Measuring Non-Forced Electoral Disproportionality

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Abstract

A new index to analyze votes-seats disproportionality involved in an electoral process is proposed. It will be called the quota index, I_q. The main feature of this index is that it just measures non-forced disproportionality: $I_q \neq 0$ if and only if there are parties that do not satisfy the quota rule (forced electoral disproportionality is not taken into account because it is inherent to the apportionment problem). Furthermore, from this index it is possible to calculate the minimum number of seats that it is necessary to transfer among the parties in order to verify the quota rule. On the other hand, relations with some other indexes appearing in the literature are proven. In order to test the implementation of I_q, it is applied to Spanish elections

Motivation

We have to distribute 6 seats among 4 electoral parties, proportionally to their obtained votes

Party	Votes
A	349,460
B	139,354
C	64,103
D	51,045

$A \to \frac{6}{603{,}952} 349{,}460 = 3.47$ $\quad B \to \frac{6}{603{,}952} 139{,}354 = 1.38$

$C \to \frac{6}{603{,}952} 64{,}103 = 0.64$ $\quad D \to \frac{6}{603{,}952} 51{,}045 = 0.51$

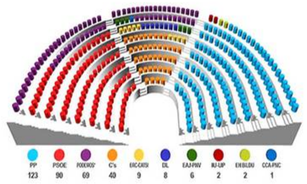
How to allocate seats in a parliament?

Notation:

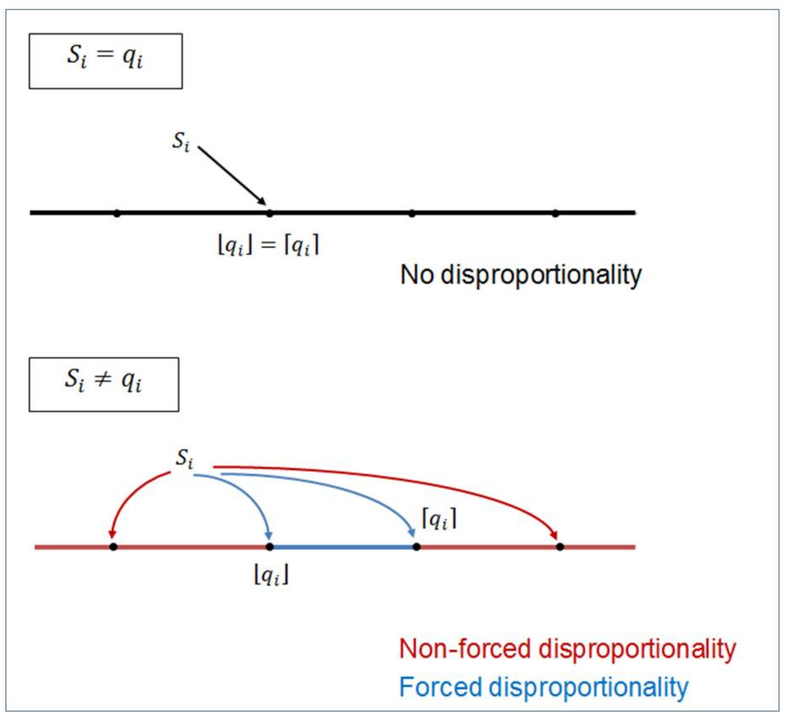
- n number of political parties
- V_i votes received by the party i; $V = V_1 + \cdots + V_n$ total amount of votes
- $v_i = \frac{V_i}{V}$ votes proportion obtained by the party i
- S_i seats assigned to the party i; $S = S_1 + \cdots + S_n$ total number of seats
- $s_i = \frac{S_i}{S}$ seats proportion obtained by the party i
- $q_i = \frac{S}{V} V_i$ quota of party i

Apportionment problem

To find integer numbers $S_1, S_2, \ldots, S_n$ being S_i close to q_i and such that $S_1 + S_2 + \cdots + S_n = S$



- Disproportionality ⟶ $q_i \neq S_i$
- Lower quota ($\lfloor q_i \rfloor$) ⟶ q_i rounded down
- Upper quota ($\lceil q_i \rceil$) ⟶ q_i rounded up
- Quota rule ⟶ $\lfloor q_i \rfloor \leq S_i \leq \lceil q_i \rceil$
- Non-forced disproportionality ⟶ $q_i \neq S_i$ and the allocation does not satisfy the quota rule



Some disproportionality indexes

- Maximum deviation index $\quad I_{MD} = \max_{1,\ldots,n} |s_i - v_i|$
- Loosemore – Hanby index $\quad I_{LH} = \frac{1}{2} \sum_{i=1}^{n} |s_i - v_i|$
- Gallagher index $\quad I_G = \sqrt{\frac{1}{2} \sum_{i=1}^{n} (s_i - v_i)^2}$

The quota index

Aim: Measure of non-forced disproportionality

$$I_q = \frac{1}{S} \max \left\{ \sum_{i,\, S_i > q_i} S_i - \lceil q_i \rceil, \sum_{i,\, S_i < q_i} \lfloor q_i \rfloor - S_i \right\}$$

$S \cdot I_q$ represents the minimum number of seats to be transferred among parties for the quota rule to be verified

Relationship with other indexes

$$I_q \leq I_{LH}$$

$$I_q \geq I_{MD} - \frac{1}{S}$$

Basic references

Karpov, A. (2008): Measurement of Disproportionality in Proportional Representations Systems. *Mathematical and Computer Modelling* 48, pp. 1421-1438

Taagepera R., Grofman B. (2003): Mapping the Indices of Seats-Votes Disproportionality and Inter-Election Volatility, *Party Politics* 9:6, pp. 659-677

Verification of properties

	Anonymity	Principle of transfers	Independence from split	Scale invariance	Zero normalizing	Variation zero and one	Insensitivity to forced dispropotionality
I_{MD}	+	(+)	–	+	+	+	–
I_{LH}	+	(+)	+	+	+	+	–
I_G	+	(+)	–	+	+	+	–
I_q	+	+	–	(–)	+	+	+

Indexes for Spanish Elections

Indexes Values **(%)**

	1993	1996	2000	2004	2008	2011	2015	2016
I_{MD}	6.33	5.39	7.04	3.97	3.92	7.89	6.21	5.86
I_{LH}	12.01	8.07	8.58	7.95	8.08	11.29	10.51	7.80
I_G	6.81	5.32	5.60	4.63	4.50	6.91	5.92	5.23
I_q	11.42	7.10	8.00	7.42	7.42	10.57	9.71	7.14

Correlation among indexes

	I_{MD}	I_{LH}	I_G	I_q
I_{MD}	1			
I_{LH}	0.7051	1		
I_G	0.9311	0.8991	1	
I_q	0.7110	0.9990	0.9033	1