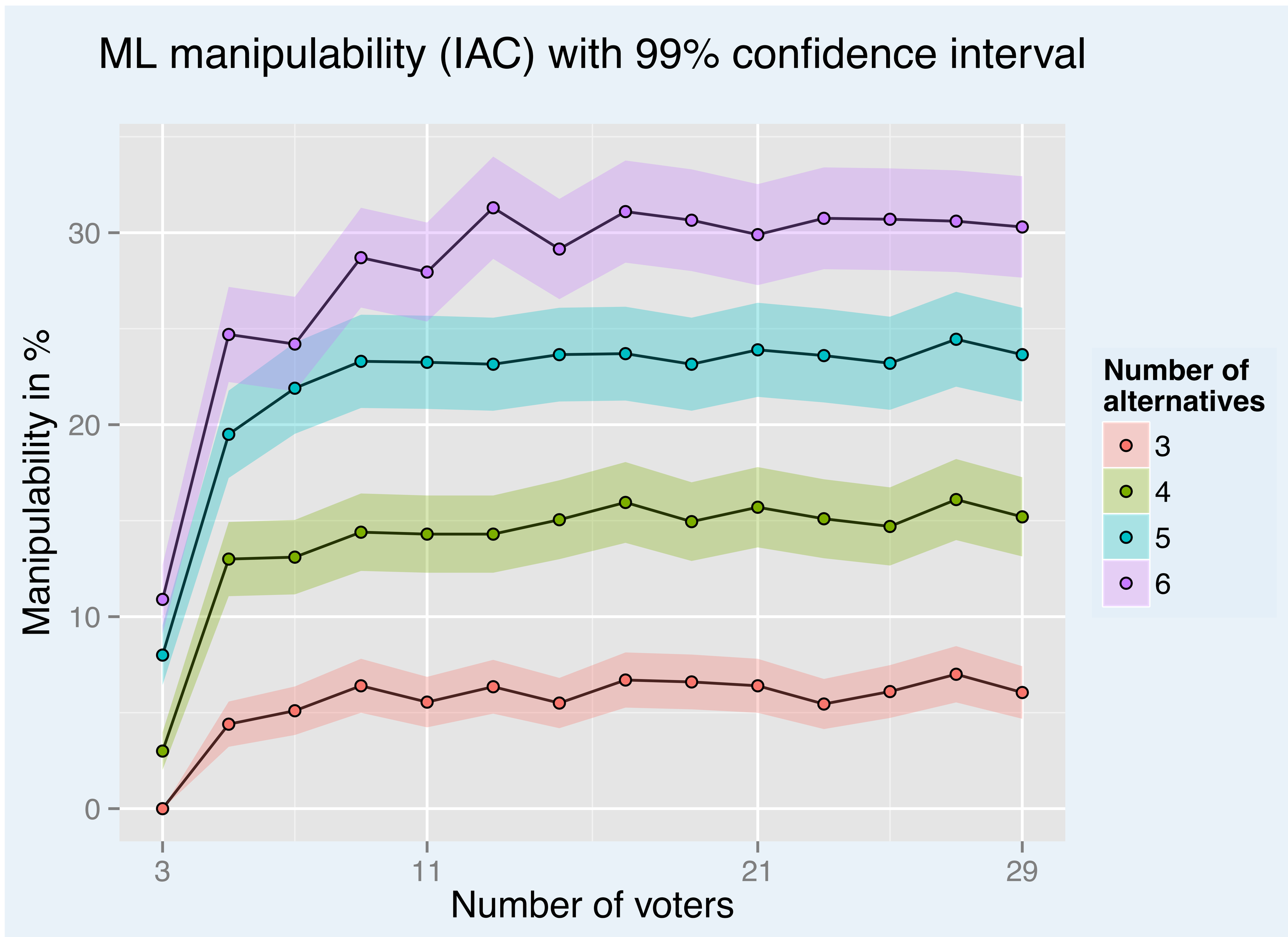
Step 1: Construct the plurality game, i.e., a two-player zero-sum game via pairwise comparisons of the alternatives.

Step 2: Return a Nash equilibrium of that game.



can manipulate by swapping and .





Conjecture: The manipulability converges for $|N| \to \infty$ to the probability that no Condorcet winner exists.

Outlook

- Monotonicity of ML
- Very strong participation of ML
- Disjointedness of variants of strict ML
- More realistic distributions of preference profiles
- Empirical analysis with real-world data
- Analytical bounds for property failure frequency