Odd choices in decision-making and logic

Hans Rott
University of Regensburg

Ernst Cassirer (1874-1945)

»... occupies a unique place in twentieth-century philosophy. His work pays equal attention to foundational and epistemological issues in the philosophy of mathematics and natural science and to aesthetics, the philosophy of history, and other issues in the 'cultural sciences' broadly conceived. More than any other German philosopher since Kant, Cassirer thus aims to devote equal philosophical attention both to the (mathematical and) natural sciences (Naturwissenschaften) and to the more humanistic disciplines (Geisteswissenschaften). In this way, Cassirer, more than any other twentieth-century philosopher, plays a fundamental mediating role between C.P. Snow's famous 'two cultures.' ... «

Overview

- Rational choice, or: Being programmed by one's preferences
- Everyday inferences based on expectations (defaults, prejudices, assumptions about normal or typical cases), argumentum ad ignorantiam ...
- Review of a thought experiment by Luce and Raiffa, and its implications for the notion of inference, viewed as a 'cognitive decisions'
- Review some well-known empirical work of Debreu, Tversky, and others that appears to call in question the existence of suitable preferences.
- Transfer of the phenomena identified by these researchers transfer to the notion of inference
- Implications of such observations for philosophy

Michael Friedman, Ernst Cassirer, in: The Stanford Encyclopedia of Philosophy, 2004

Rational choice

- Rational choice theory: The preferences of a person govern the individual actions taken by the person.
- In this picture, rational choice or action requires the existence of personal preferences that determine the action taken.
- Rational choice theory can be interpreted as claiming that agents behave as if they were programmed to maximize the satisfaction of their preferences — much in the same way as chess computers do.
- But if we are programmed or determined by our preferences, doesn't this entail that we are not free?
- No!

John Locke on determination and freedom

"To be determined by our own judgment, is no restraint to liberty. This is so far from being a restraint or diminution of freedom, that it is the very improvement and benefit of it; ... it is as much a perfection, that desire, or the power of preferring, should be determined by good, as that the power of acting should be determined by the will; and the certain determination is, the greater is the perfection. Nay, were we determined by anything but the last result of our own minds, judging of the good or evil of any action, we were not free; the very end of our freedom being, that we may attain the good we choose. And therefore, every man is put under a necessity, by his constitution as an intelligent being, to be determined in willing by his own thought and judgment what is best for him to do: else he would be under the determination of some other than himself, which is want of liberty." (1690, Essay, II.xxi.49)

The basic doctrine of rational choice theory

Rational choice is relational choice.

I.e., a choice function \( \sigma \) is rational iff there exists a preference relation that governs the person's choices using the rule of optimization

\[
\sigma(S) = \min_{\prec}(S) = \{x \in S : y < x \text{ for no } y \in S\}
\]

The preference relation \( \sigma \) is independent of the set of \( S \) options that happens to be offered to the person ("the menu").

... a question of character ...

The basic result of rational choice theory

(Samuelson, Houthakker, Arrow, Chernoff, Uzawa, Richter, Sen, Herzberger; Moulin 1985)

Let \( \sigma \) be a choice function taking as arguments all and only the finite subsets of a given domain \( X \). Then

(a) \( \sigma \) is rationalizable iff it is rationalizable by the "revealed" preference relation \( \prec \), defined by

\[
x < y \text{ iff } x \in \sigma((x, y)) \text{ and } y \notin \sigma((x, y))
\]

(b) \( \sigma \) is rationalizable iff it satisfies

(I) If \( S \subseteq S' \), then \( S \cap \sigma(S') \subseteq \sigma(S) \) Sen's Property \( \alpha \) and

(II) \( \sigma(S) \cap \sigma(S') \subseteq \sigma(S \cup S') \) Sen's Property \( \gamma^{\inf} \)

(c) \( \sigma \) is rationalizable by a transitive relation \( \prec \) iff it also satisfies

(III) If \( S \subseteq S' \) and \( \sigma(S') \subseteq S \), then \( \sigma(S) \subseteq \sigma(S') \) Aizerman

(d) \( \sigma \) is rationalizable by a virtually connected (modular) relation \( \prec \) iff it satisfies (I) and

(IV) If \( S \subseteq S' \) and \( \sigma(S') \cap S = \emptyset \), then \( \sigma(S) \subseteq \sigma(S') \) Sen's Pr. \( \beta^{+} \)
(I) If $S \subseteq S'$, then $S \cap \sigma(S') \subseteq \sigma(S)$  Property $\alpha$

(II) $\sigma(S) \cap \sigma(S') \subseteq \sigma(S \cup S')$  Property $\gamma^{\text{fin}}$

(III) If $S \subseteq S'$ and $\sigma(S') \subseteq S$, then $\sigma(S) \subseteq \sigma(S')$  Aizerman

(IV) If $S \subseteq S'$ and $\sigma(S') \cap S \neq \emptyset$, then $\sigma(S) \subseteq \sigma(S')$  Prop. $\beta^{+}$
The informational value of the menu

An example (Luce and Raiffa 1957)

- A: salmon
- B: steak

The situation: Choose your dinner!

The informational value of the menu (cont'd)

- Scenario 1 A manual with salmon and steak.
- Scenario 2 A manual with salmon and steak and fried snails and frog's legs.

- This violates Property $\alpha$ and Aizerman (i.e., (I) and (III)).

Informational value and multiple criteria

Another example (Rott 2004). The cast:

- Annie Andrews highly profiled, excellent linguist
- Bette Becker very good linguist, substantial logic
- Carlos Cortez brilliant logician, some linguistics
- David Donaldson the obvious candidate

The situation: A job (just one) in linguistics has been announced.

Initial expectations (prejudices, beliefs about a normal world):

$\neg a, \neg b, \neg c, d$

"A.A. / B.B. / C.C. will not get the job," "D.D. will get it."
Informational value and multiple criteria (cont'd)

New information from the Dean.
- Scenario 1 "Either A.A. or B.B. will be appointed."
- Scenario 2 "C.C. will be appointed."
- Scenario 3 "Either A.A. or B.B. or C.C. will be appointed."

This violates Property $\alpha$ and Aizerman (i.e., (I) and (III)).

Why?
- The scores: linguistics - logic
  - Andrews 97 - 0
  - Becker 92 - 50
  - Cortez 40 - 99

A basic insight of recent knowledge representation

Everyday inference is nonmonotonic.

In the sense that it violates the following classical principle:

If $F \subseteq G$, then $\text{Inf}(F) \subseteq \text{Inf}(G)$

or in a finitistic framework

$\text{Inf}(\alpha) \subseteq \text{Inf}(\alpha \land \beta)$

In the last example:
- Not $\text{Inf}(\emptyset) \subseteq \text{Inf}(a \lor b)$, since $d$ is in LHS but not in RHS.
- Not $\text{Inf}(a \lor b \lor c) \subseteq \text{Inf}(a \lor b)$, since $b$ is in LHS but not in RHS.
A basic insight of recent knowledge representation

The loss of monotonicity does NOT mean that anything is possible!

The following inference schemes can be saved, and are in fact widely considered to be very plausible and highly desirable constraints for everyday reasoning

(Or) \( \text{Inf}(\alpha) \cap \text{Inf}(\beta) \subseteq \text{Inf}(\alpha \lor \beta) \)

(Cond) \( \text{Inf}(\alpha \land \beta) \subseteq \text{Cn}(\text{Inf}(\alpha) \cup \{\beta\}) \)

(CMon) If \( \beta \in \text{Inf}(\alpha) \), then \( \text{Inf}(\alpha) \subseteq \text{Inf}(\alpha \land \beta) \)

Distinguish: \( \text{Inf} = \) everyday inference, \( \text{Cn} = \) classical deductive logic

Rational choices & logical properties

- Initial situation "D.D. will be appointed."
  \( d \in \text{Inf}(\emptyset) \) and \( \neg a, \neg b, \neg c \in \text{Inf}(\emptyset) \)

New piece of information from the dean:

- Scenario 1 "Either A.A. or B.B. will be appointed."
  \( a \in \text{Inf}(a \lor b) \) and \( \neg b, \neg c, \neg d \in \text{Inf}(a \lor b) \)

- Scenario 2 "C.C. will be appointed."
  \( c \in \text{Inf}(c) \) and \( \neg a, \neg b, \neg d \in \text{Inf}(c) \)

- Scenario 3 "Either A.A. or B.B. or C.C. will be appointed."
  \( b \in \text{Inf}(a \lor b \lor c) \) and \( \neg a, \neg c, \neg d \in \text{Inf}(a \lor b \lor c) \)

Rational choices & logical properties (cont'd)

The reasoning in our example violates Or and Conditionalization

(Or) \( \text{Inf}(\alpha \lor \beta) \cap \text{Inf}(\gamma) \subseteq \text{Inf}(\alpha \lor \beta \lor \gamma) \)

(Cond) \( \text{Inf}(\alpha \lor \beta) \subseteq \text{Cn}(\text{Inf}(\alpha \lor \beta) \cup \{(\alpha \lor \beta \lor \neg \gamma)\}) \)

\( a \lor \neg c \) and \( \neg b \) are in the LHS but not in RHS

And it violates Cumulative Monotony

(CMon) If \( \alpha \lor \beta \in \text{Inf}(\alpha \lor \beta \lor \gamma) \), then \( \text{Inf}(\alpha \lor \beta \lor \gamma) \subseteq \text{Inf}(\alpha \lor \beta) \)

\( b \) and \( \neg a \) are in the LHS but not in the RHS

Rational choices & logical properties (cont'd 2)

Claim: This corresponds to violations of choice conditions

(I) not \( \sigma(\{A,B,C\}) \cap \{A,B\} \subseteq \sigma(\{A',B\}) \) (B "lost"!)

Sen's condition \( \alpha \)— perhaps the single most plausible constraint used in the classical theory of rational choice and

(III) \( \sigma(\{A,B,C\}) \subseteq \{A,B\} \), but not \( \sigma(\{A,B\}) \subseteq \sigma(\{A,B,C\}) \) (A "lost"!)

Aizerman's condition

But what is meant by the term "corresponds to" above?
The role of choices in everyday inference

- Choosing best worlds (semantic choice)
  When presented with surprising information $\alpha$, the agent looks for the most plausible worlds that satisfy $\alpha$
  - where plausibility is measured according to the agent's doxastic preferences
    [...] and the preferences change in response to $\alpha$

- The inference function $\inf$ is generated by a choice function $\gamma$ over possible worlds, in symbols $\inf = \inf(\gamma)$, if and only if for every $\alpha$,
  $\beta$ is in $\inf(\alpha)$ if and only if $\beta$ is true in all worlds in $\gamma(\alpha)$

- Choosing 'worst' expectations (syntactic choice)
  When presented with surprising information $\alpha$, the agent looks for the least plausible sentences that entail $\neg\alpha$
  - where plausibility is measured according to the agent's doxastic preferences
    [...] and the preferences change in response to $\alpha$

- The inference function $\inf$ is generated by a choice function $\delta$ over sentences, in symbols $\inf = \inf(\delta)$, if and only if for every $\alpha$,
  $\beta$ is in $\inf(\alpha)$ if $\alpha \rightarrow \beta$ is not in $\delta(Cn(\neg\alpha))$
  $\beta$ is in $\inf(\alpha)$ if $\alpha \rightarrow \beta$ is not in $\delta(\{a \rightarrow \beta, \alpha \rightarrow \neg\beta\})$

- Bridge principles between choices over possible worlds and choices over sentences (expectations)
  - The choice function $\delta$ over sentences corresponding to the choice function $\gamma$ over possible worlds is defined by
    $\alpha$ is in $\delta(F)$ if and only if $\alpha$ is in $F$ and $\gamma(F[\alpha])$ is not in $\gamma(\alpha)$
  - The choice function $\gamma$ over possible worlds corresponding to the choice function $\delta$ over sentences is defined by
    $m$ is in $\gamma(F[\alpha])$ if and only if $m$ is in $\gamma(F[\alpha])$ and $m$ is in $\gamma(Cn(F) - \delta(Cn(F)))$
  - Almost all choice-theoretic properties are preserved by these transitions. (... only disagreement w.r.t. (II))

Rational choices and logical properties: $\inf$

<table>
<thead>
<tr>
<th>POSTULATES FOR INFERENCES</th>
<th>POSTULATES FOR CHOICES</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Clos) $\inf(\alpha) = Cn(\inf(\alpha))$</td>
<td>(log.prop.1) for syntactic choices</td>
</tr>
<tr>
<td>(RW) $\beta \in \inf(\alpha)$ and $\beta \vdash \gamma$, then $\gamma \in \inf(\alpha)$</td>
<td></td>
</tr>
<tr>
<td>(LLE) If $\neg \alpha \leftrightarrow \beta$, then $\inf(\alpha) = \inf(\beta)$</td>
<td>(log.prop.2) for syntactic choices</td>
</tr>
<tr>
<td>(Cut) $\inf(\alpha)$, then $\inf(\alpha \wedge \beta) \subseteq \inf(\alpha)$</td>
<td></td>
</tr>
<tr>
<td>(CMon) If $\beta \in \inf(\alpha)$, then $\inf(\alpha \wedge \beta) \subseteq \inf(\alpha)$</td>
<td>(I) If $S \subseteq S'$ and $\sigma(S) \subseteq S$, then $\sigma(S) \subseteq \sigma(S')$</td>
</tr>
<tr>
<td>(Or) $\inf(\alpha \vee \beta) \subseteq \inf(\alpha \wedge \beta)$</td>
<td>(III) If $S \subseteq S'$ and $\sigma(S) \subseteq S$, then $\sigma(S) \subseteq \sigma(S')$</td>
</tr>
<tr>
<td>(Cond) $\inf(\alpha \wedge \beta) \subseteq Cn(\inf(\alpha \cup {\beta})$</td>
<td>Aizerman</td>
</tr>
<tr>
<td>(RMon) $\neg \beta \not\in \inf(\alpha)$, then $\inf(\alpha \wedge \beta) \subseteq \inf(\alpha \wedge \beta)$</td>
<td>(IV) If $S \subseteq S'$ and $\gamma(S) \not\subseteq \gamma(S')$, then $\sigma(S) \subseteq \sigma(S')$</td>
</tr>
</tbody>
</table>

Sen's Property $\alpha$

Sen's Property $\beta$
Violations of choice constraints by real agents

In all cases to be considered we have

\[ \text{A} \text{ and } \text{B} \]

and similarly for the pairs \{A,A’\} and \{A’,B\}

- incomparabilities and ties

Violations of choice constraints by real agents 1

Effects of indistinguishable options (Debreu 1960, Tversky 1972)

\[ \text{A} \quad \text{A'} \quad \text{B} \]

background intuition “A and A’ are very similar to each other”

\[ \sigma(\{A,A',B\}) = \{B\} \quad \text{B is salient, non-arbitrary} \]

\[ \sigma(\{A,B\}) = \{A,B\} \]
Compromise effect (Simonson 1989)

background intuition “B is a compromise between the extreme cases A and A’” (needs at least two criteria)

\[
\sigma(\{A,A',B\}) = \{B\} \quad \text{— B is least salient}
\]

\[
\sigma(\{A,B\}) = \{A,B\}
\]

Effects of dominating options (Huber, Payne and Puto 1982, Shafir, Simonson and Tversky 1992ff)

background intuition “A is clearly better than A’”

\[
\sigma(\{A,A',B\}) = \{A\} \quad \text{— A gives a reason for the choice}
\]

\[
\sigma(\{A,B\}) = \{A,B\}
\]

Now Annie Andrews (A) has a sister, Alice (A').

Variation 1 ("indistinguishability", for the Debreu/Tversky case).

Annie and Alice are actually twins, and they have been sharing everything all their life. Actually, Alice is just as outstanding in linguistics as Annie.

Variation 2 ("compromise", for the Simonson case).

Alice is the younger sister of Annie and is the exact opposite of her. Her expertise and achievements are complementary to Annie's. Bette strikes a good balance between the two.

Variation 3 ("reason", for the Huber et al./Shafir et al. case).

Alice is the younger sister of Annie and is very well comparable in her expertise and achievements with her sister. But she is just not as good as Annie.
The counterexample varied (cont'd 2)

- In (at least two of) the three variations of the story, it does not make much sense to compare Bette Becker (B) with either of the Andrews sisters.

  Her scientific interests and achievements are very different, too different to pass a reasonable judgement as to whether she is better or worse than the Andrews sisters.

- This time there are several jobs in philosophy announced. Still none of A, A' and B is expected to get the job.

- In various hypothetical scenarios, various pieces of information (from the dean, for example) are given.

Variation 1: Indistinguishability

- Scenario 1. From $a \lor a' \lor b$, we infer that $b$.
- Scenario 2. From $a \lor b$, we do not infer that $b$ (we do not infer more than $a \lor b$).
- Scenario 3. From $a' \lor b$, we do not infer that $b$ (we do not infer more than $a' \lor b$).

This violates

- Cumulative Monotony (since $a \lor b$ is in $\text{Inf}(a \lor a' \lor b)$)
- Disjunctive Rationality (since $b$ is neither in $\text{Inf}(a \lor b)$ nor in $\text{Inf}(a' \lor b)$)
- Very Weak Disjunctive Rationality (since $b$ is not even in $\text{Cn}(\text{Inf}(a \lor b) \cup \text{Inf}(a' \lor b)) = \text{Cn}((a \land a') \lor b)$)

Variation 1: Indistinguishability (cont'd)

- This corresponds to

Violations of choice conditions

(III) $\sigma(\{A,A',B\}) \subseteq \{A,B\}$, but not

$$\sigma(\{A,B\}) \subseteq \sigma(\{A,A',B\})$$ (A "lost"!)

and

(II) not $\sigma(\{A,A'\}) \cap \sigma(\{A,B\}) \subseteq \sigma(\{A,A',B\})$ (A "lost"!)

Variation 2, Compromise, is analogous to variation 2.

Variation 3: "Reason"

- Scenario 1. From $a \lor a' \lor b$, we infer that $a$.
- Scenario 2. From $a \lor b$, we do not infer that $a$ (we do not infer more than $a \lor b$).
- Scenario 3. From $a' \lor b$, we do not infer that $a$ (we do not infer more than $a' \lor b$).

This violates

- Cumulative Monotony (since $a \lor b$ is in $\text{Inf}(a \lor a' \lor b)$)
- Disjunctive Rationality (since $a$ is neither in $\text{Inf}(a \lor b)$ nor in $\text{Inf}(a' \lor b)$)
- Very Weak Disjunctive Rationality (since $a$ is not even in $\text{Cn}(\text{Inf}(a \lor b) \cup \text{Inf}(a' \lor b)) = \text{Cn}((a \land a') \lor b)$)
Variation 3: "Reason" (cont’d)

- This corresponds to

Violations of choice conditions

(III) $\sigma(\{A,A',B\}) \subseteq \{A,B\}$, but not

$$\sigma(\{A,B\}) \subseteq \sigma(\{A,A',B\}) \quad (B \text{ "lost"!})$$

and

(II) not $\sigma(\{A',B\}) \cap \sigma(\{A,B\}) \subseteq \sigma(\{A,A',B\}) \quad \text{(B "lost"!)}$

Conclusion: Rational or irrational?

- The theory of rational choice can be applied to the realm of everyday inference, such inferences thus being conceived as 'cognitive decisions'.

- Problems of rational choice theory transfer to the realm of everyday inference, and the transfer is straightforward.

- It is not easy to prove that real (as opposed to idealized) persons are irrational.

- Even if money pumps could be constructed in theory, real persons are not likely to be money-pumped.

- There are various ways of explaining away the apparent violations of rationality constraints for choices.

Rational or irrational?

- Restaurant example: Menu changes completely ("having ... in a bad restaurant"/"having ... in a good restaurant")

- Job example (violation of $\alpha$ and $\beta$): Information received is richer; it includes the source ("... says that $p$" rather than just "$p$")

- Indistinguishibility: Salience as a help for reaching a decision in the absence of determining preferences; a method that combines the tasks of
  - choosing (in a principled, preference-guided way) and
  - picking (taking one among multiple best options in an arbitrary, random way, just in order to overcome the indecision)

- Compromise and "reason": like indistinguishibility

References


References (cont'd)