WEAK vs. STRONG READINGS OF DONKEY SENTENCES AND MONOTONICITY INFERENCE IN A DYNAMIC SETTING

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Weak vs. Strong Readings of Donkey Sentences and Monotonicity Inference in a Dynamic Setting*

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Abstract

In this paper, I show that the availability of what some authors have called the weak reading and the strong reading of donkey sentences with relative clauses is systematically related to monotonicity properties of the determiner. The correlation is different from what has been observed in the literature in that it concerns not only right monotonicity, but also left monotonicity (persistence/antipersistence). I claim that the reading selected by a donkey sentence with a double monotone determiner is in fact the one that validates inference based on the left monotonicity of the determiner. This accounts for the lack of strong reading in donkey sentences with ↑MON↑ determiners, which have been neglected in the literature. I consider the relevance of other natural forms of inference as well, but also suggest how monotonicity inference might play a central role in the actual process of interpretation. The formal theory is couched in dynamic predicate logic with generalized quantifiers.

1 Introduction

Of the two types of donkey sentences, I confine myself to donkey sentences with determiners and relative clauses, which I will simply call ‘donkey sentences’ in what follows. (1)–(3) are standard examples involving farmers and donkeys, which share the general form (4) (Q stands for a quantificational determiner):

(1) Every farmer who owns a donkey beats it.
(2) No farmer who owns a donkey beats it.
(3) Most farmers who own a donkey beat it.

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(4) $Q$ farmer who owns a donkey beats it.

For convenience, let us use (4) as the general form of donkey sentences with relative clauses in the following preliminary discussion. Of course, the choice of specific lexical items farmer, own, donkey, and beat is not important.

Two questions about (4) concern us in this paper:

(Q1) What is the correct descriptive generalization about the interpretation of sentences of the form (4)?

(Q2) Why do sentences of the form (4) mean what they mean?

Previous treatments of donkey sentences have generally regarded them as a challenge to compositional semantics; the main problem has been to devise appropriate semantic rules for indefinites, quantifiers, etc., that assign donkey sentences the interpretation that they are supposed to have. However, even as a descriptive issue, the truth conditions of sentences of the form (4) have not been given a fully general, satisfactory characterization. Our first question (Q1) addresses this problem. To give a definitive answer to it is not a simple matter, as even native speakers’ intuitions about specific donkey sentences are often less than clear-cut; nevertheless, an interesting generalization does seem to emerge.

Our second question (Q2) seeks an explanation for the answer given to the first. It is not the sort of question one usually asks in semantics, but in the context of donkey sentences, it turns out to be an interesting and important one, as we shall see below.

In the literature, one can recognize at least four proposals as to the interpretation of sentences of the form (4):

(5) E-type reading:
    $Q$ farmer who owns a donkey beats the donkey he owns.

(6) Pair quantification reading:
    $Q \{ \langle x, y \rangle \mid \text{farmer}(x) \land \text{donkey}(y) \land \text{own}(x, y) \} \{ \langle x, y \rangle \mid \text{beat}(x, y) \}$.

(7) Weak reading:
    $Q$ farmer who owns a donkey beats a donkey he owns.

(8) Strong reading:
    $Q$ farmer who owns a donkey beats every donkey he owns.

Note that these four ‘readings’ do not always conflict with each other. Depending on the choice of $Q$ and the type of situation under consideration, they may yield the same truth conditions/truth value.

The E-type reading (5) is the interpretation assigned to (4) by the approach originally proposed by Evans (1977). It is intuitively correct when the uniqueness condition for the donkey pronoun is met, i.e., when each farmer who owns a donkey owns no more than one donkey. Otherwise, it often yields counterintuitive
results.¹ Although Kadmon (1990) strongly argues for the presence of uniqueness presupposition in sentences like (1) and (3), i.e., the presupposition to the effect that the uniqueness condition is satisfied, her claim that donkey sentences with ‘asymmetric quantification’ generally come with such a uniqueness presupposition seems to have gained relatively little support.²

The pair quantification reading (6) is the interpretation assigned to (4) by the classical DRT/file change semantics (Kamp 1981, Heim 1982). Here, the quantifier Q counts farmer-donkey pairs: (6) is true if and only if Q holds of the set consisting of farmer-donkey pairs which stand in the owning relation and the set consisting of pairs which stand in the beating relation. This seems to give the correct truth conditions for sentences like (1) and (2), but, as is well-known, it does not work with (3). Consider the following situation:

(9)  
\[
\begin{align*}
[farmer] &= \{f_1, f_2\} \\
[donkey] &= \{d_1, d_2, d_3\} \\
[own] &= \{\langle f_1, d_1 \rangle, \langle f_1, d_2 \rangle, \langle f_2, d_3 \rangle\} \\
[beat] &= \{\langle f_1, d_1 \rangle, \langle f_1, d_2 \rangle\}
\end{align*}
\]

In this model, there are three farmer-donkey pairs that stand in the owning relation, and two of them, the majority, stand in the beating relation. However, (3) is intuitively false in such a situation. In the literature, this is known as the ‘proportion problem’ for the classical DRT/file change semantics account of quantification and indefinites.³

In fact, the term ‘proportion problem’ is a misnomer, as it suggests that the problem is limited to donkey sentences with proportional quantifiers like most.⁴ On the contrary, the problem is quite general. The following sentence, involving a cardinality quantifier at least two, is intuitively false in the above model (9), although the pair quantification reading would render it true:

(10) At least two farmers who own a donkey beat it.

Here, we might say that we have a ‘cardinality problem’. As we see below, almost all quantifiers give rise to a problem of this sort.

The interpretations (7) and (8) have been recognized by various authors, including Rooth (1987), Schubert and Pelletier (1989), Heim (1990), and Chierchia (1990, 1992). The terms ‘weak reading’ and ‘strong reading’ are apparently due

¹Depending on how the phrase ‘the donkey he owns’ is analyzed, the E-type reading assigns (4) either a counterintuitive truth value or no truth value at all when the uniqueness condition is not met.
²In addition to Heim’s (1982) celebrated sage plant example, which is treated by Kadmon, donkey sentences with no like (2), and sentences like those considered by Schubert and Pelletier (1989) are noteworthy counterexamples. See the references cited above and Rooth (1987), Heim (1990), Neale (1990), and Chierchia (1992).
³See, for example, Rooth (1987), Heim (1990), or Kadmon (1987, 1990).
⁴The term was coined by Kadmon (1987, 1990), and has since become standard.
to Chierchia (1990). The weak reading (7) seems to give the intuitively correct truth conditions for (2) and (10). (2) and (10) can be paraphrased as ‘No farmer who owns a donkey beats a donkey he owns’ and ‘At least two farmers who own a donkey beat a donkey they own’, respectively. On the other hand, the truth conditions standardly associated with (1) are given by the strong reading ‘Every farmer who owns a donkey beats every donkey he owns’. Note that when $Q$ is no, the weak reading is equivalent to the pair quantification reading, and when $Q$ is every, the strong reading is.

Turning to (3), we see that neither the weak reading (11a) nor the strong reading (11b) is clearly the right interpretation for this sentence.

(11) a. Most farmers who own a donkey beat a donkey they own.

   b. Most farmers who own a donkey beat every donkey they own.

Heim (1982) reports that her intuitions ‘vacillate’ between the two readings. Rooth (1987) is ‘simply not sure’ whether (3) means (11a) or (11b), or even

(12) Most farmers who own a donkey beat most of the donkeys they own,

and his informants ‘have not expressed strong or consistent opinions.’ According to Kamp (1991), (3) ‘makes clear sense [to many speakers] only in circumstances in which no farmer owns more than one donkey’.6

In contrast to (3), however, intuitions about the following sentences with most seem fairly clear:7

(13) Most people that owned a slave also owned his offspring.

(14) Most men that have a quarter will put it in the parking meter.

(15) Most men that have a nice suit will wear it to church tomorrow.

The natural interpretation of (13) seems to be the strong reading, i.e., ‘Most people that owned a slave owned the offspring of every slave they owned’. For the other two examples, the weak reading seems to give the right interpretation. (14) seems to say that most men that have a quarter will put (at least) one of their quarters in the parking meter, and (15) seems to say that most men that have a nice suit will wear one of their nice suits to church tomorrow.

Perhaps donkey sentences with most are all vague or ambiguous, and pragmatics selects the most sensible interpretation. Whether or not this is so, the

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5The terms are actually misleading, since when the determiner is downward monotone in the second argument, as is the case with (2), the weak reading is the logically stronger of the two. Apparently for this reason, Chierchia (1992) switches to ‘3-reading’ and ‘4-reading’. I do not adopt this terminology because of the difficulty in pronunciation. Van Eijck and de Vries (1992) and de Swart (1991) also use ‘strong reading’ and ‘weak reading’.

6As has been mentioned, Kadmon (1987, 1990) strongly argues for the presence of uniqueness presupposition in (3).

7(13) is originally Heim’s (1990) example. (14) is modeled on a sentence used by Schubert and Pelletier (1989). (15) is a slight modification of an example from Chierchia (1990).
point here is that intuitions about clear cases like (13), (14), and (15) are captured by either the weak reading or the strong reading.

It is important to observe that, unlike the other two readings, the weak reading and the strong reading have the virtue of accounting for the clear intuitions people seem to have about donkey sentences with respect to a certain class of situations. These are situations where each farmer beats either every donkey he owns or no donkey he owns. The relevant observation is explicitly stated in Rooth (1987): ‘people have firm intuitions about situations where farmers are consistent about their donkey-beating.’ In such situations, the weak reading and the strong reading turn out to be equivalent, and they seem to match the intuitions. For example, (9) is an instance of a consistent donkey-beating situation. Even though the exact truth conditions of (3) may be unclear, it is certainly false in (9), and this is the truth value assigned to it by both the weak reading and the strong reading in this situation. In general, we have the following

(16) Empirical observation: People’s intuitions about donkey sentences with respect to consistent donkey-beating situations accord with the truth conditions given by the weak reading and the strong reading.\(^8\)

The E-type reading can account for only a small subset of the consistent donkey-beating situations, namely, situations where the uniqueness condition is met. Otherwise, it diverges from both the weak reading and the strong reading, and consequently fails to capture (16).

The pair quantification reading goes against (16) in another dimension. It can capture (16) only with respect to a small set of determiners. As we have seen, the pair quantification reading is equivalent to the strong reading if \(Q\) is every, and to the weak reading if \(Q\) is no, so it conforms to (15) as far as donkey sentences with determiners like every and no are concerned. However, consistent donkey-beating situations like (9) show that this is not the case with donkey sentences with determiners like most and at least two. In fact, the ‘proportion problem’ is intimately connected to the observation (16), as it is typically illustrated by the unintuitive truth value that the pair quantification reading yields in consistent donkey-beating situations. It is the pair quantification reading’s failure to capture the intuitions stated in (16) that is responsible for the proportion problem. As was anticipated above, determiners like every and no that do not give rise to the proportion problem are among a few exceptions. It is easy to prove that the pair quantification reading contradicts (16) with respect to all quantifiers \(Q\) satisfying Permutation Invariance, Conservativity, and Extension (see van Benthem 1986 or

\[^{8}\text{It is worth pointing out that for some sentences, every situation is a consistent donkey-beating situation. Such is the case with a sage-plant example like (i) and a sentence like (ii):}

(i) Most men who bought a beer bought five others along with it.

(ii) Most farmers who own exactly one donkey beat it.

(For (ii), every situation satisfies the uniqueness condition.) Intuitions about these sentences seem accordingly robust.\]
Westerståhl 1989) except those that can be expressed as Boolean combinations of some and every.\textsuperscript{9}

To sum up the preceding discussion, none of the four putative readings (5)–(8) can account for all cases. The E-type reading (5) and the pair quantification reading (6) are only of a limited use in describing the interpretation of donkey sentences, as they fail to capture the important observation (16) in full generality. When intuitions are clear about the truth conditions of donkey sentences, they seem to always accord with either the weak reading or the strong reading.

Thus, let us henceforth disregard the E-type reading and the pair quantification reading, and assume the following:

\begin{equation}
(17) \text{ The interpretation of a donkey sentence is given by either the weak reading or the strong reading.}
\end{equation}

Our question (Q1) now becomes the following:\textsuperscript{10}

\begin{equation}
(Q1') \text{ How are the weak reading and the strong reading distributed among different donkey sentences? What determines which reading is the right one in specific cases?}
\end{equation}

\section{Factors That Determine the Interpretation}

In this section, we identify and discuss some possible factors that are relevant to the interpretation of donkey sentences. First, as should be clear from the examples in Section 1, the choice of determiner is a particularly important factor (Section 2.1). In Section 2.2, I claim that, contra Schubert and Pelletier (1989), genericity has little to do with the availability of the strong reading. In Section 2.3, we briefly discuss other possible factors.

\subsection{Choice of Determiner}

The choice of determiner is probably the most important factor that affects the possible reading(s) of donkey sentences. The only difference among (1), (2), (3), and (10) was the determiner.

\begin{equation}
(1) \text{ Every farmer who owns a donkey beats it.}
\end{equation}

\begin{equation}
(2) \text{ No farmer who owns a donkey beats it.}
\end{equation}

\begin{equation}
(3) \text{ Most farmers who own a donkey beat it.}
\end{equation}

\begin{equation}
(10) \text{ At least two farmers who own a donkey beat it.}
\end{equation}

\textsuperscript{9}Namely, some, every, no, not every, some but not every, either no or every, etc. Up to equivalence, there are only 16 of them.

\textsuperscript{10}In this form, the question has also been asked by Heim (1990), Chierchia (1992), and Gawron, Nerbonne, and Peters (1991).
As we have seen, (1) seems to have the strong reading, and (2) and (10) have the weak reading, while (3) is vague or perhaps ambiguous. Let us look at a wider range of data to see the effects of different determiners.

2.1.1 Every

First, let us consider sentences with *every*. The first thing to note is that, although the strong reading is the one standardly ascribed to (1), people’s intuitions about (1) seem to be a bit shaky.\textsuperscript{11} Chierchia (1990, 1992) even claims that (1) can have the weak reading as well in appropriate contexts.

The following example, modeled on one of Geach’s (1965), clearly has the strong reading:

(18) Every student who borrowed a book from Peter eventually returned it.

(18) is false if there is one book some student borrowed from Peter but didn’t return. Likewise, the following sentence is false if there is one girl in the neighborhood who has a younger brother taller than her, even if she is taller than her other younger brothers.

(19) Every girl in this neighborhood who has a younger brother is taller than him.

Not all examples with *every*, however, take the strong reading. The following example is modeled on one in Schubert and Pelletier 1989 and is now standard:

(20) Every man who had a quarter put it in the parking meter.

The natural interpretation of (20) is the weak reading.\textsuperscript{12}

In Section 1, we saw that the interpretations of examples with *most* vary: some have the weak reading (e.g., (14) and (15)), and others the strong (e.g., (13)), while there are some unclear cases as well (e.g., (3)). The examples we just saw above show that there is a similar variation among donkey sentences with *every*. As compared to *most*, however, *every* seems to have a stronger bias toward the strong reading. Consider the following minimal pair:

(21) Every student who took a course from Peter last year liked it.

\textsuperscript{11}A couple of my informants gave me mixed judgments about (1), while another judged the the sentence to have the strong reading. Rooth (1987) considers a situation in which one farmer owns ten donkeys and beats exactly nine of them, while all other farmers beat every donkey they own. He reports that ‘[i]nformants have given me varied and guarded judgments about this case’, although the strong reading dictates that (1) is false in such a situation.

\textsuperscript{12}I should perhaps mention that one native speaker has informed me that he does not get the weak reading for (20), insisting that one should instead say ‘Every man who had a quarter put one in the parking meter’. He does not entirely like (20) (or *Most men who had a quarter put it in the parking meter*) if the men have more than one quarter, and if he thinks about the fact that they have more than one. (The same speaker accepts (18) without the uniqueness condition.)
(22) Most students who took a course from Peter last year liked it.

Responses I got from native speakers indicate that while (21) clearly requires every student to like every course he or she took from Peter, (22) can be judged true even in situations where half of the students who took a course from Peter didn’t like some of the courses they took from him.\textsuperscript{13} In general, it seems that when a donkey sentence with \textit{every} is felt to have the strong reading, the interpretation of the corresponding sentence with \textit{most} tends more towards the weak reading, or at least is less clear.

Thus, compared to other determiners, \textit{every} seems to have the effect of favoring the strong reading.\textsuperscript{14}

2.1.2 \textit{No} and Rooth’s Generalization

Let us turn to sentences with \textit{no}. As for (2), there seems to be no problem about the accuracy of the weak reading, and the strong reading ‘No farmer who owns a donkey beats every donkey he owns’ is simply unavailable. (2) is clearly falsified by a single farmer-donkey pair such that the farmer owns and beats the donkey. This seems to hold generally with donkey sentences with \textit{no}. The following sentences clearly have the weak reading:\textsuperscript{15}

(23) No student who borrowed a book from Peter returned it.

(24) No parent with a son still in high school has ever lent him the car on a weeknight.

Intuitions about donkey sentences with \textit{no} are remarkably robust.\textsuperscript{16}

Rooth (1987) observes that this property of \textit{no} is shared by other determiners which are downward monotone in the second argument (MON\textsubscript{I}), like \textit{few} and \textit{at most} \textit{n}.\textsuperscript{17} The following sentences seem to have the weak reading, rather than

\textsuperscript{13}Responses from my informants did not indicate that (22) has the weak reading, however. The exact truth conditions of (22) seem unclear.

\textsuperscript{14}My sense is that donkey sentences with \textit{every} have a default preference for the strong reading (see Sections 2.3 and 2.4 below). As an anonymous referee and the subject editors of the journal have suggested, however, such a claim is perhaps dubious, since the bare facts are that there are clear examples of both readings. Nonetheless, it seems safe to conclude that the effect of \textit{every}, \textit{relative to} other determiners like \textit{most}, \textit{no}, and \textit{at least two}, on the interpretation of donkey sentences is to make the strong reading more readily available.

\textsuperscript{15}(24) is due to Rooth (1987).

\textsuperscript{16}A sentence like the following might perhaps be an exception to this robustness of intuitions:

(i) No man who had a credit card failed to use it.

This does not seem to mean ‘No man who had a credit card had at least one credit card he failed to use’.

\textsuperscript{17}A determiner is said to be downward monotone in the second argument if its denotation \(Q_M\) in any model (with universe \(M\)) satisfies the following:

\[
\text{For all } A, B, B' \subseteq M, Q_M AB \text{ and } B' \subseteq B \text{ imply } Q_M AB'.
\]
the strong reading. (The intuition may be somewhat less clear than in the case of *no*.)

(25) Few farmers who own a donkey beat it.

(26) Few students who borrowed a book from Peter returned it.

(27) At most three farmers who own a donkey beat it.

(28) At most ten students who borrowed a book from Peter returned it.

Although Rooth does not explicitly endorse it, it is convenient for our purposes to express his observation in the form of a principle:

(29) Rooth's Generalization: Donkey sentences with a $\downarrow$MON determiner only have the weak reading.

It should be noted that the determiners that Rooth considered are all symmetric determiners which are $\downarrow$MON as well as $\uparrow$MON.$^{18}$ Therefore, it would be interesting to see how $\uparrow$MON $\downarrow$ determiners behave in donkey sentences. Unfortunately, there is no lexical $\uparrow$MON $\downarrow$ determiner in English. We have to use complex determiners for this purpose.

(30) Not every farmer who owns a donkey beats it.

(31) Not every man who had a quarter put it in the parking meter.

(32) Not all students who borrowed a book from Peter returned it.

These sentences seem to have the same type of interpretation as the corresponding sentences with *every* or *all*—that is, (30) and (32) seem to have the strong reading (‘Not every farmer who owns a donkey beats every donkey he owns’; ‘Not all students who borrowed a book from Peter returned every book they borrowed from Peter’), and (31) the weak reading (‘Not every man who had a quarter put one of their quarters in the parking meter’). To the extent that intuitions about (30) and (32) are clear, *not every* and *not all* constitute counterexamples to Rooth’s Generalization.

$^{18}$It is not entirely clear whether *few* is $\downarrow$MON; it could be argued that it has a ‘proportional’ meaning similar to *most*:

$$\text{few}AB \iff |A \cap B| < \varepsilon |A|$$

(where $\varepsilon$ is some small constant). Partee (1989) argues that *few* and *many* are in fact ambiguous between the ‘cardinal reading’ and the ‘proportional reading’.
2.1.3 MON↑ Determiners

Rooth’s Generalization does not say anything about determiners which are not MON↓. Let us now look at how MON↑ determiners behave.

*Every*, which is ↓MON↑, favors the strong reading, as we have seen, although it does not exclude the weak reading. *All* seems to behave the same way as *every*.\(^{19}\) *Most*, which is MON↑, but not monotone in the first argument, does not seem to have a clear preference for either reading. As for ↑MON↑ determiners, we have seen one example, namely, (10), which has the weak reading. A notable fact is that sentences with ↑MON↑ determiners in general have only the weak reading, just like ↓MON↓ ones.

(33) A farmer who owned a donkey beat it.
(34) A student who borrowed a book from Peter returned it.
(35) Some farmers who own a donkey beat it.
(36) Some students who borrowed a book from Peter returned it.
(37) Several farmers who own a donkey beat it.
(38) Several students who borrowed a book from Peter returned it.
(39) At least five farmers who own a donkey beat it.
(40) At least three students who borrowed a book from Peter returned it.
(41) Many farmers who own a donkey beat it.
(42) Many students who borrowed a book from Peter returned it.\(^{20}\)

(33) and (34) are not normally considered donkey sentences, since giving the indefinite *a donkey* or *a book* wide scope yields the desired reading, but there is no reason to think that the indefinite in (33) and (34) *must* take wide scope. Rather, the semantics should predict that the ‘donkey anaphora’ reading of (33) and (34) is equivalent to the wide scope existential reading. For (35)–(42), the wide scope analysis of the relevant reading is not available. The reading of (35) that we are interested in does not imply the existence of a single donkey owned by multiple farmers, and likewise for the other examples. Strangely enough, donkey sentences like these with ↑MON↑ determiners have not been discussed in the literature. It seems fairly clear that all of the above examples, except perhaps ones with *many*, lack the strong reading.

\(^{19}\)Free choice *any*, though limited in distribution, seems to show a similar behavior. *Any man who owns a donkey beats it* was Geach’s (1962) original example.

\(^{20}\)Intuitions about *many* may be less clear, but the following sentence seems to sound OK to the speakers I have consulted.

(i) Many students who borrowed a book from Peter returned it, but only a few returned every book they borrowed.

Like *few*, left monotonicity of *many* might be questioned. See footnote 18.
2.1.4 Summary

Rooth’s generalization, even if valid as far as it goes, seems to be only part of the story about the effects of determiners on the interpretation of donkey sentences. The data considered in this section suggest that it is not just the monotonicity property in the second argument, but monotonicity properties in both arguments taken together that are relevant to determining the interpretation of donkey sentences. In Table 1, I summarize the data we have seen so far about the effects of determiners.

<table>
<thead>
<tr>
<th>Interpretation</th>
<th>Determiners</th>
</tr>
</thead>
<tbody>
<tr>
<td>⌈MON⌉Weak reading only</td>
<td>a, some, several, at least n, many</td>
</tr>
<tr>
<td>⌈MON⌉Strong reading preferred?</td>
<td>not every, not all</td>
</tr>
<tr>
<td>⌈MON⌉Strong reading preferred</td>
<td>every, all, FC any</td>
</tr>
<tr>
<td>⌈MON⌉Weak reading only</td>
<td>no, few, at most n</td>
</tr>
<tr>
<td>⌉MON⌉Both</td>
<td>most</td>
</tr>
</tbody>
</table>

Table 1: Monotonicity of determiners and interpretations of donkey sentences.

2.2 Genericity

Schubert and Pelletier (1989) were the first to point out that donkey sentences like the following can have weaker truth conditions than was standardly assumed:

(43) Everyone who has a donkey must donate its services for one day during the festival.

(44) Everyone who owns a donkey will ride it to town tomorrow.

They observe that the standard strong reading (what they call the Universal Reading) is a pretty implausible reading for these sentences.\(^{21}\) They claim that the weak reading (what they call the Indefinite Lazy Reading) ‘expresses the correct truth conditions (under certain assumptions about constraints imposed by the context of utterance)’ for donkey sentences in general, and ‘although . . . there are some types of readings for donkey sentences other than the Indefinite Lazy Reading, . . . the Universal Reading is just plain wrong.’ According to them, ‘what makes it seem correct in certain cases is its confounding with a “generic” or “habitual” or “gnomic” understanding of these sentences’ (p. 201).

Their claim is two-fold: (i) only when a donkey sentence is understood ‘generically’ can it have anything other than the weak reading, and (ii) even when a donkey sentence is understood generically, the truth conditions given by the universal quantification in the strong reading is not accurate, and the actual reading involves quantification which allows exceptions, as is always the case with generic sentences. We shall see that (i) is untenable.

\(^{21}\)According to one informant, the strong reading is quite plausible for (43).
Schubert and Pelletier's claim is meant to apply to all donkey sentences, but the analysis they propose is limited to conditional donkey sentences. I think conditional donkey sentences often allow generic reading, and some standard examples, which appear to have the strong reading, are such cases. However, some conditional donkey sentences which are not understood generically do possess the strong reading. Consider the non-generic reading of the following sentence:  

(45) If I have a quarter in my pocket, I will keep it.

Also, more importantly for our purposes, there is no obvious way to extend Schubert and Pelletier's claim and analysis to donkey sentences with relative clauses, our main concern in this paper. To take (1) as an example,

(1) Every farmer who owns a donkey beats it

even though beats is in the simple present tense and expresses a habitual disposition, there is no obvious sense in which one can say that the sentence (1) itself is a generic statement. It is a universal statement, which does not allow any exceptions with respect to farmers who own a donkey. It is true that (1) can (but need not) be understood as an 'eternal' statement about all donkey-owning farmers of all times, or as a statement about all donkey-owning farmers in the 'extended present'. However, if the apparent strong reading of (1) were to be reduced to a generic reading, the generic reading would have to involve 'generic quantification' over donkeys, and it is not at all clear how Schubert and Pelletier's analysis can be extended to achieve it in a non-ad-hoc way.

At any rate, it is clear that generic understanding is out of the question for some donkey sentences with relative clauses which appear to have the strong reading. Earlier examples (18), (19), and (21) are cases in point.  

(18) Every student who borrowed a book from Peter eventually returned it.
(19) Every girl in this neighborhood who has a younger brother is taller than him.
(21) Every student who took a course from Peter last year liked it.

I conclude that genericity has little to do with the interpretation of donkey sentences with relative clauses.  

22Dekker (1993) considers a similar example and reaches the same conclusion.
23Neale (1990) also denies that the universal force associated with donkey anaphora only arises in sentences with a generic reading. An example of his is Every man who bought a donkey vaccinated it.
24Schubert and Pelletier's claim about the inaccuracy of the universal quantification in the strong reading is plausible for the generic reading of conditional donkey sentences, and might also apply to some relative clause donkey sentences. Recall that native speakers' intuition about (1) is not entirely clear (see footnote 11). However, not all relative clause donkey sentences with the strong reading are like (1). Sentences like (18), (19), and (21), as well as the following, do not seem to allow exceptions.
2.3 Other Factors

That the head determiner is not the sole factor that influences the interpretation of donkey sentences is clear from the variation we saw among sentences with most and every. I do not fully understand exactly what other factors there are. In this section, I will try to make some pertinent observations, focusing on the variation among donkey sentences with every. I proceed under the assumption that the effect of every is to favor the strong reading, and some other factors are responsible for the apparent weak reading of some donkey sentences with every.

An example like (44) has a straightforward explanation:

(44) Everyone who owns a donkey will ride it to town tomorrow.

Under normal circumstances, a person who has two or more donkeys will not ride all of them to town, so the strong reading of (44) will always be false if somebody has more than one donkey. This is probably why the sentence is felt to have the weak reading rather than the strong reading.

It is tempting to apply the same explanation to (20):

(20) Every man who had a quarter put it in the parking meter.

It would be bizarre for every man who had a quarter to put all of his quarters in the parking meter, no matter how many he had; a rational man would use just as much money as is necessary to park his car without getting a ticket. Even so, it is quite possible for the strong reading of (20) to happen to be true. Moreover, the following variations of (20) still seem to take the weak reading:

(46) Every man who had a coin put it in the charity box.
(47) Every man who had a coin put it in the slot machine.

For these sentences, the situation described by the strong reading is not so unlikely.

Compare (20), (46), and (47) with the following sentences, which clearly have the strong reading:

(48) Every man who had a quarter kept it.
(49) Every boy who had a quarter got it from his father.

Apart from the difference in pragmatic plausibility of the claim made by the two putative readings of the sentences, it seems to me that there is something about the semantic properties of the main verb that is at least partly responsible for the difference between (20), (46), and (47), on the one hand, and (48) and (49), on the other, but it is difficult to pinpoint it.

\[\text{(i) Every graduating student who has a book checked out from the library must return it by June 13 to avoid penalty.}\]
Note that, although intuitions about the weak reading of sentences like (20), (46), and (47) seem pretty clear, it is possible to make the context force the strong reading on these sentences. The following dialogue is due to David Beaver (p.c.):

(50) A: John has a silver dollar. He didn’t put it in the charity box.

B: No, everybody who had a coin put it in the box.

Here, the preceding linguistic context makes the strong reading of the donkey sentence the only sensible interpretation.

In general, when a sentence is vague or ambiguous, context can make clear what is intended, and there is no question that this principle applies to some donkey sentences.\(^{25}\) However, the interpretation of some donkey sentences with every seems to resist manipulation by context. Compare (50) with the following:

(51) A: John doesn’t have any quarters. He used all his quarters to buy a Coke.

B: No, everybody who had a quarter kept it, so he must have at least one quarter left.

Although the surrounding context demands it, it is difficult to interpret ‘everybody who had a quarter kept it’ in this dialogue in the weak sense of ‘everybody who had a quarter kept one’, and consequently the dialogue sounds odd.

To summarize this inconclusive discussion, we observed the following about donkey sentences with every: (i) the absurdity of the situation described by the strong reading gives rise to the weak reading, (ii) context can force the strong reading on sentences which otherwise take the weak reading, and (iii) some sentences, which have the strong reading out of context, are simply unhappy in contexts that require the weak reading. I take these observations to be consonant with the assumption that the default interpretation of donkey sentences with every is the strong reading.

2.4 Summary of the Data

In an area like the semantics of donkey sentences where intuitions are often less than clear-cut, a certain amount of idealization of the data is inevitable before one can try to give a theoretical account of it. With a certain bias, I summarize the facts as follows.

\(^{25}\)Gawron, Nerbonne, and Peters (1992) discuss the following example:

(i) Anyone who catches a Medfly should bring it to me.

The interpretation is supposed to be different depending on whether the speaker is ‘a biologist looking for samples on a field trip to Northern California’ (weak reading) or ‘a health department official engaged in eradicating the Medfly from Northern California’ (strong reading).
(52) i. The only possible interpretation of a donkey sentence whose Det is ▼MON▼ or ▼MON▼ is the weak reading, while the default interpretation of a donkey sentence with a ▼MON▼ or ▼MON▼ determiner is the strong reading. *Most*, which is not monotone in the first argument, favors neither the weak reading nor the strong reading.

ii. There are pragmatic and possibly other factors associated with specific donkey sentences that favor the weak reading (in particular contexts). When the default interpretation of a donkey sentence determined by the determiner is the strong reading, but those other factors favoring the weak reading are present, they may override the effect of the determiner, and the sentence can be understood in the sense of the weak reading.

I will express the correlation between the monotonicity properties of the determiners and the interpretation of donkey sentences mentioned in (52i) by saying that the interpretation is *selected* by the determiner. Thus, I assume that ▼MON▼ and ▼MON▼ determiners by default select the strong reading, and the weak readings of donkey sentences with these determiners are exceptions, triggered by idiosyncratic properties of particular sentences and contexts. This way of presenting the facts may perhaps be controversial, but it seems to be at least consistent with the data we have looked at so far (see Sections 2.1.1 and 2.3). Since there are so few ▼MON▼ and ▼MON▼ determiners, the generalization about them mentioned under (i) may be questioned.²⁶

I take (52) to be an answer to our first question (Q1), modulo the lack of good understanding of the nature of factors other than the monotonicity properties of the determiner. Our next step is to answer (Q2), that is, to give an explanation for the observed patterns summarized in (52). In doing so, I will concentrate on the aspect that has to do with the determiner, namely (52i). The main thread of our explanation will be that the interpretation that is selected is the natural one in that it shares some logical characteristics of the interpretation of ordinary quantified sentences. This takes up Sections 3 and 4.

The ‘naturalness account’ is still short of an ultimate explanation; it has no theoretical commitment and can be fleshed out in different ways. In the last part of the paper (Section 5.3.2), I will briefly describe a highly speculative model of how it is that donkey sentences get interpreted the way they are.

What to make of (52ii) will be a vexing problem in the explanations I provide. It is difficult to give a satisfying picture of how the effect of the determiner interacts with other factors. Also, the asymmetry between ▼MON▼ and ▼MON▼ determiners, on the one hand, and ▼MON▼ and ▼MON▼ determiners, on the other, calls for an explanation. Why is it that the former group of determiners infallibly selects the weak reading, while the latter group of determiners only

²⁶One might include in the list, however, complex determiners formed with all, like *all but three* (in the sense of *all but at most three*), as in *All but three farmers who own a donkey beat it*. I am not clear about the data here.
makes the strong reading the default interpretation? I will briefly touch on these two issues in Section 5.

3 Monotonicity Inference and Interpretations of Donkey Sentences

How can one explain the correlation between the monotonicity properties of determiners and the interpretations of donkey sentences, given in (52i) in the previous section? The key to answering this question is understanding of what monotonicity means in donkey sentences. This section presents the intuitive idea; a precise formulation appears in the next section.

3.1 Monotonicity Inference in Dynamic Contexts

Monotonicity is defined in general terms as follows. Let $Y(\ldots X \ldots)$ be an expression containing $X$ as a subexpression. We say that $X$ occurs in an upward (downward) monotone position in $Y(\ldots X \ldots)$, if for any two expressions $X_1$ and $X_2$ of the same category as $X$, $[X_1] \subseteq [X_2]$ implies $[Y(\ldots X_1 \ldots)] \subseteq [Y(\ldots X_2 \ldots)]$ ($[Y(\ldots X_2 \ldots)] \subseteq [Y(\ldots X_1 \ldots)]$). (We also say that $Y$ is upward (downward) monotone in the position occupied by $X$.) Here, $[X]$ stands for the interpretation of $X$, and $\subseteq$ is a suitable generalized notion of inclusion, covering material implication in the truth value $(t)$ domain and the subset relation in the $(e, t)$ domain as special cases.

Monotonicity properties of determiners are correlated with certain patterns of inference. Suppose everyone who owns a garden owns a house, so that $[\text{man who owns a garden}] \subseteq [\text{man who owns a house}]$ (the set of men who own a garden is a subset of the set of men who own a house). Then the downward monotonicity of no in the first argument licenses inference from No man who owns a house is poor to No man who owns a garden is poor. However, a naive application of monotonicity inference of this kind can fail in the presence of an anaphoric link. For example, under the same assumption, the following inference is invalid (cf. van Benthem 1987):

$$\begin{align*}
(53) & \quad \text{No man who owns a house sprinkles it} \\
& \quad \text{No man who owns a garden sprinkles it}
\end{align*}$$

Despite this kind of failure, monotonicity inference can still make good sense in donkey sentences. Let us look at the monotonicity behavior of subexpressions of N' in sentences of the form Det N' VP.

Monotonicity marking (see van Benthem 1986 for complete exposition). There is a syntactical method of computing the monotonicity property of positions within a given expression. Figure 1 is an example involving no donkey anaphora.

Explanation. If the semantics of a phrase structure rule is function application, the functor gets +.
Figure 1: Monotonicity Marking.

\[
S \rightarrow NP \ VP, \ NP \rightarrow Det \ N', \ VP \rightarrow V \ NP
\]

(In the last case, \((e, t, t)\) must ‘type-shift’ to \((e, (e, t)), (e, t)\).) In phrase structure rules with ‘conjunctive’ semantics, both daughters get +.

\[
N' \rightarrow N' + S'
\]

(The remaining rules, \(S' \rightarrow who + VP\), and \(N' \rightarrow N\) are trivial.) Also, the semantic type of certain lexical items have markings in some argument positions, and these markings get transferred to actual arguments they combine with. (For example, \(no\) has type \((e, t), ((e, t), t)\), so that the \(N'\) and the \(VP\) it combines with get −.) Finally, the global marking on a lexical item indicating its monotonicity property is computed according to the following rules, tracing the path from the root node down to the lexical node.

\[
++ = +, \ + - = -, \ - - = +.
\]

We see \(farmer\), \(owns\), and \(donkey\) in the above example all get negative marking. In the ordinary setting, this procedure of monotonicity marking is sound in the sense that positive (negative) marking implies upward (downward) monotonicity. Thus, the following inferences are valid:
(54) No farmer who owns a donkey is poor
\[\text{No farmer who owns a donkey is poor}\]

(55) No farmer who owns a donkey is poor
\[\text{No farmer who owns and feeds a donkey is poor}\]

(56) No farmer who owns a donkey is poor
\[\text{No farmer who owns a female donkey is poor}\]

These use the fact that \([X \land Y] \subseteq [X]\), etc.

Now let us look at a donkey sentence, \(\text{No farmer who owns a donkey beats it}\). If we simply apply the above procedure, the lexical markings on \text{farmer}, \text{owns}, and \text{donkey} remain the same.

\[\text{No farmer who own a donkey beats it}\]

This means that the following inferences should be valid:

(57) No farmer who owns a donkey beats it
\[\text{No young farmer who owns a donkey beats it}\]

(58) No farmer who owns a donkey beats it
\[\text{No farmer who owns and feeds a donkey beats it}\]

(59) No farmer who owns a donkey beats it
\[\text{No farmer who owns a female donkey beats it}\]

And indeed they are intuitively valid. Thus, monotonicity marking on lexical items produces the right results, even in donkey sentences.

The difference between the earlier invalid case of monotonicity inference (53) and the present valid ones can be understood as follows.\(^\text{27}\) In (53), there is a sense in which the meanings of the substituted expressions, \text{man who owns a house} and \text{man who owns a garden}, do not really stand in the appropriate \(\subseteq\) relation, because of their different ‘dynamic’ effects. When there is an anaphoric link of the donkey variety, sentences of the form \(\text{Det N' VP}\) can no longer be understood in terms of the simple scheme \([\text{Det} ([N'], [VP])]\), where the denotations \([N']\) and \([VP]\) of \(N'\) and \(VP\) are sets of individuals. In a donkey sentence, the contribution of the \(N'\) to the truth conditions of the sentence is not exhausted by the set of individuals which is ordinarily taken to be its denotation; part of the semantic function of \(N'\) is to provide, for each individual \(x\) in this set, the set of ‘witnesses’ for \(x\), as the range of possible values for the subsequent donkey pronoun. For example, in \(\text{No farmer who owns a donkey beats it}\), the \(N'\), \text{farmer who owns a donkey}, not only specifies the set of farmers who own a donkey, but also, for each

\(^{27}\text{Notice that it will not do to simply say that the antecedent of the donkey pronoun must remain intact; witness (59). Nor will it do to assume that the logicality of the understood premise of (57)–(59) is essential. In all models with \([\text{keeps}] \subseteq [\text{owns}]\), \text{No farmer who owns a donkey beats it}\) will imply \text{No farmer who keeps a donkey beats it}.\)
donkey-owning farmer, the set of donkeys owned by that farmer, from which it can pick its values. In the case of No man who owns a house sprinkles it and No man who owns a garden sprinkles it, even though the ordinary ‘static’ denotation of man who owns a garden and man who owns a house may stand in the \( \subseteq \) relation, the whole ‘dynamic’ meaning of them does not, since they specify disjoint sets of entities as the possible range of values for the subsequent donkey pronoun. In this sense, the necessary premise of monotonicity inference is not fulfilled here.

In the above valid cases of monotonicity inference (57)–(59), on the other hand, the substituted expressions (farmer/young farmer, owns/owns and feeds, donkey/female donkey), containing no noun phrase to serve as a potential antecedent of a pronoun, have no dynamic effect. Their ordinary denotation (set of/relation between individuals) simply exhausts their semantic content. Hence the necessary premise of monotonicity inference ([X\(_1\)] \( \subseteq [X_2]\)) holds, and the inference goes through. (Note that (53) cannot be thought of in terms of replacement of house by garden, because the required premise \([\text{garden}] \subseteq [\text{house}]\) is absent.)

The inferences in (57)–(59) can also be viewed as substitution of the first argument of the determiner, licensed by its left monotonicity. Thus, farmer who owns a donkey is replaced by young farmer who owns a donkey in (57), by farmer who owns and feeds a donkey in (58), and by farmer who owns a female donkey in (59). Here, unlike the earlier invalid case (53), the ‘dynamic effect’ of the substituted expressions is also being restricted. To take (58) for example, for each farmer \( x \) who owns a donkey, the set of donkeys owned and fed by \( x \) is a subset of the set of donkeys owned by \( x \), so the dynamic denotation of farmer who owns and feeds a donkey stands in the appropriate \( \subseteq \) relation to the dynamic denotation of farmer who owns a donkey.

Let us distinguish two notions of denotation, which we already made an intuitive use of in the above discussion. The static denotation of an expression is what is usually considered its denotation. If an expression is of type \((e, t)\), its static denotation is a set of individuals, if it is of type \((e, t), t\), its static denotation is a set of sets of individuals, etc. A more enriched notion of meaning is the dynamic denotation of an expression. The dynamic denotation of an expression determines its static denotation as well as the expression’s anaphoric properties. For example, the dynamic denotation \([\alpha]\) of an expression \(\alpha\) of type \((e, t)\) can be taken to be something like a binary relation consisting of pairs of the form \((x, \langle x, y_1, \ldots, y_n \rangle)\), where \(n\) is the number of indefinite noun phrases in \(\alpha\) which can potentially serve as antecedents of upcoming pronouns. The domain of this relation is the static denotation of \(\alpha\). The range is its ‘dynamic effect’. The general, precise characterization of the notion of dynamic denotation is not given here (see Section 4).

The distinction between static and dynamic denotations has repercussions for the notion of monotonicity, defined in general terms earlier in this section. If we take \([X]\), the interpretation of \(X\), to be its dynamic denotation, we get a more
restricted notion of monotonicity than the usual one where \([X]\) is taken to be static. It is the dynamic notion of monotonicity that makes sense in the presence of anaphora. The earlier non-inference (53) from \textit{No man who owns a house sprinkles it} to \textit{No man who owns a garden sprinkles it} is a failed case of static monotonicity inference, which is in general valid only in non-donkey contexts. (57)–(59) are valid instances of dynamic monotonicity inference.\(^{28}\)

### 3.2 Explanation in Terms of Monotonicity Inference

Now, what does this all have to do with our problem, the correlation between the monotonicity properties of the determiner and the interpretation of the donkey sentence? What is of crucial importance here is the fact that the dynamic monotonicity inferences based on the left monotonicity of the determiner (left monotonicity inferences, for short) are validated only by one of the two interpretations, and that interpretation coincides with the reading that we observed to be selected by the monotonicity properties of the determiner.

Thus, of the two putative interpretations of donkey sentences with \textit{no}, only the weak reading accords with the validity of the inferences in (58) and (59).\(^{29}\)

\begin{align*}
\text{(58) } & \quad \text{No farmer who owns a donkey beats it} \\
& \quad \text{No farmer who owns and feeds a donkey beats it} \\
\text{(59) } & \quad \text{No farmer who owns a donkey beats it} \\
& \quad \text{No farmer who owns a female donkey beats it}
\end{align*}

The following model would invalidate (58) and (59) if the sentences involved had the strong reading.

\begin{align*}
\text{[farmer]} & = \{\text{Pedro, Hans}\} \\
\text{[donkey]} & = \{d_1, d_2, d_3, d_4\} \\
\text{[owns]} & = \{\langle \text{Pedro, } d_1\rangle, \langle \text{Pedro, } d_2\rangle, \langle \text{Hans, } d_3\rangle, \langle \text{Hans, } d_4\rangle\} \\
\text{[beats]} & = \{\langle \text{Pedro, } d_2\rangle, \langle \text{Hans, } d_4\rangle\} \\
\text{[feeds]} & = \{\langle \text{Pedro, } d_2\rangle, \langle \text{Hans, } d_3\rangle\} \\
\text{[female]} & = \{d_1, d_4\}
\end{align*}

In this model, neither Pedro nor Hans beats every donkey he owns. However, Pedro beats every donkey he owns and feeds, and Hans beats every female donkey he owns.

\(^{28}\)Our fundamental assumption in the paper is that a donkey pronoun is like a variable. It is not considered to be a shorthand for some contextually recoverable definite description or quantifier expression, to which it must be ‘rewritten’ at some level. Otherwise, when we replace a certain part of the N’ of the donkey sentence, we are in effect changing the VP as well, and the inference in question cannot be explained in terms of monotonicity with regard to a single position.

\(^{29}\)The inference in (57) goes through under both interpretations.
In general, if the Det of a donkey sentence is $\uparrow$MON$\downarrow$ or $\uparrow$MON$\uparrow$, the weak reading preserves the validity of inferences based on the left monotonicity of the determiner, but the strong reading does not. That is, the interpretation of donkey sentences with $\uparrow$MON$\uparrow$ and $\downarrow$MON$\downarrow$ determiners is the one that validates left monotonicity inferences. On the other hand, if the Det of the donkey sentence is $\uparrow$MON$\downarrow$ or $\downarrow$MON$\uparrow$, it is the strong reading that validates left monotonicity inferences. And our assumption was that the strong reading is the interpretation selected by these determiners.

(61) Every farmer who owns a donkey beats it
    Every farmer who owns and feeds a donkey beats it

(62) Every farmer who owns a donkey beats it
    Every farmer who owns a female donkey beats it

Given the strong reading, both of the above inferences are valid. However, (60) would serve as a countermodel for (61) and (62) if the sentences involved had the weak reading.

Thus, the correlation between the monotonicity properties of the determiner and the interpretation of the donkey sentence is explained if we postulate that the interpretation selected by a left monotone determiner is the one that validates dynamic monotonicity inferences based on the left monotonicity of the determiner.\textsuperscript{30} I contend that this is the basic principle about the interpretation of donkey sentences.

(63) Left Monotonicity Principle: A determiner upward (downward) monotone in the first argument selects the reading of a donkey sentence that makes the N$'$ position upward (downward) monotone in the dynamic sense.

This principle makes no prediction about non-left-monotone determiners like most. This is consistent with the fact that most does not seem to favor either interpretation.\textsuperscript{31}

We can, if we wish, formulate our claim in more general terms as ‘preservation of monotonicity’, since monotonicity inferences based on the right monotonicity of the determiner are validated by both the weak reading and the strong reading, and thus do not discriminate between them.

(64) Monotonicity Principle: Donkey sentences are (in the default case) interpreted in such a way that the monotonicity property of the determiner in the usual sense guarantees the dynamic monotonicity of the corresponding argument position.

\textsuperscript{30}Note that even though we are interested in inferences licensed by the left monotonicity of the determiner, which reading validates such inferences also depends on the determiner’s monotonicity property on the right.

\textsuperscript{31}Also, if the Det of a donkey sentence is left monotone but not right monotone, the N$'$ position will not be dynamically monotone on either reading. However, no such determiner seems to exist in English.
In fact, we may talk more abstractly of ‘preservation of inferential patterns’: 
*inferential patterns that hold in non-donkey sentences are generally preserved in donkey sentences* (see the discussions in Section 5.3.1).

In Section 4, we turn to the formalization of the idea of this section, using dynamic predicate logic with generalized quantifiers. As we shall see in Section 5.1, when the determiner is double monotone (monotone in both arguments), the effect of the Monotonicity Principle (64) is to pick out a unique interpretation out of all logically conceivable ones that satisfy some reasonable conditions related to the earlier empirical observation (16).

## 4 Dynamic Predicate Logic, Generalized Quantifiers, Monotonicity

Dynamic predicate logic (DPL) of Groenendijk and Stokhof (1991) augmented with generalized quantifiers turns out to be convenient in formalizing the ideas in Section 3. Perhaps the contents of the Left Monotonicity Principle (63) and the Monotonicity Principle (64) are already clear, but formalization will have an additional virtue of allowing us to state some further observations. In the interest of space, the following presentation will have to be sketchy, but the technical details are completely spelled out in Kanazawa 1993, to which the reader can turn for more information.

The general idea here is to model donkey sentences using what Groenendijk and Stokhof would call *internally dynamic* generalized quantifiers, i.e., quantifiers that can create ‘indirect binding’ relation between dynamic existential quantifiers in their first argument and corresponding free variables in their second argument. Thus, donkey sentences with relative clauses of the form (65a) are modeled by formulas of DPL with generalized quantifiers of the form (65b):

\[(65) \begin{align*}
\text{a. } & \text{Det } N' [s' \ldots [NP, a N'] \ldots] [VP \ldots [NP, \text{pronoun}] \ldots] \\
\text{b. } & Qx(\varphi(x) ; \mathcal{E}y\chi(x,y), \psi(x,y))
\end{align*}\]

In (65b), $\mathcal{E}$ is the dynamic existential quantifier of DPL, and $Q$ is an internally dynamic generalized quantifier corresponding to Det in (65a). The first argument of $Q$ is the translation of $N' [s' \ldots [NP, a N'] \ldots]$ in (65a) ($\varphi(x)$ corresponds to the head $N'$ and $\mathcal{E}y\chi(x,y)$ to the relative clause), and the second argument is the translation of the VP containing the pronoun. The donkey anaphora in (65a) is captured in (65b) by the ‘indirect binding’ relation between $\mathcal{E}y$ and the occurrence of $y$ in the second argument of $Q$. Given this representation, the Left Monotonicity Principle can be stated in precise terms as a condition on $Q$.

### 4.1 Dynamic Predicate Logic

In this section, I present my version of DPL. Familiarity with Groenendijk and Stokhof 1991 will be presupposed, but I will try to explain most important ideas.
4.1.1 Language

The language of DPL is obtained by adding some new connectives to the language of first-order logic. Thus, unlike Groenendijk and Stokhof (1991), I retain all static connectives from first-order logic

\[ \neg, \land, \lor, \rightarrow, \leftrightarrow, \forall, \exists \]

with their usual meaning. Dynamic connectives of DPL are symbolized as in Groenendijk and Stokhof 1990:

\[ ;; \Rightarrow, \mathcal{E} \]

These connectives are called dynamic conjunction, dynamic implication, and dynamic existential quantifier, respectively. In addition, the language contains two more binary connectives, which I call meta connectives:

\[ \simeq, \preceq. \]

4.1.2 Semantics

The notions of models and assignments for the language of DPL are the same as in first-order logic. I adopt the convention of writing \( M \) to denote the universe of model \( M \).

The main idea of DPL semantics is to associate with each formula \( \varphi \) a binary relation \( [\varphi]_M \) on assignments to variables as its denotation in model \( M \). For our purposes, it is expedient to define the usual satisfaction conditions

\[ M \models \varphi[s] \]

at the same time as the transition conditions

\[ s \vDash [\varphi]_M s'. \]

Semantics of DPL.

1. The interpretation \( t^M,s \) of a term \( t \) with respect to a model \( M \) and an assignment \( s \) is defined in the same way as in first-order logic.

2. For \( \varphi = t_1 = t_2, R(t_1, \ldots, t_n), \neg \psi, \psi \land \chi, \psi \lor \chi, \psi \rightarrow \chi, \psi \leftrightarrow \chi, \forall x \psi, \exists x \psi, \)

   (a) \( M \models \varphi[s] \) is defined just as in first-order logic.
   (b) \( s \vDash [\varphi]_M s' \) iff \( s = s' \) and \( M \models \varphi[s] \).

3. For \( \varphi = \psi; \chi, \)

   (a) \( M \models \varphi[s] \) iff for some \( s' \), \( s \vDash [\varphi]_M s' \).
   (b) \( s \vDash [\varphi]_M s' \) iff for some \( s'' \), \( s \vDash [\psi]_M s'' \) and \( s'' \vDash [\chi]_M s'. \)
4. For $\varphi = \psi \Rightarrow \chi$,

(a) $M \models \varphi[s]$ iff for all $s'$, $s[\varphi]_M s'$ implies $M \models \chi[s']$.

(b) $s[\varphi]_M s'$ iff $s = s'$ and $M \models \varphi[s]$.

5. For $\varphi = \mathcal{E}x\psi$,

(a) $M \models \varphi[s]$ iff for some $s'$, $s[\varphi]_M s'$.

(b) $s[\varphi]_M s'$ iff for some $a \in M$, $s(a/x)[\varphi]_M s'$.

6. For $\varphi = \psi \simeq \chi$,

(a) $M \models \varphi[s]$ iff for all $s'$, $s[\varphi]_M s'$ just in case $s[\chi]_M s'$.

(b) $s[\varphi]_M s'$ iff $s = s'$ and $M \models \varphi[s]$.

7. For $\varphi = \psi \preceq \chi$,

(a) $M \models \varphi[s]$ iff for all $s'$, $s[\varphi]_M s'$ implies $s[\chi]_M s'$.

(b) $s[\varphi]_M s'$ iff $s = s'$ and $M \models \varphi[s]$.

The notions of truth and validity are defined in the usual way in terms of satisfaction: $M \models \varphi$ ($\varphi$ is true in $M$) if and only if for every $s: \text{VAR} \rightarrow M$, $M \models \varphi[s]$; and $\models \varphi$ ($\varphi$ is valid) if and only if for all $M$, $M \models \varphi$.

The semantics of the meta connectives given in clauses 6 and 7 can be seen as Groenendijk and Stokhof’s (1991) equivalence ($\simeq$) and meaning inclusion ($\preceq$) relativized to models and assignments. $\varphi \simeq \psi$ ($\varphi \preceq \psi$) in their sense if and only if $\models \varphi \simeq \psi$ ($\models \varphi \preceq \psi$) in my sense. The connective $\preceq$ is used to express dynamic monotonicity.

By clause 2, if $\varphi$ is a formula of first-order logic, $M \models \varphi[s]$ in DPL if and only if $M \models \varphi[s]$ in first-order logic.

The (b) part of clauses 2, 4, 6, and 7 shows that all formulas except those of the form $\psi; \chi$ and $\mathcal{E}x\psi$ are tests, and all connectives except $;$ and $\mathcal{E}$ are externally static, in the sense of Groenendijk and Stokhof (1991).

The following are some of the most important validities in DPL:

(66) $(\varphi; \psi); \chi \simeq \varphi; (\psi; \chi)$

(67) $\varphi; \psi \Rightarrow \chi \simeq \varphi \Rightarrow (\psi \Rightarrow \chi)$

(68) $\mathcal{E}x\varphi; \psi \simeq \mathcal{E}x(\varphi; \psi)$

(69) $\mathcal{E}x\varphi \Rightarrow \psi \simeq \forall x(\varphi \Rightarrow \psi)$

These properties of dynamic connectives are used to represent intersentential and donkey anaphora in natural language. See Groenendijk and Stokhof 1991 for examples.
The set of active quantifier variables in $\varphi$, $AQV(\varphi)$, and the set of free variables in $\varphi$, $FV(\varphi)$, are defined as in Groenendijk and Stokhof 1991. The important thing is that dynamic existential quantifiers introduce active quantifier variables, and variables in $AQV(\varphi) \cap FV(\psi)$ behave like bound variables in $\varphi \land \psi$ and $\varphi \Rightarrow \psi$ at their occurrences in $\psi$. If the main connective of $\varphi$ is externally static, $AQV(\varphi) = \emptyset$.

The following equivalences are also important.

\begin{align*}
(70) \quad \varphi \land \psi & \equiv \mathcal{E}x(\varphi \land \psi) \quad \text{if } x \notin AQV(\varphi) \cup FV(\varphi) \\
(71) \quad \varphi \land \psi & \equiv \varphi \land \psi \quad \text{if } AQV(\varphi) \cap FV(\psi) = \emptyset \\
(72) \quad \varphi \Rightarrow \psi & \equiv \varphi \rightarrow \psi \quad \text{if } AQV(\varphi) \cap FV(\psi) = \emptyset \\
(73) \quad \mathcal{E}x\varphi & \equiv \exists x\varphi
\end{align*}

### 4.2 Static Generalized Quantifiers

Static generalized quantifier symbols taking two formulas as arguments can be introduced into DPL in the following way, just like other static connectives.

8. For $\varphi = Qx(\psi, \chi)$,

   (a) $M \models \varphi[s]$ iff
   
   \[ \{ \{ a \in M \mid M \models \psi[a/x] \} \}, \{ a \in M \mid M \models \chi[a/x] \} \} \in Q_M, \]

   (b) $s [\varphi]_M s'$ iff $s = s'$ and $M \models \varphi[s]$.

The (a) part is the satisfaction conditions from ordinary first-order logic with generalized quantifiers. Here, it is assumed that each generalized quantifier symbol $Q$ is associated with a functional $\lambda M.Q_M$ mapping a universe $M$ to a two-place monadic generalized quantifier on $M$, $Q_M \subseteq \text{pow}(M) \times \text{pow}(M)$.

The above semantics makes any static generalized quantifier $Q$ behave in DPL with generalized quantifiers just like it does in first-order logic with generalized quantifiers. In particular, the following equivalence is always valid:

Equivalence (EQUI).

\[ \forall x(\varphi \leftrightarrow \varphi') \land \forall x(\psi \leftrightarrow \psi') \rightarrow (Qx(\varphi, \psi) \leftrightarrow Qx(\varphi', \psi')) \]

Moreover, if $Q$ stands for a quantifier satisfying Conservativity, the following schema is valid:

Conservativity (CONS).

\[ Qx(\varphi, \psi) \leftrightarrow Qx(\varphi, \varphi \land \psi). \]

Monotonicity properties of $Q$ correspond to validity of the following schemas:
Monotonicity. 
\[ \uparrow \text{MON} \quad \forall x(\varphi \rightarrow \varphi') \rightarrow (Qx(\varphi, \psi) \rightarrow Qx(\varphi', \psi)) \]
\[ \downarrow \text{MON} \quad \forall x(\varphi' \rightarrow \varphi) \rightarrow (Qx(\varphi, \psi) \rightarrow Qx(\varphi', \psi)) \]
\[ \text{MON}^\uparrow \quad \forall x(\psi \rightarrow \psi') \rightarrow (Qx(\varphi, \psi) \rightarrow Qx(\varphi, \psi')) \]
\[ \text{MON}^\downarrow \quad \forall x(\psi' \rightarrow \psi) \rightarrow (Qx(\varphi, \psi) \rightarrow Qx(\varphi, \psi')) \]

That is, if \( Q \) is a symbol for a \( \uparrow \text{MON} \) quantifier, the first schema in the above list is valid, etc.

Note that the weak reading and the strong reading of a donkey sentence of the form (65a), repeated here as (74), can be expressed by formulas of first-order logic with generalized quantifiers of the form (75) and (76), respectively.

(74) \( \text{Det } N' [s' \ldots [\text{NP, a } N'] \ldots] [\text{VP} \ldots [\text{NP, pronoun}] \ldots] \)
(75) \( Qx(\varphi(x) \land \exists y \chi(x, y), \exists y (\chi(x, y) \land \psi(x, y))) \)
(76) \( Qx(\varphi(x) \land \exists y \chi(x, y), \forall y (\chi(x, y) \rightarrow \psi(x, y))) \)

Here, \( Q \) is the two-place generalized quantifier corresponding to the Det in (74), and \( \varphi \) represents the head \( N' \) of the subject, \( \exists y \chi(x, y) \) the relative clause, and \( \psi \) the VP.

### 4.3 Dynamic Generalized Quantifiers

To model donkey sentences in DPL, we need to introduce dynamic generalized quantifiers. Such quantifiers must be internally dynamic in the way \( \Rightarrow \) are; they must allow ‘indirect binding’ between dynamic existential quantifiers in their first argument and corresponding free variables in their second argument. (I assume that our dynamic generalized quantifiers are externally static.)

I use script \( Q \) as a symbol for internally dynamic generalized quantifiers. It is not necessary or important for our purposes to specify the format for the semantics of formulas of the form \( Qx(\varphi, \psi) \). All we need is to state some conditions that such formulas should satisfy.

I assume that all internally dynamic generalized quantifiers must validate the following schema:\[32\]

**Dynamic Equivalence (DEQUI).**
\[ \forall x(\varphi \simeq \varphi') \land \forall x \forall y_1 \ldots \forall y_n (\psi \leftrightarrow \psi') \rightarrow (Qx(\varphi, \psi) \leftrightarrow Qx(\varphi', \psi')) \]

where \( \{y_1, \ldots, y_n\} = A\text{QV}(\varphi) \). This is the dynamic version of EQUI. The fact that the dynamic effects of the first argument of \( Q \) reach into the second argument requires ‘dynamic equivalence’ \( \simeq \) for the first argument and the extra quantifiers \( \forall y_1 \ldots \forall y_n \) for the second argument. Since the dynamic effects of the second argument should not matter (donkey anaphora works only from left to right), static equivalence \( \leftrightarrow \) suffices for the second argument.

\[32\]There is another condition concerning renaming of bound variables. See Kanazawa 1993.
4.3.1 Chierchia’s Two Definitions of Dynamic Generalized Quantifiers

It is possible to define dynamic generalized quantifiers in terms of static ones and dynamic connectives of DPL. The following two schemata, which are essentially due to Chierchia (1990), define dynamic generalized quantifiers corresponding to the two interpretations of donkey sentences.

\[(77) \quad Q_W x(\varphi, \psi) \leftrightarrow Q x(\varphi, \varphi ; \psi)\]
\[(78) \quad Q_S x(\varphi, \psi) \leftrightarrow Q x(\varphi, \varphi \Rightarrow \psi)\]

(77) is used to represent the weak reading of donkey sentences, and (78) the strong reading.

Let us see how the two dynamic generalized quantifiers $MOST_W$ and $MOST_S$, defined in terms of $MOST$ by (77) and (78), can be used to represent the two readings for (79).

(79) Most farmers who own a donkey beat it.

The two readings of (79) are represented by the following two formulas:

\[(80) \quad MOST_W x(\text{farmer}(x) ; \mathcal{E} y(\text{donkey}(y) ; \text{own}(x, y)), \text{beat}(x, y))\]
\[(81) \quad MOST_S x(\text{farmer}(x) ; \mathcal{E} y(\text{donkey}(y) ; \text{own}(x, y)), \text{beat}(x, y))\]

By definition, (80) and (81) are equivalent to (82) and (83), respectively.

\[(82) \quad MOST x(\text{farmer}(x) ; \mathcal{E} y(\text{donkey}(y) ; \text{own}(x, y)),\]
\[(83) \quad (\text{farmer}(x) ; \mathcal{E} y(\text{donkey}(y) ; \text{own}(x, y))) \Rightarrow \text{beat}(x, y))\]

By the conservativity of $MOST$ and the properties of dynamic connectives, (82) and (83) turn out to be equivalent to (84) and (85), which express the two readings of (79) in first-order logic with generalized quantifiers.

\[(84) \quad MOST x(\text{farmer}(x) \land \exists y(\text{donkey}(y) \land \text{own}(x, y)),\]
\[(85) \quad \exists y(\text{donkey}(y) \land \text{own}(x, y) \land \text{beat}(x, y)))\]

\[(\forall y(\text{donkey}(y) \land \text{own}(x, y) \Rightarrow \text{beat}(x, y)))\]

The equivalences like these hold in general. If $Q$ is conservative, $x, y \not\in Aqv(\chi(x, y))$, and $x \not\in Aqv(\varphi(x))$,

\[(86) \quad Q_W x(\varphi(x) ; \mathcal{E} y \chi(x, y), \psi(x, y))\]
\[(87) \quad Q_S x(\varphi(x) ; \mathcal{E} y \chi(x, y), \psi(x, y))\]

Compare (75) and (76).
4.3.2 Dynamic Conservativity

Given that $Q_W$ and $Q_S$ are adequate for the purpose of representing the weak and strong readings of donkey sentences, our task now is to formulate the Left Monotonicity Principle as a condition that chooses between $Q_W$ and $Q_S$ on the basis of the monotonicity properties of $Q$. Before turning to this task, however, let us briefly discuss Chierchia’s (1990) claim that $Q_W$ is to be uniformly favored over $Q_S$, for any $Q$. His reason is that $Q_W$, but not $Q_S$, satisfies the following dynamic version of Conservativity.\(^{33}\)

Dynamic Conservativity 1 (DCONS1).

$$Qx(\varphi, \psi) \leftrightarrow Qx(\varphi; \psi)$$

Since $;$ is the dynamic connective corresponding to $\land$, it might seem natural to impose DCONS1 as a basic condition on dynamic generalized quantifiers. However, his argument is inconclusive, for DCONS1 is not the only natural dynamic version of CONS.\(^{34}\)

Since there are clear cases where the strong reading given by $Q_S$ is the right interpretation, and Chierchia’s alternative explanation for those cases does not seem to work,\(^{35}\) it would be unreasonable to insist on his notion of dynamic conservativity.

In fact, there is a natural notion of dynamic conservativity which, unlike Chierchia’s, does not have the unwanted consequence of excluding $Q_S$. This is the dynamic version of Conservativity which we adopt:

Dynamic Conservativity (DCONS).

$$\forall x(\varphi \Rightarrow (\psi \leftrightarrow \psi')) \rightarrow (Qx(\varphi, \psi) \leftrightarrow Qx(\varphi, \psi'))$$

Note that if we replace $\Rightarrow$ by $\rightarrow$ and $Q$ by $Q$, we get a condition equivalent to CONS. It is easy to see that DCONS is satisfied by both $Q_W$ and $Q_S$.\(^{36}\)

Unlike DCONS1, DCONS is reasonable on intuitive grounds. In contrast to (88), an equivalence like (89), which corresponds to an immediate consequence of DCONS, seems to be undeniable, even when the exact truth conditions of the sentences involved are not clear, e.g., when $Q$ is most.

(88) $Q$ farmer who owns a donkey beats it $\leftrightarrow$

$Q$ farmer who owns a donkey owns a donkey and beats it

(89) $Q$ farmer who owns a donkey beats it $\leftrightarrow$

$Q$ farmer who owns a donkey owns it and beats it

---

\(^{33}\)There is a proviso that $A_Q(\varphi) \cap F_V(\varphi) = \emptyset$.

\(^{34}\)As I show in Kanazawa 1993, there is a natural dynamic version of CONS which is satisfied by $Q_S$, but not by $Q_W$.

\(^{35}\)Chierchia (1992) tries to explain the presence of strong reading in some donkey sentences by assuming that donkey pronouns are ambiguous between ‘dynamically bound variables’ and ‘E-type pronouns’. The inadequacy of this proposal is shown in the Appendix.

\(^{36}\)An analogue of DCONS is also satisfied by the internally dynamic connectives of DPL ($;$ and $\Rightarrow$).
4.4 Dynamic Monotonicity and Formalization of the Left Monotonicity Principle

In Section 3, the dynamic notion of monotonicity was defined in terms of the inclusion relation between dynamic denotations. Let us first consider how to express the latter in dynamic predicate logic.

When two natural language expressions $\alpha$ and $\beta$ of type $(e,t)$ are translated into DPL as formulas $\varphi(x)$ and $\varphi'(x)$ with one free variable $x$, the condition that the static denotation of $\alpha$ includes that of $\beta$ is expressed by the formula $\forall x(\varphi'(x) \rightarrow \varphi(x))$. For example, if $\alpha$ is *man who owns a house* and $\beta$ is *man who owns a garden*, then $\varphi = \text{man}(x) ; \exists y(\text{house}(y) ; \text{own}(x,y))$, and $\varphi' = \text{man}(x) ; \exists y(\text{garden}(y) ; \text{own}(x,y))$, and the following formula says that the static denotation of $\beta$ (the set of men who own a garden) is a subset of the static denotation of $\alpha$ (the set of men who own a house):

\[
\forall x(\text{man}(x) ; \exists y(\text{house}(y) ; \text{own}(x,y))) \\
\rightarrow \text{man}(x) ; \exists y(\text{garden}(y) ; \text{own}(x,y))).
\]

Note that (90) is equivalent to the following:

\[
\forall x(\text{man}(x) \land \exists y(\text{house}(y) \land \text{own}(x,y))) \\
\rightarrow \text{man}(x) \land \exists y(\text{garden}(y) \land \text{own}(x,y))).
\]

The inclusion of dynamic denotations is a stronger condition. In the above example, it should amount to the inclusion between the set of man-house pairs and the set of man-garden pairs. It turns out that this is expressed by the following formula, which is obtained from (90) by replacing $\rightarrow$ by $\preceq$.

\[
\forall x(\text{man}(x) ; \exists y(\text{house}(y) ; \text{own}(x,y))) \\
\preceq \text{man}(x) ; \exists y(\text{garden}(y) ; \text{own}(x,y))).
\]

(92) is equivalent to the following:

\[
\forall x \forall y(\text{man}(x) \land \text{house}(y) \land \text{own}(x,y)) \\
\rightarrow \text{man}(x) \land \text{garden}(y) \land \text{own}(x,y)).
\]

(93) (Of course, in the model we considered in Section 3.1, (92) does not obtain.) In general, if natural language expressions $\alpha$ and $\beta$ of type $(e,t)$ are translated into DPL as $\varphi(x)$ and $\varphi'(x)$, $\forall x(\varphi'(x) \preceq \varphi(x))$ expresses the fact that the dynamic denotation of $\alpha$ includes that of $\beta$ ($[\beta] \subseteq [\alpha]$).

These considerations lead to the formulation of Dynamic Monotonicity in terms of the validity of the following schemas in dynamic predicate logic with generalized quantifiers.\(^{37}\) Let $\{y_1, \ldots, y_n\} = \text{AQV}(\varphi)$.

\(^{37}\)Indeed, dynamic monotonicity with respect to an arbitrary position, not just for the argument positions of dynamic generalized quantifiers, can be defined along the same line.
Dynamic Monotonicity.

\[\uparrow\text{DMON} \quad \forall x (\varphi \leq ‎ \varphi') \rightarrow (Qx(\varphi, \psi) \rightarrow Qx(\varphi', \psi))\]

\[\downarrow\text{DMON} \quad \forall x (\varphi' \leq \varphi) \rightarrow (Qx(\varphi, \psi) \rightarrow Qx(\varphi', \psi))\]

\[\text{DMON}^\uparrow \quad \forall x \forall y_1 \ldots \forall y_n (\psi \rightarrow \psi') \rightarrow (Qx(\varphi, \psi) \rightarrow Qx(\varphi, \psi'))\]

\[\text{DMON}^\downarrow \quad \forall x \forall y_1 \ldots \forall y_n (\psi' \rightarrow \psi) \rightarrow (Qx(\varphi, \psi) \rightarrow Qx(\varphi, \psi'))\]

Compare these formulas with the corresponding formulas for: static monotonicity (see Section 4.2). The \(\uparrow\text{DMON}\) and \(\downarrow\text{DMON}\) formulas differ from the \(\uparrow\text{MON}\) and \(\downarrow\text{MON}\) formulas in that the first occurrence of \(\rightarrow\) in the latter formulas is replaced by \(\leq\) in the former. In \(\text{DMON}^\uparrow\) and \(\text{DMON}^\downarrow\), the extra universal quantifiers \(\forall y_1 \ldots \forall y_n\) are required because the free occurrences of \(y_1, \ldots, y_n\) in \(\psi\) ('donkey variables') become bound in \(Qx(\varphi, \psi)\). On the other hand, we do not need \(\leq\) there because the dynamic effects of the second argument do not matter.

As summarized in Table 2, if \(Q_W\) and \(Q_S\) are defined by (77) and (78) from a double monotone static quantifier \(Q\), either \(Q_W\) or \(Q_S\), but not both, turns out to be dynamically left monotone. (When we say that \(Q\) is \(\uparrow\text{DMON}\), etc., we mean that a dynamic generalized quantifier \(Q\) validates the schema \(\uparrow\text{DMON}\), etc.)

<table>
<thead>
<tr>
<th>(Q)</th>
<th>(Q_W)</th>
<th>(Q_S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\uparrow\text{MON}^\uparrow)</td>
<td>(\uparrow\text{DMON}^\uparrow)</td>
<td>(\uparrow\text{DMON}^\uparrow)</td>
</tr>
<tr>
<td>(\uparrow\text{MON}^\downarrow)</td>
<td>(\not\uparrow\text{DMON}^\downarrow)</td>
<td>(\uparrow\text{DMON}^\downarrow)</td>
</tr>
<tr>
<td>(\downarrow\text{MON}^\uparrow)</td>
<td>(\not\uparrow\text{DMON}^\uparrow)</td>
<td>(\downarrow\text{DMON}^\uparrow)</td>
</tr>
<tr>
<td>(\downarrow\text{MON}^\downarrow)</td>
<td>(\downarrow\text{DMON}^\downarrow)</td>
<td>(\not\downarrow\text{DMON}^\downarrow)</td>
</tr>
</tbody>
</table>

Table 2: Dynamic monotonicity of \(Q_W\) and \(Q_S\).

For example, since \(\text{NO}\) is \(\downarrow\text{MON}^\downarrow\), \(\not\text{NO}_W\) defined from \(\text{NO}\) is \(\uparrow\text{DMON}\). This means that (94) and (95) imply (96), which corresponds to the valid inference (58):

(94) \(\not\text{NO}_W x (\text{farmer}(x); \mathcal{E} y (\text{donkey}(y); \text{own}(x, y)), \text{beat}(x, y))\)

(95) \(\forall x (\text{farmer}(x); \mathcal{E} y (\text{donkey}(y); (\text{own}(x, y); \text{feed}(x, y)))\)

\(\leq \text{farmer}(x); \mathcal{E} y (\text{donkey}(y); \text{own}(x, y)))\)

(96) \(\not\text{NO}_W x (\text{farmer}(x); \mathcal{E} y (\text{donkey}(y); (\text{own}(x, y); \text{feed}(x, y))), \text{beat}(x, y))\).

Recall that we are representing donkey sentences of the form Det \(N^'\) VP by formulas of the form \(Qx(\varphi, \psi)\), where \(Q\) is a dynamic generalized quantifier corresponding to the Det, and \(\varphi\) and \(\psi\) are the translations of the \(N^'\) and VP in DPL, respectively. Given the above formulation of Dynamic Monotonicity, the Left Monotonicity Principle (63) can be stated in the following way:

(97) If the Det of a donkey sentence is \(\uparrow\text{MON}\) (\(\downarrow\text{MON}\)), the dynamic generalized quantifier appropriate for representing its (default) interpretation is the \(\uparrow\text{DMON}\) (\(\downarrow\text{DMON}\)) one.

The Monotonicity Principle (64) can be stated in a similar way.
5 Discussions and Speculations

5.1 The Monotonicity Principle as an Implicit Definition

So far, our concern has been to choose between the weak reading and the strong reading using dynamic left monotonicity as a criterion. We simply assumed that the correct interpretation is given by one of these two readings (cf. (17)), and did not ask why only these, not some other logically conceivable interpretations, are the possible options. In fact, the Monotonicity Principle can answer this latter question, too, if we make some additional reasonable assumptions. When the determiner is double monotone (monotone in both arguments), either the weak reading or the strong reading is the only logically possible interpretation that satisfies the Monotonicity Principle and a few other natural conditions.

As in the previous section, the relevant observation is stated in precise terms using dynamic predicate logic with generalized quantifiers. Under certain minimal conditions, the Monotonicity Principle uniquely determines a dynamic generalized quantifier \( Q \) corresponding to a double monotone quantifier \( Q \).

In addition to DEQUI, which holds of all dynamic generalized quantifiers by definition, I assume two minimal conditions on \( Q \). One is the earlier condition of Dynamic Conservativity:

Dynamic Conservativity (DCONS).

\[
\forall x (\varphi \Rightarrow (\psi \leftrightarrow \psi')) \rightarrow (Qx(\varphi, \psi) \leftrightarrow Qx(\varphi, \psi'))
\]

As we have seen, this roughly corresponds to an equivalence like the following:

(98) \( Q \) farmer who owns a donkey beats it \( \leftrightarrow \)

\( Q \) farmer who owns a donkey owns it and beats it

The other condition is what I call Agreement. It is a condition on the relation between a static quantifier \( Q \) and its dynamic counterpart \( Q \):

Agreement. If \( \text{AQV}(\varphi) \cap \text{FV}(\psi) = \emptyset \),

\[
Qx(\varphi, \psi) \leftrightarrow Qx(\varphi, \psi)
\]

This condition says that \( Q \) must agree with \( Q \) when there is no ‘indirect binding’ between the first argument and the second argument.\(^{38}\) It roughly corresponds to the requirement that the semantics of donkey sentences should also work for non-donkey sentences:

(99) \( Q \) farmer who owns a donkey is rich.

The two dynamic generalized quantifiers \( Q_W \) and \( Q_S \) defined from a conservative \( Q \) by Chierchia’s schemas satisfy DCONS and Agreement. If the E-type reading and the pair quantification reading were to be formulated in terms of

\(^{38}\) Recall that analogous equivalences hold for two dynamic connectives of DPL (cf. (71) and (72)).
dynamic generalized quantifiers, they would satisfy DCONS, but not Agreement. The dynamic generalized quantifier corresponding to the putative reading 'Most farmers who own a donkey beat most of the donkeys they own' of Most farmers who own a donkey beat it would satisfy both DCONS and Agreement.

In fact, DCONS and Agreement have the effect of excluding any interpretation that contradicts the empirical observation (16) about consistent donkey-beating stated in Section 1. This is the content of the following lemma (see Kanazawa 1993 for proof):

**Lemma 1.** If \( Q \) satisfies DCONS and Agreement, then the following holds:

\[
\forall x((\varphi \Rightarrow \psi) \lor \neg(\varphi ; \psi)) \rightarrow \\
(Qx(\varphi, \psi) \leftrightarrow Q_w x(\varphi, \psi)) \land (Qx(\varphi, \psi) \leftrightarrow Q_s x(\varphi, \psi))
\]

Note that the antecedent of the formula in Lemma 1 states the condition of consistent donkey-beating.

The Monotonicity Principle is formulated as follows:

(100) Monotonicity Principle.

If \( Q \) is \( \uparrow \text{MON} \) (\( \downarrow \text{MON} \)), then \( Q \) must be \( \uparrow \text{DMON} \) (\( \downarrow \text{DMON} \)).

If \( Q \) is MON\( \uparrow \) (MON\( \downarrow \)), then \( Q \) must be DMON\( \uparrow \) (DMON\( \downarrow \)).

The following two propositions give the combined effect of the Monotonicity Principle, DCONS, and Agreement (see Kanazawa 1993 for proof).

**Proposition 2.** Assume that \( Q \) satisfies DCONS and Agreement. If \( Q \) is moreover DMON\( \uparrow \),

\[
Q_s x(\varphi, \psi) \rightarrow Q x(\varphi, \psi) \rightarrow Q_w x(\varphi, \psi).
\]

If \( Q \) is DMON\( \downarrow \), the reverse implications hold.

**Proposition 3.** Assume that \( Q \) satisfies DCONS and Agreement. Then if \( Q \) is \( \uparrow \text{DMON} \),

\[
Q_w x(\varphi, \psi) \rightarrow Q x(\varphi, \psi) \text{ and } Q_s x(\varphi, \psi) \rightarrow Q x(\varphi, \psi).
\]

If \( Q \) is \( \downarrow \text{DMON} \), the reverse implications hold.

By the two propositions, any \( Q \) which is, say, \( \uparrow \text{DMON} \) and satisfies DCONS and Agreement implies and is implied by \( Q_w \). That is, if \( Q \) is a \( \uparrow \text{MON} \) static generalized quantifier, \( Q_w \) is the only \( \uparrow \text{DMON} \) dynamic generalized quantifier \( Q \) that satisfies DCONS and Agreement with respect to \( Q \). Similarly with the other three double monotonicity patterns: among the dynamic generalized quantifiers that satisfy DCONS and Agreement, there is a unique one picked out by dynamic double monotonicity. Thus, given DCONS and Agreement, the requirement of the
Monotonicity Principle amounts to an implicit definition of a dynamic generalized quantifier corresponding to a given double monotone static quantifier.

This formal result allows us to shift our perspective on donkey sentences. Instead of stating the possible interpretations of donkey sentences explicitly by certain paraphrases or specific dynamic generalized quantifiers (and choosing one of them by some principle), it is possible to characterize the interpretation of donkey sentences 'implicitly' by a set of conditions that it has to satisfy. In fact, the present characterization in terms of three conditions may have some advantages. First, the conditions used here—Dynamic Conservativity, Agreement, and the Monotonicity Principle—are all natural ones in that (i) they basically say that donkey sentences should share certain semantic characteristics of non-donkey sentences, and (ii) they are backed up by clear intuitions, namely, intuitions about equivalences like (98), intuitions stated in (16) with regard to consistent donkey-beating, and intuitions about the validity of monotonicity inference. Second, the characterizing conditions underspecify the interpretation of donkey sentences with non-left-monotone determiners like most; they allow much more possibilities than just the weak reading and the strong reading, while still predicting the clear intuitions about consistent donkey-beating situations. This seems to mirror people's reactions to some donkey sentences with most.

A similar perspective will be elaborated in Section 5.3.2.

5.2 Exactly \( n \)

We have so far failed to consider determiners of the form exactly \( n \), which are not monotone in either argument. Since the Left Monotonicity Principle does not apply to these determiners, we would predict that they do not favor either the weak or the strong reading when they appear as the Det of donkey sentences. The facts seem to be that they select the weak reading, and this presents a problem for our account:

(101) Exactly three farmers who own a donkey beat it.

(102) Exactly four students who borrowed a book from Peter returned it.

There are two possible lines of solution here. One approach makes an extended use of the Left Monotonicity Principle. First of all, notice that although exactly \( n \) is not itself left monotone, it is a conjunction of two left monotone determiners, namely, \( \text{at least } n \) and \( \text{at most } n \).

\[
\text{EXACTLY}_n x(\varphi, \psi) \leftrightarrow \text{AT\_LEAST}_n x(\varphi, \psi) \land \text{AT\_MOST}_n x(\varphi, \psi)
\]

Since \( \text{at least } n \) is \( \uparrow \text{MON} \uparrow \) and \( \text{at most } n \) is \( \downarrow \text{MON} \downarrow \), they are both supposed to select the weak reading; the adequate dynamic counterparts of \( \text{AT\_LEAST}_n \)

\[39\text{In particular, the putative reading 'Most farmers who own a donkey beat most of the donkeys they own' of Most farmers who own a donkey beat it satisfy the three conditions.}\]
and \( AT \cdot MOST \cdot n \) are \( AT \cdot LEAST \cdot n_W \) and \( AT \cdot MOST \cdot n_W \). Now it is not unreasonable to suppose that the above factorization of \( EXACTLY \cdot n \) into \( AT \cdot LEAST \cdot n_W \) and \( AT \cdot MOST \cdot n_W \) should be preserved in donkey sentences; i.e., the suitable dynamic version \( EXACTLY \cdot n \) of \( EXACTLY \cdot n \) should satisfy:

\[
EXACTLY \cdot n \ x(\varphi, \psi) \leftrightarrow AT \cdot LEAST \cdot n_W \ x(\varphi, \psi) \land AT \cdot MOST \cdot n_W \ x(\varphi, \psi)
\]

If this is so, it is easy to see that \( EXACTLY \cdot n \) should be \( EXACTLY \cdot n \) rather than \( EXACTLY \cdot n_s \).\(^{40}\)

There is a different, more ‘semantical’ way of looking at this same explanation, in terms of \emph{left continuity} of \( EXACTLY \cdot n \). At the level of denotation, a quantifier \( Q \) is said to be left continuous if it satisfies the following:

For all \( A \subseteq B \subseteq C \subseteq M \) and all \( D \subseteq M \),
\[ Q_M AD \text{ and } Q_M CD \text{ imply } Q_M BD. \]

Now \( EXACTLY \cdot n \) is left continuous. In fact, a quantifier is left continuous if and only if it can be expressed as a conjunction of a \( \uparrow \text{MON} \) quantifier and a \( \downarrow \text{MON} \) quantifier. Then we can suppose that a somewhat weaker principle than the Left Monotonicity Principle applies to left continuous determiners.

(103) \emph{Left Continuity Principle}: A left continuous determiner selects the reading of a donkey sentence given by a dynamic generalized quantifier which is dynamically left continuous.

The dynamic notion of Left Continuity can be defined by an obvious analogy with Dynamic Monotonicity. It is easily checked that \( EXACTLY \cdot n_W \), but not \( EXACTLY \cdot n_S \), is dynamically left continuous.\(^{41}\)

A second explanation is the following. \emph{Exactly} \( n \), like the \( \uparrow \text{MON} \uparrow \) and \( \downarrow \text{MON} \downarrow \) determiners we have considered, is \emph{symmetric}; it satisfies the following condition (Symmetry):

For all \( A, B \subseteq M \), \( Q_M AB \) iff \( Q_M BA \)

Now under CONS, Symmetry is equivalent to the following condition, which I call \emph{Intersection}:\(^{42}\)

\(^{40}\)The same explanation might be more plausible for donkey sentences with complex determiners like \emph{some but not all}:

(i) Some but not all farmers who own a donkey beat it.

Interestingly, the interpretation predicted by the factorization of \emph{some but not all} into \emph{some} and \emph{not all} would be neither the weak reading nor the strong reading, but the pair quantification reading. I do not know if this is supported by intuitions.

\(^{41}\)Even when \( Q \) is left monotone, either \( Q_W \) or \( Q_S \) generally fails to be dynamically left continuous.

\(^{42}\)This is slightly different from, but equivalent (under CONS) to, what Barwise and Cooper (1981) called ‘intersection condition’.
For all $A, B \subseteq M$, $Q_M AB \iff Q_M (A \cap B) M$

In terms of first-order logic with generalized quantifiers, it comes out as follows:

Intersection. $Qx(\phi, \psi) \iff Qx(\phi \land \psi, true)$

Unlike Symmetry, Intersection can be expressed in English very naturally as follows, assuming Barwise and Cooper's (1981) semantics for there-sentences.\footnote{According to Barwise and Cooper (1981), a sentence of the form ‘There is/are Det N’ is true in a model with universe $M$ iff $[\text{Det}][N'], M$.}

Det N' VP $\iff$ There is/are Det N' who/which VP

For example,

(104) Two boys are crying $\iff$ There are two boys who are crying

It is not an unnatural requirement that this paraphrasability should be preserved in donkey sentences. Witness:

(105) No farmer who owns a donkey beats it $\iff$

There is no farmer who owns a donkey who beats it

This leads us to a dynamic version of Intersection:

Dynamic Intersection. $Qx(\phi, \psi) \iff Qx(\phi ; \psi, true)$

Interpreting relative clause modification naturally as dynamic conjunction, as we have been, we see that Dynamic Intersection does guarantee paraphrasability by there-sentences. Our pattern of explanation should now be familiar. If $Q$ satisfies Intersection, $Q_W$, but not $Q_S$, satisfies Dynamic Intersection.\footnote{Note that, given Agreement, Dynamic Intersection uniquely determines $Q$ out of $Q$.}

The relevant principle about interpretations of donkey sentences is:

(106) Intersection Principle: A determiner satisfying Intersection selects the reading of a donkey sentence given by a dynamic generalized quantifier satisfying Dynamic Intersection.

As it turns out, the Intersection Principle has an additional virtue. Since the $\uparrow$MON$\downarrow$ and $\downarrow$MON$\uparrow$ determiners we have considered are all symmetric, the Intersection Principle provides another explanation for the weak readings of donkey sentences with these determiners. The fact that those donkey sentences never seem to allow strong readings and the intuitions about their weak readings seem much clearer than the intuitions about strong readings of some donkey sentences may be due to the fact that their weak readings are, so to speak, doubly guaranteed by the Left Monotonicity Principle and the Intersection Principle.
5.3 Inference and Interpretation

In this section, I engage in highly speculative discussions about the role of inference in the interpretation of donkey sentences.

5.3.1 Preservation of Inferential Patterns

What is the status of the Left Monotonicity Principle? The discussion in Section 5.2 suggests that it may be more fruitful to understand the Left Monotonicity Principle in the context of a more general tendency: the interpretation of donkey sentences should preserve the valid inferential patterns of non-donkey sentences. As we have seen, Monotonicity and Intersection properties of a quantifier correspond to very natural patterns of inference in English. Another natural class of inferences are those based on the Square of Opposition. Like inferences based on Monotonicity and Intersection, they are felt to be valid even in the case of donkey sentences:

(107) a. Some student who borrowed a book from Peter didn’t return it ↔
Not every student who borrowed a book from Peter returned it
b. Every student who borrowed a book from Peter didn’t return it ↔
No student who borrowed a book from Peter returned it

Indeed, the above inferences are validated by the readings we have assumed. Notice that if, for instance, all donkey sentences had the weak reading, equivalences like these would not hold. Similarly, the following sentences are intuitively felt to be contradictory:

(108) a. Every student who borrowed a book from Peter returned it
b. Some student who borrowed a book from Peter didn’t return it

Given that the first sentence has the strong reading and the second the weak reading, as we have been granting, they are indeed contradictory. In fact, it was a contradiction like this which led Geach (1962) to conclude that it is inadequate.

Note that some properties of quantifiers do not directly correspond to any natural forms of inference. For example, to express Symmetry in English, one has to resort to circumlocution (cf. van Benthem 1987):

(i) Some boy is crying ↔
Some is crying boy ↔
Someone who is crying is a boy

(The second line is what results from the first line by a simple interchange of N' and VP.) In the case of donkey sentences, Symmetry doesn’t even make sense, since conversion of N' and VP destroys the possibility of donkey anaphora:

(ii) No farmer who owns [a donkey], it, beats it, ↔
*No person who beats it, is a farmer who owns [a donkey],

Thus, there is no place for a dynamic notion of Symmetry in donkey sentences.
to paraphrase *Any man who owns a donkey beats it* by *Any man who owns a donkey owns a donkey and beats it.*

Thus, there seems to be a general tendency that certain types of inferences are preserved in donkey sentences. The types of inferences in question are monotonicity inferences (cf. (61) and (62)), paraphrasing by *there* sentences (cf. (105)), and inferences based on the Square of Opposition (cf. (107) and (108)). In fact, it would seem that the intuitions about the validity of these inferences with donkey sentences—the feeling that they must be valid, or the inclination to draw such inferences—are stronger than the intuitions about the truth conditions of individual donkey sentences.

If we look at inferences with ordinary quantified sentences based on Monotonicity, Intersection, and the Square of Opposition, we see that they are expressed in natural sentences, and involve only a simple manipulation of form. Their validity is felt obvious, and most likely people rely on such inferences in everyday linguistic activities. It would not be so unreasonable to suppose that language users are actually resorting to some sort of formal ‘calculi’ for these inferences, skipping over laborious computations involved in comparing two sets of truth conditions. For monotonicity inference, the calculus might involve something like the system of monotonicity marking sketched in Section 3.1.46 Then, the intuitions about certain inferences involving donkey sentences might perhaps be the result of mechanically applying such calculi to donkey sentences.

If the above model of how people draw certain inferences mechanically is on the right track, then the facts about the interpretation of donkey sentences might receive some sort of functional explanation: it comes as no surprise if the semantics of donkey sentences is set up in such a way that inferential mechanisms that are primarily designed to apply to ordinary quantified sentences of a certain form are also safely applicable to sentences that have the same superficial form except that they involve donkey anaphora.

### 5.3.2 Inference as a Basis of Judgment

So far, we have not proposed any concrete model of the mechanism that assigns interpretations to donkey sentences. Although it would not be difficult to extract a compositional semantics from my treatment of donkey sentences in dynamic predicate logic with generalized quantifiers, our interest in this paper is not in finding the right set of compositional semantic rules that give donkey sentences the interpretation that they actually have (in the default case). Such a set of

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46Consider the following inference, which is valid due to the left continuity of exactly three:

(i) Exactly three farmers who own a donkey are rich
Therefore, exactly three farmers who own a female donkey are rich

Unlike monotonicity inferences like (54)–(56), the validity of (i) is not immediately obvious; it requires some thought to arrive at the conclusion that it is valid. It would be unreasonable to suppose the existence of a calculus tailored for inferences like (i).
rules would not answer the main question of this paper—the question why it is that donkey sentences have the interpretation that they do have. Why is it that the weak reading and the strong reading are distributed in the way they are rather than the other way around, when it is just as easy to write an alternative set of rules that yields the opposite distribution? Such a question usually does not arise in semantics; it would be futile to ask why in English and means conjunction and or disjunction, rather than the other way around. However, the systematic correlation between the monotonicity properties of the determiner and the interpretation of donkey sentences is interesting and does seem to be the kind of phenomenon that calls for an explanation. Our finding, in abstract terms, is that given the semantics of non-donkey sentences, the actual (default) interpretation of donkey sentences is the logically natural one in that it preserves certain important characteristics of the interpretation of non-donkey sentences.

This finding, however, is in a way just a ‘meta-level’ observation; one might say that we have only observed that the pattern uncovered in the interpretations of different donkey sentences has a distinguishing characteristics that separates itself from other conceivable patterns. At the end of Section 5.3.1, I suggested that our finding might be exploited to provide some kind of functional explanation. In this section, I would like to try a completely different line, and suggest a speculative model of how people assign interpretations to donkey sentences in actual language use. The model turns out to highlight the central importance of left monotonicity after all. What follows is a pure speculation without any firm evidence. Whether or not one finds it appealing will depend on one’s taste.

The primary assumption I make is the following:

(109) The grammar rules in general underspecify the interpretation of a donkey sentence.

Thus, I assume that, for any donkey sentence, the grammar only partially characterizes its meaning, with which a range of specific interpretations are compatible. So the truth value of donkey sentences in particular situations may be left undecided by the grammar. This may not be such an outrageous idea; it may explain the lack of robust intuitions about donkey sentences.47

For the sake of concreteness, I assume that the underspecified interpretation of a donkey sentence Det N' VP assigned by the grammar can be represented using an indeterminate dynamic generalized quantifier Q which is related to the

47People often do not have a very clear idea of what donkey sentences mean—I took this to be a relatively uncontroversial point, but a few people, including an anonymous referee and the subject editors of the journal, have expressed doubt. Certainly, what is claimed by Every student who took a course from Peter liked it is less clear than what is claimed by its purported paraphrase Every student who took a course from Peter liked every course he or she took from him, isn’t it? Relative clarity of the intuitions about donkey sentences with no seems to be an exception rather than the rule, and even here, native speakers sometimes give guarded judgments like ‘I would say that this is misleading’ instead of downright ‘False’. Conversations about my work with native speakers have generally reinforced my assessment that judgments about donkey sentences are subtle.
static generalized quantifier \(Q\) denoted by Det and which satisfies certain natural properties. Using the notions from Sections 4 and 5.1, let us say that the correlation between \(Q\) and \(Q\) is given by Agreement, and \(Q\) is supposed to satisfy DEQUII and DCONS.

Even if its interpretation is underspecified, a sentence may be assigned a definite truth value in special circumstances. As shown in Section 5.1, all specific interpretations compatible with the underspecified interpretation of a donkey sentence agree on truth-value in consistent donkey-beating situations. It is not unreasonable to suppose that people are capable of assessing the truth value of a donkey sentence without resolving the 'vagueness' of the meaning given by the grammar when there is no need to do so. For our purposes, it is enough to assume that underspecification causes no problem for people in assigning a truth value to a donkey sentence in situations where the uniqueness condition for the donkey pronoun is met. These are a special class of consistent donkey-beating situations, and the uniqueness condition can be checked just by looking at the extensions of the predicates in the \(N'\) of the sentence.

Let us take (1) as an example.

(1) Every farmer who owns a donkey beats it.

Under the present assumption, the truth value of (1) in situations where all farmers own at most one donkey is clearly and immediately recognized by the language user. In those situations, (1) is assigned the same truth value as Every farmer who owns a donkey beats the donkey he owns. Now I want to suggest that, given the distribution of truth values in those situations, a certain very natural inference based on the downward left monotonicity of every allows one to assign truth values to (1) in situations where the uniqueness condition is not met.

The inference in question has to do with a correspondence between left monotonicity of a quantifier and the preservation behavior of sentences of a certain form. In first-order logic with generalized quantifiers, we have the following:

**Proposition 4.** Assume that \(Q\) satisfies CONS and Extension. Then, \(Q\) is \(\uparrow\text{MON} (\downarrow\text{MON})\) if and only if every sentence of the form \(Q x (\varphi, \psi)\), where \(\varphi\) is existential and \(\psi\) is quantifier-free, is preserved under extensions (submodels).\(^{48}\)

This means that the following patterns of 'inference from submodels' are valid. Let \(\varphi\) and \(\psi\) be as in Proposition 4, and let us indicate left monotonicity of \(Q\) by \(\uparrow Q\) and \(\downarrow Q\).

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\(^{48}\)A first-order formula \(\varphi\) is called existential if it is of the form \(\exists x_1 \ldots x_n \psi\) for a quantifier-free \(\psi\). A model \(M\) (with universe \(M\)) is called a submodel of a model \(N\) (with universe \(N\)) (in symbols, \(M \subseteq N\)) if \(M \subseteq N\) and \(P^M = P^N \cap M^n\) for all \(n\)-ary relation symbols \(P\), \(F^M = F^N \upharpoonright M^n\) for all \(n\)-ary function symbols \(F\), and \(c^M = c^N \subseteq M\) for all constant symbols \(c\). If \(M \subseteq N\), \(N\) is called an extension of \(M\). A sentence \(\varphi\) is said to be preserved under extensions (submodels) if \(M \models \varphi\) implies \(N \models \varphi\) whenever \(M \subseteq N\) (\(N \subseteq M\)).
Confirmation:  \[ \text{N} \subseteq \text{M} \quad \text{N} \models \uparrow Qx(\varphi, \psi) \]
\[ \text{M} \models \uparrow Qx(\varphi, \psi) \]

Refutation:  \[ \text{N} \subseteq \text{M} \quad \text{N} \not\models \uparrow Qx(\varphi, \psi) \]
\[ \text{M} \not\models \uparrow Qx(\varphi, \psi) \]

Confirmation and Refutation are the ‘transmodel’ version of monotonicity inference. I think people may in fact use this sort of inference when they try to determine the truth value of quantified sentences on the basis of limited information.\(^{49}\) For instance, suppose you are trying to determine the truth value of (110), which is represented in first-order logic with generalized quantifiers as (111):

(110) Every farmer who owns a donkey is rich.
(111) \( \text{EVERY} \ x(\text{farmer}(x) \land \exists y(\text{donkey}(y) \land \text{own}(x, y)), \text{rich}(x)) \)

Notice that the first argument of \( \text{EVERY} \) in (111) is equivalent to an existential formula, and the second argument is quantifier-free. Now suppose that in the given ‘model’ \( \text{M} \) you have found a farmer \( f \) and donkey \( d \) such that \( f \) owns \( d \) and \( f \) is not rich. Consider the submodel \( \text{N} \) of \( \text{M} \) which consists of just \( f \) and \( d \).\(^{50}\) You have complete information about this submodel with respect to predicates appearing in (110) = (111), and you know that the sentence is false in \( \text{N} \). Then, by Refutation, you can conclude that the sentence must be false in the entire model \( \text{M} \).

Now I want to assume that language users are disposed to apply the same sort of reasoning ‘blindly’ to donkey sentences when they are trying to evaluate their truth value. The idea here is similar to the one suggested at the end of Section 5.3.1. Given the importance and simplicity of (the natural language counterparts of) Confirmation and Refutation, it is possible that their application is governed by some mechanical procedure, sensitive only to the left monotonicity of the determiner and a certain formal property of the rest of the sentence. What is significant here is the fact that donkey sentences typically have the same superficial appearance as the sentences for which Confirmation or Refutation is valid. It would not be so unreasonable to suppose that language users are prone to apply the inferential mechanism for Refutation to (1) just like they do to (110), given the formal similarity of the two sentences—in particular, their \( \text{N} \) is ‘existential’ and their VP ‘quantifier-free’.

If this picture is true, we can understand why (1) is judged to be false when the condition required by its strong reading does not obtain. Suppose that in the ‘actual model’ \( \text{M} \) there is a farmer \( f \) and a donkey \( d \) such that \( f \) owns \( d \) but \( f \) does not beat \( d \). Now one can form the submodel \( \text{N} \) of \( \text{M} \) consisting of just

\(^{49}\)Barwise and Cooper’s (1981) discussion of ‘witness sets’ is related.

\(^{50}\)To be precise, one first needs to form the reduct \( \text{M}' \) of \( \text{M} \) to the language consisting of expressions in (111), and then take the submodel \( \text{N}' \) of \( \text{M}' \) whose universe is \( \{f, d\} \), which is guaranteed to exist. I forego such technicalities here.
This is a model in which all farmers own at most one donkey, and we know that (1) is clearly false in N. Then, by Refutation, one can infer that (1) must also be false in M, where the uniqueness condition may not be met. Something like this might come close to capturing what people do when they try to understand what (1) says and ask themselves ‘What if some farmer doesn’t beat a donkey he owns?’

A similar story can be told about other ↓MON determiners and ↑MON determiners (in the latter case, with ‘true’ and ‘false’ switched). In fact, it turns out that the assumption that Confirmation and Refutation apply has the same effect as the Left Monotonicity Principle. This is not a complete picture and requires elaboration and refinement, but I think it is a pretty attractive one.

Finally, let me very briefly address the question of how factors other than the monotonicity properties of the determiner might come into the picture. In the above speculative model, the story might go as follows. If the function of pragmatic factors is to make clear the intended interpretation or ‘speaker’s meaning’ of a donkey sentence, then they naturally take effect immediately. If the speaker’s meaning is clear from the beginning, then the person processing the sentence does not have to try to ‘figure out’ what is meant by the sentence, and consequently he or she will not go into the trouble of invoking inference.

Appendix: On Chierchia’s E-Type Strategy

Chierchia (1990, 1992) decides to discard the definition (78) for Q_S and use Q_W for all Q. He has three reasons for doing so. His first reason was explained earlier

[See footnote 50.]

[Formally, the correspondence between left monotonicity and preservation under submodels/extensions given in Proposition 4 carries over to dynamic predicate logic with generalized quantifiers. If we define a suitable dynamic notion of Extension, we have the following (see Kanazawa 1993):

PROPOSITION 5. Assume that Q satisfies DCONS and Dynamic Extension. Then Q is ↑DMON (↓DMON) if and only if every sentence of the form Q_x(φ, ψ), where φ is existential and ψ is quantifier-free, is preserved under extensions (submodels).

A DPL formula φ is existential if it is of the form \( \exists x_1 \ldots \exists x_n \exists x_{n+1} \ldots \exists x_{n+m} \psi \) for a quantifier-free ψ. Typically, if a donkey sentence is translated into dynamic predicate logic with generalized quantifiers, the translation of the N’ will be equivalent to an existential formula in this sense.

[Note that sometimes pragmatic effects having to do with plausibility of the claim are so strong as to create an illusory interpretation. The following sentence seems to say exactly the opposite of what it should say:

(i) No eye injury is too trivial to ignore.

From the meaning of too … to one would expect it to mean ‘no eye injury is so trivial that you can’t ignore it’ or ‘every eye injury is serious enough that you can ignore it’ (Gillian Ramchand (p.c.), who got the sentence from George Boolos).]
in connection with dynamic notions of Conservativity. We saw that the failure to satisfy DCONS1 did not constitute a persuasive argument to reject \( Q_S \). His second reason is the systematic gap in the distribution of the strong reading: for some determiners, the definition (78) for \( Q_S \) is just not adequate. Chierchia seems to think that enough context can always bring out the weak reading, even when the determiner involved usually favors the strong reading. By saying that \( Q_W \) always gives the ‘dynamic meaning’ of the determiner, Chierchia can predict the availability of the weak reading in such cases. Of course, he has to come up with an explanation of what makes the strong reading available when it is. For this, he adopts his ‘E-type strategy’, which I discuss below. His third reason is that having two definitions would make all determiners systematically ambiguous. I hope the foregoing discussions have made it clear that this last reason does not have much force either.

To explain the existence of strong reading, Chierchia resorts to the idea that donkey pronouns are sometimes ‘E-type pronouns’ rather than ‘dynamically bound variables’. So he shifts the locus of ambiguity from the determiner to the pronoun. He holds that postulating the E-type use of pronouns as distinct from their use as dynamically bound variables is necessary to account for other independent phenomena.

For Chierchia, E-type pronouns are something like function terms, where ‘the nature of the function is contextually specified.’ For example, (112a) is interpreted like (112b) (ignoring the irrelevant dynamic aspects):

(112) a. Every man who has a donkey beats it

\[ \text{EVERY } x(\text{man}(x) \land \exists y(\text{donkey}(y) \land \text{have}(x, y)), \text{beat}(x, f(x))) \]

\( f \) is supposed to be a contextually determined function which maps a man who owns a donkey to one of his donkeys. If every man owns at most one donkey, there is only one such function, and (112a) is assigned a definite truth value. Now when there can be more than one function from donkey-owning men to their donkeys, Chierchia (1992) interprets (112b) as if \( f \) is universally quantified (relative to the class of such functions). In essence, what he gets is:

(113) \[ \forall f (\forall x (\text{man}(x) \land \exists y (\text{donkey}(y) \land \text{have}(x, y))) \rightarrow \text{donkey}(f(x)) \land \text{have}(x, f(x))) \rightarrow \text{EVERY } x (\text{man}(x) \land \exists y (\text{donkey}(y) \land \text{have}(x, y)), \text{beat}(x, f(x)))) \]

This turns out to give the strong reading of (112a).

Let us call the reading of a donkey sentence represented by a formula like (113) (with a universally quantified function variable) the universal function reading. Schematically, the universal function reading of a donkey sentence is expressed by a formula of the following form:

(114) \[ \forall f (\forall x (\varphi(x) \land \exists y \chi(x, y) \rightarrow \chi(x, f(x))) \rightarrow Qx(\varphi(x) \land \exists y \chi(x, y), \psi(x, f(x)))) \]
As Chierchia (1992) believes, an added bonus to this E-type analysis of the
strong reading is that if the determiner (or Q in the above formula) is MON↓, the
universal function reading comes out equivalent to the weak reading—the only
reading for donkey sentences with ↓MON↓ determiners. So it seems as if he can
maintain the E-type strategy as generally available while being able to explain
the non-existence of the strong reading in certain cases.

The pitfall here is obvious from our discussion in Section 2.1. Chierchia’s
E-type strategy makes the wrong prediction when the determiner is ↑MON↑ or
↑MON↓. He would predict existence of the strong reading in the former case, and
non-existence of it in the latter case. In fact, if we define the existential function
reading to be the reading expressed by a formula of the following form,

\[(115) \quad \exists f(\forall x(\varphi(x) \land \exists y\chi(x, y) \rightarrow \chi(x, f(x)))
\land Qx(\varphi(x) \land \exists y\chi(x, y), \psi(x, f(x))))\]

we can show the following correlation between the universal and existential
function readings on the one hand and the weak and strong readings on the other.

FACT. Assume that Q satisfies CONS. If Q is MON↑,

universal function reading (114) ↔ strong reading (76)
existential function reading (115) ↔ weak reading (75)

If Q is MON↓,

universal function reading (114) ↔ weak reading (75)
existential function reading (115) ↔ strong reading (76)

This brief discussion should suffice to establish that Chierchia’s ‘weak schema
only’ thesis combined with his E-type strategy is seriously inadequate to explain
the distribution of the strong reading. Such a radical deficiency was obviously
caused by his neglect of ↑MON↑ determiners.\(^{54}\)

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\(^{54}\) Gawron, Nerbonne, and Peters (1991) make a proposal similar to Chierchia’s E-type stra-
tegy. It seems to suffer from the same problem.


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