

Verification in Incomplete Argumentation Frameworks

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What is Abstract Argumentation?

A tool for non-monotonic reasoning, where an argumentation is modelled as a **directed graph**:

atomic arguments \leftrightarrow nodes
binary attack relation \leftrightarrow vertices

The objective is to identify sets of arguments that are **simultaneously acceptable** with regard to the attack relation.

Introducing Incompleteness

We want to represent a broader set of application scenarios:

- **intermediate states** in an elicitation process
- when **merging different beliefs** about an argumentation framework's state
- cases where complete information cannot be obtained

Previous work:

- *Attack*-incomplete argumentation frameworks were introduced by Coste-Marquis et al. [3] and studied by us [1] with regard to the complexity of verification.
- *Argument*-incomplete argumentation frameworks were introduced and their complexity was analyzed by us [2].

Argumentation Frameworks [4]

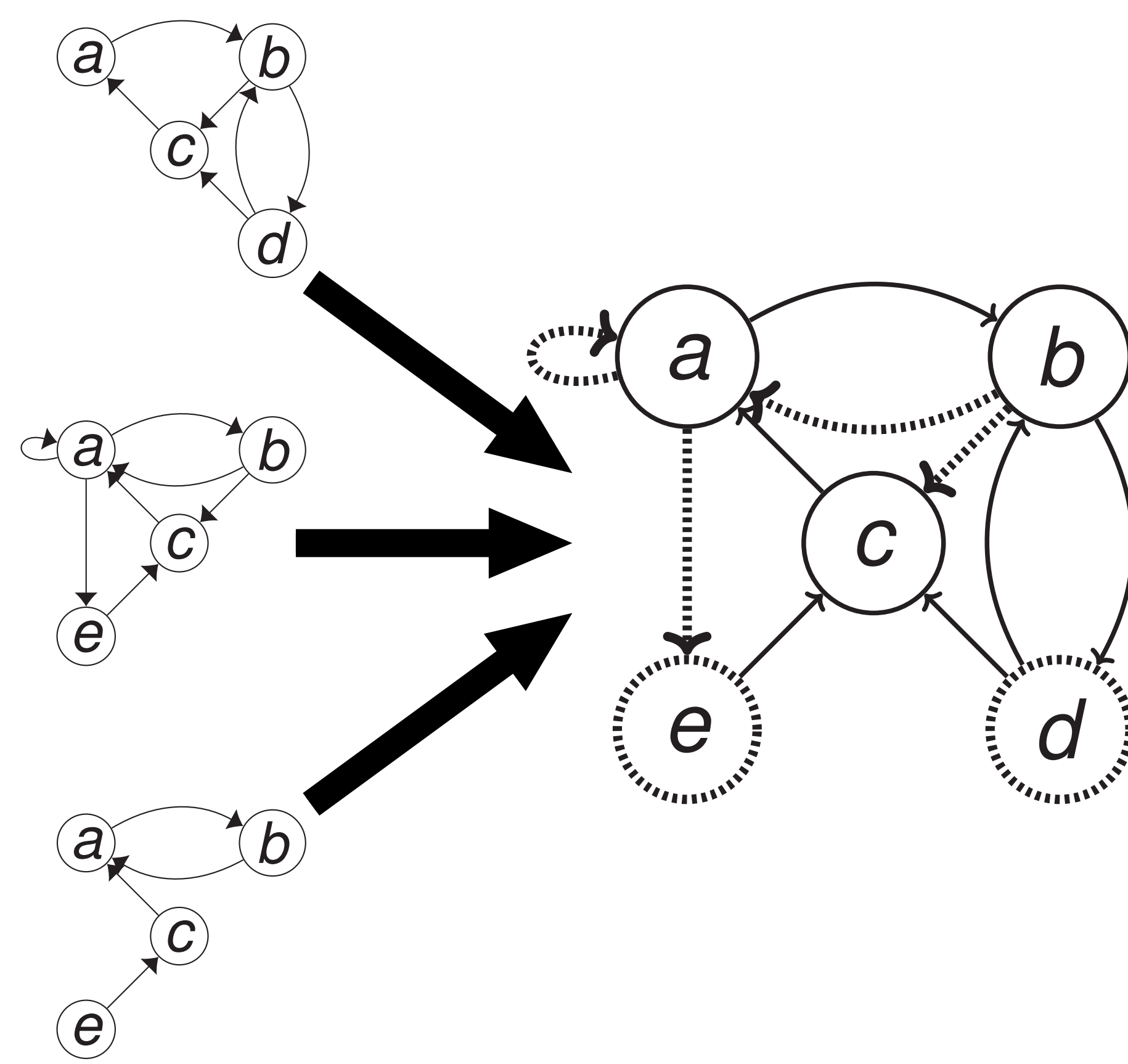
An *argumentation framework* is a pair $\langle \mathcal{A}, \mathcal{R} \rangle$, with a set of arguments \mathcal{A} and an attack relation $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$.

A subset $S \subseteq \mathcal{A}$ is

- **conflict-free** if $\forall a, b \in S : (a, b) \notin \mathcal{R}$,
- **admissible** if S is conflict-free and $\forall a \in S : a$ is acceptable with respect to S ,
- **preferred** if S is a maximal (w.r.t. set inclusion) admissible set,
- **stable** if S is conflict-free and $\forall b \in \mathcal{A} \setminus S : \exists a \in S$ with $(a, b) \in \mathcal{R}$,
- **complete** if S is admissible and contains all $a \in \mathcal{A}$ that are acceptable w.r.t. S , and
- **grounded** if S is the least (w.r.t. set inclusion) fixed point of the characteristic function of $\langle \mathcal{A}, \mathcal{R} \rangle$.

An argument $a \in \mathcal{A}$ is **acceptable** w.r.t. $S \subseteq \mathcal{A}$ if for each $b \in \mathcal{A}$ with $(b, a) \in \mathcal{R}$ there is a $c \in S$ such that $(c, b) \in \mathcal{R}$.

The characteristic function $F_{AF} : 2^{\mathcal{A}} \rightarrow 2^{\mathcal{A}}$ of $\langle \mathcal{A}, \mathcal{R} \rangle$ is defined by $F_{AF}(S) = \{a \in \mathcal{A} \mid a \text{ is acceptable w.r.t. } S\}$.



Incomplete Argumentation Frameworks

An *incomplete argumentation framework* is a quadruple $\langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, where \mathcal{A} and $\mathcal{A}^?$ are disjoint sets of arguments and \mathcal{R} and $\mathcal{R}^?$ are disjoint subsets of $(\mathcal{A} \cup \mathcal{A}^?) \times (\mathcal{A} \cup \mathcal{A}^?)$.

An argumentation framework $\langle \mathcal{A}^*, \mathcal{R}^* \rangle$ with $\mathcal{A} \subseteq \mathcal{A}^* \subseteq \mathcal{A} \cup \mathcal{A}^?$ and $\mathcal{R} \subseteq \mathcal{R}^* \subseteq (\mathcal{R} \cup \mathcal{R}^?) \upharpoonright_{\mathcal{A}^*}$ is called a **completion** of $\langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$.

\mathcal{A} : Arguments known to exist
 $\mathcal{A}^?$: Possible additional arguments
 \mathcal{R} : Attacks known to exist
 $\mathcal{R}^?$: Possible additional attacks

For a set \mathcal{A}^* of arguments with $\mathcal{A} \subseteq \mathcal{A}^* \subseteq \mathcal{A} \cup \mathcal{A}^?$, define the restriction of \mathcal{R} to \mathcal{A}^* by $\mathcal{R} \upharpoonright_{\mathcal{A}^*} = \{(a, b) \in \mathcal{R} \mid a, b \in \mathcal{A}^*\}$.

s-Verification [5]

Given: An argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and a subset $S \subseteq \mathcal{A}$.

Question: Is S an **s** extension of AF ?

S is an **s** extension of $\langle \mathcal{A}, \mathcal{R} \rangle$ if S is **s** in $\langle \mathcal{A}, \mathcal{R} \rangle$, for all $\mathbf{s} \in \{\text{conflict-free, admissible, preferred, stable, complete, grounded}\}$.

s-Inc-Possible-Verification (s-INCPV)

Given: An incomplete argumentation framework $IAF = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ and a set $S \subseteq \mathcal{A} \cup \mathcal{A}^?$.

Question: Is there a completion $AF^* = \langle \mathcal{A}^*, \mathcal{R}^* \rangle$ of IAF such that $S \upharpoonright_{\mathcal{A}^*} = S \cap \mathcal{A}^*$ is an **s** extension of AF^* ?

s-Inc-Necessary-Verification (s-INCNV)

Given: An incomplete argumentation framework $IAF = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ and a set $S \subseteq \mathcal{A} \cup \mathcal{A}^?$.

Question: For all completions $AF^* = \langle \mathcal{A}^*, \mathcal{R}^* \rangle$ of IAF , is $S \upharpoonright_{\mathcal{A}^*} = S \cap \mathcal{A}^*$ an **s** extension of AF^* ?

Results

s	VERIFICATION [4]	INCPV	INCNV
CONFLICT-FREE	in P	in P	in P
ADMISSIBLE	in P	NP-c.	in coNP
STABLE	in P	NP-c.	in coNP
COMPLETE	in P	NP-c.	in coNP
GROUNDING	in P	NP-c.	in coNP
PREFERRED	coNP-c.	DP-h., in Σ_2^P	coNP-c.

Next Steps

- Close gaps in complexity results
- Cover new semantics
- Consider other decision problems

References

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