New Foundations for Hyperintensional Semantics?

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"Hyperintensional contexts are simply contexts which do not respect logical equivalence." (Cresswell 1975; cf. Carnap 1947)

Examples: contexts of ascriptions of

- belief, aprioricity, desire, aboutness, causality, explanation, grounding,...

"The twenty-first century is seeing a hyperintensional revolution." (Nolan 2014)
$\hookrightarrow$ what is the logic and semantics for hyperintensional metaphysics!?

Worries:

- Extension $\hookrightarrow$ Intension $\hookrightarrow$ $\underbrace{\ldots}_{\text{natural stopping point?}}$ $\hookrightarrow$ (Interpreted) Syntax?
- Notions, such as aboutness and grounding, seem unclear and ambiguous. And some of the formal accounts, where existing at all, look a bit baroque.
- Don't we have hyperintensional semantics? (Situation semantics,...) Metaphysics merely needs to catch up with philosophy of language?

Is the future of philosophical logic hyperintensional?

Plan:

1. The Formal Structure of Hyperintensions
2. Basic Logical Operations on Hyperintensions
3. Example I: Relevance, Parts, Grounds
4. Example II: Fuzziness
5. Example III: Modalities
6. Conclusions

The following will be very elementary, preliminary, and programmatic.

The Formal Structure of Hyperintensions

If [subject matters] are to be introduced, the conservative choice would be Lewisian subject matters. . . equivalence relations on, or partitions of, logical space. I will argue for going one step further, to similarity relations on, or "divisions" of, logical space. . . a division's cells can overlap. . . A division's cells are incomparable, so allowance has not been made for "nested" truthmakers (Yablo 2014)

The grounding operator. . . is variably polyadic; although it must take exactly one argument to its 'right', it may take any number of arguments to its 'left'. . . all of the grounds must be relevant to conclusion (Fine 2012)

(Schaffer 2012: "grounding is something like metaphysical causation".)

Mackie (1980):

> Then in the case described above the complex formula '(*ABC* or *DGH* or *JKL*)' represents a condition which is both necessary and sufficient for *P*: each conjunction, such as '*ABC*', represents a condition which is sufficient but not necessary for *P*. Besides, *ABC* is a *minimal* sufficient condition: none of its conjuncts is redundant: no part of it, such as *AB*, is itself sufficient for *P*. But each single factor, such as *A*, is neither a necessary nor a sufficient condition for *P*. Yet it is clearly related to *P* in an important way: it is an *insufficient* but *non-redundant* part of an *unnecessary* but *sufficient* condition: it will be convenient to call this (using the first letters of the italicized words) an *inus* condition.[5]

A, B, C,... above can also be negative: e.g., $\overline{C}$ is the absence of C.

E.g.: "It may be the consumption of a certain poison conjoined with the non-consumption of the appropriate antidote which is invariably followed by death."

This motivates the following formal structure of hyperintensions:

- Let S be a finite, non-empty set of elementary states of affairs, such that for every p in S, there is a unique "negation" $\overline{p}$ in S (and $\overline{\overline{p}} = p$).

Definition

$\mathcal{X}$ is a hyperintension (over S) iff
$\mathcal{X} \subseteq \wp(S) \setminus \{\varnothing\}$, such that there are no X, Y in $\mathcal{X}$: $X \subset Y$.

- An $X \in \mathcal{X}$ is a (potential) ground of $\mathcal{X}$ / reason for $\mathcal{X}$ / way of $\mathcal{X}$ being true.
- If $\mathcal{X}$ is a hyperintension, its set $min(\mathcal{X})$ of minimal elements equals $\mathcal{X}$. Minimality is supposed to capture relevance / non-redundancy of grounds.
- Let $s \subseteq S$: $s \models \mathcal{X}$ iff there is a ground $X \in \mathcal{X}$, such that $s \supseteq X$.
- Mathematicians are studying hyperintensions under different names (for arbitrary S without $\bar{\cdot}$): "Sperner families" (combinatorics), "clutters" (optimization), "simple hypergraphs" (hypergraph theory),...
- Let $H(S)$ be the set of all hyperintensions over S.

Examples of hyperintensions:

- $\{\{a, b, c\}, \{d, e, f\}, \{j, k, l\}\}$
- $\{\{a, b, \overline{c}\}, \{a, \overline{a}, b\}\}$
- $\{\{p\}\}$
- $\{\{p, q\}, \{p, \overline{q}\}\}$

From hyperintensions one can determine intensions and (relative to the actual world) extensions.

E.g., $\{\{p\}\} \neq \{\{p, q\}, \{p, \overline{q}\}\}$ (cf. Barwise and Perry 1983), but they determine the same classical intension and extension:

for every maximally consistent $s \subseteq S$,

$$s \models \{\{p\}\} \text{ iff } s \models \{\{p, q\}, \{p, \overline{q}\}\},$$

and with respect to the actual world $@ \subseteq S$:

$$@ \models \{\{p\}\} \text{ iff } @ \models \{\{p, q\}, \{p, \overline{q}\}\}.$$

Basic Logical Operations on Hyperintensions

Which hyperintension is to count as the negation of, e.g., $\{\{p,q\},\{p,\overline{q}\}\}$?
The answer is underdetermined by classical intensions/extensions.

But here is a natural proposal:

Definition

- For all $X \subseteq S$, let $\overline{X}$ be the set $\{\overline{p} \,|\, p \in X\}$.
- For every hyperintension $\mathcal{X}$, we define

$$\neg\mathcal{X} = min(\{\overline{Y} \subseteq S \,|\, \forall X \in \mathcal{X}, Y \cap X \neq \varnothing\})$$

E.g., $\neg\{\{p,q\},\{p,\overline{q}\}\} = \{\{\overline{p}\},\{\overline{q},q\}\}$

So each ground in $\neg\mathcal{X}$ contradicts, or rules out, every ground in $\mathcal{X}$, and the grounds of $\neg\mathcal{X}$ are minimal having this property.

Apart from $\bar{\cdot}$, our negations are well known to mathematicians, in very different contexts, as ‘blockers’ or ‘transversal hypergraphs’:

E.g., if $\mathcal{X}$ is the set of arc-sets of paths from node *a* to *b* in a graph, then $\neg\mathcal{X}$ is the set of minimal cuts that separate *b* from *a* (suppressing $\bar{\cdot}$).

Theorem

For all hyperintensions $\mathcal{X}$ (on S):

1. $\neg\neg\mathcal{X} = \mathcal{X}$.
2. *For all $s \subseteq S$: $s \models \neg\mathcal{X}$ iff $\overline{S \setminus s} \not\models \mathcal{X}$.*

   (And this second property determines negation uniquely.)

The proof is easy but not completely trivial.

(Note that 2 above relates to Routley and Routley’s 1972 star operator for ¬.)

Next we turn to the conjunction and disjunction of hyperintensions.

The obvious proposal is (think of truthmaker semantics!):

Definition

For all hyperintensions $\mathcal{X}, \mathcal{Y}$ (on S), we define

- $\mathcal{X} \wedge \mathcal{Y} = min(\{X \cup Y \mid X \in \mathcal{X}, Y \in \mathcal{Y}\})$,
- $\mathcal{X} \vee \mathcal{Y} = min(\mathcal{X} \cup \mathcal{Y})$.

Theorem

For all hyperintensions $\mathcal{X}, \mathcal{Y}$ (on S):

- $\neg(\mathcal{X} \wedge \mathcal{Y}) = \neg\mathcal{X} \vee \neg\mathcal{Y}$, $\neg(\mathcal{X} \vee \mathcal{Y}) = \neg\mathcal{X} \wedge \neg\mathcal{Y}$.

Indeed, with $\mathcal{X} \leq \mathcal{Y}$ iff for every $X \in \mathcal{X}$ there is a $Y \in \mathcal{Y}$, such that $X \supseteq Y$ [iff for all $s \subseteq S$, if $s \models \mathcal{X}$ then $s \models \mathcal{Y}$]

$H(S)$ becomes the free De Morgan algebra with generators $\{\{p\}\}$ ($p \in S$).

Remark: What if we simply eliminate inconsistent grounds whenever the occur?

- Let $\mathcal{X} = \{\{p,q\},\{\overline{p},r\},\{q,r\}\},\ \mathcal{Y} = \{\{p\}\}$.
- $\neg\mathcal{X} = \{\{\overline{p},\overline{r}\},\{p,\overline{q}\},\{\overline{q},\overline{r}\}\},\ \neg\mathcal{Y} = \{\{\overline{p}\}\}$.
- $\mathcal{X} \wedge \mathcal{Y} = \{\{p,q\},\{p,\overline{p},r\}\} \overset{\text{Consistent}}{\longmapsto} \{\{p,q\}\}$.
- $\neg(\mathcal{X} \wedge \mathcal{Y}) = \{\{\overline{p}\},\{\overline{q}\}\} \neq$

  $\neg\mathcal{X} \vee \neg\mathcal{Y} = \{\{\overline{p}\},\{p,\overline{q}\},\{\overline{q},\overline{r}\}\}$.

Moral: In order to maintain both the $\{\{\ldots\},\{\ldots\},\ldots\}$ format and our standard logical laws, one needs the information that is encoded in inconsistent grounds!

Example I: Relevance, Parts, Grounds

Let $\mathcal{L}$ be the language of propositional logic with finitely many propositional variables (from a given non-empty set S) and with $\neg, \wedge, \vee$.

Definition

A hyperintensional valuation on $\mathcal{L}$ is a function $V : \mathcal{L} \to H(S)$, such that

- $V(\neg\alpha) = \neg V(\alpha)$, $V(\alpha \wedge \beta) = V(\alpha) \wedge V(\beta)$, $V(\alpha \vee \beta) = V(\alpha) \vee V(\beta)$.

Question: How do we determine the atomic case?

Here is one option (in line with truthmaker semantics and situation semantics):

- Atomic case: $V(p) = \{\{p\}\}$.

If we then finally define

- $\alpha_1, \ldots, \alpha_n \models \beta$ iff for all V: $V(\alpha_1 \wedge \ldots \wedge \alpha_n) \leq V(\beta)$,

we get precisely FDE or tautological entailment (a fragment of relevance logic).

- In a sense, this is but a different presentation of van Fraassen's (1969) truthmaker semantics for FDE (see also Kit Fine's recent work on this):

  While the grounds in our hyperintensions are minimal, van Fraassen takes certain supersets of our hyperintensions to be the sets of truthmakers.

  Advantages of the present approach: there is no need for "false-making".

- Yablo (2014) contrasts "recursive" with "reductive" ("non-excessive") truthmakers: our approach shows that there is some kind of minimality that is built already into recursive truthmaking.

- There are further natural relations to be studied:

  – cf. Yablo (2014) on "inclusive entailment":

  $\alpha \models^* \beta$ iff for all V: for every $X \in V(\alpha)$ there is a $Y \in V(\beta)$, such that $X \supseteq Y$, and for every $Y \in V(\beta)$ there is an $X \in V(\alpha)$, such that $X \supseteq Y$.

  – Close to Fine (2012) (here '$V(p) = \{\{p\}\}$' should *not* be assumed):

  $\alpha_1, \ldots, \alpha_n$ (potentially) ground β (in V) iff $V(\alpha_1 \wedge \ldots \wedge \alpha_n) \subset V(\beta)$.

Example II: Fuzziness

Theorem

- *Let V be a hyperintensional valuation on $\mathcal{L}$.*
- *Let $st : S \to [0, 1]$ measure the "strength" $f(p)$ of p, such that $st(\overline{p}) = 1 - st(p)$.*
- *Let $St : H(S) \to [0, 1]$ measure the "strength" of grounds for arbitrary hyperintensions, with*

$$St(\mathcal{H}) = max_{X \in \mathcal{H}} \, min_{p \in X} \, st(p)$$

(call this the 'Weakest Link Principle').

Then $St \circ V$ is a fuzzy valuation on $\mathcal{L}$ by which $\neg$ corresponds to $1 - x$, $\wedge$ to min, and $\vee$ to max. In particular, $St(V(\neg\alpha)) = 1 - St(V(\alpha))$.

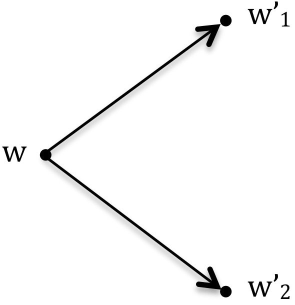
The proof relies on Edmonds and Fulkerson' (1970) Bottleneck Theorem.

Example III: Modalities

I will only give an example here:

$V(\Box(p \vee q), w) =$
$\{\{\langle w_1', p\rangle, \langle w_2', p\rangle\},$
$\{\langle w_1', p\rangle, \langle w_2', q\rangle\},$
$\{\langle w_1', q\rangle, \langle w_2', p\rangle\},$
$\{\langle w_1', q\rangle, \langle w_2', q\rangle\}\}$

$w \models_M \Box(p \vee q)$



$V(p \vee q, w_1') =$
$\{\{\langle w_1', p\rangle\}, \{\langle w_1', q\rangle\}\}$

$F(w_1') = \{p, \overline{q}\}$

$w_1' \models p \vee q$

$V(p \vee q, w_2') =$
$\{\{\langle w_2', p\rangle\}, \{\langle w_2', q\rangle\}\}$

$F(w_2') = \{\overline{p}, q\}$

$w_2' \models p \vee q$

Intensionally, such modal operators correspond to those of normal modal logic.

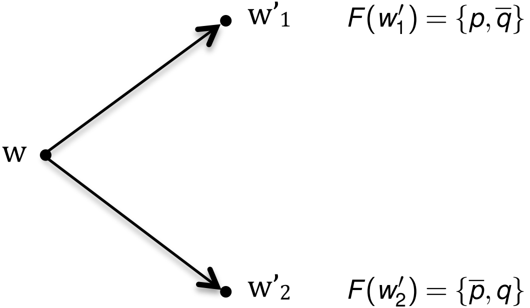
The underlying *intensional* logic for $\neg, \wedge, \vee$ is classical.

But there are alternative options—here is another example of what one can do:

$V(p \vee q, w) =$
$\{\{\langle w, p\rangle\}, \{\langle w, q\rangle\}\}$

$V(\Diamond(p \vee q), w) =$
$\{\{\langle w'_1, p\rangle, \langle w'_1, q\rangle\},$
$\{\langle w'_1, p\rangle, \langle w'_2, q\rangle\},$
$\{\langle w'_1, q\rangle, \langle w'_2, p\rangle\},$
$\{\langle w'_2, p\rangle, \langle w'_2, q\rangle\}\}$

$w \models_M \Diamond(p \vee q)$



With such a semantics, $\Diamond(p \vee q) \rightarrow \Diamond p \wedge \Diamond q$ becomes logically true.

You may take the spaghetti or the Wiener Schnitzel $\models$
You may take the spaghetti and you may take the Wiener Schnitzel

(Free Choice Permission; cf. Kamp 1973)

Conclusions

- There is a natural set-theoretic format for hyperintensions: hyperintensions are sets of minimal grounds. This allows also for natural logical operations on hyperintensions.

  (If we don't trust the metaphysics, maybe we trust the maths?)

- Hyperintensional semantics for relevance, aboutness, grounding, and modalities might solve problems of purely intensional accounts.

- There are lots of potential further applications: conditionals (e.g., counterfactuals—cf. Fine 2012), belief, knowledge (Gettier cases!), dynamic epistemic logic, quantifiers, adverbs, probability, grounded type-free truth, grounding in mathematics, implicatures,...

- There is a lot to do: The corresponding logics need to be explored. For infinite S, minimal grounds do not exist for free anymore. The relation to situation semantics, event semantics, justification logic,... should be determined. And so on.

Is the future of philosophical logic hyperintensional?

And the answer is. . . TO BE CONTINUED.

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Theorem

$johan \wedge retirement \models \bot.$

Proof: Obvious.

Appendix: Modalities

In the previous example, if the grounding operator is meant to be factive (as it is usually taken to be), then it is useful to introduce points of evaluation.

More generally, this is so if one turns to languages with modalities; e.g.:

- Let W be a non-empty set of worlds.
- Let $S_W = \{\langle w, p\rangle \mid p \in S, w \in W\}$ be our new set of elementary states of affairs, such that $\overline{\langle w, p\rangle} = \langle w, \overline{p}\rangle$.
- Let $R \subseteq W \times W$ (the accessibility relation).

Definition (TO BE CONTINUED)

$V : \mathcal{L}_\Box \times W \to H(S_W)$ is a hyperintensional valuation on $\mathcal{L}_\Box$ w.r.t. R iff

- $V(\neg\alpha, w) = \neg V(\alpha, w)$, $V(\alpha \wedge \beta, w) = V(\alpha, w) \wedge V(\beta, w)$,
  $V(\alpha \vee \beta, w) = V(\alpha, w) \vee V(\beta, w)$.
- $V(p, w) = \{\{\langle w, p\rangle\}\}$.

Definition (CONTINUED)

- $V(\Box\alpha, w) = min(\{\cup_{w':wRw'} c(w') \,|\, c : w' \in W \mapsto V(\alpha, w')\})$.
  (“A reason for $\Box\alpha$ is a conjunction of reasons for α distributed over all accessible worlds.”)
- $V(\Diamond\alpha, w) = min(\cup_{w':wRw'} V(\alpha, w'))$.
  (“A reason for $\Diamond\alpha$ is a reason for α at an accessible world.”)

V determines for each α and w the potential grounds for α at w.

Finally, in order to define truth at a world (and thus the intension mapping), one needs to equip each world with its obtaining states of affairs:

Definition

$M = \langle W, R, V, I \rangle$ is a hyperintensional model for $\mathcal{L}_\Box$ iff

- $W \neq \emptyset$, $R \subseteq W \times W$,
- $V : \mathcal{L}_\Box \times W \to H(S_W)$ is a hyperintensional valuation on $\mathcal{L}_\Box$ w.r.t. R,
- $F : W \to \{s \subseteq S \,|\, s \text{ is maximally consistent}\}$.

$w \models_M \alpha$ iff there is a ground $X \in V(\alpha, w)$, s.t. for all $\langle w', p \rangle \in X$: $F(w') \ni p$.

But there are alternative options, too.

Let us restrict the modal language so that there are no nestings of $\Diamond$.
And let us determine hyperintensions for $\Diamond$ differently now:

- For all $X = \{\langle w, p\rangle, \langle w, p'\rangle, \ldots\} \in V(\alpha, w)$, for all $w' \in W$, let

$$X^{w'} = \{\langle w', p\rangle, \langle w', p'\rangle, \ldots\}.$$

- $V(\Diamond\alpha, w) = min(\{\cup_{X \in V(\alpha,w)} X^{c(X)} \,|\, c : V(\alpha, w) \rightarrow \{w' : wRw'\}\})$.

  ("Distribute *all* reasons for α in some way over accessible worlds: the conjunction of such a distribution of reasons is a reason for $\Diamond\alpha$.")