CHAPTER V

THE PTQ-FRAGMENT

ABSTRACT

In this chapter the fragment of English described in Montague's article PTQ (MONTAGUE 1973) is presented. The method of exposition consists in starting with a very small fragment, and expanding it gradually. In each stage both the syntax and the semantics are discussed extensively. Special attention is paid to the motivation and justification of the analysis.
1. INTRODUCTION

The aim of this chapter is to present in a rigorous way the syntax and the semantics of a certain fragment of a certain dialect of English. The fragment is about the same as the one presented in MONTAGUE (1973), henceforth PTQ. On all essential points I will follow the treatment given in PTQ, in the details, however, there are some differences. The presentation, motivation and justification I will give for the treatment, differs considerably from PTQ. For the presentation I will employ a method which might be called 'concentric'. I will start with a very small fragment, and gradually expand this. For the fragments in each of the stages both the syntax and semantics are given, together with an extensive discussion. I hope that this method will make it easier to understand the sometimes difficult or subtle details of the PTQ-treatment. Certain details (concerning the problems of extension and intensity) will be discussed in appendix 1 of this book. A list of the rules of the fragment (useful as a survey) can be found in chapter 8.

In the exposition I will give special attention to algebraic and algorithmic aspects of the treatment. The algebraic considerations often provide an explication why a certain detail is as it is, and not otherwise. The algorithmic aspect concerns the method to obtain simple meaning representations. I do not like some rather abstract relation between a sentence and its meaning. For instance, I am not satisfied with a two-lines-long formula, if there is a one-line-long-formula which represents the same meaning, and if a meaning is represented by a formula which has to be interpreted in models satisfying certain meaning postulates, I would like to have a formula in which these postulates are made explicit. So I prefer concise and clear meaning representations. In order to reach this aim, several rules will be given for the reduction of formulas.

The syntax of the fragment in PTQ is treated rather poorly. In this chapter only minor improvements will be given (for more fundamental changes see chapter 8). But syntax was not Montague's main interest; he was interested primarily in semantics. The fragment is rich in semantically interesting phenomena, and it deals with several famous semantic puzzles. Below I will mention some of the sentences dealt with, together with some comments.

A first kind of phenomena dealt with concerns sentences of which it is clear what their meanings are, and how these should be represented using standard predicate logic. Their challenge lies in the aim to obtain these
meanings in a systematic way. Consider (1) and (2).

(1) *John runs.*

(2) *Every man runs.*

These two sentences are closely related in form: only the subject differs. Therefore one would like to produce the sentences along the same lines. The representations of their meanings, however, are rather different. In standard logic it would be as in (3) and (4).

(3) run(john)

(4)  \( \forall x [\text{man}(x) \rightarrow \text{run}(x)] \).

This gives rise to the question how to obtain rather divergent formulas from closely related sentences. A corresponding question arises for the ambiguity of (5).

(5) *Every man loves a woman.*

This sentence may be used when one specific woman is loved by every man, (say Brigitte Bardot), or when for each man there may be another woman (say his own mother). Sentence (5) is considered as being ambiguous between these two possibilities (for arguments, see section 6). This kind of ambiguity is called 'scope ambiguity' (of quantifiers). The two readings that will be obtained for (5) are (simplified) represented in (6) and (7).

(6)  \( \forall x [\text{man}(x) \rightarrow \exists y [\text{woman}(y) \land \text{love}(x,y)]] \)

(7)  \( \exists y [\text{woman}(y) \land \forall x [\text{man}(x) \rightarrow \text{love}(x,y)]] \).

A second kind of phenomena dealt with concerns sentences for which it is difficult to say how their meanings should be represented. Consider (8) and (9)

(8) *John seeks a unicorn.*

(9) *John finds a unicorn.*

These two sentences have about the same form, only the verbs they contain are different. One is tempted to expect that they have about the same meanings as well; the only difference being that they express another relation between John and a unicorn. This is not the case, however. The one sentence gives information about the existence of unicorns, which the other sentence does not. So an approach which says that the seek-relation is always a relation between two individuals would not be acceptable. We have
to provide a meaning for (8) from which it does not follow that unicorns exist. However, sentence (8) can also be used in a situation that unicorns exist, and it is ambiguous between these two possibilities. It has a reading from which it follows that at least one unicorn exists (the referential reading), and a reading from which this does not follow (the non-referential reading).

Some examples of the referential/non-referential ambiguity are (10), (11), and (12).

(10) John talks about a unicorn.

(11) John wishes to find a unicorn and eat it.

(12) Mary believes that John finds a unicorn and that he eats it.

Sentence (9) allows only for a referential reading. The same holds for sentence (13), see MONTAGUE 1973, p.269.

(13) John tries to find a unicorn and wishes to eat it.

The ambiguity we distinguish in sentences (8), (10), (11) and (12) is in the literature also called the 'de-direct/de-re' ambiguity, or the 'specific/non-specific' ambiguity. This terminology is not felicitous, because one might associate with it a distinction that is not covered by the formal analysis that will be provided. Nevertheless, this terminology will sometimes be used in the sequel, since it is standard for some of the examples.

2. JOHN RUNS

The fragment in this section consists of very simple sentences like John runs. It has three categories (=sorts): the category T of terms, the category IV of intransitive verb phrases, and the category S of sentences (in PTQ a t is used instead of S). There are basic expressions (=generators) of the categories T and IV. The set $B_T$ of generators of the category T contains the proper names of the PTQ-fragment, ($B_T \sim$ 'Basic expressions of category T'). Furthermore a special name is added for illustrative purposes: Bigboss. The sets $B_T$ and $B_{IV}$ are defined as follows ($B_T$ will be extended in section 4).

2.1. $B_T = \{John, Bill, Mary, Bigboss\}$

2.2. $B_{IV} = \{run, walk, talk, rise, change\}$. 
2.2. END

In the logic there is for each element of $B^I$ a corresponding constant of type $\epsilon$, except for $\textit{Bigboss}$. In PTQ these constants are called $j,m,b$ respectively, but I will use full names: $\textit{john}$ etc.. Notice the difference in the letter type used for English ($\textit{Mary}$), and the one used for logic ($\textit{mary}$). One might expect that a proper name translates into the corresponding constant, but for reasons to be explained later, the translation is a complex expression containing this constant. So among the constants in IL of type $\epsilon$, we distinguish three special ones.

2.3. \( \{\textit{john}, \textit{bill}, \textit{mary}\} \in \text{CON}_{\epsilon} \).

2.3. END

Constants of type $\epsilon$ get as interpretation (with respect to a point of reference) some element in the domain of individuals. This interpretation has to be restricted, for the following reason. If we will speak tomorrow about John, then we will mean the same individual as today (although he may have some other properties). For instance, if the world would have been different, say, if the Mount Everest would not be the highest mountain, then John would still be the same individual (although his opinion about the Mount Everest might be different). This conception about the individual corresponding with a proper name is expressed by the phrase 'proper names are rigid designators'. For an extensive discussion of this conception, see KRIPKE 1972. This idea will be incorporated in our semantics by interpreting constants like $\textit{john}$ 'rigidly', i.e. for each index it will denote the same individual. The name $\textit{Bigboss}$ is to be understood as a surname of the most powerful individual on earth. Since this will not always be the same individual, $\textit{Bigboss}$ is not treated as a rigid designator of type $\epsilon$.

The constants of intensional logic are not interpreted rigidly, on the contrary, they are interpreted index-dependent. I recall the clause for the interpretation of constants:

\[
c^{\epsilon,i}_g = F(c)(i) \quad (c \in \text{CON}).
\]

This means there is no guarantee that the constants corresponding with the proper names of PTQ are interpreted rigidly. Therefore not all possible models for the interpretation of IL are reasonable candidates for an interpretation of English. We will consider only those models in which the
constants \textit{john}, \textit{bill}, and \textit{mary} are interpreted rigidly. This is formalized as follows. The requirement of 'rigidity' is expressed by means of an IL-formula, and we will consider only those models in which this formula holds. The formula is called a Meaning Postulate (an MP). It bears index 1 because it is the first meaning postulate in PTQ. Notice that this postulate describes in fact a collection of three formulas.

2.4. \textit{Meaning postulate 1}:

\[ \exists u [u = \alpha] \quad \text{where} \ \alpha \in \{\text{john}, \text{bill}, \text{mary}\}. \]

2.4. END

This meaning postulate requires that there is one individual in the semantic domain such that the interpretation of \textit{john} equals that individual for all indices. For the PTQ fragment this postulate may be considered sufficient. In more subtle situations this formalization of rigidity is probably too absolute. If epistemological verbs like \textit{know} or \textit{believe} are analysed in detail, then the notion of rigidity may have to be weakened to something like 'in all worlds compatible with the beliefs of some individual such a constant is rigid'. I will, however, follow the PTQ formalization.

An important technical consequence of MP is that lambda-conversion is allowed when one of the constants \textit{john}, \textit{bill} or \textit{mary} occurs as argument. First I recall the notation for substitution, for a formal definition see chapter 3, definition 4.3.

2.5. \textit{DEFINITION}. \([a/z] \phi\) denotes the result of \textit{substitution} of \(a\) for all free occurrences of \(z\) in \(\phi\).

2.6. \textit{THEOREM}.

\[ \models \lambda u [\phi](\alpha) = [a/u] \phi \]

where

\[ \alpha \in \{\text{john}, \text{bill}, \text{mary}\}. \]

\textbf{PROOF}. MP says that for all \(i, g\):

\[ i, g \models \exists u [u = \alpha] \]

so there is a \(g' \models g\) such that:

\[ i, g' \models [u = \alpha] \]

hence for all \(j\):

\[ j, g' \models u = \alpha \]
Let $i_1$ and $i_2$ be arbitrary. Then:

$$V_{i_1} \delta (\alpha) = V_{i_1} \delta, (\alpha) = V_{i_1} \delta, (u) = g'(u) = V_{i_2} \delta, (u) = V_{i_2} \delta, (\alpha) = V_{i_2} \delta, (\alpha).$$

This means that the condition of theorem 6.4 from chapter 3 is satisfied, hence the theorem allows us to apply $\lambda$-conversion.

2.6. END

In the sequel $\lambda$-conversion will be used frequently for reducing a formula to a simpler form. Besides $\lambda$-conversion several other rules will be introduced for this purpose; they are called reduction rules (RR's). Together they will constitute a procedure which simplifies the formulas obtained by translating the expressions of the fragment. For each reduction rule a correctness proof has to be given, i.e. a proof that the rule transforms a formula into a logically equivalent one. Theorem 6.1 from chapter 3 then allows us to reduce a formula as soon as it is obtained. The purpose of the reduction rules is to obtain formulas which express the intended meaning as clearly and simply as possible. The rules presented in this chapter are almost identical with the rules presented in JANSSEN 1980a. Related reduction rules are discussed in FRIEDMAN & WARREN 1980a, b and INDURKHYA 1981; these authors use the reduction rules for a somewhat different purpose (e.g. to obtain the most extensionalised form), and therefore there are some differences.

The first reduction rule concerns $\lambda$-conversion. With respect to this rule the following class of formulas is important: the formulas which contain no operators $\vee$, $\land$, or $\Rightarrow$, and which contain as constants only john, mary or bill. Extending definition 6.2 from chapter 3, I will call these expressions modally closed, since they have the same properties with respect to $\lambda$-conversion.

2.7. DEFINITION. An IL formula is called modally closed if it is an element of the IL-subalgebra:

$$<[[john, mary, bill]], (\text{VAR})_{t \in Ty}, R \cup \{R^\land, R^\square\}>$$

where $R$ consists of the operators of $Ty_2$ (recall that $R^\land$ and $R^\square$ indicate prefixing with $\land$ and $\square$ respectively).
2.8. Reduction rule 1

Let \( z \in \text{VAR}_1 \), \( \alpha \in \text{ME}_1 \), and \( \beta \in \text{ME}_2 \).

Then replace \( \lambda z[\beta](a) \) by \([a/z]\beta\) if

1) no variable in \( \alpha \) becomes bound by substitution of \( z \) for \( z \) in \( \beta \) and either

2) no occurrence of \( z \) in \( \beta \) lies within the scope of \( \wedge, \vee, W \) or \( \square \) or

3) \( \alpha \) is modally closed.

**CORRECTNESS PROOF**

The difference between this rule and theorem 6.3 from chapter 3 is that condition 3 allows for the occurrence of the rigid designators *john*, *mary* and *bill*. Hence if conditions 1) and 2) are satisfied, the correctness of the \( \lambda \)-conversion follows from that theorem. Suppose now that conditions 1) and 3) are satisfied, and consider the case that \( \alpha \) contains of the constants *john*, *bill* and *mary* only occurrences of *john*.

Let \( w \) be a variable which does not occur in \( \alpha \) or \( \beta \), and let \( \alpha' \) and \( \beta' \) be obtained from \( \alpha \) and \( \beta \) by substitution of \( w \) for *john*. Consider now

\[
(A) \quad \lambda w[\lambda z[\beta'](a')](\text{john}).
\]

Since \( \alpha' \) and \( \beta' \) do not contain occurrences of *john* the old conditions for \( \lambda \)-conversion on \( z \) are satisfied (chapter 3, theorem 6.3). So (A) is equivalent with:

\[
\lambda w[(a'/z)\beta'](\text{john}).
\]

From theorem 2.6 above, it follows that \( \lambda \)-conversion on \( w \) is allowed, so this formula is equivalent with

\[
[john/w][(a'/z)\beta'].
\]

By the definition of substitution, this is equivalent with

\([a/z]\beta\).
So (A) is equivalent with this last formula. On the other hand, we may perform in (A) $\lambda$-conversion on $\omega$ because the condition of theorem 2.6 is satisfied. So (A) is also equivalent with

$$\lambda\alpha[\beta](\alpha).$$

The combination of these last two, with (A) equivalent, formulas proves the correctness of $\lambda$-conversion for the case that conditions 1) and 3) are satisfied, and that $\alpha$ contains only occurrences of $\text{john}$. For other constants and for occurrences of more than one constant, the proof proceeds analogously.

2.8. END

As said before, at different indices different persons can be Bigboss. Therefore we cannot translate Bigboss into a rigid constant of type $e$. We might translate it into a constant of type $<s,e>$, or into a constant of type $e$ and interpret it non-rigidly. I choose the former approach (thus being consistent with the examples involving bigboss given in section 7 of chapter 3). This explains the following definition

2.9 $\text{bigboss} \in \text{CON}<s,e>.$

2.9. END

The interpretation of the constant bigboss is a function from indices to individuals. Such a function is called an individual concept. Also $^\wedge \text{john}$ denotes an individual concept. The individual concept denoted by $^\wedge \text{john}$ is a constant function, whereas the one denoted by bigboss is not. One might expect that Bigboss translates into the corresponding constant. But, as for the other proper names, it will be explained later why this is not the case.

Suppose that the balance of power changes and Brezjnev becomes Bigboss instead of Reagan. Then this might be expressed by sentence (14).

(14) Bigboss changes.

The meaning of (14) is not correctly represented by a formula which says that the predicate change applies to a certain individual. Who would that be? Maybe there was a change in the absolute power of Reagan (it decreased), or in the absolute power of Brezjnev (it increased). Probably both persons changed with respect to power. Sentence (14) rather says that the concept
'Big boss' has changed in the sense that it concerns another person. So the meaning of (14) can be represented by a formula which says that the predicate \textit{change} holds for the individual concept related with \textit{big boss}. In such an analysis \textit{change} has to be of type \(<s,e>,t>\). Due to the homomorphic relation between syntax and semantics, this means that all intransitive verbs have to be of type \(<s,e>,t>\).

At this stage of the description of the fragment the only example of an argument of type \(<s,e>\) is the artificial example \textit{big boss}. In appendix 2 of this book, other examples will be given where the translation of the argument of a property has to be of this type. This discussion explains the introduction of the following constants and translations. The translation function is indicated by means of a ' (prime). Note that this is a different use of ' than in PTQ (there it distinguishes English words from logical constants).

\begin{align*}
2.10 & \quad \{\text{run, walk, talk, rise, change}\} \subseteq \text{COR} <s,e>, t> \\
2.11 & \quad \text{run}' = \text{run, walk}' = \text{walk, talk}' = \text{talk} \\
& \quad \text{rise}' = \text{rise, change}' = \text{change}.
\end{align*}

2.11. END

One might be tempted to take the constant \textit{john} as translation of the proper name \textit{John}. In the fragment consisting only of sentences like \textit{John runs} there would be no problem in doing so. But there are more terms, and the similarity of syntax and semantics requires that all terms are translated into expressions of the same type. We already met the proper name \textit{Big boss}, translating into an expression of type \(<s,e>\). One might expect ' \textit{john} as translation for \textit{John}. But in the sequel we will meet more terms: e.g. \textit{every man}. If we would translate \textit{John} into an expression denoting an individual concept (or alternatively an individual), then \textit{every man} has to be translated into such an expression as well. Would that be possible?

The idea is discussed by LEWIS (1970). He tells us that in the dark ages of logic a story like the following was told. 'The phrase every pig names a [...] strange thing, called the universally generic pig, which has just those properties that every pig has. Since not every pig is dark, pink, grey or of another color, the universally generic pig is not of any color (Yet neither he is colorless, since not every - indeed not any - pig is colorless)'. (LEWIS 1970, p.35). This illustrates that this approach is not sound. Therefore, we will forget the idea of universal generic
objects (for a proposal for a reconstruction, see Van Benthen 1981a), and we will interpret the term 'every man' as the set of properties every man has. As a consequence of the similarity of syntax and semantics, all other terms will denote sets of properties as well.

On the basis of this argumentation one might expect for John the translation \( \lambda Z(z(\hat{^\text{john}})) \), where Z is a variable of type \( \langle<s,e,t>\rangle \). But this is not adequate for the following reason. A variable of type \( \langle<s,e,t>\rangle \) denotes (the characteristic function of) a set of individual concepts. What we usually take to be a property cannot be adequately formalized in this way. Consider the property 'being a football player'. This would be formalized as a set of individual concepts. The same holds for the property of 'being a member of the football union': this is formalized as a set of individual concepts as well. Suppose now that (for a certain index) all football players are members of the football union. Then these two sets would be the same, so the two properties would be formalized in the same way. But we do not consider these two properties as being the same. In other circumstances (for other indices) there might be players who are not a member of the union. In order to formalize these differences, properties are taken to be of one intensional level higher hence a variable which ranges over properties has to be of type \( \langle<s,\langle<s,e,t>\rangle\rangle \). This explains the following translations of proper names.

2.12. Translations

\[
\begin{align*}
\text{John'} &= \lambda P ([\hat{^\text{P}}(\hat{^\text{john}})]) \\
\text{Bill'} &= \lambda P ([\hat{^\text{P}}(\hat{^\text{bill}})]) \\
\text{Mary'} &= \lambda P ([\hat{^\text{P}}(\hat{^\text{mary}})]) \\
\text{Bigboss'} &= \lambda P ([\hat{^\text{P}}(\hat{^\text{bigboss}})])
\end{align*}
\]

here \( P \in \text{VAR}_{<s,\langle<s,e,t>\rangle}>\).

2.12. END

After this discussion concerning the proper names and intransitive verbs, the rule for their combination can be given. I first quote the PTQ formulation, since this way of presentation is in the literature the standard one. The formulation of the rule contains expressions like 'a \( \in P_T \)', this should be read as 'a is a phrase of the category T'. The rule is called \( S_4 \), because it is the fourth syntactic rule of PTQ, and I wish to follow that numbering when possible.
2.13. Rule $S_4$

If $\alpha \in T_k$ and $\beta \in IV_\alpha$, then $F_4(\alpha, \beta) \in S$, where $F_4(\alpha, \beta) = a\tilde{\alpha}$ and $\tilde{\beta}$ is the result of replacing the first verb in $\beta$ by its third person singular present.

2.13. END

This formulation of the rule contains a lot of redundancy, and therefore I will use a more concise presentation. As one remembers from the previous chapters, the syntactic rules are operators in an algebraic grammar. The form of representation I will use, resembles closely the representations used in the previous chapters for algebraic operators. First it will be said what kind of function the rule is; as for $S_4$ it is a function from $T \times IV$ to $S$ (written as $T \times IV \rightarrow S$). Next it will be described how the effect of the operator is obtained. I will use a notation that suggests that some basic operations on strings are available, in particular a concatenation operator which yields the concatenation of two strings as result. The semi-colon (;) is used to separate the consecutive stages of the description of the syntactic operator; it could be read as 'and next'. Furthermore the convention is used that $\alpha$ always denotes the expression which was the first argument of the syntactic rule. If this expression is changed in some step of the syntactic operation, it will then denote the thus changed expression. For the second argument $\beta$ is used in the same way. Rule $S_4$ presented in this format reads as follows.

2.14. Rule $S_4$

\[ T \times IV \rightarrow S \]

$F_4$: replace the first verb in $\beta$ by its third person singular present; concatenate $(\alpha, \beta)$.

2.14. END

The occurrence of the name $F_4$ is a relic of the PTQ formulation, and might be omitted here. But in a context of a long list of rules it is sometimes useful to have a name for an operation on strings, because it can
then be used in the description of other rules.

The translation rule corresponding with $S_4$ reads in PTQ as follows.

2.15. $T_4$:
- If $a \in P_T$, $b \in P_{IV}$, and $a, b$ translate into $a', b'$ respectively, then $F_4(a, b)$ translates into $a'(\langle b' \rangle)$.

2.15. END

Also the translation rule contains a lot of redundant information. Let us denote by $\alpha'$ the translation of the first, and by $\beta'$ the translation of the second argument of the preceding syntactic rule. Then a translation rule can fully be described by giving just the relevant logical expression (polynomial over IL with $\alpha'$ and $\beta'$ as parameters). What the types of $\alpha'$ and $\beta'$ are, follows immediately from the sorts mentioned in the syntactic rule $T_4$ presented in this format reads:

2.16. $T_4$:
- $a'(\langle b' \rangle)$

2.16. END

Now we come to the production of sentence (15), viz. $\text{Bigboss changes}$. This sentence, containing the artificial term $\text{Bigboss}$, is given as the first example because all information needed for a full treatment of this sentence is given now; sentences like $\text{John changes}$ have to wait for a moment. Sentence (15) is obtained by application of $S_4$ to the basic term $\text{Bigboss}$ and the basic verb $\text{change}$. This information is presented in the tree in figure 1. The $S$ in $(S, 4)$ stands for the category of the obtained expression, the 4 for the number of the rule used to produce the expression (15) $\text{Bigboss changes}$.

![Figure 1](image-url)

The translation of $\text{Bigboss}$ is $\lambda x P(bigboss)$, and the translation of $\text{change}$ is $\text{change}$. If we combine $\text{Bigboss}$ and $\text{change}$ according to rule $S_4^*$, thus producing (15), then the translation of the result is obtained by application of $T_4$ to their respective translations. Since
\[ T_4(\alpha', \delta') = \alpha'(\hat{\delta}'), \text{ sentence (15) translates into (16).} \]

(16) \[ \lambda P(\text{bigboss}) (\hat{\text{change}}). \]

Now conditions 1 and 2 of reduction rule RR1 are satisfied. So this formula can be reduced to (17).

(17) \[ [\forall \text{change}](\text{bigboss}). \]

This formula can be simplified further using the following reduction rule.

2.17. Reduction Rule 2

Let be given a formula of the form \[ \forall \alpha. \] Then replace this formula by \[ \alpha. \]

CORRECTNESS PROOF. \[ \vdash \forall \alpha = \alpha \] see chapter 3, theorem 7.1.

2.17. END

Using this reduction rule formula (17) reduces to (18).

(18) \[ \text{change} (\text{bigboss}). \]

This formula expresses that the predicate \text{change} holds for the individual concept \text{bigboss}.

Instead of all this verbosity, we might present the translations immediately in the tree. Depending on the complexity of the formulas involved, these may be unreduced, partially reduced or completely reduced formulas.

An example is given in figure 2.

Another method to present the production and translation process is to write this in an algebraic way, of which the following is an example.

\[ [\text{Bigboss \ changes}]' = [S_4(\text{Bigboss, change})]' = \]
\[ = T_4(\text{Bigboss}', \text{change}') = \text{Bigboss}'(\hat{\text{change}})' = \]
\[ = [\lambda P(\hat{\text{bigboss}})][\hat{\text{change}}] = (\text{RR}_1) = [\forall \text{change}](\text{bigboss}) = (\text{RR}_2) = \]
\[ = \text{change}(\text{bigboss}) \]
The treatment of \textit{Mary walkes} proceeds, in its first stage, analogously to the treatment of \textit{Bignose changes}, see figure 3.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Figure 3}
\end{figure}

The formula obtained as translation for \textit{Mary walkes}, is not completely satisfactory. Intuitively one interprets this sentence as stating that a certain predicate (denoting the property of walking) holds for a certain individual (Mary). This is not reflected in the obtained translation; in \textit{walk(\textasciicircum{mary})}, a predicate is applied to an individual concept. Since \textit{mary} is a rigid constant, \textit{\textasciicircum{mary}} denotes a function which yields for all indices the same individual. Saying that this constant function has a certain property is tantamount to saying that the corresponding individual has a certain property (there is a \textit{1-1} correspondence between individuals and functions yielding always the same individual). However, one would like to have reflected in the translation of \textit{Mary walkes} that a predicate holds for an individual. Therefore the following notation is introduced (see PTQ, p.265).

2.18. DEFINITION. Let $\delta \in \text{CON}_{\langle s,e\rangle,t}$. Then $\delta_*$ is an abbreviation for $\lambda u \delta(\textasciicircum{u})$ (so $\delta_* \in \text{ME}_{e,t}$).

2.18. END

Consequently we have the following rule for simplifying formulas.

2.19. Reduction rule 3

Let be given a formula of the form $\delta(\textasciicircum{a})$, where $\delta \in \text{CON}_{\langle s,e\rangle,t}$ and $a \in \text{VAR}_e$ or $a \in \{\text{john}, \text{bill}, \text{mary}\}$. Then replace $\delta(\textasciicircum{a})$ by $\delta_*(a)$.

\textbf{Correctness Proof}. $\delta_*(a) = \lambda u [\delta(\textasciicircum{u})](a) = (\text{RR}) = \delta(\textasciicircum{a})$. Note that $\lambda$-conversion is allowed because the mentioned constants of type $e$ are rigid designators.

2.19. END
Using RR3 the translation of Mary walks in figure 3, reduces to (19).

(19) walk*(mary).

As last example I present the treatment of the sentence mentioned in the title of this section. For variation I use not the tree representation, but the algebraic one.

[John runs]' = [S₄(John, run)]' = John'(^run') = λP[^[^P](^John)](^[^run]) =
=(RR₁)= [^[^run](^John)] = (RR₂) = run(^John) = (RR₃) = run*(John).

In PTQ more is said about the fragment presented so far. A meaning postulate (MP₃) is introduced which says that the truth of e.g. walk(x) only depends on the extension of x, i.e. the subject position of walk is extensional. In appendix 2 of this book the problems of extension and intension will be discussed, and this postulate will be considered. For verbs of other categories the extensionality of the subject position is guaranteed by meaning-postulates as well, (see appendix 1).

3. THE WOMAN WALKS

In this section the fragment is extended with the categories of Common Nouns (CN) and of determiners (Det). The treatment of determiners given here differs from their PTQ treatment. In PTQ determiners are introduced syncategorematically, introducing each determiner by a distinct rule. Maybe the motivation for Montague to do so, was that in logic quantifiers are usually introduced syncategorematically. From a linguistic point of view it is more attractive to have determiners in a separate category (they form a group of expressions which behave syntactically in a regular way). Since I do not know any argument against treating them categorically, the PTQ approach is not followed here. The generators of the two new categories are as follows

3.1. \( \mathbf{B}_{\text{CN}} = \{ \text{man, woman, pen, fish, pen, unicorn, price, temperature} \} \)

3.2. \( \mathbf{B}_{\text{Det}} = \{ \text{every, a, the} \} \)

3.2. END

For each element in \( \mathbf{B}_{\text{CN}} \) there is a corresponding constant, and the common nouns translate into these constants. The nouns are treated semantically in the same way as the intransitive verbs we have met in section 2. Hence the nouns translate into constants of type \(<s,e,t>\). This
explains the following definitions.

3.3. \( \{\text{man}, \text{woman}, \text{park}, \text{fish}, \text{pen}, \text{unicorn}, \text{price}, \text{temperature}\} \subset \text{CON}_{\langle s, e \rangle, t} \)

3.4. \( \text{man}' = \text{man}, \text{woman}' = \text{woman}, \text{park}' = \text{park}, \text{pen}' = \text{pen}, \text{unicorn}' = \text{unicorn}, \text{price}' = \text{price}, \text{temperature}' = \text{temperature}. \)

3.4. END

An example of a formula containing the constant \( \text{bill} \) is (20), in which is expressed that Bill is a man.

(20) \( \text{man}('\text{bill}') \).

The \( \delta's \) notation (definition 2.18) is applicable to all constants of type \( \langle s,e,t \rangle \), so it can be applied to constants translating common nouns as well. So (20) may be replaced by (21).

(21) \( \text{man}_s('\text{bill}') \).

Out of a CN and a determiner a term can be formed, using the following rule.

3.5. Rule \( S_2 \)

\[
\begin{align*}
\text{Det} \times \text{CN} & \rightarrow \text{T} \\
F_2 & : \text{concatenate}(a, \beta) \\
T_2 & : a('('\beta').')
\end{align*}
\]

Example

\( F_2(a, \text{woman}) = \text{a woman}. \)

3.5. END

We wish to use the terms produced with this rule in the same way as we used the term \text{John}: rule \( S_4 \) should be applicable to the result of \( S_2 \), yielding sentences like (22), (23) and (24).

(22) A woman runs
(23) Every woman runs
(24) The woman runs.

The meanings associated with determiners are best understood by considering the meanings that we wish to assign to the above sentences (cf. the discussion concerning contextuality and compositionality in section 2 of chapter 1). Let us accept (for the moment without explanation) the quantification over individual concepts; then the translations of (22),
(23) and (24) are (25), (26) and (27) respectively.

(25) $\exists x [\text{woman}(x) \land \text{run}(x)]$

(26) $\forall x [\text{woman}(x) \rightarrow \text{run}(x)]$

(27) $\exists x \forall y [(\text{woman}(y) \leftrightarrow x = y) \land \text{run}(x)].$

The last formula is somewhat complex. It says that there is an entity $x$ which is a woman, and that for any entity $y$ which is a woman holds that it is identical to the entity $x$. In other words, (27) is false when there is no woman at all, and it is false when there is more than one woman. This kind of analysis for the is called the Russelian analysis, because it was proposed by Russell to deal with the famous example (28).

(28) The present King of France is bald.

The meanings of the terms have to be such that if they take an IV-translation as argument, the resulting translations are the ones we desired for the obtained sentences. Hence their translations have to be of the same kind as the translation of the term John: a (characteristic function of a) set of properties of an individual concept. So we wish to translate (29) by (30).

(29) a woman

(30) $\lambda P \exists x [\text{woman}(x) \land \forall y (P(y)].$

Formula (30) is interpreted as the characteristic function of those properties $P$ such that there is at least one woman which has this property $P$. Other determiners are treated analogously. As translation for the determiner a we take formula (30), but with woman replaced by a variable. This variable is of type $<s,<s,e,t>>$ (the reason for this is the same as the reason given for the type of the variable $P$, see the translation of John). This explains the following translations of determiners.

3.6. Translations of determiners

$\text{every}' = \lambda Q \lambda P \exists x [\forall x (Q(x) \rightarrow \forall \forall P(x)]$

$a' = \lambda Q \lambda P \exists x [\forall x (Q(x) \land \forall \forall P(x)]$

$\text{the}' = \lambda Q \lambda P \exists x [\forall y (Q(y) \leftrightarrow x = y) \land \forall \forall P(x)].$

3.6. END
Formulas (25), (26) and (27) are not in all respects a satisfactory representation of the meanings of sentences (22), (23) and (24) respectively. The formulas contain quantifications over individual concepts, whereas one would prefer a quantification over individuals. The conditions for application of $\text{RS}_3$ are not satisfied, so we have no ground for the elimination of the individual concepts by means of an application of this rule. On the contrary: as I will explain, the replacement of (31) by (32) would replace a formula by a non-equivalent one.

(31) $\exists x [\text{woman}(x) \land \text{run}(x)]$

(32) $\exists u [\text{woman}_u(u) \land \text{run}_u(u)]$.  

A possible choice for the value of $x$ in (31) would be to assign to $x$ the same interpretation as to bigboss, but in (32) there is not a corresponding choice. One would prefer to have (32) as the meaning representation of the meaning of (25) because intuitively (25) gives information about individuals, and not about individual concepts. Following Montague, we obtain this effect by means of the introduction of a meaning postulate. Only those models for intensional logic are possible models for the interpretation of English in which the following meaning postulate holds.

3.7. Meaning postulate 2

$$\Box [\delta(x) \rightarrow \exists u [x = u]]$$

where $\delta \in \{\text{man,woman,park,fish,pen,unicorn}\}$.  

3.7. END

This meaning postulate says that constants such as man can yield true only for constant individual concepts, i.e. for individual concepts which yield for every index the same individual. Note that the constants price and temperature are not mentioned in $\text{MP}_2$. Arguments for this, and examples involving price and temperature will be given in appendix 1 of this volume. As a consequence of $\text{MP}_2$, it can be shown that (31) and (32) are equivalent. I will not present a proof for this, because it is only one of the situations in which $\text{MP}_2$ will be used. In appendix 1, it will be investigated in general in which circumstances $\text{MP}_2$ allows us to replace a quantification over individual concepts by a quantification over individuals. For the moment it suffices to know that in all examples we will meet, such a replacement is allowed. This is expressed in the following reduction rule.
3.8. Reduction Rule 4

Let be given a formula of one of the following forms: \( \exists x[\delta(x) \land \phi(x)] \), \( \forall x[\delta(x) \rightarrow \phi(x)] \) or \( \exists x[\forall y[\delta(y) \leftrightarrow x=y] \land \phi(x)] \).

If MP holds for \( \delta \), then replace these formulas by respectively \( \exists u[\delta(u^*) \land \phi(u^*)] \), \( \forall u[\delta(u^*) \rightarrow \phi(u^*)] \) or \( \exists u[\forall v[\delta(v) \leftrightarrow u=v] \land \phi(u)] \).

**CORRECTNESS** PROOF. See appendix 2.

3.8. END

The production of the sentence mentioned in the title of this section is given in figure 4.

\[
\text{The woman walks } \{S,4\}
\]
\[
\exists u[\forall v[\text{woman}_*(v) \leftrightarrow u=v] \land \text{walk}_*(u)]
\]

\[
\text{the woman } \{T,2\}
\]
\[
\lambda u \exists u[\forall v[\text{woman}_*(v) \leftrightarrow u=v] \land \forall u[\text{walk}_*(u)]]
\]

\[
\text{walk } \{IV\}
\]

\[
\lambda \exists u[\forall v[\text{woman}_*(v) \leftrightarrow u=v] \land \forall u[\text{walk}_*(u)]]
\]

\[
\text{woman } \{CN\}
\]

\[
\lambda \forall \exists u[\forall v[\text{woman}_*(v) \leftrightarrow u=v] \land \forall u[\text{walk}_*(u)]]
\]

\[
\text{woman}
\]

**Figure 4**

Note how in this simple example RR\(_4\) and RR\(_3\) are used in order to simplify the translation of \( \text{the woman} \), and RR\(_1\) and RR\(_3\) to simplify the translation of \( \text{the woman walks} \). In the sequel such reductions will often be performed without any further comment.

4. MARY WALKS AND SHE TALKS

In this section the fragment is extended with rules for disjunction and conjunction, and with a rule for co-referentiality. The rules for producing conjoined sentences are as follows.

4.1. Rule S\(_{11a}\):

\[
\text{S \times S + S}
\]

\( F_{11a} \): concatenate \((a, and, \beta)\)

\( T_{11a} \): \( a' \land \beta' \)
4.2. Rule $S_{11b}$:

$$S \times S \rightarrow S$$

$F_{11b}$: concatenate ($a$, $or$, $b$)

$T_{11b}$: $a' \lor b'$.

4.2. END

Notice that the words and and or are not members of a category of connectives; they are introduced syncategorematically. It would be possible to have a three-place rule for sentence conjunction, with for the connective and as translation $\lambda x[\lambda x[y \land y]]$. This categorical approach is not followed here because there are rules for disjunction and conjunction for other categories as well. Furthermore, the situation is complicated by the fact that there is term disjunction in the fragment, but no term conjunction (in order to avoid plurals). In this situation it would not be a simplification to use a categorical treatment of connectives. For a categorical treatment in a somewhat different framework, see GAZDAR 1980.

The rules for forming conjoined phrases of other categories than sentences are as follows.

4.3. Rule $S_{12a}$:

$$IV \times IV \rightarrow IV$$

$F_{12a}$: concatenate ($a$, and $b$)

$T_{12a}$: $\lambda x[a'(x) \land b'(x)]$.

4.4. Rule $S_{12b}$:

$$IV \times IV \rightarrow IV$$

$F_{12b}$: concatenate ($a$, or $b$)

$T_{12b}$: $\lambda x[a'(x) \lor b'(x)]$.

4.5. Rule $S_{13}$:

$$T \times T \rightarrow T$$

$F_{13}$: concatenate ($a$, or $b$)

$T_{13}$: $\lambda p[a'(p) \lor b'(p)]$.

4.5. END
The production of (33) is given in figure 5.

(33) *John walks and talks.*

\[
\begin{align*}
&\text{walk}_a(john) \land \text{talk}_a(john) \\
&\text{John } \{T\} \\
&\lambda P[\overset{\uparrow}{P}(^\wedge \text{john})] \\
&\text{walk and talk } \{\text{IV, 12a}\} \\
&\lambda x[\text{walk}(x) \land \text{talk}(x)] \\
&\text{walk } \{\text{IV}\} \quad \text{talk } \{\text{IV}\} \\
&\text{walk} \quad \text{talk}
\end{align*}
\]

*Figure 5*

Note that the produced sentence is not identical with (33). The treatment presented in figure 5 obeys the formulation of $S_8$, and, therefore, only the first verb is conjugated. For an improved treatment see chapter 8, or FRIEDMAN 1979b.

An example of term disjunction is given in (34).

(34) *John or Mary talks.*

First (35) is formed according to $S_{13}$. Its unreduced translation is (36).

(35) *John or Mary*

(36) $\lambda P[\overset{\downarrow}{\lambda P}[\overset{\uparrow}{P}(^\wedge \text{john})](P) \lor \lambda P[\overset{\uparrow}{P}(^\wedge \text{mary})](P)]$.

Formula (36) contains several occurrences of the variable $P$, and three binders for $P$ (viz. three occurrences of $\lambda P$). However, due to the different scopes of the lambda operators, it is uniquely determined which variables occur in the scope of each of the lambda operators. The conditions for $\lambda$-conversion are satisfied, and after two applications of $\text{RR}_1$, formula (36) reduces to (37).

(37) $\lambda P[\overset{\downarrow}{\lambda P}(^\wedge \text{john}) \lor \overset{\downarrow}{\lambda P}(^\wedge \text{mary})]$.

Application of $S_8$ to term (35) and the verb *talk*, yields (34), which has as unreduced translation (38). This formula reduces by application of $\text{RR}_1$ and $\text{RR}_2$ to (39), and using $\text{RR}_3$ to (40).

(38) $\lambda P[\overset{\downarrow}{\lambda P}(^\wedge \text{john}) \lor \overset{\downarrow}{\lambda P}(^\wedge \text{mary})](^\wedge \text{talk})$

(39) $\text{talk}(^\wedge \text{john}) \lor \text{talk}(^\wedge \text{mary})$

(40) $\text{talk}_a(john) \lor \text{talk}_a(mary)$. 
In sentences containing conjunctions or disjunctions pronouns occur often which are coreferential with some other term in that sentence. An example is the coreferentiality of she and Mary in (41).

(41) Mary walks and she talks.

In order to account for coreferentiality, a collection of new artificial terms is introduced. Since they have a relationship with logical variables, they are called syntactic variables. These variables are not words of English, and might be represented by means of some artificial symbol. Since the variables are related to pronouns, it has some advantages, to give them a representation exhibiting this relationship. The variables are written as male pronouns provided with an index (e.g. he\_n). Their translations contain logical variables x\_n of type <s,e>. The syntactic variables he\_n are generators of sort T.

4.6. \{he\_1, he\_2, ... \} = B\_T.

4.7. he\_1' = \lambda p[p(x\_1)],  he\_2' = \lambda p[p(x\_2)], ...

4.7. END

One of the most important rules of PTQ is S\_14. As for the syntax it removes the syntactic variables. As for the translation, it binds the corresponding logical variables. This rule enables us to deal with most of the ambiguities mentioned in the introduction, but in this section we will only deal with its use for coreferentiality. In fact S\_14 is not a rule, but rather a rule-scheme which for each choice of the index n constitutes a rule. This aspect will be indicated by using the parameter n in the description of the rule scheme.

4.8. Rule S\_14,n:

\[T \times S \rightarrow S\]

F\_14,n: If \(a = he\_k\) then replace all occurrences of he\_n/him\_n in \(\beta\) by he\_k/him\_k respectively.

Otherwise replace the first occurrence of he\_n in \(\beta\) by \(a\), and replace all other occurrences of he\_n in \(\beta\) by he/she/it and of him\_n by him/her/it according to the gender of the first CN or T in \(a\).
\[ T_{14,n} : \alpha' (\lambda x_1 [S']) \].

4.8. END

An example of the use of (an instance of) \( S_{14,n} \) arises in the production of (41), as presented in figure 6.

\[
\begin{align*}
\text{Mary walks and she talks } & \{S, 14,1\} \\
\lambda P \forall P(\wedge \text{mary}) \big[ (\forall x_1 [\text{walk}(x_1) \land \text{talk}(x_1)]) \\
\lambda P \forall P(\wedge \text{mary}) & \big[ \text{walk}(x_1) \land \text{talk}(x_1) \big] \\
\text{Mary[T]} & \quad \text{He}_2 \text{ walks and he}_2 \text{ talks } \{S, 11a\} \\
\lambda P \forall P(\wedge \text{mary}) & \big[ \text{walk}(x_1) \land \text{talk}(x_1) \big] \\
\text{He}_2 \text{ walks } \{S, 4\} & \quad \text{He}_2 \text{ talks } \{S, 4\} \\
\text{walk}(x_1) & \quad \text{talk}(x_1) \\
\text{He}_2 \text{[T] } \quad \text{walk } \{IV\} & \quad \text{He}_2 \text{[T] } \quad \text{talk } \{IV\} \\
\lambda P \forall P(x_1) & \big[ \text{walk}(x_1) \big] \quad \lambda P \forall P(x_1) & \big[ \text{talk}(x_1) \big] \\
\end{align*}
\]

Figure 6

The translation for (41) given in figure 6 can be reduced, using RR3, to (42).

\[ (42) \ [\forall \lambda x_1 [\text{walk}(x_1) \land \text{talk}(x_1)]](\wedge \text{mary}). \]

By application of RR2 and RR1 this reduces to (43), and by RR3, further to (44).

\[ (43) \text{walk}(\wedge \text{mary}) \land \text{talk}(\wedge \text{mary}) \]

\[ (44) \text{walk}_\wedge (\text{mary}) \land \text{talk}_\wedge (\text{mary}). \]

Some syntactic details of \( S_{14,n} \) give rise to problems. The rule for term disjunction allows us to produce term phrases like \( \text{he}_2 \) and \( \text{Mary} \), and \( \text{he}_2 \) or \( \text{he}_2 \). In both cases it is not clear what is to be understood by the gender of the first T or CN in such a term. And if the term \( \text{John or Mary} \) is formed, it is not correct to use the pronoun \( \text{he} \), but one should use \( \text{he or she} \), witness the following example (FRIEDMAN, 1979).

\[ (45) \text{John or Mary walks and he or she talks.} \]

It would require a more sophisticated syntax than we have available here in order to account correctly for these problems (see FRIEDMAN 1979 for an improved treatment).

The detail of \( S_{14,n} \) that the first occurrence of \( \text{he}_n/\text{him}_n \) has to be
replaced, is explained as follows. A pronoun may always be coreferential with a common noun or term occurring earlier in the sentence, but it may not always refer forward to terms or nouns occurring later. So it is a safe strategy to put the coreferential noun phrase always in a position which is as leftmost as possible. It is a difficult, and not completely solved task, to characterize the situations in which a pronoun may refer to a term occurring later in the sentence. Therefore \( S_{14} \) describes only reference to terms occurring earlier than the pronoun. Even this safe procedure does not avoid all problems. In some cases a personal pronoun is produced, where a reflexive pronoun is required. Sentence (46) has, according to the rules described here, a translation which expresses that John loves himself. This result is, of course, incorrect.

(46) John loves him.

Our aim was to deal with certain semantic problems, and therefore I will not consider here proposals for dealing with this syntactic problem (one of the proposals from the literature, viz. PARTEE 1973, will be considered in chapters 5 and 6 although not from the present point of view).

5. JOHN FINDS A UNICORN

In this section the category \( TV \) of transitive verb phrases is introduced. The generators of this category are as follows.

5.1. \[ B_{TV} = \{find, loose, eat, love, date, be, seek, conceive\}. \]

5.1. END

Corresponding with these TV's (except for be), there are constants in the logic. They denote higher order functions which take as argument the intension of a term translation, and yield an element of the same type as the translations of IV-phrases. The translations of the basic verbs of the category TV are the corresponding constants; the translation of be is a compound expression of the same type. Let us indicate by \( \tau(C) \) the type of the translation of an expression of category C. Then \( \tau(TV) = <<s, \tau(T)>, \tau(IV)>>. \) This explains the following definitions

5.2. \[ \{find, loose, eat, love, date, seek, conceive\} \in CON <<s, \tau(T)>, \tau(IV)>> \]
5.3. \( \text{find}' = \text{find}, \text{loose}' = \text{loose}, \text{eat}' = \text{eat}, \text{love}' = \text{love}, \)

\( \text{be}' = \lambda x (\forall y (\text{P}(\lambda z [(x = y)])) \quad \text{where } P \in \text{VAR}_{<s,T>} \)

\( \text{seek}' = \text{seek}, \text{conceive}' = \text{conceive}. \)

5.3. END

Out of a TV and a Term and IV can be formed according to the following rule.

5.4. Rule \( S_5' \):

\( \text{TV} \times T \rightarrow IV \)

\( S_5': \text{concatenate } (a, b) \)

\( T_5': a' (\text{\textasciitilde} b'). \)

5.4. END

An example of the use of this rule is the production of (47), partially presented in figure 7.

(47) John seeks a unicorn.

\[
\text{John seeks a unicorn (S,4)} \\
\text{\quad seek(\text{\textit{\textlambda}} u. [\text{\textit{\textlambda}} x [u (u) \lor P(u)]] (\text{\textit{\textlambda}} john))} \\
\text{John (T)} \quad \text{\quad seek a unicorn (IV,5)} \\
\text{\quad \text{\quad seek(\text{\textit{\textlambda}} u. [\text{\textit{\textlambda}} x [u (u) \lor P(u)]])} \\
\text{\quad \text{\quad seek (TV)} \quad a \text{ unicorn (T,2)}} \\
\text{\quad \text{\quad seek} \quad \text{\textit{\textlambda}} u. [u (u) \lor P(u)]} \\
\]

Figure 7

The translation obtained in figure 7 is not the traditional one: one would like to consider seek as a two-place relation. Therefore the following convention is introduced.

5.5. DEFINITION. \( \gamma(\alpha, \beta) = \gamma(\beta)(\alpha) \), where \( \gamma \) is an expression translating a TV.

5.5. END
In PTQ (p.259) this convention is defined for all $\gamma$. It is however only useful for $TV$'s (see section 11). The above definition gives rise to the following reduction rule.

5.6. Reduction rule 5

Let be given a formula of the form $\gamma(\beta)(\alpha)$, where $\gamma$ is the translation of some transitive verb. Then replace this formula by $\gamma(\alpha,\beta)$.

**CORRECTNESS PROOF**

See definition 5.5.

5.6. END

Using RR5, the formula obtained in figure 7 reduces to (48).

\[(48) \ \text{seek}^{(\wedge)}(\text{john}, \ \lambda F \exists u [\text{unicorn}(u) \wedge F(\wedge u)]).\]

This translation describes the de-dicto reading of (47). The de-re reading will be considered in section 6. Below I will discuss whether the formula expresses a relation between the right kinds of semantic objects.

The first argument of *seek* is a constant: individual concept. One might wish to have an individual as first argument. In analogy of the $\delta_s$ notation for intransitive verbs, we might introduce a notation for transitive verbs in which the $\wedge$ in front of *john* disappears. ParTee (1975, p.290) has proposed such a notation, but it is not employed in the literature, therefore I will not use it here. Notice that the interpretation of (48) is tantamount to a relation of which the first component is an individual (see section 2).

The second argument in (48) is the intension of a collection of properties. So *seek* is not treated as a relation between two individuals, and therefore (48) does not allow for the conclusion that there is a particular unicorn which John seeks. In this way the problem mentioned in section 1 is solved, so in this respect the formula is satisfactory. But one might ask whether this effect could be obtained by means of a simpler formula, viz. one without the intension sign. The need for this intension in the second argument is explained as follows (Janssen 1978b, p.134). Suppose that *seek* is considered as a relation between an individual and (the characteristic function of) a set of properties. Consider a world in which there exist no unicorns. Then for any property $p$ it is true that $\exists u [\text{unicorn}_s(u) \wedge \forall F(\wedge u)]$. Thus in these circumstances $\lambda F \exists u [\text{unicorn}_s(u) \wedge \forall F(\wedge u)]$ is the
characteristic function of the empty set of properties. The semantic interpretation of \textit{John seeks a unicorn} then states that the seek-relation holds between John and this empty set. Suppose moreover that in this world also no centaurs exist. Then the semantic interpretation of

(49) \textit{John seeks a centaur}

also expresses that the seek-relation holds between John and the empty set of properties. But this contradicts our intuition that (47) and (49) have different meanings. When we wish to describe the difference between centaurs and unicorns we cannot restrict our attention to the present state of the present world. We should also consider other worlds (or other states of the present world) for instance, those in which unicorns or centaurs do exist.

In other worlds the set \( \lambda P \exists u [\text{unicorn}_u (u) \land \nu P (\nu u)] \) might be different from \( \lambda P \exists u [\text{centaur}_u (u) \land \nu P (\nu u)] \). Therefore the seek-relation will be considered as a relation between individuals and intensions of sets of properties.

Since these intensions are different, \textit{seek a unicorn} will get an interpretation different from the one for \textit{seek a centaur} (even if both are extinct).

In the same way as we produced \textit{John seeks a unicorn}, we may produce (50) with as reduced translation (51).

(50) \textit{John seeks Mary}

(51) seek(\( \land \text{john}, \land \lambda P [\nu ] (\text{mary}) \)).

This formula expresses that the seek relation holds between an individual concept and the collection of properties of Mary. But sentence (50) expresses that the seek-relation holds between two individuals: between John and Mary. One would like to have this aspect expressed by the obtained formula. Therefore the following definition (PTQ, p.265).

5.7. DEFINITION. \( \delta_5 = \lambda \nu \lambda u \delta (\land \text{u}, \land \lambda P [\nu] (\nu)) \), where \( \delta \in \text{CON}_1 (TV) \).

5.7. END

On the basis of this definition we have the following reduction rule.

5.8. Reduction rule 6

Let be given an expression of the form \( \delta(\land \alpha, \land \lambda P [\nu] (\nu \beta)) \), where \( \alpha, \beta \in \text{VAR} \cup \text{CON} \), and \( \delta \in \text{CON}_1 (TV) \). Then replace this expression by \( \delta_5 (\alpha, \beta) \).
CORRECTNESS PROOF

\[ \delta(a, \beta) = \delta_\beta(\alpha) = \lambda\nu\lambda\beta(\lambda u, \lambda P \nu P(\lambda v).)(\beta)(\alpha) = \{RR1\} = \]
\[ = \delta_\alpha(\lambda P \nu P(\lambda v).) \].

Note that \( \lambda \)-reduction is allowed because the constants of type \( e \) in the
fragment are rigid.

5.8. END

Using RR6 we may reduce (51) to (52).

(52) \textit{seek}_a(\textit{john,mary}).

In the same way as we produced the sentence \textit{john seeks a unicorn}, we
may produce (53), with translation (54).

(53) \textit{john finds a unicorn}

(54) \textit{find}_\lambda\textit{john,}\lambda P(\exists u \textit{unicorn}_a(u) \land \nu P(\lambda u)).

This result is not precisely what we would like to have. Sentence (53) gives
the information that there exists at least one unicorn, and (54) does not
express this information. In order to deal with this aspect we restrict
our attention to those models for IL in which the following meaning postu-
late is satisfied.

5.8. Meaning Postulate \( \delta \)

\[ \Box \exists y \forall x [\delta(x, P) \leftrightarrow P(\lambda y. \nu s(\lambda x, \lambda y))]
\]

where \( \delta \in \{\textit{find}, \textit{loose}, \textit{eat}, \textit{love}, \textit{date}\} \) and \( \gamma \in \text{VAR}_{s, \tau(T)}. \)

5.8. END

This meaning postulate expresses that if the relation \( \delta \) holds between
an individual concept and a collection of properties, then there is a cor-
responding relation which holds between individuals. This relation is index
dependent: the set of pairs which consist of a 'finder' and a 'found object',
may be different for different indices. Therefore the existence of a rela-
tion between finders and found objects is formalized by means of an existen-
tial quantification over a variable which is of one intension level higher
than the relation itself. An equivalent alternative would be (55), where
the quantification \( \exists s \) is within the scope of \( \Box \) (this variant is due to
P. van Emde Boas).
\[\exists x y z [\delta(x,F) \leftrightarrow F(\lambda y z (x, y))]\]

A notation for the relation between finder and found object is already provided by the \(\delta_x\) notation. This notation is introduced in the following rule.

5.9. Reduction rule 7

Let be given an expression of the form \(\delta(a,\beta)\) where \(\delta \in \{\text{find, loose, eat, love, date}\}\) and \(a \in \text{ME}_{<s,e,\alpha}\), \(\beta \in \text{ME}_{=T}\). Then, replace this expression by \(\forall \beta(\lambda y [\delta_x(v, y)])\).

**Correctness Proof**

From \(\text{ME}_{<s}\) follows that for all \(g\), there is a \(g' \sim_S g\) such that

\[g' \models \Box [\delta(x, F) \leftrightarrow F(\lambda y z (x, y))].\]

This means that for all expressions \(a \in \text{ME}_{<s,\alpha}\), \(\beta \in \text{ME}_{<s,\alpha}(T)\) holds that

\[g' \models \delta(a,\beta) \leftrightarrow \forall \beta(\lambda y z (a, y)).\]

For this \(g'\) the following equalities hold:

\[\forall \beta(\lambda y \delta_x(v, y)) = \text{(Def. 5.5)} = \beta(\lambda y \delta_x(v)(a)) = \text{(Def. 5.7)} = \lambda y \lambda z [\delta_x(v)](\lambda y z (u, v)) = \text{(choice of } g') = \beta(\lambda y \lambda z [\delta_x(v)](\lambda y z (u, v)))](v)(\alpha) = \text{(RR 2.1)} = \beta(\lambda y \lambda z [\delta_x(v)](u, v)](v)(\alpha) = \text{(RR 2.1)} = \lambda y \lambda z [\delta_x(v)](a)(\alpha) = \text{(RR 2.1)} = \beta(\lambda y \lambda z [\delta_x(v)](a)(\alpha)) = \text{(choice of } g') = \delta(a,\beta).\]

Since \(S\) does not occur in the first and last formula, these expressions are equivalent for all \(g\). From these equalities the reduction rule follows.

5.9. END

After the introduction of \(\text{RR}_7\) we return to our discussion of (54). Application of \(\text{RR}_7\) to (54) yields (56).

\[\forall \lambda [P \exists u \text{unicorn}_x(u) \land \forall v (v)](\lambda y [\text{find}_x(v, \alpha)])\]
This reduces further to (57), and that is the kind of formula we were looking for: it expresses that the find-relation holds between two individuals

(57)  \( \exists u [\text{unicorn}_n(u) \land \text{find}_n(\text{john}, u)] \).

The fragment contains one single verb be, which is used both for the be of identity, and for the copula be. An example of the be of identity is given in (58).

(58)  \text{John is Mary}.

The first step in its production is to combine be with Mary according to \( S_5 \). This yields the IV-phase be Mary. The translation of this phrase reduces by several applications of RR and RR to \( \lambda x [v = \text{mary}] \). Combining this with \text{john} according to \( S_6 \) yields (58), and the corresponding translation reduces by applications of RR and RR to \text{john} = \text{mary}. One observes that the final result is an identity on the level of individuals. This shows why there is no meaning-postulate like \( N_{\phi_4} \) introduced for \text{be}: its translation already applies to the level of individuals rather than the level of individual concepts.

Next I give an example of the copula use of be.

(59)  \text{John is a man}.

First the IV-phase be a man is formed. Its translation reduces to the formula \( \lambda x \exists u [\text{man}_n(u) \land v = u] \). Combining this with the translation of \text{John} yields as translation (60), which reduces to (61).

(60)  \( \lambda P (P(\text{john})) \lambda x [\exists u [\text{man}_n(u) \land v = u]] \)

(61)  \( \exists u [\text{man}_n(u) \land \text{john} = u] \).

In this situation one could perform one further simplification replacing (61) by (62); below I will explain why I will not do so.

(62)  \( \text{man}_n(\text{john}) \).

It would of course be possible to introduce a new reduction rule performing this last reduction. But it is difficult to cover the reduction from (61) to (62) by a general rule. Suppose that a rule \( R \) would say when the occurrence of a subformula \text{john} = u implies that all occurrences of \( u \) may be replaced by \text{john}. In order to decide whether reduction is possible, \( R \) has to take the whole formula into consideration. Reduction from (61) to (62) is allowed, but if in (61) connective \( \land \) would be replaced by \( \rightarrow \) the
reduction is not allowed. This supposed rule \( R \) would have a different character than all other reduction rules. The other rules are 'local': the question whether they may be applied, can be answered by inspecting a context of fixed length. But \( R \) would not be local because the whole formula has to be taken into account. I will not try to design such a rule \( R \) because I prefer to have only local reduction rules. Moreover, the set of reduction rules is incomplete, even with such a rule \( R \), and only a partial solution of the reduction problem is possible. This one sees as follows.

Suppose that we would define in each class of logically equivalent formulas one formula as being the simplest one (say some particular formula with shortest length). Then there exists no algorithm which reduces all formulas to the simplest in their class, since otherwise we could decide the equivalence of two formulas by reducing them to their simplest form and looking whether they are identical. Such a decision procedure would contradict the undecidability of IL (see also chapter 5, section 4).

6. EVERY MAN LOVES A WOMAN

The rules introduced in the previous sections allow us to produce sentence (63).

(63) Every man loves a woman.

In the introduction (section 1) I have described the two readings of this sentence. On the one reading, the same woman is loved by every man (say Brigitte Bardot), and on the other reading it might for every man be another woman (say his own mother). These two readings are represented by (64) and (65) respectively.

(64) \( \exists v [\text{woman}_x (v) \land \forall u [\text{man}_x (u) \rightarrow \text{love}_x (u, v)]] \)

(65) \( \forall u [\text{man}_x (u) \rightarrow \exists v [\text{woman}_x (v) \land \text{love}_x (u, v)]] \).

Note that the difference between (64) and (65) is a difference in the scope of the quantifiers \( \forall \) and \( \exists \). Therefore this ambiguity is called a scope ambiguity. A well known variant of this scope ambiguity is (66).

(66) Every man in this room speaks two languages.

A remarkable aspect of the two readings of (63) is that the one reading has the other as a special case: from (64) it follows that (65) holds. Therefore one might doubt whether the two formulas really constitute
an ambiguity we should deal with. One might say that the weaker formula
(viz. (65)) describes the meaning of (63), and that, with additional infor-
mation from the context, this can be narrowed down to the stronger one.
This argument holds for (63), but I will illustrate, that it is not general-

(67) Every schoolboy believes that a mathematician wrote 'Through the
looking glass'.

This sentence is (at least) twofold ambiguous. On the one reading there is
one mathematician of which every schoolboy believes that he wrote 'through
the looking glass', but not every schoolboy necessarily believes that the
person was a mathematician. On the other reading every schoolboy has the
belief that some mathematician wrote the book, without necessarily having
a special mathematician in mind. The rules needed for the production of
sentences like (67) will be given in section 9. The formulas we will obtain
then, are presented below in a somewhat simplified form. Formula (68) cor-
responds with the first reading (the believes concern the same mathe-
matician), the second reading is represented by (69).

(68) ∃v[mathematician(v) ∧ ∀u[ Believes(u, wrote(v,
'Through the looking glass'))]]

(69) ∀u[Believes(u,∃v[mathematician(v) ∧ wrote(v,'Through
the looking glass'))]].

These two readings are logically independent: the one can be true while the
other is false. The same situation arises for the well known example (66):
if we read in that sentence two as precisely two, then the different scope
readings are logically independent. These examples show that for variants
of the scope ambiguity, both readings have to be produced by the grammar.
Then it is not clear why (63) should get only one reading.

A part of the production of reading (65) of sentence (63) is given in
figure 8. This production is called the direct production (because no
quantification rule is used).

```
Every man loves a woman (S, 4)
∀u[man(u) → love(∧P∃u[woman(u) ∧ ∀(u)])](u)]

Every man (T, 2) love a woman (IV, 5)
∧P∀[man(u) → ∀(u)] love(∧P∃u[woman(u) ∧ ∀(u)])
```

Figure 8
The translation obtained in figure 8 can be reduced further by an application of RR₆, yielding (70).

(70) \( \forall u (\text{man}_*(u) \rightarrow \text{love}^*(u, \wedge \forall \phi [\text{woman}_*(u) \wedge p(u)])). \)

Application of RR₆ yields (71), and twice application of RR₂ yields (72).

(71) \( \forall u (\text{man}_*(u) \rightarrow [\forall \phi [\text{woman}_*(u) \wedge p(u)]]) \rightarrow [\lambda y [\text{love}_*(u, y)]) \)

(72) \( \forall u (\text{man}_*(u) \rightarrow [\lambda \forall \phi [\text{woman}_*(u) \wedge p(u)]]) \rightarrow [\lambda y [\text{love}_*(u, y)]) \).

Further application of lambda conversion is not allowed because this would bring the \( u \) in \( \text{love} (u, y) \) under the scope of \( \exists u \). In order to simplify this formula further, we first have to replace the variable \( u \) bound by \( \exists u \) by another variable.

6.1. Reduction rule 8

Let be given an expression of the form \( \lambda z \phi, \exists z \phi \) or \( \forall z \phi \). Let \( \omega \) be a variable of the same type as \( z \), but which does not occur in \( \phi \). Then replace \( \lambda z \phi, \exists z \phi, \forall z \phi \) by respectively \( \lambda \omega [\omega / z] \phi \), \( \exists \omega [\omega / z] \phi \), and \( \forall \omega [\omega / z] \phi \).

CORRECTNESS PROOF

Evident from the interpretation of these formulas.

6.1. END

Application of RR₆ to (72) yields (73). Applications of RR₁ and RR₂ yield then (74), which reduces further to (75).

(73) \( \forall u (\text{man}_*(u) \rightarrow [\lambda \forall \phi [\text{woman}_*(\nu) \wedge p(\nu)]] \rightarrow [\lambda y \text{love}_*(u, y)]) \)

(74) \( \forall u (\text{man}_*(u) \rightarrow [\forall \phi [\text{woman}_*(\nu) \wedge [\lambda y \text{love}_*(u, y)](\nu)]) \)

(75) \( \forall u (\text{man}_*(u) \rightarrow [\forall [\text{woman}_*(\nu) \wedge \text{love}_*(u, \nu)])]. \)

A part of the production of reading (64) of sentence (63) is given in figure 9. The production uses \( S_{14}, n \), and it is called (for this reason) an indirect production of (63).
Every man loves a woman \( \{S, 14,1\} \)
\[
\lambda P[\exists u \; \text{woman}_s(u) \land \forall x_1 [\forall u [\exists x_1 [\text{man}_s(u) \Rightarrow \text{love}_s(\forall x_1, u)]]] \]
\]
Every man loves him \( \{S, 4\} \)
\[
\lambda P[\exists u \; \text{woman}_s(u) \land \forall x_1 [\forall u [\exists x_1 [\text{man}_s(u) \Rightarrow \text{love}_s(\forall x_1, u)]]] \]
\]
Every man loves him \( \{IV, 5\} \)
\[
\lambda P[\forall u [\exists x_1 [\text{man}_s(u) \Rightarrow \text{love}_s(\forall x_1, u)]]] \quad \text{love}(\forall \lambda P. P(x_1))
\]

Figure 9

The translation obtained in figure 9 reduces by application of RR\_1 and RR\_2 to (76).

(76) \( \exists u [\text{woman}_s(u) \land \lambda x_1 [\forall u [\exists x_1 [\text{man}_s(u) \Rightarrow \text{love}_s(\forall x_1, u)]]] \).

After change of bound variable (RR\_p) we apply RR\_1, and obtain (77).

(77) \( \exists u [\text{woman}_s(u) \land \forall u [\exists v [\text{man}_s(v) \Rightarrow \text{love}_s(v, u)]]] \).

In the introduction I have already said that sentence (78) is ambiguous; its ambiguity is called the de-dicto/de-re ambiguity. From the de-re reading (79) it follows that unicorns exist, whereas this does not follow from the de-dicto reading (80).

(78) John seeks a unicorn

(79) \( \exists u [\text{unicorn}_s(u) \land \text{seek}_s(john, u)] \)

(80) \( \text{seek}(\forall \lambda P[\exists u \; \text{unicorn}_s(u) \land P(u)]) \).

This ambiguity can be considered as a scope ambiguity: the difference between (79) and (80) is the difference in scope of the existential quantifier. Note that formulas (79) and (80) are logically independent, hence we have to produce both readings. These productions are analogous to the productions of the different scope readings of Every man loves a woman. The de-dicto reading (80) is obtained by a direct production. We have considered this production in the previous section. The de-re reading, viz. (79), is obtained by an indirect production. As a first stage of the indirect production sentence (81) is formed, which has (82) as translation.

(81) John seeks him

(82) \( \text{seek}(\forall \lambda P. P(x_1)) \).
Combination according to $S_{14,1}$ of (81) with the term a unicorn yields (78), and combination of their translations according to $T_{14,1}$ yields (83), reducing to (84).

(83) $\lambda P\exists u[\text{unicorn}_x(u) \land \forall^v P(x_1)](\forall^x_1[\text{seek}(\forall^v \text{john}, \lambda P[\forall^v P(x_1)])])$

(84) $\exists u[\text{unicorn}_x(u) \land \text{seek}(\forall^v \text{john}, \lambda P[\forall^v P(x_1)])]$. Application of RR$_6$ reduces this formula to (85).

(85) $\exists u[\text{unicorn}_x(u) \land \text{seek}_x(\text{john}, u)]$.

Sentence (86) can be produced using the same syntactic rules as in the production of (78).

(86) *Mary finds a unicorn.*

This sentence is not ambiguous; it only has a referential reading. In the previous section it was explained how the translation of the direct production reduces to such a reading. The indirect production yields, of course, a referential reading as well. An interesting aspect of the indirect production is the way in which the obtained formulas can be reduced. For this reason I will consider this production in more detail. A first stage of the indirect production of (86) is (87), which has (88) as translation.

(87) *Mary finds him._

(88) $\text{find}(\forall^v \text{mary}, \lambda P[\forall^v P(x_1)])$.

One method to reduce (88) is to apply the same reduction rules as used in the reduction of (82). Then as last step RR$_6$ is applied, see the reduction of (84). But another reduction process is possible as well. We might apply RR$_7$ to (88) because meaning postulate 4 holds for find. Thus we obtain (89), reducing to (90).

(89) $\lambda P[\forall^v P(x_1)](\forall^y \text{find}_x(\text{mary}, y))$

(90) $\text{find}_x(\text{mary}, x_1)$. Combination, according to $S_{14,1}$ of (90) with the translation of a unicorn yields (91), which reduces to (92).

(91) $\lambda P\exists u[\text{unicorn}_x(u) \land \forall^v P(x_1)](\forall^x_1[\text{find}_x(\text{mary}, x_1)])$

(92) $\exists u[\text{unicorn}_x(u) \land \text{find}_x(\text{mary}, u)]$. 
This shows that there are two methods to reduce the formulas obtained in the indirect production of (86).

In general it makes no difference in which order we apply the reduction rules. Sooner or later we have to apply the same rule to the same (sub)expression. An exception is the introduction of $\delta$ for constants to which meaning postulate 4 applies. Once we have applied the meaning postulate (i.e. RR$_{\delta}$), we cannot apply the definition for $\delta$ (i.e. RR$_{\delta}$) any more. The reason for this is that both applications consume an occurrence of $\delta$, and produce an occurrence of $\delta$. As practice learns, these two ways of reduction always yield the same result. I have, however, not a formal proof of some formal version of this observation. The situation is difficult due to the interaction of RR$_{\delta}$ and RR$_{\gamma}$ with many other reduction rules. In FRIEDMAN & WARREN (1979) related reduction rules are considered, and they provide several examples of the complex interactions of the rules (they have no formal theorem for their system either).

Finally I consider a sentence which is not ambiguous. For sentence (93) the de-re reading is the only possible reading, and it is the only reading produced by the grammar.

(93) John seeks a unicorn and Mary seeks it.

The occurrence of it requires an application of $S_{14,n}$. A part of the production of (93) is given in figure 10.

\[
\begin{align*}
\exists u [\text{unicorn}(u) \land \lambda x_1 [\text{seek}(\text{john}, \lambda P \land P(x_1))] \land \text{seek}(\text{mary}, \lambda P \land P(x_1))] u] \\
\text{a unicorn} & \\
\lambda P [\exists u [\text{unicorn}(u) \land P(u)]] & \text{seek}(\text{john}, \lambda P \land P(x_1)) \land \text{seek}(\text{mary}, \lambda P \land P(x_1)) \\
\text{John seeks him, and Mary seeks him} & \\
\text{John seeks him,} & \text{Mary seeks him,} \\
\text{seek}(\text{john}, \lambda P \land P(x_1)) & \text{seek}(\text{mary}, \lambda P \land P(x_1)) \end{align*}
\]

Figure 10

The obtained translation for (93) reduces to (94).

(94) $\exists u [\text{unicorn}(u) \land \text{seek}(\text{john}, u) \land \text{seek}(\text{mary}, u)]$. 
7. BILL WALKS IN THE GARDEN

In this section the fragment is extended with the categories Prep of prepositions, and IAV of IV-modifying adverbials. In PTQ the category 'IAV' is also called 'IV/IV'. For the basic elements of IAV there are corresponding constants of type \(<<s, \tau(IV)\>, \tau(IV)>\). The definitions concerning IAV are as follows.

7.1. \(B_{IAV} = \{\text{slowly, voluntarily, allegedly}\}\)

7.2. \(\{\text{slowly, voluntarily, allegedly}\} \subseteq \text{CON}_{\tau(IV)}\)

7.3. \(\text{slowly}' = \text{slowly, voluntarily}' = \text{voluntarily, allegedly}' = \text{allegedly}\).

7.3. END

An adverb forms with an IV-phrase, according to \(S_{10}\), a new IV-phrase.

7.4. Rule \(S_{10}\):

\[
\begin{align*}
\text{IAV} \times \text{IV} & \rightarrow \text{IV} \\
F_{10} & : \text{concatenate } (a, b) \\
T_{10} & : a'(\cdot b').
\end{align*}
\]

7.4. END

An example of a sentence containing an IAV is (95).

(95) John voluntarily walks.

The production of (95) is presented in figure 11.

```
John voluntarily walks (S,4) [voluntarily('walk')]('john')
John (T) voluntarily walk (IV,10) voluntarily('walk')
\lambda P [P(john)] voluntarily (IAV) walk (IV)
voluntarily walk
```

Figure 11

In PTQ the convention was introduced to write all expressions of the form \(\gamma(a)(\beta)\) as \(\gamma(s,a)\). This example shows that the PTQ formulation was too
liberal: it would allow to write voluntarily as a relation: voluntarily(\text{^john}, \text{^walk}). This result is not attractive because traditionally one does not consider voluntarily as a relation. Therefore in reduction rule 5 this convention was only introduced for \gamma being a verb.

The translation obtained for (95) does not allow for the conclusion that \text{John walks}, although this would be a correct conclusion from sentence (95). Not all adverbs allow for such a conclusion. From (96) it does not follow that \text{John walks}.

(96) \text{John allegedly walks}.

This means that the adverb allegedly creates an intensional context for the object of a verb. Also sentence (97) does not allow to conclude to the existence of a unicorn.

(97) \text{John allegedly loves a unicorn}.

One might expect the introduction of a meaning postulate that expresses the extensional character of slowly and voluntarily. Such a meaning postulate is not given in PTQ. I expect that it would be of a different nature than the postulates we have met before: it would be an implication, and I expect that it would not give rise to simplifications of the formulas involved.

The fragment contains two prepositions, and from these new adverbial-phrases can be formed. Prepositions translate into constants of type \left<s, \tau(T), \tau(IAV)\right>.

7.5. \quad \mathbf{B_{\text{Prep}}} = \{\text{in, about}\}

7.6. \quad (\text{in, about}) \subseteq \text{CON}_{\tau}(\text{Prep})

7.7. \quad \text{in'} = \text{in, about'} = \text{about}.

7.7. END

The rule for creating new adverbs is as follows.

7.8. Rule S_6:

\begin{align*}
\text{Prep} \times T & \rightarrow IAV \\
\mathbf{F_6} & : \text{concatenate (a, B)} \\
\mathbf{T_6} & : a'(\text{^B'}).
\end{align*}

7.8. END
An example of an application of this rule is given in figure 12, where sentence (98) is produced.

(98) John talks about a unicorn.

\[
\begin{align*}
\text{John talks about a unicorn [S,4]} & \quad \text{about}(\lambda \exists u [\text{unicorn}_u(u) \land \forall \text{P}(u)])(\lambda \text{talk}(\lambda \text{john}) \\
\text{John [T]} & \quad \text{talk about a unicorn [IV,10]} \\
\forall \text{P}(\lambda \text{john}) & \quad \text{about}(\lambda \exists u [\text{unicorn}_u(u) \land \forall \text{P}(u)])(\lambda \text{talk}) \\
\text{about a unicorn [IAV,6]} & \quad \text{talk [IV]} \\
\text{about}(\lambda \exists u [\text{unicorn}_u(u) \land \forall \text{P}(u)]) & \quad \text{talk} \\
\text{about} & \quad \text{a unicorn [T]} \\
\text{about} & \quad \lambda \exists u [\text{unicorn}_u(u) \land \forall \text{P}(u)]
\end{align*}
\]

Figure 12

The translation obtained here does not imply that there is a unicorn John talks about: about creates an intensional context. This is the result we aimed at (see section 1).

In the same way as we produced (98), we may produce (99) with as translation (100).

(99) Bill walks in the park

(100) \(\exists \forall x [\text{in}(\lambda \exists u [\text{park}_u(v) \leftrightarrow u=v] \land \forall \text{P}(u))](\lambda \text{walk}(\lambda \text{bill})

This result is not completely satisfactory. If Bill walks in the park, then one may conclude that there exists a park, and if the park is the Botanical garden, then from (99) it may be concluded that Bill walks in the Botanical garden. So the locative preposition in does not create an intensional context. This property of in is formalized in the following meaning postulate.

7.9. Meaning postulate 8

\[
\exists \forall x [\text{in}(\lambda \exists u [\text{park}(v) \leftrightarrow u=v] \land \forall \text{P}(u)](\lambda \text{walk}(\lambda \text{bill})
\]

7.9. END

In order to be able to give a reduction rule on the basis of this meaning postulate, a notation for the predicate denoted by in MP8 is
introduced (such a notation for prepositions is not defined in PTQ). This notation is chosen in analogy of the notation $\delta_*$ for verbs.

7.10. **DEFINITION.**

$$\delta_* = \lambda x \lambda y [\delta^{\langle^\lambda P \mapsto P(\widehat{\lambda})\rangle} (\Psi) (x)] \text{ where } \delta \in \text{CON}_r(\text{Prep}).$$

7.10. **END**

On the basis of this definition we have the following reduction rule.

7.11. **Reduction rule 9**

Let be given an expression of the form $\text{in}(\eta)(\xi)(\gamma)$, where $\eta \in \text{MET}_{s_n, \tau (I)}$, $\xi \in \text{MET}_{s_n, e}$, $\gamma \in \text{MET}_{s_n, e}$. Then replace this expression by $\eta \mapsto \lambda y [\text{in}_x (\gamma) (\xi)(\gamma)]$.

**CORRECTNESS PROOF.** Let $\varphi \in \text{VAR}_x$, $\chi \in \text{VAR}_{s_n, e}$ and $\Psi \in \text{VAR}_{s_n, \tau (I)}$. Then for all $g$

$$g \vdash \text{in}_x (\varphi)(\Psi)(x) = \text{in}^{\langle^\lambda P \mapsto P(\widehat{\lambda})\rangle} (\varphi) (x).$$

We now apply MP8 to the right hand side of the equality: this meaning postulate says that there is a $g' \sim g$ such that

$$g' \vdash \text{in}_x (\varphi)(\Psi)(x) = [\lambda y^{\langle^\lambda P \mapsto P(\widehat{\lambda})\rangle} (\gamma) (\xi) (\gamma)(\chi)(x)].$$

The expression to the right of the equality sign reduces by means of several applications of RR$_1$ and RR$_2$. Thus we obtain

$$g' \vdash \text{in}_x (\varphi)(\Psi)(x) = \langle^\lambda G \rangle (\varphi) (x).$$

Consequently $g' \vdash \text{in}_x = \langle^\lambda G \rangle$. This means that from MP8 it follows that

$$\vdash \forall \varphi \forall \chi \forall \Psi [\text{in}(\varphi)(\Psi)(x) \leftrightarrow P(\widehat{\lambda} y [\text{in}_x (\gamma)(\chi)(\Psi)(x)])].$$

7.11. **END**

Formula (100) can be reduced, using RR$_9$, to (101) and further to (102)

(101) $[\lambda y^{\langle^\lambda P \mapsto P(\widehat{\lambda})\rangle} (\gamma) (\xi) (\gamma)(\chi)(x)]$
\[(102) \exists u \forall v \left[ \text{park}_u(v) \leftrightarrow u=v \right] \land \text{in}_u(\text{walk}(b\text{ill})).\]

In PTQ no examples concerning the meaning postulate for \textit{in} are given. This example illustrates the consequence of the meaning postulate: if one stands in the relation of walking in with 'a collection of properties', then there is an 'individual' with which one has this relation.

8. JOHN TRIES TO FIND A UNICORN

In this section a new category of IV-modifiers is introduced. This new category is called IWIV (IV modifying verbs) and contains verbs taking verbs as complements. The fragment has only two of such verbs (\textit{try to}, \textit{wish to}), although there are a lot more in English. The syntactic treatment of these verbs is rather primitive: \textit{try to} is considered as a single word containing a space (so \textit{to} is not treated as a word). But our main interest is semantics, and the verbs are interesting in this respect. They create intensional contexts even when the sentence without such a verb would only have a de-re reading. An example is (103); this sentence does not necessarily have the implication that unicorns exist.

\[(103) \text{John tries to find a unicorn.}\]

Corresponding with the verbs of category IWIV there are constants in the logic of the type \(\langle s, \tau(\text{IV}), \tau(\text{IV}) \rangle\). The verbs translate into these constants.

8.1. \( B_{\text{IWIV}} = \{ \text{try to}, \text{wish to} \} \)

8.2. \( \{ \text{try to}, \text{wish to} \} \subset \text{CON}_{\tau(\text{IWIV})} \)

8.3. \( \text{try to' = try to, wish to' = wish to.} \)

END

The members of IWIV are used in the following rule.

8.4. Rule \( S_0 \):

\[\text{IWIV} \times \text{IV} \rightarrow \text{IV}\]

\( F_0: \text{concatenate } (a, b) \)

\( T_0: a'(b'). \)

END
The production of (103) is partially presented in figure 13.

\begin{align*}
\textit{John tries to find a unicorn (§4)} \\
\text{try to} (\arrow{\wedge \text{John}}, \arrow{\lambda p \exists u [\text{unicorn}_u (u) \land \neg p (u)])} \\
\textit{John} \\
\lambda p \, p (\arrow{\wedge \text{John}}) \\
\text{try to} (\arrow{\lambda p \exists u [\text{unicorn}_u (u) \land \neg p (u)])} \\
\text{try to} (\lambda p \exists u [\text{unicorn}_u (u) \land \neg p (u)])
\end{align*}

\textbf{Figure 13}

The formula obtained in this production process does not reduce further, and it does not allow to conclude for the existence of a unicorn which John tries to find. So the de-dicto aspect is dealt with adequately. But sentence (103) can also be used in a situation in which there is a unicorn which John tries to find. For reasons related to the ones given concerning \textit{John seeks a unicorn}, the reading involving a particular unicorn has to be obtained as an alternative translation for (103). That reading can be obtained using $S'_{14,n}$.

The translation obtained for (103) in figure 13 is, however, not in all respects satisfactory. We do not get information concerning the relation between John and the property expressed in the second argument of \textit{try to}. In particular it is not expressed that what John tries to achieve is that John (he himself) finds the unicorn, and not that someone else finds the unicorn. For verbs like \textit{promise} and \textit{permit} the relation between the subject and the complement is much more complex. A correction of this disadvantage of the PTQ treatment can be found along the lines of DOWTY (1978) and BARTSCH (1978b), see also section 4.1 in chapter VII.

In section 4 we introduced the rules for IV conjunction and disjunction. The verb phrases involved may concern two coreferential terms as in (104).

\textbf{(104) John finds a unicorn and eats it.}

The coreferentiality can be dealt with by means of quantifying in the term a \textit{unicorn}. This yields the reading (105).

\textbf{(105) }\exists u [\text{unicorn}_u (u) \land \text{find}_u (\text{john}, u) \land \text{eat}_u (\text{john}, u)].
This formula expresses that there is a particular unicorn which John finds and eats.

The conjoined verb phrase underlying (106) can be embedded in a try to
construction.

(106) John tries to find a unicorn and eat it.

This sentence does not allow for the conclusion that there is a unicorn.
The occurrence of a pronoun, however, invites us to produce this sentence
with quantification rule S_{14}{^*}, and that would result in a referential reading, viz. (107)

(107) \exists u[\text{un} \text{icorn}_u(u) \land \text{try to}(\wedge \text{john}_u, \wedge \text{find}(\lambda p[\vee p(\wedge u)])) \land \text{eat}(\lambda P[\vee P(\wedge u)])].

A new quantification rule makes it possible to produce (106) in a reading
which does not imply the existence of a unicorn. The following rule scheme
describes the quantification of a Term into an IV-phrase.

8.5. Rule S_{16,16}:

$$T \times IV \rightarrow IV$$

\(F_{16,16} : \) If \(\alpha\) does not have the form \(he_{\eta}\),
then replace in \(\beta\) the first occurrence of \(he_{\eta}\) or \(him_{\eta}\)
by \(\alpha\), and all other occurrences of \(he_{\eta}\) by \(he/she/it\) and of
\(him_{\eta}\) by \(him/her/it\) according to the gender of the first \(T\)
or CN in \(\alpha\)
else replace all occurrences of \(he_{\eta}\) by \(he_{\kappa}\) and of \(him_{\eta}\) by \(him_{\kappa}\).

\(T_{16,16} : \lambda y[\alpha'(^\wedge \lambda x[\beta'(y)])].\)

8.5. END

In order to produce (106) we first produce the verb phrase (108).

(108) find a unicorn and eat it.

The production of (108) is partially given in figure 14.
find a unicorn and eat it \( \{IV, 16, 1\} \)
\[ \lambda y p \exists u[\text{unicorn}_u(y) \land p(y)](\lambda x_1[\text{find}(y, \lambda p^y p(x_1)) \land \text{eat}(y, \lambda p^y p(x_1))]) \]

a unicorn \( \{\text{T}, 2\} \)
\[ \text{find him}_1 \text{ and eat him}_1 \ (\text{IV, 12a}) \]
\[ \lambda x_1 \exists u[\text{unicorn}_u(x_1) \land p(x_1)](\lambda x_2[\text{find}(x_1, \lambda p^x_1 p(x_2)) \land \text{eat}(x_1, \lambda p^x_1 p(x_1))]) \]
\[ \text{find him}_1 \ (\text{IV}) \]
\[ \text{eat him}_1 \ (\text{IV}) \]
\[ \text{find}(\lambda p^x_1 p(x_1)) \]
\[ \text{eat}(\lambda p^x_1 p(x_1)) \]

Figure 14

Now we return to the production of sentence (106). Its production from (108) is presented in figure 15.

\[ \text{John tries to find a unicorn and eat it} \ (S, 4) \]
\[ \text{try to} (\lambda \text{john}, \lambda y z u[\text{unicorn}_u(y) \land \text{find}(z, \lambda y^z y, u)]) \]

\[ \text{John} \ (\text{T}) \]
\[ \text{try to find a unicorn and eat it} \ (IV, 8) \]
\[ \lambda p^x \text{p}(\lambda \text{john}) \]
\[ \text{try to} (\lambda y z u[\text{unicorn}_u(y) \land \text{find}(z, \lambda y^z y, u) \land \text{eat}(z, \lambda y^z y, u)]) \]
\[ \text{try to} (\text{IV} \text{IV}) \]
\[ \text{find a unicorn and eat it} \ (\text{IV}) \]
\[ \text{try to} \]
\[ \lambda y z u[\text{unicorn}_u(y) \land \text{find}(z, \lambda y^z y, u) \land \text{eat}(z, \lambda y^z y, u)] \]

Figure 15

A sentence related with (106) is (109).

(109) John tries to find a unicorn and wishes to eat it.

Montague argues that only a referential reading of this sentence is possible (except for the case that the pronoun it is considered as a pronoun of laziness). A production of sentence (109) might be given in which \( S_{14} \) is used. Then it is not surprising that a referential reading is obtained. But this is also the case for a production using \( S_{16} \), as will be shown below. The first step is to form verb phrase (110), with translation (111).

(110) try to find him and wish to eat him

\[ \lambda x \text{try to}(x, \lambda \text{find}(\lambda p^x p(x))) \land \text{wish to}(x, \lambda \text{eat}(\lambda p^x p(x))) \].

Combination of (110) with a unicorn according to \( S_{16} \) yields (112). The translation is (113), which reduces to (114).

(112) try to find a unicorn and wish to eat it
(113) \( \lambda y[\lambda P\exists u[\text{unicorn}(u)] \land \forall v(P(v))] \land \lambda x_1[\text{try-to}(y, \forall \text{find}(\lambda P \forall P(x_1))) \land \text{wish to}(y, \forall \text{eat}(\lambda P \forall P(u)))]. \)

(114) \( \lambda y[\exists u[\text{unicorn}(u)] \land \text{try to}(y, \forall \text{find}(\lambda P \forall P(u))) \land \text{wish to}(y, \forall \text{eat}(\lambda P \forall P(u))). \]

Combination of (112) with John according to \( S_4 \) yields sentence (109). The translation is (115).

(115) \( \exists u[\text{unicorn}(u) \land \text{try-to}(\forall \text{john}, \forall \text{find}(\lambda P \forall P(u))) \land \text{wish-to}(\forall \text{john}, \forall \text{eat}(\lambda P \forall P(u))). \]

The formula obtained here can be simplified by replacing \( \delta(\lambda P \forall P(u)) \) by \( \lambda y \delta(\lambda P \forall P(u))(y) \), where \( \delta \) is the translation of a transitive verb. The advantage of this replacement is that now \( \text{RR}_2 \) and \( \text{RR}_6 \) can be used. In this way (115) reduces to (116).

(116) \( \exists u[\text{unicorn}(u) \land \text{try-to}(\forall \text{john}, \exists y \text{ find } (y, y) \land \text{wish to}(\forall \text{john}, \exists y \text{ eat } (y, u)). \]

This method is formulated in a reduction rule as follows.

8.6. Reduction rule 10

Let be given an expression of the form \( \delta(\lambda P \forall P(u)) \), where \( \delta \) is the translation of a TV for which \( \text{MP}_4 \) holds. Then replace this expression by \( \lambda y \delta(y, u) \).

**Correctness proof.** By definition of interpretation the two expressions are equivalent.

8.6. END

The possibilities for application of \( \text{RR}_{10} \) are limited by mentioning explicitly the argument of \( \delta \). One might omit this argument; then the rule would be applicable in many more circumstances, for instance to the formula obtained in figure 13. I have not used this more general version because it would not give rise to simpler formulas (in the sense of more concise formulas), but one might judge that the general rule would give rise to more understandable formulas.
9. JOHN BELIEVES THAT MARY WILL RUN

A new construction considered in this section arises from verbs of the category IV/S; i.e. verbs taking a sentence as complement. There are several such verbs, but only two of them are incorporated in the fragment.

9.1. \( B_{IV/S} = \text{\{believe that, assert that\} } \)

9.2. \( \text{\{believe that, assert that\}} \subset \text{CON}_{<s\text{,}\tau(s)\text{,}\tau(IV)>>} \)

9.3. \( \text{\text{believe that}}' = \text{\text{believe that}, assert that}}' = \text{\text{assert that}}. \)

9.3. END

The rule producing IV phrases from these verbs reads as follows.

9.4. Rule \( S_4) :

\[
\begin{align*}
\text{IV/S} \times S & \rightarrow IV \\
F_4 & : \text{concatenate (a,}\hat{\beta}) \\
T_4 & : \text{a'}(\hat{\beta'}). \\
\end{align*}
\]

9.4. END

An example of a sentence with a verb of category IV/S is (117).

(117) John believes that Mary runs.

Part of the production of (117) is given in figure 16.

![Figure 16](image)

Believe is considered as a relation between an individual concept and a proposition (i.e. a function from indices to truth values). It is not said what kind of relation this is. There are several proposals in the
literature analyzing the believe relation is more detail (e.g. LEWIS 1970), but Montague did not analyze it any further.

The formula obtained in figure 16 expresses that believe is a relation with as first argument \( ^\wedge \text{john} \). To this, the same comment applies as to the first argument of the seek-relation: there is no generally accepted notation which expresses that for this argument believe can be considered as a relation with as first argument an individual. The second argument is an expression of type \(<s,t>\). Would it have been an expression of type \( t \), then we could replace it by any other expression which denotes (for the current index) the same truth value. So if someone would believe a truth, he would believe all truths (for the current index). Now that the second argument of the believe-relation is a proposition, this is not the case. If John and Mary walk, then one may believe that John walks, without having the formal implication that one believes that Mary walks. Nevertheless, the use of a proposition is not completely satisfactory. It implies that in case John believes a tautology, he believes all tautologies. This is a fundamental shortcoming of this kind of approach; there is, however, not a generally accepted alternative.

The aspect that makes the introduction of believe and assert interesting in the present fragment, even with the present semantics, is that these verbs introduce intensional contexts in which a de-re reading is impossible. Sentence (118) does not allow for the conclusion that there exists a unicorn.

(118) Mary believes that John finds a unicorn and he eats it.

The relevant part of the production of sentence (120) is given in figure 17.

\[
\begin{align*}
\text{Mary believes that John finds a unicorn and he eats it} \\
\text{believe that}( ^\wedge \text{mary}, ^\exists u \text{ unicorn}(u) \land \text{find}_{u} (\text{john}, u) \land \text{eat}_{u} (\text{john}, u)) \\
\text{Mary believes that John finds a unicorn and he eats it} \\
\lambda P ( ^\wedge \text{mary}) \text{ believe that}( ^\wedge \exists u \text{ unicorn}(u) \land \text{find}_{u} (\text{john}, u) \land \text{eat}_{u} (\text{john}, u))
\end{align*}
\]

Figure 17

A further extension of the fragment are the restrictive relative clauses: terms will be produced like \textit{Every man such that he runs}. This \textit{such that} form is not the standard form of relative clauses, but it avoids the syntactic complications arising from the use of relative pronouns. The
following rule scheme describes how relative clause constructions are formed out of a CN and a sentence.

9.5. Rule \( S_{3,n} \):

\[
\text{CN} \times S \rightarrow \text{CN}
\]

\( F_{3,n} \): replace in \( \beta \) all occurrences of \( h_{n} \) by \( he/\text{he/it} \) and \( h_{m} \) by \( \text{him/her/it} \) according to the gender of the first CN in \( \alpha \).

\( T_{3,n} \) := \( \lambda x_{n}[a'(x_{n}) \wedge \beta'] \).

9.5. END

An example is the production of term (119), which is given in figure 18.

(119) a man such that he runs.

\[
\begin{align*}
\lambda \exists x[\text{man}(x) \wedge \text{run}(x) \wedge \neg p(x)]
\end{align*}
\]

\[
\begin{align*}
\lambda \forall \lambda \exists x[\neg p(x) \wedge \forall p(x)]
\end{align*}
\]

(119) a man such that he runs \{T,2\}

\[
\begin{align*}
\lambda \exists x[\text{man}(x) \wedge \text{run}(x) \wedge \neg p(x)]
\end{align*}
\]

\[
\begin{align*}
\lambda \forall \lambda \exists x[\neg p(x) \wedge \forall p(x)]
\end{align*}
\]

Figure 18

The obtained translation can be reduced, using \( RR_{4} \), to (120).

(120) \( \lambda \exists u[\text{man}_{u}(u) \wedge \text{run}_{u}(u) \wedge \neg p(u)] \).

Rule \( S_{3,n} \) takes a CN as one of its arguments, and yields a CN as result. This means that the rule can be applied more than one time in succession. Then terms are obtained like the one in (121)

(121) Every man such that he walks such that he talks.

In (121) both the relative clauses are attached to the head \( \text{man} \); this phenomenon is called 'stacking'. A situation that may arise in connection with stacking is as follows. The second relative clause contains a pronoun which is coreferential with a term in the first relative clause, whereas the pronoun is (semantically) within the scope of the determiner of the whole term. An example, due to Bresnan (PARTEE 1975, p.263) is (124).
Every girl who attended a woman's college who gave a donation to it, was put on the list.

Sentence (124) exhibits co-reference within the compound CN phrase: it in the second relative clause refers to the college in the first relative clause. The whole term has a reading in which the college needs not to be the same for all girls. Suppose that we obtained coreferentiality by means of quantifying in the term a woman's college for him in sentence (123).

Every girl who attended him who gave a donation to him was put on the list.

In that production process a reading would be obtained with for the existential quantifier wider scope than for the universal quantifier. That is not the intended reading. In order to obtain the intended reading, a new quantification rule is introduced: quantification into a CN phrase.

9.6. Rule $S_{15,n}^*$

$T \times CN \rightarrow CN$

$F_{15,n}^*$: Replace the first occurrence of he/him in $\beta$ by a.

Replace all other occurrences of he/him by he/she/it, and of him by him/her/it, according to the gender of the first CN or T in $\alpha$.

$T_{15,n}^*$: $\lambda y \eta' ((\lambda x_n [\beta'(y)]))$.

9.6. END

An extensive discussion of relative clause formation will be given in chapter 8; examples in which rule $S_{15}$ is used, will be considered in appendix 1. There also will be solved a problem that I neglected above: reduction rule $R_4^*$ applies to the translation of terms like (120), but not to such terms with the determiners every or the.

In the remainder of this section I mention some rules which are introduced only to incorporate the complete PTQ fragment. The first rule concerns the sentence modifier necessarily.

9.7. $S_{9'/S}^*$ = necessarily

9.8. necessarily' = $\lambda \tilde{p} [^{\tilde{y}}p]$. 
9.9. Rule S_9:

\[
S/S \times S \rightarrow S
\]

\[F_9: \text{concatenate } (α, β)\]
\[T_9: α'(^Aβ').\]

9.9. END

An example is the production of (124) which gets as its translation (127).

(124) \text{Necessarily John runs.}

(125) □ \text{run}_x(\text{john}).

This example illustrates how sentence modifiers can be incorporated in the fragment. The translation of (126) is not correct since that sentence cannot mean that John always runs. For an alternative of the semantics of necessarily see e.g. VELTMAN 1980.

Up till now we have met sentences in the positive present tense. PTQ has rules for some other tenses as well. These rules have several shortcomings, and I will mention them without further discussion.

9.10 Rule S_{17a}:

\[
T \times IV \rightarrow S
\]

\[F_{17a}: \text{replace the first verb in } β \text{ by its negative third person singular present; concatenate}(α, β)\]
\[T_{17a}: \sim α'(^Aβ')\]

9.11 Rule S_{17b}:

\[
T \times IV \rightarrow S
\]

\[F_{17b}: \text{replace the first verb in } β \text{ by its third person singular future; concatenate}(α, β)\]
\[T_{17b}: Wa'(^Aβ')\]

9.12 Rule S_{17c}:

\[
T \times IV \rightarrow S
\]

\[F_{17c}: \text{replace the first verb in } β \text{ by its negative third person singular future; concatenate}(α, β)\]
\[T_{17c}: \sim Wα'(^Aβ')\]
9.13 Rule $S_{17d}$:
$T \times IV \rightarrow S$

$F_{17d}$: replace the first verb in $\beta$ by its third person singular perfect.
concatenate($\alpha, \beta$)

$T_{17d}$: $H[a'(\hat{\beta}')]$

9.14 Rule $S_{17e}$:
$T \times IV \rightarrow S$

$F_{17e}$: replace the first verb in $\beta$ by its negative person singular present perfect; concatenate($\alpha, \beta$)

$T_{17e}$: $\overline{H}[a'(\hat{\beta}')]$.

9.14. END

This completes the exposition of the PTJ fragment. One should realize that the sentences we have discussed, constitute a special selection of the sentences of the fragment. Besides those rather natural examples, there are a lot of remarkable sentences in the fragment. An example is (126).


Whether this is a shortcoming or not, depends on the opinion one has about the acceptability of (126). And how this should be dealt with, depends on the opinion one has about the question which component of the grammar should deal with such phenomena. Since these questions are completely independent of the problems we were interested in, I have not discussed this aspect. Several more fundamental aspects of the system which were not completely satisfactory, have been mentioned in the discussions. Other such aspects will arise in the discussion in later chapters, for instance in appendix. As for the main aim of the enterprise, I conclude that Montague has for the problematic sentences mentioned in section 1 indeed provided an analysis which has the desired semantic properties, and which is in accordance with the compositional framework.