CHAPTER IX

RELATIVE CLAUSE FORMATION

ABSTRACT

Does the principle of compositionality compel us to a certain analysis of relative clause constructions? Answers given by Partee and Bach & Cooper will be investigated, and new arguments will be put forward. The question will be generalized and answered on the basis of algebraic properties of the framework. The investigations give rise to a restriction on the use of variables in Montague grammar: the variable principle.
1. INTRODUCTION

Our framework, which formalizes the principle of compositionality of meaning, says that the syntax and semantics are similar algebras, and that the meaning assignment function is a homomorphism. Now one may ask to what extent this organization of the grammar restricts the options we have in the syntax to describe a particular phenomenon. This question was raised by Partee (1973) with respect to relative clause constructions, and her answer was that we have to use a particular analysis. She concluded that the framework puts very strong constraints on the syntax, with the consequence that 'it is a serious open question whether natural language can be so described' (Partee 1973, p.55). Her argumentation is used by Chomsky (1975) to support his ideas of an autonomous syntax in transformational grammars. Partee's conclusion about relative clause formation has been disputed by Bach & Cooper (1978), who give an alternative construction.

In chapter 2 it has been proven that every recursively enumerable language can be described by means of a finite algebraic grammar. Hence Partee's question, as quoted above, has already been answered positively. But we will curtail it to the question whether the framework constrains the way in which natural language phenomena can be described. More in particular, we will investigate the thematic question: does the framework of Montague grammar compel us to a particular syntactic analysis of restrictive relative clauses? The arguments given in the literature will be considered, and new arguments will be put forward. In the course of the discussion positive and negative answers to the thematic question will alternate. An answer to the general version of the question is obtained as well. It will turn out that syntactic variables (like he1) play an important role in relative clause constructions. This role is investigated, and this gives rise to the introduction of a new principle for Montague grammar: the variable principle.

This chapter is a slightly revised version of Janssen (1981a).

2. THE CN-S ANALYSIS

2.1. The discussion by Partee

Partee (1973) considers three kinds of analyses of relative clause constructions which were proposed in the literature in the framework of transformational grammar. She investigates which of them constitutes a good
basis for a compositional semantics. The comparison is carried out in the way described in chapter 7, section 2.1: the derivational histories from Montague grammar are compared with the constituent structures proposed in transformational grammars. As was explained there, this is not the most felicitous way to compare the two approaches. Our thematic question, however, does not concern a comparison but is a question about the present framework itself: the structures from transformational grammar merely constitute a starting point. Hence all trees under discussion have to be taken as representing derivational histories, even in case they originate from transformational grammar as constituent structures. In the sequel I will use the categorial terminology from the previous chapters, and not the transformational terminology used in the proposals under discussion.

Below I summarize Partee's argumentation. She discusses three kinds of analysis for the restrictive relative clause construction. They are named after the configuration in which the relative clause is introduced. These analyses (of which the second was the most popular among transformational grammarians) are

1. CN-S : the Common Noun-Sentence analysis (Figure 1)
2. T-S : the Term-Sentence analysis (Figure 2)
3. Det-S: the Determiner-Sentence analysis (Figure 3).

In the analysis presented in Figure 1, the common noun boy can be interpreted as expressing the property of being a boy, and the phrase who runs as expressing the property of running. The conjunction of these properties is expressed by the noun phrase boy who runs. The determiner the expresses that there is one and only one individual which has these two properties. So the CN-S analysis provides a good basis for obtaining the desired meaning in a compositional way.
In the T-S analysis as presented in Figure 2, the term the boy is interpreted as expressing that there is one and only one individual with the property of being a boy. Then the information that the individual is running can only be additional. So in a compositional approach to semantics who runs has to be a non-restrictive relative clause. Therefore Partee's conclusion is that the T-S analysis does not provide a good basis for a compositional semantics of restrictive relative clauses.

The Det-S analysis from Figure 3 does not provide a good basis either. The phrase dominated by the uppermost Det-node (i.e. the who runs), expresses that there is one and only one individual with the property of running, and the information that this individual is a boy, can only be additional.

Of course, these arguments do not constitute a proof that it is impossible to obtain the desired meanings from the T-S and Det-S analyses. It is, in general, very difficult to prove that a given approach is not possible, because it is unlikely that one can be sure that all variants of a certain approach have been considered. This is noted by Partee when she says: 'I realize that negative arguments such as given against analyses 2. and 3 can never be fully conclusive. [...] [PARTEE 1973, p.74 - numbers adapted T.J.). She proceeds: 'The argument against 3. is weaker than that against 2., since only in 2 the intermediate constituent is called a T.' (ibid.). Her carefully formulated conclusion is 'that a structure like 1, can provide a direct basis for the semantic interpretation in a way that 2 and 3 cannot' (ibid. p.54).

2.2. The PTQ-rules

Accepting the argumentation given in Section 2.1, is not sufficient to accept the claim that one should use the CN-S analysis. It remains to be shown that such an analysis is indeed possible, and this means providing explicit syntactic and semantic rules. Partee does not need to do so because in her discussion she assumes the rules for relative clause formation which are given in PTQ. Although these rules do not produce literally the same string as she discusses, the same argumentation applies to them.

I recall the rule for relative clause formation given in chapter 4.

\[ S_{3,n} : CN \times S \to CN \]

\[ F_{3,n} : \text{Replace } he_n \text{ in } \beta \text{ by } he/aha/it \text{ and him}_n \text{ by him/her/it, according to the gender of the first CN in } \alpha; \text{ concatenate } (a, \text{ much that, } \beta) \]

\[ T_{3,n} : \lambda x [\alpha'(x) \land \beta'] \]
According to this rule, the derivational history of _boy who runs_ has the structure presented in figure 1. The phrase can be produced from the noun _boy_ and the sentence _he_ runs by an application of instance $S_{3,3}$ of the above scheme. The corresponding translation reads

(1) $\lambda x_3[\text{boy}(x_3) \land \text{run}(x_3)]$.

This expression is interpreted as the property which holds for an individual if it both is a boy and is running. This is completely in accordance with the interpretation sketched for figure 1.

I recall that $S_{3,n}$ can be applied two times in succession (or even more). Then sentences are obtained like (2) (due to Bresnan, see PARTEE 1975, p.263) and (3) (due to PARTEE 1975: ibid).

(2) Every girl who attended a women's college who made a large donation to it was included in the list.

(3) Every man who has lost a pen who does not find it will walk slowly.

In these sentences two relative clauses are attached to a single head noun. This construction is known under the name stacking (of relative clauses). In Dutch and German stacking is not a grammatical construction.

Rules $S_{3,n}$ and $T_{3,n}$ do not give a correct treatment of all phenomena which arise in connection with relative clauses. Some examples are:

1. The rule produces the such-that form of relative clauses, and this is not their standard form. A rule which produces a form with relative pronouns cannot be obtained by means of a straightforward reformulation of $S_{3,n}$, since complications arise (see ROMAN 1976).

2. In certain circumstances $T_{3,n}$ may give rise to an, unintended, collision of variables. This problem was discussed in section 5.3 of chapter 6; see also section 6.1.

3. Some famous problematic sentences do not get a proper treatment with this rule. Examples are the so called 'Bach-Peters sentences' and the 'Donkey sentences'. There are several proposals for dealing with them.

   For instance HAUSSER (1979c) presents a treatment for the Bach-Peters sentence (4), and COOPER (1979) for the donkey sentence (5).

(4) The man who deserves it gets the price he wants.

(5) Every man who owns a donkey beats it.

For a large class of sentences, however, the PTQ rule yields correct results, and I will restrict the discussion to this class. The class
contains the relative clause constructions in the such-that form, the relative clause is a single (i.e. unconjoined) sentence, and stacking is allowed. Bach-Peters sentences and Donkey sentences are not included. For this class, the CN-S analysis gives a correct treatment in a compositional way, whereas for the T-S and Det-S analyses it is argued that this is not the case. So in this stage of our investigations, the answer to the thematic question has to be positive: the compositionality principle compels us to a certain analysis of relative clause constructions.

2.3. Fundamental problems

The PTQ rule for relative clause formation is essentially based on the use of variables in the syntax \( (h_n^a) \), and the use of unbound variables in the logic \( (x_n^p) \). This device gives rise to two problems which are of a more fundamental nature than the problems mentioned in Section 2.2. The latter concerned phenomena which were not described correctly by the given rule, but it is thinkable that some ingenious reformulation might deal with them. The fundamental problems I have in mind are problems which arise from the use of variables as such. It is essential for the entire approach to obtain a solution for these problems, since in case they are not solved satisfactorily we cannot use the tool at all. This aspect distinguishes them from the problems mentioned in Section 2.2. The problems also arise in connection with other rules dealing with variables \( (S_{14,n}, ..., S_{17,n}) \). Note that the epithet 'fundamental' is not used to make a suggestion about the degree of difficulty of the problem, but to indicate the importance that some answer is given to it. The two fundamental problems are the following.

1) 'left-over'

The first problem is: what happens in case a variable is introduced that is never dealt with by \( S_{3,n} \) or any other rule. On the syntactic side it means that we may end up with a sentence like \( h_n \) runs. Since \( h_n \) is not an English word, this is not a well-formed sentence, and something has to be done about it. On the semantic side it means that we may end up with an expression containing an unbound logical variable. From the discussion in Section 5 it will appear that it is not obvious how we should interpret the formulas thus obtained.

2) 'not-there'

The second problem is: what happens when a rule involving variables with a given index is applied in case such variables are not there. I give
two examples of such situations. The first is obtained if one applies $S_{3,1}$ to the common noun *man*, and the sentence *Mary talks*. Then the noun-phrase (6) is produced, which is ill-formed because there is no pronoun which is relativized.

(6) *man such that Mary talks.*

On the semantic side (6) gives rise to a lambda operator which does not bind a variable. The second example (GROENENDIJK & STOKHOFF 1976b) is obtained by an application of $S_{3,1}$ to *man* and *he$_2$ walks*. Then the common noun phrase (7) is formed, out of which (8) can be obtained.

(7) *man such that he$_2$ walks.*

(8) *He$_2$ loves the man such that he$_2$ walks.*

By an application of $S_{14,2}$ we finally obtain

(9) *John loves the man such that he walks.*

This sentence has just one reading, viz. that John loves a walking man. The translation rules of PTQ however, yield (10) as reduced translation for (9).

(10) $\exists u \forall v [\text{man}_u(v) \land \text{walk}_u(\text{john})] \leftrightarrow u = v \land \text{love}_u(\text{john}, u)$.

This formula expresses that the one who walks is John. THOMASON (1976) makes a related observation by counting the number of ambiguities of (11).

(11) *Bill tells his father that John resembles a man such that he shaves him.*

For the first problem it is evident that it is the use of variables which creates it, and that it are not the phenomena themselves: if there were no variables in the syntax, they could not be 'left-over', nor remain 'unbound' in their translation. For the second problem it is rather a matter of conviction that it is the use of variables that creates the problem. Even if (6) would be well-formed, I would consider its production in the way sketched above, as an undesirable side effect of the use of variables, because it does not exhibit a phenomenon for which variables are required.

In the literature there are some proposals for dealing with these two fundamental problems. One proposal (implicitly given in RODMAN 1976) is of a purely syntactic nature and simply says: the 'left-over' and 'not-there' constructions are not acceptable, and in case such a construction threatens to arise, it is filtered out. This approach is not considered here in detail, because it played no role in the discussion concerning our thematic
question. In the approach of COOPER (1975) the 'left-over' constructions are accepted, an answer is given to the semantic questions, and the 'not-there' constructions are dealt with in the semantics. In the next sections his proposal will be discussed in detail. A proposal combining syntactic and semantic aspects (JANSSEN 1980b) will be considered in Section 5.

3. THE T-S ANALYSIS

3.1. Cooper on Hittite

COOPER (1975) considers the construction in Hittite which corresponds to the relative clause construction in English. In Hittite the relative clause is a sentence which is adjoined to the left or the right of the main sentence. For this and other reasons, Cooper wishes to obtain such constructions by first producing two sentences and then concatenating them. A simplified example is the Hittite sentence which might be translated as (12), and has surface realization (13). The sentence is produced with the structure given in figure 4. For ease of discussion English lexical items are used instead of Hittite ones. 'Genitive' is abbreviated as 'gen', 'plural' as 'pl', 'particle' as 'prt', and 'which' as 'wh'. The example is taken from BACH & COOPER (1978) (here and in the sequel category names are adapted).

(12) And every hearth which is made of stones costs 1 shekel.

(13) SA NA4 HI.A-ta kues GUNNI.MES nu kuisaa 1 GIN gen.stone-pl.-and which hearth-pl. ptc. each(one) 1 shekel

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![Diagram of sentence structure](image-url)

**Figure 4**
Sentence (13) is assumed to have the same meaning as the corresponding English sentence (12). There seems to be a conflict between the arguments in favor of a CN-S analysis as given in section 2, and the wish to use the S-S analysis for Hittite. Cooper's solution is to allow the Term-phrase *each (one)* to denote the set of properties possessed by every entity having property \( R' \) (BACH & COOPER 1978, p.147). Which property \( R \) is, is specified by the relative clause \( S_1 \). The translations of \( S_2 \) and \( S_1 \) are (14) and (15), respectively (here and in the sequel \( \forall, \land, \) and \( \ast \) symbols are added).

(14) \( \forall x \, R(x) \rightarrow \text{Cost-one-shekel}(x) \)

(15) \( \text{Hearth}(z) \land \text{Made-of-stone}(z) \).

The syntactic rule which combines \( S_1 \) and \( S_2 \) to a phrase of the category \( \bar{S} \), has as corresponding translation rule

\[ \lambda z[S_1, S_2][\lambda z[S_1 \uparrow]] \.
\]

Here \( S_1 \uparrow \) and \( S_2 \uparrow \) are the translations of \( S_1 \) and \( S_2 \), respectively. When this rule is applied to (14) and (15), we obtain (16) as reduced translation.

(16) \( \forall z[\text{Hearth}(x) \land \text{Made-of-stone}(x) \land \text{Cost-one-shekel}(x)] \).

Since \( \bar{S} \) is of another category than \( S_1 \) and \( S_2 \), this production process does not allow for stacking, what is claimed to be correct for Hittite.

3.2. Bach & Cooper on English

BACH & COOPER (1978) argue that the treatment of COOPER (1975) of Hittite relative clauses can be used to obtain a T-S analysis for English relative clause constructions which is consistent with the compositionality principle. Terms are treated analogously to (the Hittite version of) each (one). The term *every man* is assumed to denote, in addition to the PTQ interpretation, the set of properties possessed by *every man* which has the property \( R \). Then the term-phrase *every man who loves Mary* is obtained from the structure given in figure 5.

![Figure 5](image-url)
The rule for combining the translation of the term and the relative clause is:

$$\lambda x[x', \lambda y[\lambda z (\forall x \forall z \forall y (x, z, y) \land \forall z (x, y, z) \land \forall z (y, x, z)]].$$

Here $T'$ and $S'$ are the translations of the term phrase and the relative clause, respectively. If we take (17) as translation of every man, and (18) as translation of the relative clause $S$, then we obtain (19) as translation of the whole term (after reduction).

(17) $\lambda y[\forall z\forall x (x, z, y) \land \forall y (y, x, z)]$

(18) $\lambda z (\forall x (x, z, y) \land \forall y (y, x, z)]$.

Thus a T-S analysis is obtained for relative clause constructions, of which the translation is equivalent to the translation in the case of a CN-S analysis.

As Bach and Cooper notice, if we follow this approach, a complication has to be solved, since English allows for indefinite stacking of relative clauses. The proposal sketched so far provides for one relative clause for each $T$. The complication can be taken care of by allowing an alternative interpretation not only for Terms, but also for relative clauses. 'Thus, for example, the relative clause who loves Mary can denote not only the property of loving Mary but also the property of loving Mary and having property $R'$ (BACH & COOPER 1978, p.149).

Bach and Cooper remark that their compositional treatment of the T-S analysis clearly is less elegant and simple than the alternative CN-S analysis. They conclude: 'Our results seem to indicate, however, that such an analysis cannot be ruled out in principle, since any constraint on the theory that would exclude the T-S analysis, would seem to exclude the Hittite analysis as well. [...] or the happy discovery of some as yet unknown principles will allow the one, but not other.' (ibid. p.149).

The conclusion which prompts itself in this stage of our investigations is that the answer to the thematic question is a negative one: the principle of compositionality does not compel us to a special analysis of English relative clauses.
3.3. Fundamental problems

As a matter of fact, the discussion in BACH & COOPER (1978) does not provide the evidence that a T-S analysis is indeed possible for English relative clauses. They do not present explicit rules, and neither is it immediately clear what the details would look like (e.g. what is the role of S and COMP in the system of categories, and what is the translation rule which combines the translations of S and COMP). Nevertheless, the main point of their approach has become clear from their exposition.

The kernel of the approach of Bach and Cooper is to let the translations of terms and relative clauses contain a free variable R. For this variable the translation of some relative clause will be substituted. However, this variable R gives rise to the same kind of problems as mentioned in section 1 with respect to the variables x_n.

1. 'Left-over'

We may select for a term the translation with free variable R, whereas we do not use in the remainder of the production a rule which deals with this variable. Since R has no syntactic counterpart, the produced sentences are not per se ill-formed, but the question concerning the interpretation of unbound variables remains to be answered.

2. 'Not-there'

There may be an occurrence of the term-phrase every man with the translation without R, nevertheless appearing in a structure where a relative clause is attached to it. Then an incorrect meaning is obtained.

Only when these fundamental problems are solved, we may hope that the idea of Bach and Cooper leads to rules for the T-S analysis. Notice that the proposal of RUDMAN (1976) for solving the two fundamental problems by filtering them out, cannot be followed here because in the syntactic expressions there is no variable which may control the filter. A solution has to be found on the semantic side. These problems for the Bach-Cooper idea, are signalized for the case of Hittite by COOPER (1975). He has proposed some solutions which are assumed by Bach and Cooper. In order to obtain further justification for the answer to the thematic question given in Section 3.2, we have to check the details of Cooper's proposals for these problems. This will be done in the next section.
4. THE PROPOSALS OF COOPER

4.1. Not-there

A translation rule which usually binds a certain variable, may be used in a situation where no occurrences of such a variable are present. To avoid problems, Cooper proposes to give no semantic interpretation to expressions of intensional logic which contain a vacuous abstraction. According to his proposal the interpretation of $\lambda x$ is undefined in case $x$ has no occurrences of $\beta$.

Let us first consider in which way this idea might be formalized. At first glance it seems easy to obtain the desired effect. One just has to look into the expression $x$ in order to decide whether $\lambda x$ is defined or not. However, this is not acceptable. Such an approach would disturb the homomorphic interpretation of intensional logic: for each construction of the logical language there is a corresponding interpretation instruction. To obtain the interpretation of a compound logical expression, the interpretations of the parts of that compound are relevant, but not their actual forms. An important consequence of this is that two semantically equivalent expressions are interchangeable in all contexts. If we would have a condition like 'look into $x$' in the definition of interpretation, this basic property of logic would no longer be valid. Two $\theta$-expressions $a$ and $b$ might be semantically equivalent, whereas $a$ satisfies the 'look into'-condition, and $b$ not. Consequently, the interpretation of just one of $\lambda x$ and $\lambda y$ would be defined. Such a violation of the fundamental law of substitution of equivalents is of course not acceptable, and therefore, a 'look into' clause has to be rejected. One has to respect the homomorphic interpretation of logic, and therefore, the situations in which $\lambda x$ should receive no interpretation have to be characterized in terms of the semantic properties of $a$ (i.e. in terms of the interpretation of $a$ with respect to a point of reference and a variable assignment). Cooper follows this strategy.

Cooper's first step towards a characterization consists of adding a restriction to the usual definition of the interpretation of $\lambda u$.

'... the function denoted by the abstraction expression $\lambda u$ is only defined for entities within its domain if a different assignment to the variable $u$ will yield a different denotation for $a$' (COOPER 1975, p.246). As he notes, this definition has as a consequence that $\lambda u$ is 'undefined' not only if $a$ does not contain a free occurrence of $u$, but also if $a$ is
a tautology. Thus for instance, according to this definition \( \lambda u [u = u] \) represents a function which is undefined for any entity. However, the technique of supervaluation [...] will show these expressions to be defined but not those where \( \alpha \) is not a tautology' (ibid.). This definition is Cooper's final one, but it is not the one we need. It implies that now \( \lambda R [x = x] \) is defined. This has the following consequence for relative clause formation. One might produce some sentence expressing a tautology, while its translation does not contain an occurrence of the variable \( R \). Syntactically there needs not, in Cooper's approach, be anything which can prevent us from using this sentence in a relative clause construction, whereas, contrary to his intention, the interpretation of the translation is defined. So Cooper's definition does not provide a solution to the 'not-there' problem.

Cooper's aim was to give a semantic characterization of the IL-syntactic property 'contains an occurrence of the variable \( R \). I expect that there is no semantic property coinciding with the syntactic one. This is suggested by the observation that almost always a semantic irrelevant occurrence of a certain variable can be added to a given IL-expression. (\( \phi \) and \( R \models \phi \) are semantically indiscernable). Therefore, I expect that no solution in this direction can be found. Moreover, I consider the whole idea underlying Cooper's approach to be unsound. The standard interpretation of \( \lambda R \alpha \) is, in case \( \alpha \) does not contain an occurrence of \( R \), a function that delivers for any argument of the right type, the interpretation of \( \alpha \) as value. So \( \lambda R \alpha \) denotes a constant function. Following Cooper's idea, one would loose this part of the expressive power of IL, a consequence I consider to be undesirable.

4.2. Left-over, proposal 1

The translation of a completed syntactic production of a sentence may contain an occurrence of a free variable. The second fundamental problem was what to do with variables that are 'left over'. Cooper proposes to assign no interpretation to such an expression, and to follow this approach for special variables only. Let \( z \) be such a variable (of the type of individuals). As was the case with the first problem, discussed in Section 4.1, one has to respect the homomorphic interpretation of IL. The desired effect should not be obtained by looking into the formula, but by changing the definition of interpretation. Cooper claims that the desired effect is obtained 'by restricting the assignments to variables so that \( z \) is always
assigned some particular non-entity for which no predicate is defined' (COOPER 1975, p.257). This proposal gives rise to a considerable deviation from the model for IL as it is defined in PTQ. In that model, there are for every entity predicates which hold for it, e.g. the predicate of being equal to itself (viz. \( \lambda u (u = u) \)). This property is lost in Cooper's approach. He does not define a model which has the desired properties, nor does he give other details. For the discussion concerning the thematic question, this point is not that relevant, because BAKH \& COOPER (1978) do not propose to follow this proposal in the case of English relative clause constructions, but another one, which will be discussed in Section 4.3.

4.3. Left-over, Proposal 2

A second proposal of COOPER (1975) for the treatment of unbound variables which occur in the translation of a completed production of a sentence is to let the unbound variables be interpreted by the variable assignment function, and to give some linguistic explanation of how to understand the results thus obtained. This approach assumes that in complete sentences indices of variables can be neglected, or that there is some final 'cleaning-up' rule which deletes the indices. For our discussion of relative clause formation the syntactic details of this proposal are irrelevant because the variable \( R \) leaves no trace in the syntax.

The unbound relative clause variable \( R \) only occurs in subexpressions of the form \( R(x) \). These subexpressions are understood by Cooper as 'a way of representing pragmatic limitations on the scope of the quantifier [binding \( x \)]. [...]. Thus assigning a value to \( R \) in this case has the same effect as adding an unexpressed relative clause to show which particular set we are quantifying over' (COOPER 1975, p.258-259). The same strategy is employed in COOPER (1979a,b) for indexed pronouns. A pronoun \( h_{n}^{0} \) that has not been dealt with by a relative clause formation rule or some other rule, is considered as a personal pronoun referring to some contextually determined individual. Its translation has introduced a variable \( x_{n} \), which remains unbound, and is interpreted by the variable assignment. This idea for dealing with free variables is also employed in GROENENDIJK \& STOKHOF (1976b). In one respect the idea leads to a deviation from PTQ. There, an expression of type \( t \) is defined to be true in case it denotes 1 for every variable assignment (MONTAGUE 1973, p.259). So, \( run(x) \) would mean the same as its universal closure. In the proposal under discussion this definition
has to be dropped, but this does not cause any difficulties.

I have several objections against this proposal of Cooper. The first
one is that it yields incorrect results; the other three argue that the
whole approach is unsound. My objections are explained below.

1. If the translation of a phrase contains two occurrence of \( R \), and a re-
   lative clause is combined with that phrase, then the translation of the re-
   lative clause is, by \( \lambda \)-conversion, substituted for both occurrences of \( R \).
   As Cooper mentions, this phenomenon arises in his grammar for Hittite for
   (the Hittite variant of):

   (20) \textit{That(one) adorns that(one)}.

   Here the translation of both occurrences of \textit{that(one)} contains an occurrence
   of the variable \( R \). If this sentence is combined with a sentence containing
   two occurrences of a \textit{wh}-phrase, semantically strange things happen. Cooper
   notes this problem and he says: 'My intuition is, however, that if there
   were such sentences, they would not receive the interpretation assigned in
   this fragment. [...] As it is not clear to me what exactly the facts of
   Hittite are here I shall make no suggestions for improving the strange
   predictions of the fragment as it is.' (COOPER 1975, p.260).

   Unfortunately, the proposal for English of BACH \\& COOPER (1978) runs
   into a related problem. Consider the structure for the term phrase given in
   Figure 6. It is an example taken from their article, and exhibits stacking
   of relative clauses (the structure is simplified by omitting Comp's).

   \begin{figure}[h]
   \centering
   \includegraphics[width=0.5\textwidth]{figure6.png}
   \caption{Figure 6}
   \end{figure}

   The translation of \textit{every man} has to contain a variable for the relative
   clause. Recall that, in the conception of Bach \\& Cooper, the proposal dis-
   cussed in Section 4.1 deals with the situation that we have the translation
   not containing \( R \). Let us assume that we have taken the translation (21),
   which contains an unbound variable \( R \).
(21) $\lambda Fx[\text{man}(x) \land xR(x) \rightarrow F(x)]$.

Suppose now that the referent of a girl is to be contextually determined (this possibility is not considered by Bach & Cooper). Then the translation of a girl has to contain the variable $R$. Besides this variable the translation of (22) has to contain a variable $R$ for the second relative clause. So the translation of (22) has to be (23).

(22) who loves a girl
(23) $\lambda z \exists y [\text{girl}(y) \land yR(y) \land \text{love}_x(y, y) \land xR(z)]$.

Consequently, the translation of (24) has to be (25).

(24) every man who loves a girl
(25) $\lambda Fx[\text{man}(x) \land \exists y [\text{girl}(y) \land yR(y) \land \text{love}_x(y, y) \land xR(z)] \rightarrow F(x)]$.

The translation of who lives in Amherst roughly is indicated in (26).

(26) $\lambda x[\text{live-in-Amherst}(x)]$.

The translation of the entire term-phrase in figure 6 is described by

(27) $\lambda R[\text{every man who loves a girl}] (\text{who lives in Amherst'})$.

This yields a logical expression which says that both the man and the girl live in Amherst, which is not the intended reading of the construction with stacked relative clauses.

These incorrect predictions are not restricted to stacking. The same problems arise in case a relative clause like who runs is combined with a disjoined term phrase like the man or the woman. Then semantically both terms are restricted, whereas syntactically only the second one is. The source of all these problems is that a single variable is used for relative clauses and for contextual restrictions. These two functions should, in my opinion, be separated. But then the left-over/not-there problem for relative clause variables arises with full force again.

2. As a motivation for interpreting the $R$'s as contextual restrictions, the argument was given that when we speak about every man, we in fact intend every man from a contextually determined set. But this argument applies with the same force in case we speak about every man who runs. It is not true that terms sometimes are contextually determined, and sometimes not. If one wishes to formalize contextual influence, then every term should be restricted. This suggests (as under 1) a system of variables for context
restrictions which is independent of the system of variables for relative clauses.

3. Variables of which the interpretation is derived from the context have to receive a very special treatment. This can be shown most clearly by considering a sentence which has as translation a formula containing an occurrence of an unbound variable of the type of individuals or individual concepts: *he runs*, obtained from the sentence *he runs.* These sentences have as translation *(\(x_n\))*. For every variable assignment this translation gets an interpretation. One of the possible assignments is that \(x_n\) is the person spoken to, so *he runs* would have the same truth conditions as *You run.* Some female person might be assigned to \(x_n\), so the sentence may have the same truth-conditions as *she runs.* These are incorrect results, so there has to be some restriction on the variable assignments for \(x_n\). There are also semantic arguments for such a restriction. A pronoun *he* usually refers to individuals from a rather small group (e.g. the person mentioned in the last sentence, or the person pointed at by the speaker). So again some restriction has to be given. These two sources of inadequacy can be dealt with by not evaluating a complete sentence with respect to all variable assignments, but only to a subset thereof. In the light of the arguments given above, this subset is rather small. So the contextually determined variables are not so variable at all; they behave more like constants.

4. A rather fundamental argument against the use of variables for formalizing contextual influence is the following. In PTQ the contextual factor of the reference point under consideration (a time world pair), is formalized by means of the so called indices I and J. Several authors have proposed to incorporate other factors in the indices. LEWIS (1970), for instance, mentions as possible indices: speaker, audience, segment of surrounding discourse, and things capable of being pointed at. These indices constitute an obvious way to formalize contextual influence. In the light of this, it is very important to realize that in IL the interpretation of constants is 'index dependent', whereas variables have an 'index independent' interpretation:

\[
c^A,\overline{1},\overline{1},\overline{1},\overline{1} = F(c)(i, j), \quad x^A,\overline{1},\overline{1},\overline{1},\overline{1} = g(x).
\]

This means that in IL it is very strange to use logical variables for the purpose of encoding contextual restrictions. The obvious method is by means
of constants. This is precisely the method employed in MONTAGUE (1968) and BENNETT (1978).

4.4. Conclusion

We considered Cooper's proposals concerning the solution of the 'not-there/left-over' problems. His idea to give a semantic treatment of the 'not-there' problem was not successfully formalized. His treatment of the variables 'left-over' led to incorrect results for English sentences. We have to conclude that the technical details of the Bach & Cooper proposal are such that their approach does not work correctly. This means that at the present stage of our investigations concerning the thematic question we are back at the situation of the end of Section 2: only the CN-S analysis seems to be possible.

I have not formally proved that it is impossible to find some treatment in accordance with Cooper's aims. As I said in Section 2, such a proof is, in general, difficult to give. But I have not only showed that the proposals by Bach & Cooper do not work correctly, I have also argued that they have to be considered as unsound. They constitute a very unnatural approach, and in my opinion one should not try to correct the proposals, but rather give up the idea underlying them altogether. Since I consider such proposals as unsound, I will in the next section put forward a principle which prohibits proposals of these kinds.

5. THE VARIABLE PRINCIPLE

In the previous section we have considered some attempts to deal with the 'not-there/left-over' problems. These attempts do not give me the impression that the considered situations they deal with are welcome; rather they seem to be escapes from situations one would prefer not to encounter at all. In my opinion these attempts arise from a neglect of the special character of syntactic variables. Syntactic variables differ from other words in the lexicon since they are introduced for a special purpose: viz. to deal with coreferentiality and scope. In this respect they are like logical variables, and in fact they can be considered as their syntactic counterpart. One would like to encounter syntactic variables only if they are used for such purposes. This special character of syntactic variables is expressed by the variable principle, of which a first tentative version is given in (29).
(29) Syntactic variables correspond closely to logical variables.

The intuition behind this statement is not completely new. THOMASON (1976) draws attention to the analogy between 'that-complement' constructions in English, and the λ-abstraction operator in logic. PARTEE (1979b) proposes the constraint that any syntactic variable must be translated into an expression of the logic containing an unbound logical variable. Partee does not accept this constraint the other way around, precisely because she does not want to disallow Cooper's treatment of free variables.

The formulation of the principle given in (29) is vague, and one might be tempted to strengthen it to (30).

(30) An expression contains a syntactic variable if and only if its unreduced translation contains a corresponding unbound logical variable.

This is intuitively an attractive formulation. However, a major drawback is that it does not fit into the framework of Montague grammar. It would give the unreduced translation of an expression a special status which it does not have in the framework as it is. The unreduced translation, would no longer be just one representation among others, all freely interchangeable. It would become an essential stage since the principle would have to function as a filter on it. It would no longer be allowed to reduce the intermediate steps in the translation process since then a semantically irrelevant occurrence of a logical variable might disappear, and thereby a translation that had to be rejected, might become acceptable. Therefore, I will give a formulation which turns the principle into a restriction on possible Montague grammars. The formulation below has the same consequences for the unreduced translation as (30), but it is not a filter on the unreduced translations and it leaves the framework untouched. This formulation is slightly more restrictive than (30), and than the formulation in JANSSSEN (1980b).

The VARIABLE PRINCIPLE is defined as consisting of the following 6 requirements:

1a) A syntactic variable translates into an expression which contains a free occurrence of a logical variable, and which does not contain occurrences of constants.

1b) This is the only way to introduce a free occurrence of a logical variable.
2a) If a syntactic rule removes all occurrences of a certain syntactic variable in one of its arguments, then the corresponding translation rule binds all occurrences of the corresponding logical variable in the translation of that argument.

2b) If a translation rule places one of its arguments within the scope of a binder for a certain variable, then its corresponding syntactic rule removes all the occurrences of the corresponding syntactic variable from the syntactic counterpart of that argument.

3a) The production of a sentence is only considered as completed if each syntactic variable has been removed by some syntactic rule.

3b) If a syntactic rule is used which contains instructions which have the effect of removing all occurrences of a certain variable from one of its arguments, then there indeed have to be such occurrences.

This formulation of the variable principle is not what I would like to call 'simple and elegant'. I hope that such a formulation will be possible when the algebraic theory of the organization of the syntax is further developed. Suppose that we have found which basic operations on strings are required in the syntax (following PARTEB (1979a,b, see chapter 8)), and that a syntactic rule can be described as a polynomial over these basic operations. Then we may hope to formulate the variable principle as a restriction on the relation between the syntactic and semantic polynomials. We might then require that these polynomials are isomorphic with respect to operations removing/binding variables.

Requirement 1a) is a restriction on the translation of lexical elements. It can easily be checked whether a given grammar satisfies the requirement. It is met by all proposals in the field of Montague grammar that I know of; e.g. the PTQ translation of he\(_n\) is \(\lambda x. P(x)\), and the translation of the common noun variable one\(_n\) (HAUSSEMr 1979c) is the variable \(P_n\).

For reasons of elegance, one might like to have formulation 1a') instead of formulation 1a).

1a') A syntactic variable translates into a logical variable.

In order to meet 1a') in the PTQ fragment, one could introduce a category of Proper Names containing John, Mary, he\(_{1}\), he\(_{2}\),... (with translations john, mary, x\(_1\), x\(_2\), respectively). Out of these Proper Names, Terms could be produced which obtain the standard translation (\(\lambda x. P(john)\), etc.). Since I do not know of a phenomenon, the treatment of which would be simplified using this approach, and since the variable principle then still
would not have a simple formulation anyhow, I will not use it here. Re-
requirement 1a) has as a consequence that the translation of a syntactic
variable is logically equivalent to a logical variable. If constants are
allowed to occur, then this would no longer be true (e.g. it is not true
that for every c the formula $\exists x(x=c)$ is valid).

Requirement 1b) is a restriction both on the translation of lexical
elements, and on the translation rules. This requirement is met by PTQ.
It is not met by the proposals of BACH & COOPER (1978) which allow free
variables to occur which do not have a syntactic counterpart. Since they
do not present explicit rules, I do now know at which stage the context
variable $R$ is introduced, as a lexical ambiguity of the noun, or by means
of some syntactic rule.

Requirements 2a) and 2b) are conditions on the possible combinations
of a syntactic rule with a translation rule. Whether a grammar actually
meets them is easily checked by inspection (PTQ does). Requirement 2b) is
not met by the Bach & Cooper proposal since their approach in some cases
gives rise to the introduction and binding of logical variables without
any visible syntactic effect.

Requirements 3a) and 3b) we have already mentioned in chapter 6. They
are not met by PTQ, nor by Bach & Cooper. In a certain sense they con-
stitute the kernel of the principle. They express that certain configura-
tions (described with respect to occurrences of variables) should not arise.
When these requirements are met, the fundamental problems described in
Section 1 disappear. As such, the two requirements are closely related to
two instructions in JANSSEN (1980a, p.366), and to two conventions in
RODMAN (1976, one mentioned there on p.176, and one implicitly used on
p.170). Requirements 3a) and 3b) alone, i.e. without 1) and 2), would suf-
fice to eliminate the syntactic side of the two fundamental problems, but
then the close relationship between syntactic and logical variables would
not be enforced. That freedom would give us the possibility to abuse syn-
tactic variables for other purposes than coreferentiality and scope. An ex-
treme case is given in JANSSEN (1980b), where some rules which obey 3a) and
3b), but violate 1) and 2), are defined in such a way that the information
that a rule is obligatory is encoded in the syntactic variables. I intend
to prohibit this and other kinds of abuse of variables by combining the
third requirement with the first and second. In chapter 6, section 5.3,
it is discussed how we might incorporate requirements 3a) and 3b) by filters
and partial rules or by total rules (using a refined system of categories)
For the present discussion it is irrelevant how these requirements are
exactly incorporated in the system. Since we are primarily interested in
the effects of the principle, it suffices to know that it can be done in
some way.

Let me emphasize that the principle is intended to apply to the stand-
ard variables of intensional logic and their corresponding syntactic
variables. For instance, the argument concerning the use of unbound vari-
ables for contextual influence does not apply if we do not translate into
IL but into Ty2. If Ty2 is used, the variable principle does not simply
apply to all the variables of type s. Neither does the principle apply to
so called 'context variables' of HAUSSE (1979c), or the 'context expressions
of GROENENDIJK & STOKHOF (1979), which both are added to IL for the special
purpose of dealing with contextual influence.

The principle eliminates the basic problems from section 2 and dis-
allows the treatment of variables aimed at in COOPER (1975), and COOPER
(1979a,b). Another example of a treatment which is disallowed is the pro-
posal of OH (1977). For a sentence without discourse or deictic pronouns
he gives a translation containing a unbound variable! A consequence of the
principle is that the denotation of a sentence is determined completely by
the choice of the model and the index with respect to which we determine
its denotation. In other words, the denotation is completely determined by
the choice of the set of basic entities, the meaning postulates, the index,
and the interpretation function for constants (i.e. the interpretations of
the lexical elements in the sentence). In determining the denotation the
non-linguistic aspect of an assignment to logical variables plays no role.
This I consider to be an attractive aspect of the principle. What the im-
port of the principle is for the answer on the thematic question will be
investigated in the next section.

6. MANY ANALYSES

6.1. The CN-S analysis for English

Do the rules for the CN-S analysis of relative clauses obey the vari-
able principle?
Recall the PTQ rules from Section 2.1.
$S_{3,n} \quad \text{CN} \times \text{S} \rightarrow \text{CN}$

$F_{3,n} \quad$ Replace $\text{he}_n$ in $\beta$ by $\text{he/she/it}$ and $\text{him}_n$ by $\text{him/her/it}$, according to the gender of the first CN in $\alpha$; concatenate $(\alpha, \text{such that }, \beta)$.  

$T_{3,n} \quad$ (PTQ) $\lambda x_n [\alpha'(x_n) \land \beta']$.  

This combination of $S_{3,n}$ and $T_{3,n}$ does not obey the variable principle since possible occurrences of $x_n$ in $\alpha'$ are, by $\lambda x_n$, bound in the translation, whereas the occurrences of the corresponding syntactic variable $\text{he}_n$ in $\alpha$ are not removed. This aspect is the source of the 'collision of variables' mentioned in Section 3.1. (For details see section 3.4 of chapter 5). A reformulation of $T_{3,n}$ which avoids such a collision is given by THOMASON (1974, p.261).

$T_{3,n} \quad$ (THOMASON)  

$\lambda x_m [\alpha'(x_m) \land \beta']$  

where $\beta'$ is the result of replacing all occurrences of $x_n$ in $\beta'$ by occurrences of $x_m'$ where $m$ is the least even number such that $x_m$ has no occurrences in either $\alpha'$ or $\beta'$.  

The syntactic rule $S_{3,n}$ removes the occurrences of $\text{he}_n$ in $\beta$. Thomason's reformulation has the effect that the unbound logical variables $x_n$ in $\beta'$ do not occur free in the translation of the whole construction, whereas the same variables in $\alpha$ remain unbound. Nevertheless, Thomason's reformulation does not obey the variable principle since in the syntax occurrences of $\text{he}_n$ in $\alpha$ are removed, whereas in the translation the occurrences of the corresponding variable (i.e.$x_n$) are not bound, but of a variable $x_m$ (where $n \neq m$).

Another kind of objection against Thomason's rule is that it is not a polynomial over IL. This objection was considered in chapter 5, section 3.4. The formulation proposed there for the translation rule is the following.  

$T_{3,n} \quad \lambda p [\lambda x_n [\gamma p(x_n) \land \beta'](\alpha')]$.  

This formulation has as a consequence that only those occurrences of $x_n$ are bound, of which the syntactic counterparts are removed in $S_{3,n}$.  

6.2. The S-S analysis for Hittite

Is an analysis of Hittite relative clause constructions possible which on the one hand satisfies the variable principle, and on the other hand produces such a construction out of two sentences?
Below I will describe an analysis which shows that the answer is affirmative. I will only deal with the example discussed in Section 3, and not with all other cases of Hittite relative clauses which are treated by COOPER (1975). My analysis is intended mainly as an illustration of the kinds of technique which are available if one obeys the variable principle.

The treatment described in Section 3 violates the variable principle because both subsentences in Figure 4 have a translation which contains an unbound variable, whereas the sentences themselves do not contain a syntactic variable. Given the principle, in both sentences there has to be an occurrence of a syntactic variable as well. The English variant of sentence S_2 gives a hint on how to do this. It contains in a CN-position the word (one) - probably added for explanatory reasons. This word suggests the introduction in the syntax of CN variables one, one', two, ... , which are translated into logical variables P_1, P_2, ... , respectively (such CN-variables are discussed in HAUSSEER (1979c)). The rule which combines S_1 with S_2 will then give rise to a translation in which (by λ-conversion) the relevant property is substituted for P_n. In case one prefers not to introduce a new constituent one, a new variable of category T might be introduced alternatively:

(31) each

(32) λx[∀x[v_p_n(x) → v_0(x)]]

The variable in the translation of the relative clause can be introduced by the translation of the determiner which. Therefore, the category of determiners (which contains the Hittite version of every, etc.) is extended with a variable (33), translating as (34).

(33) which

(34) λxλP[v_0(z_n) ∧ v_p_n(z_n)]

We have to combine a relative clause containing a free variable z_n with a main sentence containing a free variable P_n. This can be done by means of a single rule binding both logical variables and performing the relevant operations on both syntactic variables, or by means of two rules, each dealing with one variable at a time. The former method would yield the tree from figure 4, but it would implicate that a new kind of rules is introduced (rules with two indices). I will follow the two-rules approach.

First the relative clause is transformed into an expression of the new
category Prop (=τ/ε), being a set of expressions denoting properties. We do this by means of the following rule (the numbers in the 800-series are numbers of newly proposed rules).

\[ S_{801,n} \quad S \rightarrow \text{Prop} \]

\[ F_{801,n} \quad \text{Replace } \omega h \text{ in } a \text{ by } \omega h \]

\[ T_{801,n} \quad \lambda x_n[\alpha'] \]

The rule combining a property with a sentence is

\[ S_{802,n} \quad \text{Prop} \times S \rightarrow S \]

\[ F_{802,n} \quad \text{delete all occurrences of } one_n \text{ from } \beta; \text{ concatenate } (\alpha, \beta) \]

\[ T_{802,n} \quad [\lambda \beta''](\alpha') \]

Using these rules, the Bach & Cooper example is obtained in the way indicated in figure 7. Its translation is equivalent to the one given in Section 3 for figure 4. Since we assume that it is guaranteed that the variable principle is obeyed, no problems arise with the syntactic variables. The principle guarantees that rule \( S_{802,1} \) is applied only in case the main sentence contains an occurrence of \( one_j \) and that rule \( S_{801,2} \) is applied only when the sentence contains an occurrence of the variable \( \omega h \). Furthermore, it guarantees that all syntactic variables finally will have disappeared.

![Diagram](image)

**Figure 7**

6.3. **The T-S analysis for English**

As shown in Section 6.2, an S-S analysis can be obtained simply by introducing a variable in the syntax, when such a variable is required in
the translation. The same idea can be used to obtain a T-S analysis for relative clauses. In this case, we need a variable of the category Prop, written as \( \text{of kind}_n \). It translates into the variable \( k_n \).

A property and a common noun phrase combine to a new common noun phrase as follows:

\[
\begin{align*}
S_{803} & \quad \text{CN} \times \text{Prop} \to \text{CN} \\
F_{803} & \quad \text{concatenate} \ (a, b) \\
T_{803} & \quad \lambda y[a'(y) \land b'(y)].
\end{align*}
\]

A category RC of relative clauses (RC = t/int) is introduced because RC's and Prop's will occur in different positions. The expressions of the category RC are made out of sentences as follows:

\[
\begin{align*}
S_{804,n} & \quad S \to \text{RC} \\
F_{804,n} & \quad \text{delete the index } n \text{ from all pronouns in } a; \\
& \quad \text{concatenate } (\text{such that}, a) \\
T_{804,n} & \quad \lambda x_n [a'].
\end{align*}
\]

A relative clause may be quantified into a term phrase by substituting the relative clause for a property variable:

\[
\begin{align*}
S_{805,n} & \quad T \times \text{RC} \to T \\
F_{805,n} & \quad \text{substitute } b \text{ for of-kind}_n \text{ in } a \\
T_{805,n} & \quad \lambda x_n [a'(b')] \\
\end{align*}
\]

An example of a production using these rules is given in figure 8.

![Figure 8](image)

The translation of the lower term phrase in figure 8 is (35), the translation of the RC phrase (36), and of the upper term phrase (after
(35) \( \lambda\forall x \exists y [ \text{boy}(x) \land \forall y_3(x) \rightarrow \forall y_4(x)] \)

(36) \( \lambda x_2 [ \text{run}(x_2)] \)

(37) \( \lambda\forall x [ \text{boy}(x) \land \text{run}(x) \rightarrow \forall y_4(x)] \).

Note that the intermediate stage of an RC is not required if \( S_{805} \) is a double indexed rule, dealing both with \( h_{e_n} \) and \( v_{\text{kind} m'} \).

6.4. The Det-S analysis for English

Is a Det-S analysis possible which obeys the variable principle? Recalling the pattern underlying the S-S and T-S analyses, one might try to find such an analysis as a variant of the CN-S analysis by introducing new variables. It appeared, to my surprise, that it is possible to obtain a Det-S analysis which is not a variant of the CN-S analysis, but which is a pure Det-S analysis (recall the proviso by Partee for her argumentation concerning the Det-S analysis). I will not discuss the heuristics of this analysis, but present the rules immediately.

\( S_{806,n} \)  \( \text{Det} \times S \rightarrow \text{Det} \)

\( F_{806,n} \)  remove all indices \( n \) from pronouns in \( \beta \); concatenate (\( \alpha', \text{such that}, \beta \))

\( T_{806,n} \)  \( \lambda R \alpha' (\lambda y [\forall R(y) \land \lambda x_4 [\beta^f(y)]]) \).

Maybe the following explanation of the translation is useful. A determiner \( \delta \) is, semantically, a function which takes as argument the property \( \eta \) expressed by a noun and delivers a collection of properties which have a certain relation with \( \eta \). \( S_{806} \) produces a determiner which takes a noun property \( \eta \) and delivers a set of properties which has that relation with the conjunction of \( \eta \) and the property expressed by the relative clause.

The combination of a CN with a Det-phrase, requires that the CN is placed at a suitable position in the determiner phrase. In the present fragment this position is the second position (if we had determiners like \( \text{all} \, \text{thea} \), then also other positions might under certain circumstances be suitable). The rule for this reads as follows:

\( S_{807} \)  \( \text{Det} \times \text{CN} \rightarrow \text{CN} \)

\( F_{807} \)  insert \( \beta \) after the first word of \( \alpha \)

\( T_{807} \)  \( \alpha' (\beta^f) \).
The combination of the determiner *every* with the sentence *he runs* yields determiner (38), with (39) as unreduced, and (40) as reduced translation.

(38) *every such that he runs*

(39) \( \lambda R \forall x \forall y \forall z \forall \varphi (x) \to \varphi (x) \) \( \lambda y \forall \varphi (y) \land \forall x \exists z [ \text{run}(z) ] (y) \) \( \lambda x \forall \varphi (x) \).

(40) \( \lambda R \forall x \forall y \forall z \forall \varphi (x) \land \text{run}(x) \to \varphi (x) \).

The combination of (38) the common noun *man* yields the term phrase (41), which has the (usual) reduced translation (42).

(41) *every man such that he runs*

(42) \( \lambda M \forall \varphi (x) \land \text{run}(x) \to \varphi (x) \).

The techniques which are used to obtain a T-S analysis from a CN-S analysis can be used as well to obtain a T-S analysis which is a variant of the Det-S analysis: introduce in the Det-S analysis the variable \( of^{kind_n} \), but now within the determiner. This means that at least two kinds of T-S analyses are available.

6.5. Conclusion

In Section 5 a new principle was introduced: the variable principle. Obeying this principle we designed rules for relative clause constructions. It turned out that for English besides the CN-S analysis both the T-S and the Det-S analysis are possible in at least two essentially different variants. And for Hittite an S-S analysis is possible. So at the present stage of our investigations a negative answer to the thematic question has to be given: several analyses of relative clauses are possible.

Consider the T-S analysis of 6.3 again. Is it the kind of T-S analysis meant by Partee? I do not think so. At a certain level we indeed have a T-S analysis, but on another level in the production tree there is a CN-Prop analysis which is nothing but a variant of the CN-S analysis. The opposition between the two analyses was, however, the main point in the discussion of ParTee (1973). So one could say that her conclusion that the pure T-S analysis cannot be used, in a certain sense still holds. For the case of Hittite however, the discussion primarily aimed at obtaining an S-S analysis at some level, rather than at avoiding the CN-S analysis on all levels. In Section 2 I quoted Bach & Cooper who expressed the hope for the 'happy discovery of yet unknown principles' which exclude the
T-S-analysis, but allow for the S-S-analysis. It seems reasonable to interpret this as the desire for a principle which prohibits the pure T-S analysis, but allows some variant of the S-S analysis. The variable principle has such an effect. But if it is interpreted as the hope for a principle which excludes all kinds of T-S analyses, or which allows a pure S-S analysis, then the variable principle is not such a principle. So the answer to the thematic question I gave above, has to be relativized: although several analyses are available, not all analyses are possible.

The answer to the thematic question obtained in this section, was based upon an investigation of the relative clause construction as such. Interaction with other phenomena was not taken into consideration. In the next section I will leave this isolation and consider the interaction of relative clause formation with some other phenomena.

7. OTHER ARGUMENTS

7.1. Syntax: gender agreement

The relative pronoun has to agree in gender with the antecedent noun-phrase. In the Det-S analysis, this poses a problem. The rule which combines a determiner with a relative clause has to specify what is to be done with the syntactic variable. The formulation I gave of rule $S_{806,n}$ just deletes the index, so it gives a correct result if the noun has male gender. But in the same way as we produced every boy such that he runs, we may produce every girl such that he runs. It is not possible to formulate $S_{806}$ in such a way that this kind of ill-formedness is avoided, because the information which gender the noun has, is not available at the stage at which the determiner and the relative clause are combined. Not removing the index would, according to the variable principle, require a free variable in the translation of the term phrase; but I do not see how this approach might work.

The T-S analysis gives rise to a similar problem. The rule which makes the relative clause (RC) out of a sentence ($S$), has to specify what has to be done with $he_n$. The formulation I gave of $S_{804}$ works correctly for masculine nouns only. Again, information about the gender of the noun is not yet available, and not removing the index would constitute a break with the principle. This argument does not apply to the T-S analysis in which a double indexed rule is used. In the CN-S analysis, no problems arise from
gender agreement, since at the stage at which the index has to be removed, the gender of the noun is known.

One should not conclude from this discussion that it is impossible to obtain correct gender agreement in case of the Det-S or T-S analysis under discussion. I expect that it can be done by means of further subcategorization. One has to distinguish feminine, masculine, and neuter relative clauses, and feminine, masculine, and neuter determiners, and probably one needs to make similar distinctions in other categories. Then the subcategory system provides the information needed to obtain precisely the correct combinations of relative clause, determiner and noun.

There is the hidden assumption in this discussion that gender agreement has to be handled within the syntax. If we do not assume this, then a phrase as a girl such that he runs, is no longer considered to be syntactically ill-formed. COOPER (1975) argues in favor of dealing with gender in the semantics (at least for English). Others might prefer to handle gender in pragmatics (Karttunen, according to PARTEE (1979a)). Then the arguments given here are no longer relevant. But in languages with grammatical gender (e.g. Dutch, German), this escape is not available. Here one might adopt one of the solutions I mentioned: refined subcategorization, a T-S analysis with a double indexed rule, or simply the CN-S analysis for relative clauses.

7.2. Semantics: scope

Consider the following sentence (exhibiting stacking on the head man):

(43) Every man such that he loves a girl such that he kisses her is happy.

This sentence has a possible reading in which every has wider scope than a. In a PTQ like approach (so with the CN-S construction for relative clauses), this reading is obtained by quantification of a girl into the CN phrase

(44) man such that he loves him\textsubscript{n} such that he kisses him\textsubscript{n}.

The corresponding translation of the sentence (44) reduces to

(45) ∀y[∃x[girl(x) ∧ man(y) ∧ love\textsubscript{x}(y, x) ∧ kiss\textsubscript{x}(y, x)] → happy(y)].

Can this reading be obtained in other analyses of relative clauses?

In the T-S analysis this stacking of relative clauses can be obtained by means of a process indicated in figure 5. In order to obtain coreferentiality between both occurrences of the term him\textsubscript{n}, the term a girl has
to be substituted at a stage in which both relative clauses are present. The earliest moment at which this is the case, is immediately after the uppermost term has been formed. Using a rule analogous to the standard quantification rules would assign the existential quantifier wider scope than the universal quantifier, thus not yielding the desired reading. So it seems to be impossible to obtain in such a T-S analysis coreferentiality and correct scope at the same time.

Figure 9

In the Det-S analysis the earliest stage at which the coreferentiality of she and a girl can be accounted for, is when the determiner phrase (46) has been formed.

(46) *every such that he loves him₃ such that he kisses him₃.*

Some later stage (e.g. the term level), might be selected as well. But in all these options, the quantification rule would give wider scope to a than to *every*, thus not yielding the desired reading.

Underlying this discussion is the assumption that there is something like stacking of relative clauses. If there is stacking, then the rule for quantification into a CN is essential for the PTQ fragment (FRIEDMAN & WARREN (1979b)). But is stacking indeed a phenomenon of natural language? As for Hittite, BACH & COOPER (1975) inform us that no stacking occurs, and in Dutch and German stacking is not possible. As for English, no author expresses doubts, except for PARTEE (1979b). She states that the evidence for stacking is spurious. If we accept this, it would leave a rather small basis for our argumentation concerning an answer on the thematic question.

There is another phenomenon, however, that requires quantification
into CN's. It might be the kind of examples meant by PARTEE (1975, p.236).

Example (47) assumes that there are common nouns in the fragment of the form friend of.

(47) Every picture of a woman which is owned by a man who loves her is a valuable object.

Here the intended reading is the one in which every has wider scope than a, and in which there is coreferentiality between a woman and her. This reading can easily be obtained by means of substitution of a woman into the CN-phrase (48).

(48) picture of he₁ such that it is owned by a man such that he loves him₁.

So even if we do not accept stacking as a phenomenon of English, a CN-S analysis would be required.

It is remarkable to notice that the variable principle plays no role in the discussion concerning scope. The occurrences of the Prop variables, which form a practical consequence of the principle, were not relevant. If they were omitted, which would bring us back to the original Bach & Cooper approach, then still the same problems would arise with respect to scope. So even without the variable principle a CN-S analysis appears to be required. This conclusion has to be relativized immediately. I have not given a formal proof that it is impossible to obtain a correct treatment of scope in the other analyses. I just showed that the CN-S analysis provides a direct basis for a semantic treatment of scope phenomena in a way that the considered T-S and Det-S analyses can not. This conclusion mentions another argument for relativizing. We only considered the three analyses which had our main interest. A lot more analyses are possible, and for some a correct treatment of scope may be possible. For instance, a correct treatment of scope might be possible if the category of determiners contains variables for which a determiner can be substituted in a later stage.

7.3. Conclusion

In the previous section we observed that the framework of Montague grammar hardly restricts the possible syntactic analyses of relative clauses. In this section we investigated the possibilities for incorporating the available options in a somewhat larger fragment. It turned out that from the three main options only one was suitable. From this we learn that it is important to consider phenomena not only in isolation, but to design
grammars for larger fragments. The fact that for each isolated phenomenon there are many syntactic options available, gives us a firm basis for the hope that it is indeed possible to find a combination of syntactic constructions that fits together in a system yielding the correct semantics for the constructions involved. Thus we see that extending fragments is a fruitful step which has impact on the description of isolated phenomena. This can be considered as a reaction to be a remark Van Berthem (1981, p.31) who denies the use of generalization and the combination of partial theories.

8. THE GENERAL QUESTION

In this section I answer the general version of the thematic question. We employ a framework in which the syntax and semantics have to be algebras, and in which meaning assignment is a homomorphism. The general version of the thematic question was to what extent this organization of the grammar restricts the options we have available for describing a particular phenomenon in the syntax.

For the special case of relative clause formation we obtained in section 6 the answer that any kind of analysis can be obtained, but that certain kinds of analysis cannot be avoided. This practical result will be explained below on the basis of the algebraic properties of the framework, and the result will be generalized to an answer on the general question.

Let us suppose that we have found a semantic operation $T_{888}$ which takes two arguments, and delivers the meaning of a certain construction. So in the semantics we have the construction step $T_{888}(\alpha',\beta')$. Due to the homomorphism relation, there has to be a corresponding operation $F_{888}(\alpha,\beta)$ in the syntax, and the two semantic arguments have to correspond with the two syntactic arguments. Instead of the semantic step $T_{888}(\alpha',\beta')$, several variants are possible, each with its own consequences for the syntax. These variants amount to a construction process with two stages. We may first have $T_{888}(\alpha',R)$, where $R$ is a variable, and introduce in a later stage a $\lambda$-operator for $R$ taking $\beta'$ as argument:

$$\lambda R[...T_{888}(\alpha',R)...](\beta').$$

This means that the syntactic expression $\beta$ can be introduced in an arbitrary later stage of the syntactic production process. Consequently, a lot of variants of the original syntactic construction can be formed. These variants
are based on the use of the construction step $T_{888} (α', R)$ in the logic. Due to the variable principle, the variable $R$ has to be introduced by the translation of some syntactic variable. Let us suppose that $ν$ is such a variable. Due to the homomorphic relation between syntax and semantics, this means that in the syntax there has to be a step $F_{888} (α, ν)$. So whereas we have gained the freedom to introduce $β$ in a later stage of the syntactic construction process, step $F_{888}$ is not avoided. The same argumentation applies when the first argument of $T_{888}$ is replaced by a variable. It is even possible to replace both arguments by a variable, thus obtaining a large freedom in the syntax concerning the stage at which $α$ and $β$ are introduced. But in all these variants $F_{888}$ is not avoided. Application of this argumentation to the case of relative clauses (where two basic constructions are found) means that we cannot avoid both the CN-S and the Det-S construction at the same time.

So on the basis of the compositionality principle, formalized in an algebraic way, many relative clause constructions are possible. This is due to the power of $λ$-abstraction. This operation makes it possible that on the semantic side the effect is obtained of substituting the translation of one argument on a suitable position within the other argument, whereas in the syntax a completely different operation is performed. Referring to this power Partee once said 'Lambdas really changed my life' (Lecture for the Dutch Association for Logic, Amsterdam, 1980).

The above argumentation is not completely compelling: there is (at least) one exception to the claim that it is not possible to make a variant of a given semantic construction which avoids the corresponding syntactic construction step. An example of such an exception arose in the S-S analysis for Hittite. In the main sentence we had the Det-CN construction $each_{ν′}$, where $one_{ν}$ was a variable. We obtained a variant in which there is no Det-CN construction: the logical variable introduced by $one_{ν}$, could be introduced by a new variable $each_{ν}$ (see (34)). The algebraic description of this method is as follows. Consider again $T_{888} (α', R)$. The variable $R$ might, under certain circumstances, be introduced by the translation of $α$, thus allowing to replace $T_{888}$ by a related semantic operation which takes only one argument. That the translation of $α$ introduced the variable $R$, means that in the syntax $α$ is to be replaced by some variable, say an indexed variant of $α$. Its translation is then a compound expression (being a combination of the old translation $α'$ with the variable $R$). This process,
which avoids to have \( F_{888} \) in the syntax, is possible only if \( a \) is a single
word with a translation which does not contain a constant (e.g. if \( a \) is a
determiner). If the translation of \( a \) would contain a constant, then require-
ment 1a) of the variable principle would prohibit that its translation in-
troduces a variable. If \( a \) is not a single word, then it cannot be replaced
by a syntactic variable (maybe one of its parts can then be indexed). This
method of creating exceptions would be prohibited when requirement 1a) of
the variable principle would be replaced by the more restrictive version
1a'). In order to prove that the exception described here is the only one
by which a given analysis can be avoided, the details of the relation be-
tween operations in the semantics or in the syntax have to be formalized
algebraically (see also Section 3).

These algebraic considerations explain the results of our practical
work. On the basis of these considerations it would be possible to explain
that a Det-S analysis which is variant of the CN-S analysis, is not to be
expected (in any case the described method for obtaining variants does not
work). The algebraic considerations also answer the general question whether
the principle of compositionality restricts the options available for
descriptive work. On the basis of a given construction step, a lot of
variants are possible, but due to the variable principle and the homomorphic
relation between syntax and semantics, this construction step cannot be
avoided in these variants. So the answer to the general question is that
there are indeed restrictions on the syntactic possibilities, but only in
the sense that a basic step cannot, generally speaking, be avoided. But
these restrictions are not that strong that only a single analysis is pos-
sible. Formal proofs for these considerations would require, as I said be-
fore, a further algebraization of the syntax.