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**Reasoning, games, action, and
rationality**

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Reasoning, games, action, and rationality

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Reasoning, games, action and rationality

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Course Syllabus

Eric Pacuit Olivier Roy

Welcome to *Reasoning, games, action and rationality*. The course will consist of five 90 minute lectures. This document contains a general introduction to the course and, for each lecture, a short description and a bibliography. The website for the course is

<http://staff.science.uva.nl/oroy/~wordpress>

On this website you will find the detailed lecture notes and slides (updated each day).

Introduction and Motivation

Starting from the work of Ramsey (1926); de Finetti (1937); von Neumann and Morgenstern (1944) and Savage (1954), the formal analyses carried in decision and game theory have provided important insights for the theory of rational decision making. More recently, the *epistemic program* in game theory (Harsanyi, 1967-68; Aumann, 1999; Brandenburger, 2007) has highlighted the importance of *mutual expectations* for the understanding of *interactive* rationality, that is for rational decision making in situation of social interaction.

Game theory has inherited from decision theory its *instrumental* stance on rationality. In both disciplines to choose rationally is to choose, in the light of one's expectations, the best *means* to achieve one's *ends*. Decision theory studies instrumental rationality in situations where *one* agent chooses among various actions on the basis of their expected consequences. Crucially, in decision theoretic scenario it is the agent's *environment*, or "Nature", which determines the consequences of his actions. Game theory, on the other hand, is concerned with the interaction of *many* rational decision makers. Here the

consequences of one agent's decision depends on the choices of *all* the agents involved in the situation. The expectations of an individual each decision makers are thus no more about a “passive” or “external” environment, but rather about the *choices* and thus also expectations of other rational decision makers.

Acknowledging this apparently small difference, one vs many agents, complicates the picture of instrumental rationality. In games the players' expectations become *interrelated*: what one expects from his opponents depends on what one thinks the others expect from him, and what the others expect from a given player depends on what they think his expectations about them are. *Dynamic epistemic logic* provides here a fruitful environment to study such entangled expectations. It allows for an elegant analysis of information and information *about* information, that is of *higher-order information*.

In this course we will study various foundational issues that arise from the epistemic outlook on games, and show how dynamic epistemic logic (Plaza, 1989; Baltag et al., 1998; van Benthem, 2003; van Ditmarsch et al., 2007) sheds new lights on them. We will, in other words, take a “logical” perspective on conceptual problems regarding the notion of *rationality*, *expectations* and *choices* in *interactive* situations. The kind of problems that we are interested in and the methods we draw from make the present course a contribution to contemporary *formal epistemology* (Fitelson, 2006; Hendricks, 2006), while our emphasis on interaction and is also relevant for *social software* Parikh (2002).

Lecture 1 : Decision, games, game playing situations and Rationality

In this first lecture we will lay the conceptual and formal foundations for the whole course.

Important concepts and literature overview:

- Decision theory: *endogenous* and *exogenous* uncertainty, *Bayesian rationality* and *maximization of expected utility*. See: Savage (1954, Chap.2-3), Anscombe and Aumann (1963), Jeffrey (1965, chap.1), Myerson (1991, chap.1), Joyce (2004), Bradley (2007).
- Games: *extensive* and *strategic* representations, *strategy* and *higher-order expectations*. See: von Neumann and Morgenstern (1944), Myerson (1991, chap.2), Osborne and Rubinstein (1994, chap.2 and 6).

- Game playing situation: *information, knowledge, beliefs, belief hierarchy* and *rationality; relational models* and *type structures*. See Harsanyi (1967-68), Fagin, Halpern, Moses, and Vardi (1995), Aumann (1999), de Bruin (2004), Brandenburger (2007).

Lecture 2: Reasoning with Mutual Expectations

In the second lecture we will look in more details at the role of higher-order expectations in strategic reasoning. Important concepts and literature overview:

- Epistemic logic for relational models: *modal languages, expressive power, axiom systems* for knowledge, beliefs preferences and actions ($K, D, 4, 5, S$). Logics for probabilistic reasoning. See: Blackburn, de Rijke, and Venema (2001), Heifetz and Mongin (2001), Bonanno (1991), van Ditmarsch, van de Hoek, and Kooi (2007), van Benthem, van Otterloo, and Roy (2005), Gerbrandy, van Benthem, and Pacuit (2007), Baltag and Smets (2008),
- Rationality and iterated removal of strongly dominated strategies: *common belief in rationality, strict dominance, iterated elimination, solution concepts, announcement of rationality, dynamic* and *static characterization*. See: Brandenburger and Denkel (1987); Tan and Werlang (1988); van Benthem (2003)

Lecture 3: Refining Mutual Expectations

In the third lecture we will examine how the formal frameworks presented in the earlier lectures can be used to provide epistemic analyses of various solution concepts. Important concepts and literature overview:

- Epistemic characterization of solution concepts for strategic games: Semantic approaches: eg. Bonanno and Battigalli (1999); Brandenburger (2007), syntactic approaches: eg. Bonanno (2007); de Bruin (2004), rationality and removal of weakly dominated strategies: Apt (2007); Brandenburger, Friedenberg, and Keisler (2008)
- Reasoning about impossible events: Halpern (2001b); Baltag and Smets (2008), universal type spaces: eg. Moss and Viglizzo (2005); Mertens

and Zamir (1985) , Brandenburger-Keisler Paradox: Brandenburger and Keisler (2006)

Lecture 4: Hard Knowledge and Prior Beliefs

In the fourth lecture we will look at the effect of “hard information”, that is genuine *knowledge* on interactive reasoning, and at how one can use exogenous information to understand how it emerges.

Important concepts and literature overview:

- Knowledge and Nash Equilibrium: *mutual knowledge of strategy, equilibrium, strategy announcements*. See: Nash (1950), Aumann and Brandenburger (1995).
- Exogenous information: *prior and posterior beliefs, the common prior assumption and the “Harsanyi doctrine”, mixed strategies, correlated and Nash equilibria*. See: Aumann (1987); Aumann and Dreze (2005), Bonanno and Nehring (1997, 1999), Morris (1995), Halpern (2002).

Lecture 5: Extensive Games, Belief Revision and General Conclusion

In the last lecture, we will examine epistemic analyses for extensive games and conclude with general comments about the use of logical frameworks to study foundational issues in game theory.

- Epistemic analysis of solution concepts for extensive games. See: Aumann (1998, 1994); Stalnaker (1998); Halpern (2001a); Halpern and Moses (2007); Feinberg (2005)

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