

Realism and Anti-Realism on (Double-)Truth

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- **Weak paraconsistency:** because inferences are commonly made from inconsistent bodies of information (theories, sets of beliefs, etc., including both α and $\neg\alpha$), but people are clearly not at liberty to conclude anything (any β) they like.
- Paraconsistent models are useful mathematical tools, but we can reject (EFQ) without having to claim that contradictions can be *true*.

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- **Strong paraconsistency**: because some contradictions are *true* (although not everything is). This is also called *dialetheism* (sub-distinctions in Beall 2004).
- Some contradictions are *provably* true, in the sense that they are entailed by plain facts concerning natural language and our thought processes (say, extended Liar paradoxes like “This sentence is not true”).

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- Priest (1987, 2000, and 2006): dialetheism is not by itself tied to a specific conception of truth; not more to the coherence theory of truth ("typically those who endorsed the [coherence] theory have held that it makes no sense to define truth in terms of some objective reality, independently of our cognitive functioning") than to the correspondence theory (which is "likely to appeal to a metaphysical realist": Priest 2006, 53).

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- **Realist dialetheism**: the truth of some contradictions entails the real existence of inconsistent objects and/or states of affairs: those that make the contradictions true.
- It is quite common in the current literature on (pro and against) dialetheism to straightforwardly speak of inconsistent *objects*, *states of affairs*, and entire inconsistent *worlds*.

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- The charge of committing the Russellian Fallacy can be raised against realist dialetheism by resorting to the *intuitive reading* of the semantics underlying paraconsistent logics.
- “Intuitive reading” can be clarified via a well-established distinction between *pure* and *applied* semantics, which is nowadays common even in textbooks of logic and philosophy of logic (Plantinga 1974, Kirwan 1978 and Haack 1978).

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- “The (algebraic, set-theoretic) structures that provide models for your logics become applied semantics by receiving an intuitive interpretation. But the things those structures are interpreted *as* are *not* entities that objectively exist apart from, and antecedently to, anyone’s thought of them.”

3. Pure semantics, applied semantics

- **a) Pure semantics:** A model for a QML is a sequence $M = \langle W, a, R, D, q, \nu \rangle$ of set-theoretic constructions: W is a non-empty set, $a \in W$, R is a binary relation on W , D is a non-empty set, q is a function from W to subsets of D , ν is a function taking $\langle n$ -ary predicate, W -member \rangle pairs to n -tuples of members of D , taking each \langle closed sentence, W -member \rangle pair to one member of $\{1, 0\}$, etc. etc.

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- This is a piece of *algebra*: in order to accept it, one does not need to believe in “possible worlds”, however the expression is interpreted. One just has to believe in sets and set-theoretic constructions out of sets.

3. Pure semantics, applied semantics

- **b)** We move to an **applied semantics**, to begin with, by specifying what the set-theoretic entities are supposed to **represent** – and now PW-talk is needed:
 - W is the set of all possible worlds,
 - a is the actual world,
 - R is a relation of accessibility on the set of all possible worlds,
 - D is the set of all possible individuals,
 - q is a function assigning to each possible world the domain of things existing at that world, etc. etc.

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- In this informal explanation of the intuitive reading of the semantics, we quantify on possible worlds and individuals, and we are liable to be asked the question, *What are these entities we are ontologically committed to?* An answer to this is a part of what it takes to have an applied semantics.

4. Applied paraconsistent semantics

4.1. Relevant logic and impossible worlds

- A *Routley-Meyer semantics for relevant logic(s)* is a quintuple $\langle W, O, a, *, R \rangle$, where W is a set of "situations (or worlds)" (Routley 1979, 309); O is a subset of W including normal or "theorem-regular" situations, that is to say, worlds in which all the theorems of relevant logic hold; a is an element of O , representing the actual world; $*$ is a unary operation on W , i.e., a function from worlds to worlds (the *Routley star*, sometimes called "involution"); R is a ternary accessibility relation on W .
- The most controversial part of the story has to do with the intuitive reading of R and $*$, which are used in the semantic clauses for relevant negation and the relevant conditional. I will talk only of $*$.

4. Applied paraconsistent semantics

4.1. Relevant logic and impossible worlds

- Given a world w , the “involution” operation produces a world w^* (on whose nature I shall say something in the following). The semantic clause for negation is:

$$(S_{\neg}) \mathcal{T}_w(\ulcorner \neg \alpha \urcorner) \Leftrightarrow \text{Not } \mathcal{T}_{w^*}(\ulcorner \alpha \urcorner).$$

- The standard clause for classical negation claims that $\neg \alpha$ is true (at w) if and only if α is not true (at w , i.e., at that very world). The Routley-Meyer negation is characterized by saying that $\neg \alpha$ is true at w if and only if α is not true at w^* (Routley and Meyer 1976; Routley 1979).

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- This provides a counter-example to (EFQ): α is true at w , β is not true at w and α is not true at w^* : both α and $\neg\alpha$, then, are true at w while β is not.

4. Applied paraconsistent semantics

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- But what kind of world is w^* with respect to w ? Here is a little bit of help from the literature:

“One way of thinking of $[w]$ and $[w^*]$ is to regard them as ‘mirror images’ one of another reversing ‘in’ and ‘out’. Where one is inconsistent (containing both A and $\neg A$), the other is incomplete (lacking both A and $\neg A$), and vice versa (when $[w] = [w^*]$, $[w]$ is both consistent and complete and we have a situation appropriate to classical logic). Viewed this way the Routleys’ negation clause makes sense, but *it does require some anterior intuitions about inconsistent and incomplete set-ups.*” (Dunn 1986, 191; my italics)

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4.1. Relevant logic and impossible worlds

- **Is this just pure semantics?** The relevant problems for relevance come exactly at the level of the intuitive reading of the semantics. Several authors have claimed that the Routley-Meyer relevant semantics remains *pure*, not reaching the status of applied semantics (van Benthem 1979, Copeland 1986, Smiley 1993).
- This holds particularly for the Routley star $*$, which should give the semantics of relevant negation:

“By itself this ‘star rule’ is merely a device for preserving a recursive treatment of the connectives [...] and it does nothing to explain their tilde until supplemented by an explanation of $[w^*]$ ” (Smiley 1993, 17-8).

4. Applied paraconsistent semantics

4.1. Relevant logic and impossible worlds

- **What are these (im)possible relevant worlds?** In their 1972 paper the Routleys introduced non-standard worlds realizing inconsistencies as “set-ups”, characterizing them quite syntactically as sets of *sentences* (see Routley and Routley 1972, 335). In the Routley and Meyer 1973 paper, set-ups became what is *described by* a set of sentences.
- Such slips obviously attract the charge of Russellian Fallacy: “this is dialectic rhetoric *par excellence*. It takes us from doxastic inconsistency to ontic inconsistency, that is, from the fact that beliefs [theories, sets of sentences, etc.] are sometimes inconsistent to the possibility that inconsistent beliefs are sometimes true, and so to the possibility that sometimes objects or states of affairs are inconsistent.” (Woods 2003, 89-90).

4. Applied paraconsistent semantics

4.1. Relevant logic and impossible worlds

- A plausible reading of the Routley-Meyer semantics for relevant negation has been proposed in terms of *information flow*, within the framework of *situation semantics* (Barwise and Perry 1993; Mares 1996; Bremer and Cohnitz 2004).
- The basic idea is to model circuits hosting regulated flows of information: "Talking in situation semantics language we can say that information about one situation is derived from another situation by some channel. The basic building blocks of information are *infons*, which resemble Russellian propositions." (Bremer 2005, 68-9).

4. Applied paraconsistent semantics

4.1. Relevant logic and impossible worlds

- **An intuitive characterization of the Routley star** (Restall 1999): Two pieces of information or infons can be compatible or incompatible with each other, thereby bringing in a certain partial compatibility or incompatibility between worlds containing them. We can accordingly introduce a binary *compatibility* relation C between worlds and characterize relevant negation via such a relation:

$$(S_{\neg}) \quad T_w(\ulcorner \neg \alpha \urcorner) \Leftrightarrow \forall w_1 (C(w, w_1) \Rightarrow \text{Not } T_{w_1}(\ulcorner \alpha \urcorner)),$$

that is to say: $\neg \alpha$ is true at w iff α is not true at all compatible worlds.

4. Applied paraconsistent semantics

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- We can read " $\mathcal{C}(w, w_1)$ " as saying that nothing given by w is rejected by w_1 , so the set of worlds compatible with w may represent what is not excluded by what holds at w (Restall 1999, 61-2).
- The "reverse twin" of w , w^* , is the most informative, or comprehensive, among the worlds w_1 for which $\mathcal{C}(w, w_1)$ holds: "the Routley star is a simplification of our compatibility clause for negation when we assume that \mathcal{C} is symmetric, directed and convergent"; and "given that the compatibility semantics makes sense and is an applied semantics, it follows that its simple retelling, involving the Routley star also makes sense, and it too is an applied semantics" (Restall 1999, 63; see also Bremer 2005).

4. Applied paraconsistent semantics

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But what is an *infor*?

- If worlds are considered as make-ups of infons (not simple sets, but something with more structure imposed on them: say, structures that act themselves as *conduits* for information in some way); and if an infon is information *on* something; then relevant impossible **worlds are *informational structures***.
- In this case, worlds retain the right to be inconsistent and/or incomplete, since it is out of question that *information* can be inconsistent and/or incomplete. But again, we are dealing with theories, beliefs, etc., that is to say, *representations*.

4. Applied paraconsistent semantics

4.1. Relevant logic and impossible worlds

- If a world w of the Routley-Meyer semantics is characterized as a set of sentences or beliefs, a collection of hypotheses, infons, an informational structure, etc., then the property of ***being true at a world*** actually means that “the sentence A is a member of the set of sentences $[w]$, this set representing the beliefs of a given individual, or a given mathematical or physical theory” (Copeland 1986, 486-7).
- Now, ***membership of a set of beliefs*** is a property quite different from ***truth***. The predicate “is a member of belief set w ” does not in general satisfy the Tarskian schema; whereas many (including Priest) have considered the T-schema as a minimal constraint on a truth predicate (even independently of the issue whether the schema itself should be interpreted “deflationistically”, or in a robust sense).

4. Applied paraconsistent semantics

4.2. Reinterpreting dialethic logics

- **The preservationist interpretation of FDE and of Priest's LP** (Brown 2004): the main aim of preservationist logics (see Schotch and Jennings 1989) is to provide accounts of consequence in which the property preserved by the consequence relation is not *truth* at all, but some other desirable feature (e.g., the level or degree of incoherence, or the number of ambiguous sentential variables) we want to maintain in situations of inconsistent information, without having to assume that the inconsistencies in question are true.
- The FDE consequence relation can be read "without adopting a semantics that allows truth or any other 'designated' [...] values to be assigned to inconsistent sets of sentences or to contradictions" (Brown 2004, 141).

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- This shows that a strongly dialethic reading of LP, or of FDE, is not mandatory at all: we can interpret the consequence relations of LP and FDE as preserving some (classically perfectly acceptable) semantic properties, rather than preserving dialethic truth-values. And this:

“blocks the slippery slope argument in Priest [2000] that aims to drive us from the second level of paraconsistency (at which we accept the existence of interesting or valuable inconsistent theories that can be modelled with the help of paraconsistent consequence relations) to the third (at which we accept the possibility that some such theories are true).”
(Brown 2004, 146)

5. Dialetheism as an idealism?

(Temporary) conclusion:

- The applied semantics for mainstream paraconsistent logics may legitimate only a *weak* paraconsistent approach; not a strong or dialetheic one, promising genuine counterexamples to the Law of Non-Contradiction.

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- The applied semantics for mainstream paraconsistent logics may legitimate only a *weak* paraconsistent approach; not a strong or dialethic one, promising genuine counterexamples to the Law of Non-Contradiction.
- Contradictions are said to be sometimes “true”; once an interpretation of the semantics has been produced, though, the intuitive meaning of “true” seems to have turned in our hands into something like “included in a theory or a system of beliefs”.

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- **a)** If she claims to have exhibited true contradictions, she is liable of committing the Russellian Fallacy: the “worlds” which should realize contradictions actually are sets of infons, theories, etc.
- **b)** If she bites the bullet and takes the shift from “true (of the world)” to “included in a theory” at face value, she appears to commit herself to some equivalence between *worlds* and *theories*, thereby taking the road of some constructivism or idealism: the view according to which *the world is made of theories*, or conceptual schemes, etc.

5. Dialetheism as an idealism?

- If one chooses **b)** the very distinction between *weak* and *strong* paraconsistency seems to fade. This was put forward as a difference between the following two claims:
 - (1) there can be inconsistent but non-trivial theories (sets of beliefs, etc.);
 - (2) there can be true contradictions, thus – given a realist stance on “true” – inconsistent objects and/or states of affairs that make the contradictions true.
- However, such a difference could not be easily maintained, when we make the distinction between *theories* and (sets of) *states of affairs* collapse. And this might be a welcome result for Priest, supporting in a roundabout way his “slippery slope” from weak paraconsistency to dialetheism.

5. Dialetheism as an idealism?

- Graham: would you be(come) an anti-realist dialetheist? 😊