Another Approach to Truthmaker Semantics

MSc Thesis *(Afstudeerscriptie)*

written by

Stephen Pastan
(born December 22, 1989 in Indiana, USA)

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Date of the public defense:	**Members of the Thesis Committee:**
March 31, 2016	Dr. Maria Aloni
	Prof. Franz Berto
	Dr. Luca Incurvati
	Prof. Benedikt Löwe
	Dr. Floris Roelofsen
	Prof. Frank Veltman

INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Abstract

This thesis concerns truthmaker semantics, introduced by Stephen Yablo in his book Aboutness and by Kit Fine in a series of papers. In large part, what I have accomplished here is filling in gaps. I not only fill in gaps between truthmaker semantics and related theories, I also fill in gaps within truthmaker semantics itself. The literature makes many assumptions but makes only a few explicit, leaving the reader to provide their own account of things like facts, of explanation and even, to some degree, of truthmaking itself. Aside from this, I attempt a contribution of my own. In particular, I present arguments for an account of truthmaker semantics on which truthmakers must be consistent in a precise sense.

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Introduction

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In the first chapter I provide motivation for truthmaker semantics from hyperintensionality, which I define and demonstrate the importance of. In Chapter II, I assess various perspectives on facts, explanation, and truthmaking, each of which is central to truthmaker semantics and none of which have been discussed in adequate depth in the field. In Chapter III I argue for my own account of truthmaker semantics, consistentism, and show its advantages over the competing views.

I. Motivation

The goal of this chapter is to motivate truthmaker semantics with hyperintensionality. The problems of a non-hyperintensional theory of content, the truth conditional theory, are listed and discussed. It is argued that an adequate theory of content must not only be hyperintensional, but strongly hyperintensional in a specific manner. Truthmaker semantics is one such theory.

1. Introduction

The truth conditional theory of content, which says that contents are individuated by the possible facts that imply their truth, encounters a few serious difficulties. A hyperintensional theory of content—in particular, a *strongly* hyperintensional theory of content—avoids these worries, and truthmaker semantics is one such theory. In this way, the present chapter uses hyperintensionality to motivate truthmaker semantics.

Section 2.1 defines the truth conditional theory of content and section 2.2 describes the problems it encounters. Section 3.1 defines hyperintensionality as a general phenomenon and section 3.2 provides the schematic hyperintensional theory of content to match. Section 3.3 distinguishes between strongly and weakly hyperintensional theories of content, arguing that a strong hyperintensionality is needed to avoid the problems faced by the truth conditional theory. The section concludes by explaining how hyperintensionality provides a springboard for the truthmaker semantics I develop in the following chapters.

2. The Truth Conditional Theory of Content

2.1 Its Definition

The present section does not offer much in the way of a history of the truth conditional theory of content, which some (e.g. Kemp 1998) take to have originated with Frege. The aim, simply, is to summarize it. This theory comes with a kind of motto,

"Contents are individuated by the possible facts that imply their truth."

My first move here is to explain this motto, which is replete with undefined terms, and present the truth conditional theory of content in so doing.

While I save discussion of what facts and possible facts are until Chapter II section 2, I can say some here. The facts are semantic primitives that one can view as demands that reality be a certain way. A fact is possible, roughly, if reality can meet the fact's demands. The possible facts inhabit the set $\mathfrak{P}$.

Content, as I view it here, goes by many different names: 'proposition', 'meaning', 'intension'. Contents are semantic entities that are the primary bearers of truth and falsity, among other duties. There is much disagreement about the nature and status of content, but on some of the popular views contents are structured entities, e.g. (King 2007), or sets of things, e.g. possible worlds semantics and any of the published work on truthmaker semantics (Fine 2014, 2015a, 2015b, 2016c, 2016a, 2016b; Yablo 2014a, 2014b), or even *sui generis* entities, e.g. (Bealer 1998).

I do not come down on what contents are. To avoid total quietism, I will say that each sentence expresses a content and that each sentence's content roughly corresponds to "what is said" by that sentence. However this is not all there is to content. As pointed out by Yablo (2014a, §3.3) and earlier by Kaplan (1968, 207–208), there may be contents no sentence expresses. To maintain as general an account as possible, I say no more. For the time being, I cavalierly corral the contents of the truth conditional theory into $\mathfrak{C}_{\text{tct}}$.

Whether one is considering the truth conditional theory of content or not, facts bear an important truth-theoretic relation to contents: implication. For a fact to imply a content is just for the content to be true given reality is as the fact demands. In the context of possible worlds semantics, for instance, facts are possible worlds and contents are sets of possible worlds; possible worlds imply contents by being elements of them.

In the context of the truth conditional theory, implication is a bit more general: it is a relation $\models \subseteq \mathfrak{P} \times \mathfrak{C}_{\text{tct}}$ such that for $s \models S$ to hold, is just for $S \in \mathfrak{C}_{\text{tct}}$ to be true if things are the way $s \in \mathfrak{P}$ demands they be. I say that s implies S or, somewhat redundantly, that s implies the truth of S just in case

$s \models S$. Implication gives the truth conditional theory a way of individuating contents. If the same possible facts imply contents $S, T \in \mathfrak{C}_{\text{tct}}$ then they are the same, $S = T$. If different possible facts imply S and T then they are different, $S \neq T$.

Now that we know what the truth conditional theory of content is, where can one find examples of it? There are some who call Jon Barwise and John Perry's situation semantics (Barwise and Perry 1983) truth conditional, including Scott Soames (1987, 2008, 2014). This doesn't seem quite right though, for Barwise and Perry allow for contents to be individuated by more than just the possible facts. Donald Davidson is often accused of promoting a truth conditional theory in his 1967 'Truth and Meaning,' (Davidson 1967), but this can't be right either. After all, Davidson wanted to do away with contents (in his terminology, meanings) as entities altogether, banishing them for their uselessness (Davidson 1967, 307). Who, then, can rightly be said to support a truth conditional theory of content? David Lewis, I argue. He takes contents (in his terminology, propositions) to be individuated by the possible worlds in which (or "at which") they are true (Lewis 1986b, 53–4). Taking possible worlds to be facts, it can truly be said that Lewis' is a truth conditional theory of content.

So, then, the above provides a run-through of possible facts, contents, implication, and individuation. With all this in place, the motto of the truth conditional theory of content should start to make some sense. This subsection closes with a definition of the truth conditional theory, summarizing all that has been touched on, for reference later on. The definition should be particularly useful in the next section, where problems with the truth conditional theory are discussed at length.

Definition 1 (Truth Conditional Theory of Content).
The truth conditional theory of content consists of three items, $\mathfrak{P}$, $\mathfrak{C}_{\text{tct}}$, and $\models$, that obey the following.

a. $\mathfrak{P}$ is a non-empty set of possible facts.

b. $\mathfrak{C}_{\text{tct}}$ is a non-empty the set of contents.

c. $\models \subseteq \mathfrak{P} \times \mathfrak{C}_{\text{tct}}$ is the implication relation.

d. Contents are individuated by the possible facts that imply their truth.

$$\text{If } s \models S \iff s \models T \text{ for all } s \in \mathfrak{P}, \text{ then } S = T$$

for any $S, T \in \mathfrak{C}_{\text{tct}}$.

2.2 Problems Encountered

Here I describe three problems faced by the truth conditional theory of content, each of which falls into its own area: agent–agent interaction, agent–content interaction, and content–reality interaction. These problems justify the use of hyperintensionality in theories of content, and ultimately serve to bolster truthmaker semantics. I'll start with agent–agent interaction, and move to the other two in turn.

When agents use language to interact, they do more than make declarative statements. Agents, in their ignorance, occasionally use language interrogatively to state a lack of information, to ask questions or, generally, to raise issues. This is not news. Anyone who has answered a question knows that sometimes language is declarative and sometimes it is interrogative. Any theory of content aiming to capture the way agents interact must address interrogative sentences.

The truth conditional theory of content unfortunately does not provide an adequate treatment of interrogatives. Why is this so? Consider the following sentences.

(1) 'Does Jamie live in north or south Atlanta?'

(2) 'Does Jamie live in east or west Atlanta?'

It seems clear that their contents must be different, regardless of what they turn out to be, but on the truth conditional theory this is not so.

There are two ways of seeing this: the quick way and the slow way. Starting with the former, one assumes that sentences are true exactly when their contents are. As a result, no fact implies the truth of either sentence's content since questions like these are never true. For example, 'Jamie lives in east or west Atlanta' may be true but 'Does Jamie live in east or west Atlanta?' cannot. Since the same possible facts imply the content of (1) and (2)—none—the truth conditional theory takes their contents to be the same. Repeating this line of reasoning the truth conditional theory, absurdly, takes all questions to have the same content. This plainly isn't the case.

The latter, slow, correct way of seeing that the truth conditional theory does a bad job with interrogatives assumes, instead, that a content may be true while the sentence expressing it is not. But how could one say that the contents of either (1) or (2) are true?

Inquisitive semantics, as described in (Ciardelli, Groenendijk, and Roelofsen 2013), answers by distinguishing between two different aspects of content (in their terminology, meaning): informative and inquisitive. A sentence's informative content is the information, if any, it has the potential to convey. (1)

and (2) have the same informative content: the fact that Jamie lives in Atlanta. A sentence's inquisitive content is identified with the different ways, if any, of resolving the issue it has the potential to raise. The fact that Jamie lives in north Atlanta, for example, resolves (1) but not (2) for, assuming the fact that Jamie lives in the north, it is still left open whether Jamie lives in the northeast or the northwest.

The informative and inquisitive aspects of content are each individuated by their own relation between facts and contents.[1] Informative content is individuated by $\models_{\text{inf}}$, the informative implication relation, while inquisitive content is individuated by $\models_{\text{inq}}$, the support relation.

The informative implication relation tells us when informative contents are true: a fact s informatively implies a content S just when $s \models_{\text{inf}} S$. The support relation tells us when inquisitive contents are resolved: a fact s resolves a content S just when $s \models_{\text{inq}} S$. The fact that Jamie lives in north Atlanta, therefore, supports (1)'s content but not (2)'s. The same fact also informatively implies the contents of both (1) and (2) and so, assuming the fact that Jamie does indeed live in north Atlanta, (1) and (2)'s informative content is true.

The truth conditional theory of content, then, is inadequate because it can only accommodate informative implication, not support, and flattens the contents of (1) and (2) into their informative aspects as a result. Having done this, the truth conditional theory still wrongly takes (1) and (2) to have the same content and, therefore, fails to account for interrogatives.

Content is not only useful to semanticians but also to epistemologists who, in particular, often investigate the ways in which agents interact with contents. For example, in epistemology one provides accounts of what are often called 'propositional attitudes,' or just 'attitudes' for short. Examples include knowledge, belief, acceptance, consideration, and many more. Different attitudes are different relations agents can have to contents.[2] So say Sam believes that a certain ball is red. Belief, an attitude, relates Sam, an agent, to the content that the ball is red.

On the truth conditional theory of content, however, one cannot provide this analysis of attitudes. To see why this is so, consider another example.

(3) 'Sam believes that Superman is stronger than Clark Kent.'

(4) 'Sam believes that Clark Kent is stronger than Superman.'

1. As far as I know, inquisitive semantics has not been presented in this way. However, it is equivalent to the presentation in (Ciardelli, Groenendijk, and Roelofsen 2015, 37) where $\models_{\text{inf}}$ and $\models_{\text{inq}}$ are defined as relations between contents, instead of between facts and contents.

2. There are those who do not view attitudes this way. Schiffer (2012) provides a nice overview of what's at stake here.

It should be possible for (3) to be true and (4) to be false. If, for instance, Sam is another unsuspecting Metropolitan fooled by a pair of glasses, then Sam might be under the impression that Superman has superhuman strength and Clark Kent is just a lowly human reporter.

However, this combination of truth values is not allowed by the truth conditional theory. Beliefs relate Sam to contents and so, assuming (3) is true, Sam is related by belief to S_C, the content that Superman is stronger than Clark Kent. On the truth conditional theory, S_C is identical to C_S, the content that Clark Kent is stronger than Superman, because both are implied by the same possible facts: none. Therefore belief also relates Sam to C_S and, as a result, (4) must be true even though it should be capable of falsity.

The third and final drawback of the truth conditional theory of content that I mention involves the way contents relate to reality. Specifically, contents that are implied by all the same possible facts sometimes appear to be about different things. Consider the following sentences.

(5) 'If Jon Stewart loves sandwiches, Jon Leibowitz does not.'

(6) 'If Mark Twain hates baseball, Samuel Clemens does not.'

Clearly, the contents of (5) and (6) are implied by the same possible facts since, after all, Jon Stewart is Jon Leibowitz and Mark Twain is Samuel Clemens. However, the sentences simply seem to say different things about different pieces of reality. While (5) is about Jon Stewart's food preferences, (6) is about Mark Twain's sports preferences. Since contents are supposed to capture "what is said" by a sentence, this difference in what (5) and (6) are about should amount to a difference in their contents. On the truth conditional theory, however, their contents are the same. Aboutness has been most recently explored in Stephen Yablo's book *Aboutness* and, in §2.1, Yablo provides a nice summary of prior explorations (2014). These include (Lewis 1988b; Goodman 1961; Ryle 1933), but Yablo does not cite the equally exciting doctoral dissertation of J.M. Dunn (1966), which discusses aboutness in his Chapter IX, or Lewis' slightly later 'Relevant Implication' (1988).

The truth conditional theory of content indeed has some shortcomings. When it comes to agent–agent interaction, it cannot provide an account of interrogatives; when it comes to agent–content interaction, it cannot support a common account of attitudes; when it comes to content–world interaction, it cannot capture our intuitions of aboutness. Why is this theory so problematic? The reason, I claim, is individuation: it is doomed because it individuates contents with possible stuff (in this case: facts) and a truth theoretic relation (implication, that is).

Hyperintensionality, defined in the next section, roughly says that individuation requires more than truth theoretic relations to the possible stuff. In order to avoid the problems faced by the truth conditional theory of content a theory of content must not only be hyperintensional but also strongly hyperintensional, as defined in section 3.3. Strong hyperintensionality, we shall see, is a distinct strength of truthmaker semantics.

3. Hyperintensionality

Vaguely put, the core of hyperintensionality is questioning the common pairing of truth and possibility and asking for more. “More” might be more than just truth (cf. support in inquisitive semantics), “more” might be more than just the possible (cf. impossible worlds), and “more” might come in both forms simultaneously as it does in some forms of truthmaker semantics, as we see in Chapter III.

The goal of the present section is to introduce hyperintensionality and explain its ties to theories of content. Section 3.1 clears up this vague language, saying a bit about what hyperintensionality is. Section 3.2 defines the schematic hyperintensional theory of content, which embodies the hyperintensionality found in various theories of content. Section 3.3 then shows the advantages of strong hyperintensionality, explaining the ways in which it avoids the problems encountered by the truth conditional theory.

All this talk of hyperintensionality serves to motivate truthmaker semantics, which is itself hyperintensional. In the end, however, hyperintensionality does not go far enough. I argue that a theory of content should not only be hyperintensional, but also strongly hyperintensional or, roughly, provide distinct necessarily false contents. Strong hyperintensionality appears as a central theme in Chapter III when deciding how truthmaker semantics should individuate contents.

3.1 Its Definition

One might expect a definition of hyperintensionality to be easy to come by, especially since the word ‘hyperintensional’ is thrown around often in many different genres of philosophy. However, this is not so. There are few definitions of hyperintensionality in its own right, and the definitions one tends to find lack important details. In particular, hyperintensionality is often defined in terms of necessary equivalence,[3] which itself is undefined.[4]

Bjørn Jespersen and Marie Duži, for example, define hyperintensionality in terms of necessary equivalence in their introduction to a special issue of *Synthese* on hyperintensionality.

> Hyperintensionality concerns the individuation of non-extensional entities such as propositions and properties, relations-in-intension and individual roles, as well as, for instance, proofs and judgments and computational procedures… *A principle of individuation qualifies as hyperintensional as soon as it is finer than necessary equivalence.* A hyperintensional principle of individuation bars necessary equivalence from entailing identity, making logically possible the cohabitation of necessary equivalence and non-identity between a pair of fine-grained entities A, B…
>
> Jespersen and Duži 2015, 525

where the emphasis is added.

In defining hyperintensionality, Jespersen and Duži assume a definition of necessary equivalence, but is this really such a big deal? It is, in fact, no small matter. In order to define necessary equivalence one must already have accounts of possibility (which puts the 'necessary' in 'necessary equivalence') and of truth (which puts the 'equivalence' in 'necessary equivalence'); however, many of the areas in which hyperintensionality appears do not. Metaphysical studies of, for instance, "non-extensional entities such as propositions and properties" do not come packaged with accounts of possibility and truth because their goal, in many cases, is to provide them. In cases like these, definitions of hyperintensionality like Jespersen and Duži's are of little help.

The fact that hyperintensionality appears throughout the literature does not, then, mean it has a solid and agreed-upon definition. In some ways, the common appearance of hyperintensionality complicates the task of defining it, for any good definition has to apply to its variegated appearances in disparate corners of the literature.

3. It should also be remarked that hyperintensionalty is often defined in terms of logical equivalence. I do not discuss logical equivalence here for it, too, is difficult to define. Not to say too much on the issue, but logical equivalence is either a model-theoretic or proof-theoretic notion. If it's proof-theoretic, then logical equivalence depends upon a particular set of axioms and rules and, therefore, logical equivalents may change drastically depending on who you ask. Say, for instance, that A and B are not logically equivalent sentences and neither is logically equivalent to the negation of the other. In classical logic, $A \vee \neg A$ is logically equivalent to $B \vee \neg B$ but in intuitionistic logic this logical equivalence need not hold. If logical equivalence is model-theoretic then similar problems arise, for what kinds of models are under consideration is open as well.

4. Thanks to Floris Roelofsen on this point.

To list a few of these appearances, hyperintensionality is described as a property of logics (Cresswell 1975), of metaphysical accounts (Nolan 2014), of propositional attitudes (Soames 2014), of sentential operators (Jago 2015a), of theories of intrinsicality (Bader 2013), of theories of properties themselves (Eddon 2011), and, relevantly, of theories of content. Here, I provide a method of finding a definition of hyperintensionality *ad hoc*. All one has to do is find the situationally appropriate definition of necessary equivalence.

Necessary equivalence seems to say something about truth and possibility, something like two things, be what they may, are necessarily equivalent just in case they bear a truth theoretic relation, of some sort, to all the same possible stuff, whatever that might be. This is only decapitating hydras, now there are more unanswered questions. What are these two necessarily equivalent things? What is this truth theoretic relation? What is this possible stuff?

Let's take possible worlds semantics contents as an example. Contents are necessarily equivalent iff they are true in all the same worlds. What are the necessarily equivalent things in this example? Contents, e.g. S. What is the possible stuff? Possible worlds, e.g. a world w. What is the truth theoretic relation? S's truth in w, i.e. $w \models S$.[5]

Let's take properties as another example, e.g. from Eddon (2011). Properties are necessarily equivalent iff, in each world, they hold of all the same objects. What are the necessarily equivalent things in this example? Properties, e.g. P. What is the possible stuff? Possible objects, e.g. an object a that inhabits a world w. What is the truth theoretic relation? Being true in w that the object a has the property P, i.e. $w \models Pa$.

To get a necessary condition for necessary equivalence of any sort, one can generalize these instances of necessary equivalence away from contents and properties.

Remark 1 (Necessary Condition for Necessary Equivalence).
Let $R \subseteq X \times Y$ be a relation between sets X and Y, and let $y, y' \in Y$. If y is necessarily equivalent to y', then

$$xRy \iff xRy'$$

for all $x \in X$.

What must one do in order to turn this necessary condition into a full-fledged definition of necessary equivalence? Pick the right X and R, given the Y.

5. This phrasing of possible worlds semantics might look familiar, but it might not. Possible worlds semantics is most often described as taking contents *to be sets of* worlds, not *to be individuated by* worlds. However, it is more general not to come down on what contents are and have them relate to worlds with $\models$ instead of $\in$. There are those who prefer this means of presenting possible worlds semantics, particularly in discussing hyperintensionality, notably Carl Pollard in, e.g., (Pollard 2008) and Max Cresswell in (Cresswell 1975).

It is not hard to find the appropriate values for X and R to churn out the familiar instances of necessary equivalence, and we already gave two examples of how to do so above. If Y contains possible worlds semantics' contents, the Xs are possible worlds and the R is truth in a possible world. If Y contains properties, then the Xs are possible objects and R is being true in a possible world that the object has the property. Necessary equivalence gets tied to possibility and truth here: X always has something to do with possibility and R always has something to do with truth.

What, then, is hyperintensionality? It's when necessary equivalence, whatever that may be in the context of a particular theory, does not amount to equality. It can now be seen how hyperintensionality rejects the pairing of possibility and truth: in being unsatisfied with necessary equivalence as a guide to identity, it is unsatisfied with the possibility and truth inherent in necessary equivalence.

Although this does not amount to a definition of hyperintensionality, it gives one a recipe for finding such a definition *ad hoc*. In order to see whether a theory of something (properties, content, etc.) is hyperintensional, put all the "things", be what they may, in Y. Next find the right possible stuff—put it all in X—and the right truth theoretic relation—call it $R \subseteq X \times Y$—given the particular Y. Now refer back to Remark 1 to get a definition of necessary equivalence and try to find distinct $y, y' \in Y$ that are necessarily equivalent. If there are distinct necessarily equivalent elements of Y, the theory is hyperintensional; otherwise, it is not.

One can see that the truth conditional theory of content, for instance, is not hyperintensional. Given that Y is $\mathfrak{C}_{\text{tct}}$, the set of truth conditional contents, have X be $\mathfrak{P}$, the set of possible facts, and R be $\models$, the truth theoretic implication relation. By Definition 1d,

$$\text{If } s \models S \iff s \models T \text{ for all } s \in \mathfrak{P}, \text{ then } S = T$$

for all $S, T \in \mathfrak{C}_{\text{tct}}$. Since X and R have been chosen correctly given Y, and since the truth conditional theory has necessarily equivalent contents be identical, we get the expected result: the truth conditional theory of content is not itself hyperintensional.

I have not provided a grand unified theory of hyperintensionality, but I has given good reasons why one cannot do so. Specifically, a definition of hyperintensionality requires a definition of necessary equivalence that not everybody is prepared to give. The account of hyperintensionality given here, then, is schematic and depends upon how one conceives of possibility, of truth, and hence of necessary equivalence. Sure, schematicitiy brings an unappealing lack of specificity, but it also has the appealing result of an account of hyperintensionality that can be made to fit a variety of situations. The account is broad but not deep.

In the following, the phrase 'necessary equivalence' is thrown around, but this is not done ham-handedly. It is true that one cannot employ this terminology without assuming a background theory, without specifying the right values for X and R given the value of Y. In what follows, however, this should pose no problem. One should now have some idea of what the right values for X and R look like: X is the set of some possible stuff, and R is some truth theoretic relation along the lines of implication.

3.2 Hyperintensional Theories of Content

The reason why the truth conditional theory of content fails to be hyperintensional is that it individuates contents with possible facts and implication. Any theory of content that individuates contents with either (i) more than just *possible* facts, or (ii) something more discerning than implication is hyperintensional.

Impossible worlds semantics, for example, takes the former route. Instead of individuating contents with something more discerning than implication, it individuates them with more than just the possible worlds. What these additional worlds, sometimes called 'non-normal' worlds, turn out to be is different on different accounts. I choose not to enter this debate, simply calling them 'impossible'.

Definition 2 (Impossible Worlds Semantics' Theory of Content).
In impossible worlds semantics, contents are defined using four items: $\mathfrak{W}', \mathfrak{W}, \mathfrak{C}_{\text{iws}}$, and $\models_{\text{iws}}$. These collectively obey the following.

a. $\mathfrak{W}'$ is the non-empty set of all worlds, both possible and impossible.

b. $\mathfrak{W} \subseteq \mathfrak{W}'$ is the non-empty set of possible worlds.

c. $\mathfrak{C}_{\text{iws}} \subseteq 2^{\mathfrak{W}'}$ is the non-empty set of impossible worlds contents.

d. $\models\ \subseteq \mathfrak{W}' \times \mathfrak{C}_{\text{iws}}$ is the implication relation, where

$$x \models S \iff x \in S$$

for all $x \in \mathfrak{W}'$ and $S \in \mathfrak{C}_{\text{iws}}$.

With the fair assumption that sets are individuated by their members, this definition results in the following.

Proposition 1 (Property of Impossible Worlds Semantics' Contents).
Let $\mathfrak{W}', \mathfrak{C}_{\text{iws}}$, and $\models$ be as they are in Definition 2. On the impossible worlds semantics' theory of content, contents are individuated by the

worlds, either possible or impossible, in which they are true,

$$\text{if } x \models S \iff x \models T \text{ for all } w \in \mathfrak{W}', \text{ then } S = T$$

for any $S, T \in \mathfrak{C}_{\text{iws}}$.

Contents are the same, on this view, if they are true at all the same possible and impossible worlds. Impossible worlds semantics is therefore hyperintensional because contents may be necessarily equivalent by being true in the same possible worlds and simultaneously distinct by having different truth values at impossible worlds.

Inquisitive semantics takes the other route to hyperintensionality. While it individuates contents with only the possible facts, it also individuates them with a support relation that is more discerning than implication. Before phrasing this formally in Proposition 2, I state the theory of content that comes with inquisitive semantics in the following definition.[6]

Definition 3 (Inquisitive Semantics' Theory of Content).
In inquisitive semantics, contents are defined using five items:
$\mathfrak{W}, \mathfrak{P}_{\text{inq}}, \mathfrak{C}_{\text{inq}}, \models_{\text{inf}}$, and $\models_{\text{inq}}$. These collectively obey the following.

a. $\mathfrak{W}$ is the non-empty set of possible worlds.

b. $\mathfrak{P}_{\text{inq}} \subseteq 2^{\mathfrak{W}} \setminus \{\varnothing\}$ is the non-empty set of possible inquisitive facts.

c. $\mathfrak{C}_{\text{inq}} \subseteq 2^{2^{\mathfrak{W}} \setminus \{\varnothing\}}$ is the non-empty set of inquisitive contents, each of which is downward closed,

$$\text{if } s \subseteq t \text{ and } t \in S, \text{ then } s \in S$$

for any $s \subseteq \mathfrak{W}$, any $t \in \mathfrak{P}_{\text{inq}}$, and any $S \in \mathfrak{C}_{\text{inq}}$.

d. $\models_{\text{inf}} \subseteq \mathfrak{P}_{\text{inq}} \times \mathfrak{C}_{\text{inq}}$ is the informative implication relation,

$$s \models_{\text{inf}} S \text{ if and only if } s \subseteq \bigcup S$$

for all $s \subseteq \mathfrak{W}$ and $S \in \mathfrak{C}_{\text{inq}}$.

e. $\models_{\text{inq}} \subseteq \mathfrak{P}_{\text{inq}} \times \mathfrak{C}_{\text{inq}}$ is the support relation,

$$s \models_{\text{inq}} S \text{ if and only if } s \in S$$

for all $s \in \mathfrak{P}_{\text{inq}}$ and $S \in \mathfrak{C}_{\text{inq}}$.

6. It should be said that inquisitive semantics does make use of a unique impossible fact: the empty set of worlds. However, this impossible fact is not used to individuate contents because every content, in fact, is supported by the impossible fact.

From this definition, it is not hard to prove the following proposition.

Proposition 2 (Properties of Inquisitive Semantics' Contents).
Let $\mathfrak{P}_{\text{inq}}, \mathfrak{C}_{\text{inq}}, \models_{\text{inf}}$, and $\models_{\text{inq}}$ be as they are in Definition 3. The following follow from inquisitive semantics' theory of content.

a. Contents are individuated by the possible inquisitive facts that support them,

$$\text{if } s \models_{\text{inq}} S \iff s \models_{\text{inq}} T \text{ for all } s \in \mathfrak{P}_{\text{inq}}, \text{ then } S = T$$

for any $S, T \in \mathfrak{C}_{\text{inq}}$.

b. Support is more discerning than informative implication,

$$\text{if } s \models_{\text{inq}} S, \text{ then } s \models_{\text{inf}} S$$

for any $s \in \mathfrak{P}_{\text{inq}}$ and $S \in \mathfrak{C}_{\text{inq}}$.

It is not necessary to recount the whole view to make the present point, it is enough for now to just say that inquisitive semantics is hyperintensional because it allows contents that are necessarily equivalent—by being informatively implied by the same possible inquisitive facts—to be distinct—by being supported by distinct possible inquisitive facts.

If a theory of content is supposed to embody all hyperintensional theories, it must allow for both routes. On such a theory, call it the schematic hyperintensional theory of content, contents are individuated by facts, which may be possible, impossible, and beyond, and the explication relation, $||\!-$, which is at least as discerning as truth-theoretic implication and perhaps even more so.

Definition 4 (The Schematic Hyperintensional Theory of Content).
Contents are defined using five items, $\mathfrak{F}, \mathfrak{P}, \mathfrak{C}, ||\!-$, and $\models$. These collectively obey the following.

a. $\mathfrak{F}$ is the set of facts.

b. $\mathfrak{P} \subseteq \mathfrak{F}$ is the set of possible facts.

c. $\mathfrak{C}$ is the set of contents.

d. $\models \subseteq \mathfrak{F} \times \mathfrak{C}$ is the implication relation.

e. $||\!- \subseteq \mathfrak{F} \times \mathfrak{C}$ is the explication relation.

f. Contents are individuated by the facts that explicate them.

$$\text{If } s \;||\!-\; S \iff s \;||\!-\; T \text{ for all } s \in \mathfrak{F}, \text{ then } S = T$$

for any $S, T \in \mathfrak{C}$.

g. Explication is at least as discerning as implication.

$$\text{If } s \mid\!\vdash S, \text{ then } s \models S$$

for any $s \in \mathfrak{F}$ and $S \in \mathfrak{C}$.

In order to really understand this definition, it would not be a bad idea to go back over it slowly. Facts relate to contents in two sorts of ways. First, there is plain old truth-theoretic implication, $\models \subseteq \mathfrak{F} \times \mathfrak{C}$. For a fact $s \in \mathfrak{F}$ to imply a content $S \in \mathfrak{C}$, i.e. for $s \models S$ to hold, is just for S to be true if things are as s demands. Second, there is the more involved explication relation, $\mid\!\vdash \subseteq \mathfrak{F} \times \mathfrak{C}$. Whenever a fact $s \in \mathfrak{F}$ explicates a content $S \in \mathfrak{C}$, i.e. whenever $s \mid\!\vdash S$, s must also imply S.

Explication is not supposed to be some one thing in the way that implication is truth profferance. Explication is simply the relation that individuates contents. Different theories of content might have very different ways of interpreting explication, and there is no problem with this. In inquisitive semantics, explication would be support; in truthmaker semantics, it would be truthmaking. As discussed in Chapter II section 3, truthmaker semantics requires that a fact both imply a content and explain its truth in order for its "explication" to occur. In a general setting, however, this need not be the case; the relations $\models$ and $\mid\!\vdash$ may coincide as they do in impossible worlds semantics, for example.

The central notion of the schematic hyperintensional theory is that contents are individuated by the facts that explicate them.[7] Contents that are explicated by all the same facts are the same; contents explicated by some different facts are different.

It is not hard to see that familiar theories of content are special cases of the schematic hyperintensional theory of content. To get inquisitive semantics: Let the facts be the possible inquisitive facts ($\mathfrak{F} = \mathfrak{P}_{\text{inq}}$), let the contents be the inquisitive contents ($\mathfrak{C} = \mathfrak{C}_{\text{inq}}$), let implication be informative implication ($\models \; = \; \models_{\text{inf}}$), and let explication be support ($\mid\!\vdash \; = \; \models_{\text{inq}}$). The moves for impossible worlds semantics are similar: let the facts be the worlds ($\mathfrak{F} = \mathfrak{W}'$), let the contents be the impossible worlds contents ($\mathfrak{C} = \mathfrak{C}_{\text{iws}}$), let implication be, well, implication, and let explication coincide with implication ($\mid\!\vdash \; = \; \models$).

In sum, the schematic hyperintensional theory of content has five parts: the set of facts $\mathfrak{F}$, the set of possible facts $\mathfrak{P}$, the set of contents $\mathfrak{C}$, and the implication and explication relations $\models$ and $\mid\!\vdash$ respectively. Two properties of explication have been described as well: it individuates contents and it is at least as discerning as implication although the two may coincide. This is all one needs for the schematic hyperintensional theory of content.

7. Falsism, discussed in Chapter III section 2, actually has two explication relations instead of one. Falsism is found unfit for truthmaker semantics, however, so I needn't account for it here.

3.3 Problems Avoided

In section 2.2, three different problems for the truth conditional theory of content were raised: it cannot distinguish between the contents of interrogatives, it cannot abide a common treatment of attitudes, and it cannot accommodate our intuitions of aboutness. Hyperintensionality avoids these problems. By permitting more content-individuators, it allows for a much more versatile notion of content.

Take, for instance, the second problem from subsection 2.2,

(3) ‘Sam believes that Superman is stronger than Clark Kent.’

(4) ‘Sam believes that Clark Kent is stronger than Superman.’

It has been argued that a good theory of content allows (3) to be true and (4) to be false. In order to do so, such a theory has to allow the following contents to be distinct.

S_C the content that Superman is stronger than Clark Kent

C_S the content that Clark Kent is stronger than Superman

The schematic hyperintensional theory of content allows for two different options for individuating contents: have explication differ from implication, $|\!|\!\!- \neq \models$, or have contents be individuated by more than just the possible facts, setting $\mathfrak{F} \neq \mathfrak{P}$. Is either of these options enough to individuate S_C from C_S?

On their own, they are not. Although both these options lead to hyperintensional theories of content, neither is guaranteed to lead to a theory of content on which S_C is distinct from C_S. The resulting hyperintensional theories individuate more contents than the truth conditional account, surely. Not all the resulting theories, however, individuate S_C from C_S.

Something can be said about what a theory that individuates S_C from C_S looks like. First notice that the two contents share a property in common: neither is implied by any possible facts. In other words, they're both necessarily false.

Definition 5 (Necessarily False Content).
Let $(\mathfrak{F}, \mathfrak{P}, \mathfrak{C}, \models, |\!|\!\!-)$ be the schematic hyperintensional theory of content. $S \in \mathfrak{C}$ is necessarily false if and only if

$$s \not\models S$$

for all $s \in \mathfrak{P}$.

In order for a theory of content to individuate S_C and C_S, it must individuate necessarily false contents. Such a theory would not only be hyperintensional, allowing necessarily equivalent contents to be distinct, it would also be some thing stronger, allowing necessarily equivalent and necessarily false contents to be distinct. Call this *strong* hyperintensionality.

Definition 6 (Strong Hyperintensionality).
Let $(\mathfrak{F}, \mathfrak{P}, \mathfrak{C}, \models, ||\!\!-)$ be the schematic hyperintensional theory of content. If there exist distinct necessarily false $S, T \in \mathfrak{C}$, then the theory of content is strongly hyperintensional; otherwise it is weakly hyperintensional.

Strong hyperintensionality is useful for far more than just treating attitudes. Looking back at the third problem for the truth conditional theory, which highlighted its inability to account for aboutness, one was also called to individuate necessarily false contents, the contents of (5) and (6). Strong hyperintensionality pops up everywhere and plays a vital role in Chapter III.

4. Conclusion: A Step To Truthmaker Semantics

The truth conditional theory, which is not hyperintensional, is too restrictive. The schematic hyperintensional theory of content, which is intended to embody all hyperintensional theories of content, makes up for the flaws of the truth conditional theory. What, however, does this have to do with truthmaker semantics?

It should first be noted that hyperintensionality is a primary motivation for truthmaker semantics. Yablo's account (Yablo 2014a) sprung from the inherently hyperintensional goal of treating aboutness, and Fine, in his first full articulation of truthmaker semantics, wrote

> Instead of starting with an intensional account of propositions and trying as best one can to convert it into a satisfactory hyperintensional account, one should start with a hyperintensional account and then see how best it might be accommodated within an intensional framework.
>
> Fine 2015a, 19

Fine went so far as to build hyperintensionality into his formalism for its own sake.

Even if hyperintensionality and truthmaker semantics are the kindred spirits I claim them to be, why would one adopt truthmaker semantics instead of the schematic hyperintensional theory of content given above, which serves our purposes so well? The answer: it cannot properly be adopted because it isn't properly a theory. In fact, it employs two purely formal notions that have not been given definitions: $\mathfrak{F}$, the set of facts, and $||\!-$, the explication relation. What has been said is that facts and explication individuate contents, but nothing has been said on the topic of what facts are or which facts individuate contents, and nothing has been said on the topic of what explication really is.

Truthmaker semantics, as found in the literature, takes a stand on explication: it is to be interpreted as truthmaking, even if not much is said on what truthmaking actually is. As found in the literature, truthmaker semantics does not take a stand on what facts are or on which facts individuate contents. I say a bit more about what truthmaking is in Chapter II. Also in Chapter II, I give an idea of what facts should be and in Chapter III I argue that the consistent facts individuate contents.

II. Truthmaker Semantics: Facts and Truthmakers

The goal of this chapter is to provide accounts of facts and of truthmaking fit for truthmaker semantics. I do not come down on what facts are, although I give some ideas of what they might be. I do come down on what truthmaking is: a fact's implying and explaining the truth of a content. A preliminary account of explanation is given.

1. Introduction

There are many different versions of truthmaker semantics.[1] No matter the version, two tenets are agreed upon.

Pertinence Contents are individuated by the pertinent facts.

Truthiness Contents are individuated by their truthmakers.

Together, pertinence and truthiness say that contents are individuated by the pertinent facts that make them true.

Pertinence is different things to different people. To me it is consistency, but more on this in Chapter III. For now, what's important are facts, mentioned in pertinence, and truthmakers, mentioned in truthiness. Even though truthmaker semantics is characterized by facts and truthmaking, neither have been discussed in significant depth in the truthmaker semantics literature.

This chapter aims to partially correct this omission. In section 2 I discuss facts and in section 3 I provide an account of truthmaking which, in large part, means providing an account of explanation. The section concludes with

1. For an overview of the different versions of truthmaker semantics, see Fine 2016c.

a summary of the notions discussed. With this in place, we have enough to argue that the pertinent facts are the consistent ones in the next chapter.

2. Facts

Facts are the center of truthmaker semantics because they provide it with contents. If truthmaker semantics' contents are individuated at all, facts are doing the individuating. This section discusses facts from two different angles. First it gives a short history of facts in related theories of content. I do not ultimately come down on what facts are, but this history does provide some examples of what they might be. Second, it gives a way of formalizing the facts. This formalization is the largest part of the chapter and occupies subsection 2.2 and subsection 2.3. The former defines some basic properties of facts: parthood, possibility, fusion, and a few more. The latter defines two more involved properties of facts: contrariety and consistency.

2.1 A Brief History of the Facts

There are two principal accounts of truthmaker semantics, one developed by Stephen Yablo in his book *Aboutness* (Yablo 2014a) and its supplement (Yablo 2014b), the other developed by Kit Fine in a series of papers, particularly (Fine 2015a, 2015b). Even though content's dependence on facts is at the core of all accounts of truthmaker semantics, neither Yablo nor Fine says very much about what facts are. Instead, both go instrumentalist by focusing on what facts do.

One cannot find any explicit mention of such instrumentalism and it would be surprising, if not oxymoronic, for Yablo and Fine to discuss what their accounts lack. Nonetheless one can still pick up on it in their discussions of particular kinds of facts. In *Aboutness*, Yablo's approach to truthmakers, which are themselves facts, is quite explicitly instrumentalist. To quote,

> I will not be trying to tell you "what truthmakers are," because we can afford to be flexible; it is only their behavior that matters.
>
> Yablo 2014a, 54

Fine, when discussing impossible facts (which reappear in Chapter III section 3 of the present work), referred to as "impossible worlds or the like", elaborates on Yablo's point.

> The central question is whether impossible worlds or the like are of any use, especially for the purposes of semantic enquiry. If they

> are of no use, then who cares whether they exist or what they are like? And if they are of some use, then we should be able to find a place for them within our ontology, if only as a convenient fiction.
>
> Fine 2016a, 4

Although instrumentalism's focus on the use of facts is practicable, something about it is a bit unsatisfying. It seems that there should be something to say about what facts are. For some help with this, I turn to a discussion of facts in related semantic theories.

Possibility semantics, due originally to Humberstone (1981) and recently worked on by Holliday (2014, 2015), takes facts to be *possibilities*. Possibilities are much like possible worlds except in that they lack total specificity. On this view, facts are partially ordered by how specific they are and so, for instance, the fact that the ball is mauve is greater (with respect to the partial order) than the fact that the ball is red.

In situation semantics, facts are *situations*. As Devlin (2008) points out, situations are thought of in two different sorts of ways. To some, situations are parts of possible worlds and bear some resemblance to possibilities. This position is espoused by Perry (1986, esp. 100–6) and, in a slightly different form, by Kratzer (1989, 2002) who adds an ontological flavor by including particulars among the facts.

Stalnaker (1986) resists this view of situations but, to others, situations are something else. As discussed in detail by Barwise (1989, ch. 2), situations are also viewed as information states. Inquisitive semantics, too, takes facts to be information states, each of which is represented formally as a set of possible worlds (Ciardelli, Groenendijk, and Roelofsen 2015).

How do Yablo and Fine fit in to this backdrop? Quite well, formally speaking. Yablo takes after the inquisitive semanticians at times, having facts be sets of possible worlds (Yablo 2014b, 5). Fine takes facts to be partially ordered by a parthood relation, resembling the possibility semanticians. Conceptually speaking, however, instrumentalism distinguishes Yablo's and Fine's accounts from the others. What is of central importance to Yablo and Fine is that facts can play the role of truthmaker, they do not care as much about what facts must be—world-parts, information states, etc.—in order to do so.

Even though it is unclear what Yablo and Fine think facts are, the historical context should give some idea of the kinds of things they might be. What's more, there is now a clear way of formalizing their conceptions of facts: Yablo's facts are sets of possible worlds, Fine's facts are elements of a partial order.

The present work follows in the footsteps of Yablo and Fine by being noncommittal. To provide the most generality possible, it is best to be flexible about the nature of facts. They might be possible, impossible, total like worlds or partial like possibilities. The precedent for such open posturing goes all the way back to what might be considered the first work in truthmaker semantics, Bas van Fraassen's 'Facts and Tautological Entailments' (1969). One of the above accounts of facts may be correct, but the matter is neither settled here nor changes what is settled here.

Since the following mainly discusses the work of Yablo and Fine, claims about facts are often illustrated with talk of information states or sets of possible worlds or, generally, demands made of reality. This language is appropriate for sets possible worlds and information states both can be thought of as making demands. A set of possible worlds demands that reality must be the way it is in one of its members. Information states demand that reality conform to the information they embody. These approaches need not be distinct, however, since sets of possible worlds can be used to model information, which has already been seen in inquisitive semantics and has roots going back to Stalnaker's context sets (Stalnaker 1975).

In sum, there is nothing lost and much ease of explanation gained by talking about facts as demands made on reality. But, in the van Fraassenian manner, I hope "that engaging in such discourse does not involve ontological commitment" (1969) to any particular sort of fact. Facts, here, are simply the elements of $\mathfrak{F}$.

2.2 A Few Facts about Facts

Although I am not committed to a view of what facts are, I do commit myself to a few different properties they are generally taken to have. This subsection describes some of these properties and the next describes two more.

I have already mentioned that one can view facts as demands of reality, and this proves extremely useful in the present subsection. To start, facts can hold or not hold. Just as a set of possible worlds holds if reality (or, as the case may be, if the actual world) is one its members and just as an information state holds if the information it embodies reflects reality, a fact holds if reality meets its demands. Even though I have already described contents as the primary bearers of truth and falsity, I say here that a fact is true if it holds and false if it does not. This choice of terminology is not intended to make any substantial claims about what truth is or what falsity is or about the relation between facts and contents.[2] I simply intend to make exposition a bit easier.

Next, some facts are part of others. Take, for instance, the following.

2. It should be mentioned that Yablo, interestingly, takes truthmaking to be horizontal in that truthmakers, facts, are "of the same category of at least *some* of what they make true, namely other propositions" (Yablo 2014a, 74) or, in the present terminology, contents.

(1) The fact that the ball is red

(2) The fact that the ball is mauve

There is something intuitively right about (1) being a part of (2). It just seems that something's being red is part of its being mauve, and the literature agrees. Although I have borrowed Fine's term 'part' to describe this relationship between facts, the idea is found in other semantic theories under other names. In inquisitive semantics, for instance, instead of saying "facts are part of" one another one would say "states enhance" one another (Ciardelli, Groenendijk, and Roelofsen 2015, 19; Ciardelli 2016, 5). Yablo says "facts imply" one another (cf. Yablo 2014a, 133) while Humberstone says that "possibilities are more specific than" one another (Humberstone 1981, §1). In the end these different terms all roughly amount to the relation described above: for one fact to be part of another is just for it to have looser demands on reality.

With demands in mind, let's look again at the above example concerning the ball's reddish color. For one fact to be part of another is just for it to ramp up the demands made of reality. (1) demands something, namely that the ball be red, while (2) ups these demands, requiring not only that the ball be red but that it be a certain red, that it be mauve. Thought of in terms of information retention (like Fine and the inquisitive semanticians), $s \in \mathfrak{F}$ is part of $t \in \mathfrak{F}$ if and only if t retains all the information present in s. If one prefers possible worlds (like Yablo and the inquisitive semanticians), fact parthood can equally be thought of as supersethood: s is part of t if and only if $s \supseteq t$.

Under whichever conception, a little thought makes it clear that fact parthood is indeed a partial order, i.e. transitive, reflexive, and antisymmetric. One can formalize it as a relation $\leq \subseteq \mathfrak{F} \times \mathfrak{F}$.

Definition 7 (Fact Parthood).
Let $\mathfrak{F}$ contain the facts. $\leq \subseteq \mathfrak{F} \times \mathfrak{F}$ is a partial order called fact parthood.

If $s \leq t$ one says that s is part of t or, when there is no ambiguity, that s is smaller than t. $(\mathfrak{F}, \leq)$, then, is a partially ordered set or, more familiarly, a poset.

Using fact parthood, one can define the downward closure operator $(\cdot)^{\downarrow} : 2^{\mathfrak{F}} \to 2^{\mathfrak{F}}$. When given a set of facts $X \subseteq \mathfrak{F}$, its downward closure $X^{\downarrow}$ is the result of supplementing X with anything less than some of its elements, anything that X "looks down on". Dually, one can define $(\cdot)^{\uparrow} : 2^{\mathfrak{F}} \to 2^{\mathfrak{F}}$, the upward closure operator, in the expected way.

Definition 8 (Upward/Downward Closure).
Let $\mathfrak{F}$ contain the facts. The downward closure operator, $(\cdot)^{\downarrow} : 2^{\mathfrak{F}} \to 2^{\mathfrak{F}}$,

and upward closure operator, $(\cdot)^{\uparrow} : 2^{\mathfrak{F}} \to 2^{\mathfrak{F}}$, obey the following,

$$X^{\downarrow} = \{s \in X \mid s \leq t, \text{ for some } t \in X\}$$
$$X^{\uparrow} = \{s \in X \mid s \geq t, \text{ for some } t \in X\}$$

for any $X \subseteq \mathfrak{F}$. If $X = X^{\downarrow}$, then X is called 'downward-closed'; if $X = X^{\uparrow}$, then X is called 'upward-closed'.

Fact parthood can also be used to define an operator $\max_{\leq}(\cdot) : 2^{\mathfrak{F}} \to 2^{\mathfrak{F}}$ which, when given a set of facts $X \subseteq \mathfrak{F}$, returns the set $\max_{\leq}(X)$ of maximal facts in X, i.e. the set of elements of X that are each part of no other element of X. Once again, one can dually define $\min_{\leq}(\cdot) : 2^{\mathfrak{F}} \to 2^{\mathfrak{F}}$, the minimality operator, in the expected way.

Definition 9 (Minimality/Maximality Operator).
Let $\mathfrak{F}$ contain the facts. The maximality operator, $\max_{\leq}(\cdot) : 2^{\mathfrak{F}} \to 2^{\mathfrak{F}}$, and the minimality operator, $(\cdot)^{\uparrow} : 2^{\mathfrak{F}} \to 2^{\mathfrak{F}}$, obey the following,

$$\max_{\leq}(X) = \{s \in X \mid \text{if } t \in X \text{ and } s \leq t, \text{ then } s = t\}$$
$$\min_{\leq}(X) = \{s \in X \mid \text{if } t \in X \text{ and } s \geq t, \text{ then } s = t\}$$

for any $X \subseteq \mathfrak{F}$.

Upward and downward closure operators, minimality and maximality operators are all instances of what are called closure operators on the powerset of $\mathfrak{F}$.

Definition 10 (Closure Operator).
Let $\mathfrak{F}$ contain the facts. $c : 2^{\mathfrak{F}} \to 2^{\mathfrak{F}}$ is a closure operator on the powerset of $\mathfrak{F}$ if and only if the following hold.

a. c is inflationary.
$X \subseteq c(X)$.

b. c is monotonic.
If $X \subseteq Y$ then $c(X) \subseteq c(Y)$.

c. c is idempotent.
$c\,(c(X)) = c(X)$

for all $X, Y \subseteq \mathfrak{F}$.

I do not show that upward closure, downward closure operators, minimality and maximality operators are all closure operators, but this is routine to check.[3]

Another property of facts is possibility. If one analyzes possibility in terms of possible worlds, it seems right to say that a fact is possible just in case it is part of a possible world. However, there are those who take possibility to be a basic notion that does not depend upon worlds. One such example is Fine in his "Constructing the Impossible," discussed in section 3 of Chapter III, another being Humberstone (1981). Humberstone, whom we'll discuss here, calls the existence of possible worlds into question. He posits that every possible fact is "capable of being further specified" (Humberstone 1981, 315) and therefore labels possible worlds "inadmissible" because they cannot be further specified. However Humberstone still provides an account of possibility. After all, his is a form of possibility semantics.

Given that some analyze possibility without possible worlds, the most agreeable account of facts should provide an analysis that does not rely upon them either. With agreeability in mind, the present account places two constraints on the set of possible facts, $\mathfrak{P}$. First, that something be possible, $\mathfrak{P} \neq \varnothing$. Second, the set of possible facts is closed under taking parts.

Why this second constraint? If one identifies a fact with demands made on the world, then this is not so hard to see. Let's say a fact t is possible. That means the demands t makes can be met, that reality is capable of doing what t asks. Now let s be part of t. This means that s's demands are looser than t's and thus that s's demands can be met if t's can. Since t's demands can be met, so can s's; hence t is possible. Consider again the example of (1), the fact that the ball is red, being part of (2), the fact that it's mauve. Surely if it's possible for the ball to be mauve, it's possible for it to be red.

Definition 11 (Possible Facts).
Let $\mathfrak{F}$ contain the facts. $\mathfrak{P} \subseteq \mathfrak{F}$ is the set of possible facts and has the following properties.

a. $\mathfrak{P}$ is downward-closed.
If $t \in \mathfrak{P}$ and $s \leq t$, then $s \in \mathfrak{P}$.

b. $\mathfrak{P}$ is non-empty.
$\mathfrak{P} \neq \varnothing$.

Possible worlds, if there are any, serve as examples of maximal possible facts. On the intuitive description, one fact has another as a part by making stronger demands of reality. From this perspective, possible worlds make the

3. A question naturally arises: does $\mathfrak{F}$ have maximal facts? There are some who say so, including the inquisitive semanticians and Yablo, and their existence even plays a vital role for Rumfitt (2012, p. 109) and Fine (2016b). This issue is is interesting, but not treated here. Generally, however, it is not guaranteed that any subset of $\mathfrak{F}$ has a maximal element.

strongest demands that reality can still meet. Each possible world w demands that reality be a single way—that it be precisely as w says. If the demands of some fact were strictly greater they could not be met and the fact in question would not be possible. This, then, is another property of facts: the maximal possible facts, if there are any, are the possible worlds.

Definition 12 (Possible Worlds).
Let $\mathfrak{P}$ contain the possible facts. $\mathfrak{W} \subseteq \mathfrak{P}$ is the set of possible worlds, and obeys the following definition.

$$\mathfrak{W} = \max_{\leq}(\mathfrak{P})$$

where $\max_{\leq} : \mathfrak{F} \to \mathfrak{F}$ is the maximality operator from Definition 9.

Facts can combine in various ways as well. The fusion of facts s and t, written $s \vee t$, is the smallest fact that has both s and t as parts. Viewed in terms of demands on reality, $s \vee t$ is the least demanding fact that strengthens the demands of both s and t. Fusion is a least upper bound or supremum, and is a kind of conjunction for facts. To provide an example,

$$\begin{aligned} s = &\ \text{the fact that Pat is smart} \\ t = &\ \text{the fact that Quinn is clever} \\ s \vee t = &\ \text{the fact that Pat is smart and Quinn is clever} \end{aligned}$$

Aside from the binary $\vee : \mathfrak{F} \times \mathfrak{F} \to \mathfrak{F}$, there is a unary $\bigvee : 2^{\mathfrak{F}} \to \mathfrak{F}$ that, when given a subset X of $\mathfrak{F}$, returns $\bigvee X$, the fusion of all the elements in X. Dually one can define an overlap operation, corresponding to an infimum and a sort of disjunction of facts, in both binary ($\wedge : \mathfrak{F} \times \mathfrak{F} \to \mathfrak{F}$) and unary ($\bigwedge : 2^{\mathfrak{F}} \to \mathfrak{F}$) versions. Occasionally I refer to $\bigvee X$ as the fusion of X, but this is not strictly correct. $\bigvee X$ is the fusion of *the elements* of X; the same goes for overlap.

Definition 13 (Fusion of Facts).
Let $X \subseteq \mathfrak{F}$ be the non-empty set of facts ordered by fact parthood, $\leq$. The supremum of X with respect to $\leq$ is called 'the fusion of X', written $\bigvee X$. The infimum of X with respect to $\leq$ is called 'the overlap of X', written $\bigwedge X$.

Notice, importantly, that the set of facts $\mathfrak{F}$ has not been defined in such a way that either fusion or overlap is always defined. Explicitly, one is not guaranteed that $\bigvee X$ or $\bigwedge X$ exist for all non-empty $X \subseteq \mathfrak{F}$.

To connect some of the definitions in this section, note that one can offer alternative definitions for fusion and overlap.

Proposition 3.

Let $\mathfrak{F}$ be the set of facts. Whenever the operations are defined,

$$\left\{\bigvee X\right\} = \min_{\leq}\left(\bigcap\left\{x^{\uparrow} \in 2^{\mathfrak{F}} \mid x \in X\right\}\right)$$
$$\left\{\bigwedge X\right\} = \max_{\leq}\left(\bigcap\left\{x^{\downarrow} \in 2^{\mathfrak{F}} \mid x \in X\right\}\right)$$

for any $X \subseteq \mathfrak{F}$.

The proof is routine and omitted. The proposition does highlight an important point about fusion and overlap: just as maximality and minimality are not defined for all sets of facts, so too for fusion and overlap.

That about does it for the initial properties of facts. The next subsection is devoted to two more properties, consistency and contrariety, that prove themselves invaluable in the sequel.

2.3 Consistency and Contrariety

The present subsection defines consistency and contrariety. Each serves a purpose for my account. It is my contention that contents are individuated by the consistent facts, and this is argued in Chapter III. Contrariety is useful in that it is needed to define consistency, and can also be useful on some accounts for defining negation.

Aside from the ends they serve, what are consistency and contrariety and what does it mean for a fact to be contrary or consistent? I address these questions with an inspection of inconsistency, looking to Ray Sorensen's 'The Art of the Impossible' (2002) for some guidance. By poking and prodding in the right places, it becomes clear what consistent and contrary facts should be.

In the article Sorensen offers a $100 reward to anyone who can present him with a picture of an inconsistency. To make reading a bit simpler, I'll just call these 'inconsistent pictures'.[4] Ultimately he offers a picture of his own and is optimistic that he himself deserves the reward. The picture contains five Penrose triangles arranged into a five-pointed star.

It does not matter for the present purposes why there must be five triangles. What matters is why the picture is inconsistent. In Sorensen's words,

> The vertices are each possible, but not *co-possible*. One sees this without relying on labels or captions. The inconsistency is within the picture itself.
>
> Sorensen 2002, 358

4. In his words, Sorensen seeks a picture of a "logical impossibility". However, he uses the words 'inconsistent', 'logically impossible', and 'contradictory' interchangeably, so this rephrasing of his ideas is an acceptable deviation.

where the emphasis is added. A picture is inconsistent, then, if it has parts that are not "co-possible". I have a grievance here, but I'll save it for later.

One might be wondering why a philosopher would care enough about a picture to pay $100 for it. Sorensen describes two different sorts of uses for it in his §3 and §4 (Sorensen 2002). The former, which is the only one of present interest, is that pictures have a role in content.

I agree that pictures play a role in content, and Sorensen provides two good examples. First, some pictures provide evidence for contents the same way that photographs provide evidence for events. Second, some pictures are objects in contents, e.g. the content that the color of Sam's hair is *blank*, where *blank* is some picture.

However, Sorensen does not say much about how the existence or non-existence of an inconsistent picture could affect one's theory of content. I, at least, cannot see how this would be. What difference does it make if inconsistent pictures can provide evidence for, or be objects in, contents?

Since I'm already challenging Sorensen, I might as well air my earlier grievance. It seems to me that non-co-possibility is not sufficient for the kind of inconsistency Sorensen aspires to. One can have a diagram, itself a kind of picture, which shows the ways an ice cube can develop over time. From an original solid ice cube one can simply two draw arrows, themselves pictorial, one pointing to a puddle of water and another to a solid ice cube.

The parts of the picture at the end of these arrows are not co-possible, one and the same ice cube cannot be both melted and unmelted. However the picture does not depict an inconsistency. Rather, it depicts a truism. Sometimes ice melts, sometimes it doesn't.

Sorensen encounters two problems. First, it is unclear how the existence of inconsistent pictures has an affect on theories of content. Second, if inconsistency is not having non-co-possible parts, then what is it? I think both these problems can be solved.

The first problem can be solved by, as I call it, "going Tractarian": take pictures to be facts.[5] How does the existence or non-existence of inconsistent facts affect content? If there are inconsistent facts then they individuate contents; if not then not. It is now clear how the existence or non-existence of inconsistent facts affect a theory of content. If they do exist, then the theory of content is more fine-grained, more readily individuating contents. If they do not exist, the theory of content is less fine-grained.

The second problem is still unsolved and the definition of inconsistency remains open. It has been argued that inconsistency requires more than having non-co-possible parts. There is a lot more to say about what makes a fact

5. Refer to the *Tractatus* itself, "Das Bild ist eine Tatsache," (Wittgenstein 2014, 2.141) meaning that all pictures are facts.

inconsistent (and hence what makes it consistent), and the rest of the subsection is devoted to doing so.

So far, co-possibility has been a property of parts of pictures, but when are facts $s, t \in \mathfrak{F}$ co-possible? Whenever they are both part of some possible fact $u \in \mathfrak{P}$. s and t are non-co-possible, then, if no possible fact is an upper bound for both.

Facts may be non-co-possible in a number of different ways, but which version of non-co-possibility is right for defining inconsistency? The hunt is on for a relation between facts such that, when it holds between $s \in \mathfrak{F}$ and $t \in \mathfrak{F}$ and s and t are parts of $u \in \mathfrak{F}$, then u is inconsistent. The proposed answer is contrariety. Two facts are contrary if they are non-co-possible and each is a reason why the other cannot hold. This conception of contrariety is a little non-standard, but I'll be as clear as I can be: facts are contrary, to a first approximation, if they stand in a natural opposition to one another. When a fact has parts that conflict one another as strongly as this, it is fitting to call it inconsistent.[6]

It is not easy to get past a first approximation. Contraries are difficult to capture and there has been interesting work on the subject in recent years (e.g. Humberstone 2013). I argue here that contrariety is best captured by a basic relation between facts, as proposed by Dunn (1996, 10). To do so, I show that the three main analyses of contrariety do not suffice. Contrariety is neither constituent-based, minimal non-co-possibility (also known as pseudo-complementarity), nor maximal non-co-possibility. A basic relation is the only way out.

I'll start by offering some motivating examples.

(3) The fact that the apple is red.

(4) The fact that the apple is green.

(5) The fact that the tomato is the only red thing.

Both (4) and (5) are contrary to (3). Clearly, each is non-co-possible with (3), but something more is going on. Contrary facts must also be related in some, as of yet undefined, way.

I'll start by demonstrating that contrariety is not captured by pseudo-complementarity, but what is pseudo-complementarity? One facts is the pseudo-complement of another if it is minimally non-co-possible. So s is the pseudo-complement of t if and only if s is the smallest fact that is non-co-possible

6. Notice, too, that this is what the picture describing the development of an ice cube left out. It did have non-co-possible parts, the depictions of the same ice cube as a solid and a liquid, but these parts did not give reasons for why the other could not be so. The fact that an ice cube is presently liquid (or solid) is not a reason for it never to be solid (or liquid) again. These parts of the picture are not adequately opposed to count as pictorially contrary, in sum.

with t, i.e. if s is non-co-possible with t and s is a part of any u that is also non-co-possible with t.

Pseudo-complementarity is often used to define negation, as in the case of Heyting algebras (Bezhanishvili 2006, §2.2) or some algebraic approaches to inquisitive semantics (Roelofsen 2013, §3).[7] The following, then, is an example of a fact that is a pseudo-complement of (3).

(6) The fact that the apple is not red.

To show that this truly is a pseudo-complement, take any fact that is non-co-possible with (3). For the sake of illustration, let's consider the fact that the apple is colorless. Is (6) part of the fact that the apple is colorless? It indeed seems to be so. To demand that the apple not be red leaves open which color the apple is and whether the apple has a color at all. This example strengthens the demands of (6) by asking, additionally, that the apple have no color. Thinking of parthood in terms of demands, it follows that (6) indeed is the pseudo-complement of (3).

Contrariety is not pseudo-complementarity, and (4) serves as an example of a fact that is contrary to, but is not a pseudo-complement of, (3). If (4) were a pseudo-complement of (3) then it would be part of all facts non-co-possible with (3), including the following.

(7) The fact that the apple is yellow.

Since (4) is not part of (7) even though (7) is non-co-possible with (3), one may conclude that (4) is not a pseudo-complement of (3).

One cannot run in the opposite direction and define contrariety as maximal non-co-possibility. Consider again the example that (3) is contrary to (4), i.e. the fact that the apple is red is contrary to the fact that it's green. If contrariety is maximal non-co-possibility, then (3) is not contrary to (4): (4) is not maximally non-co-possible with (3). Consider, for instance,

(8) The fact that the apple is chartreuse.

Clearly (8) is non-co-possible with (3), and clearly (4) is smaller than (8) since chartreuse is a shade of green. Thus, if contrariety is maximal non-co-possibility, (4) is not contrary to (3). Since we can't have that, we can't have this perspective on contrariety.

7. Technically, in the two cited examples negation is defined to be maximal, not minimal, non-co-possibility. This does not present a great concern, for the definitions amount to the same thing when one flips the direction of the order on facts. This is done, for example, in Fine's semantics for intuitionsitic logic (Fine 2014). See (Reyes and Zolfaghari 1996) for a discussion of the differences between what I am calling maximality and minimality.

Contrariness is neither minimal nor maximal non-co-possibility. Aside from assuming it to be basic, there is one last option: have contrariety be constituent-based and depend upon objects and properties. Two facts, on this view, are contrary if they ascribe incompatible properties to the same object. (3), the fact that the apple is red, is contrary to (4), the fact that the apple is green from this perspective, because the same object cannot be both red and green.

This last option is also inadequate. In order for (3) to be contrary to (5) on the constituent-based account, i.e. in order for the fact that the apple is red to be contrary to the fact that only the tomato is red, these two facts must ascribe incompatible properties to the same object. However, this is not so. They ascribe compatible properties—something can both be red and the only red thing—to different objects—the apple and the tomato.

Contrariety isn't constituent-based. Neither is it maximal or minimal non-co-possibility. Contrariety is some kind of non-co-possibility in between maximal and minimal. In between, but where? How is one supposed to find out where? One has no leg to stand on in this matter, there is no apparent way of reducing contrariety to things that are already familiar. The only option left is to assume the relation as basic.

This is not as drastic or draconian as it might appear. In fact it has a rather well-researched formal precedent in the literature on negation. The study of such relations seems to have first caught on in quantum logic (Foulis and Randall 1969; Goldblatt 1974) although Dunn (1993, 331) claims it can be found as early as (Birkhoff 1940). They have applications in relevance logic (Goldblatt 2011, §6.4) and, as pointed out by Dunn in the same work, in linear logic (Girard 1987). Berto (2015) has shown such relations to have useful applications in defining a zoo of negations and Brandom (2008, esp. §5.3) has even used them as the basis for an entire semantics.

Following (Dunn 1996), contrariness is formalized as a relation between facts.

Definition 14 (Contrariety Relation).
Let $\mathfrak{F}$ contain the facts. $\perp \subseteq \mathfrak{F} \times \mathfrak{F}$ is the contrariety relation. Accordingly, s is contrary to t if and only if

$$s \perp t$$

for any $s, t \in \mathfrak{F}$.

It seems right to say that $\perp$ is symmetric and irreflexive. Better descriptions of contrariety would say even more, perhaps that $\perp$ is increasing in both arguments. The details do not matter so much at present. The thing to take away is that contrariness is used to define inconsistency: a fact is inconsistent if and only if it has contrary parts.

Definition 15 (Inconsistent Facts).
Let $\mathfrak{F}$ contain the facts and let $\perp \subseteq \mathfrak{F} \times \mathfrak{F}$ be the contrariety relation on $\mathfrak{F}$. $t \in \mathfrak{F}$ is inconsistent if and only if there are $s, s' \in \mathfrak{F}$ such that

a. $s \leq t$ and $s' \leq t$;

b. $s \perp s'$.

Each fact is either consistent or inconsistent. Consistency, therefore, obeys the following definition.

Definition 16 (Consistent Facts).
Let $\mathfrak{F}$ contain the facts and let $\perp \subseteq \mathfrak{F} \times \mathfrak{F}$ be the contrariety relation on $\mathfrak{F}$. $t \in \mathfrak{F}$ is inconsistent if and only if

$$\text{If } s \perp s' \text{ then either } s \not\leq t \text{ or } s' \not\leq t.$$

for all $s, s' \in \mathfrak{F}$. In sum, a fact is consistent exactly when it's not inconsistent.

To get a feel for how consistency and contrariness interact, consider the following facts.

$$\begin{aligned} s &= \text{ the fact that the ball is red} \\ t &= \text{ the fact that the ball is blue} \\ u &= \text{ the fact that the ball is round} \end{aligned}$$

Of the above facts only s and t are contrary, so $s \perp t$ and therefore $s \vee t$ is inconsistent. The fusion of all these facts, $s \vee t \vee u$, is also inconsistent for it has the same contrary parts: s and t. Since u is contrary to neither s nor t, each of $s \vee u$ and $t \vee u$ is consistent.

The above, I think, provides an adequate sketch of a general theory of facts for use in truthmaker semantics. For clarity and reference, Remark 2 in the conclusion of the present chapter provides a summary of the main points touched on in formalizing the facts.

3. Truthmakers

Following Kit Fine in his chapter on truthmaker semantics for the upcoming Blackwell companion to the philosophy of language (Fine 2016c), I would like to open this section by commenting on the connection between truthmakers in semantics and truthmakers in metaphysics. Although truthmakers in metaphysics are usually taken to be objects (Armstrong 2004; Merricks 2007, esp. ch. 3), though there are notable exceptions (e.g. Rodriguez-Pereyra 2002), in semantics this need not be the case. In the truthmaker semantics developed by Yablo and Fine, truthmakers and ontology are not a packaged deal.

This section puts the truthmaker in truthmaker semantics. To do so, I inspect the two major accounts of truthmaking in semantics due, respectively, to Yablo and Fine. I find faults with both accounts, the main problem being that neither Yablo nor Fine says enough about what truthmakers should be.

Yablo for instance offers two accounts of truthmaking, one recursive (Yablo 2014a, §4.2) the other reductive (Yablo 2014a, §4.3), and says that one may need to employ one rather than the other depending on the goals at hand. Although he does offer some good examples of goals that require recursive rather than reductive truthmakers and vice versa (Yablo 2014a, §4.6), Yablo himself wishes he "had more to say about how the two models interact," (Yablo 2014a, 69).

I won't say more on this recursive versus reductive issue. My criticisms of Yablo in subsection 3.1 focus on the recursive approach specifically and the gaps he has left unfilled. When discussing Fine in subsection 3.2 I focus on his notion of exact verification. My own opinion is that truthmakers, as found in semantics, imply and explain the truth of contents. I say more on this in subsection 3.3, where I give my own proposal.

3.1 Yablo and Truthmakers

Although truthmakers are the basis of Yablo's account, there is still an air of mystery about them. In particular, one wonders when a fact counts as a truthmaker for a content. Truthmaking, for Yablo, comes down to implying truth and explaining truth.

> s is a truthmaker for S if and only if:
>
> (i) s implies S's truth;
>
> (ii) s explains S's truth.

where s is a fact and S is a content.

Now it is more or less clear what implication is from Chapter I section 2. We still don't know, however, what it means for a fact to explain a content. Yablo doesn't offer much in the way of an intuitive discussion of explanation *per se*, although he does reduce explanation to two qualities.

> s explains S if and only if it strikes the right balance between:
>
> (i) naturalness, and

(ii) proportionality.

where, again, s is a fact and S is a content.

What about naturalness and proportionality? Let's start with naturalness. The intuition behind naturalness is straightforward: sometimes there are two well-qualified candidates for the role of truthmaker but one just seems more, well, natural. The example given by Yablo (2014b, 3),

(9) the content that nobody in that chair is ten feet tall.

(10) the fact that nobody is in that chair.

(11) the fact that nobody is ten feet tall.

Naturalness comes in here by helping to arbitrate between (10) and (11) as truthmakers for (9). Since (10) seems more natural than (11), (10) is a truthmaker for (9) while (11) is not. The example, although it makes some sense, doesn't say all that much about what naturalness really is. What is it for one fact to seem more natural than another?

The short answer is that Yablo doesn't really say, but I'll give the long answer too. Naturalness is a restriction on facts, which Yablo takes to be sets of possible worlds. In order for one set of possible worlds to be a more natural truthmaker than another, it must obey the following definition Yablo (2014b, 5).

With respect to S, s is more natural than t if and only if:

(i) s is a more *compact* set of possible worlds than t, and

(ii) t is a more *principled* set of possible worlds than t.

where S is a content and s and t are facts or, to Yablo, sets of possible worlds. As for what it means to be compact or principled, Yablo has nothing to say.

Having finished with naturalness, our investigation of explanation moves on to proportionality. The more proportional a truthmaker is, the fewer "irrelevant extras" (Yablo 2014a, 75; 2014b, 5) it brings along or, from another angle, the most proportional truthmakers don't explain more than is needed (Yablo 2014a, 76). The idea is something like this: if s and t both meet all other standards for being a truthmaker for the content S but $s \lneq t$, then s is preferred because t adds more than is needed.

Let's trace the breadcrumbs back. First Yablo starts with truthmaking, which boils down to implication and explanation. Then he moves on to explanation, which he reduces to a balance of naturalness and proportionality. Saying something concrete about proportionality, he defines naturalness in terms of compactness and principledness. The trail of breadcrumbs ends here, without knowing what it means for a fact to be compact or principled.

This is not a good situation to be in, for without knowing what it means to be compact or principled, one cannot be sure what naturalness is. Without knowing what naturalness is, one cannot be sure how to balance it with proportionality. Without knowing how to balance naturalness and proportionality, one cannot be sure when a fact is explanatory. Without knowing when a fact is explanatory, one cannot say when it is a truthmaker. Without an account of truthmakers, one does not have a truthmaker semantics. This, it seems, is a serious problem. Because he does not provide an account of truthmakers, Yablo's account is incomplete.

3.2 Fine and Truthmakers

For Fine, truthmaking is exact verification: in order for a fact to be a truthmaker for a content it must exactly verify it. Verification and exactness, then, are the key. What is verification? What is exactness? Fine doesn't really say. He does, however, give two ways in which one might come at exact verification.

First, a content's exact verifiers are "wholly relevant" (Fine 2014, 551) to it. Fine provides an example (Fine 2016c, 3): the presence of rain exactly verifies the content that it is rainy, the presence of rain and wind does not. Second, he mentions that exact verification corresponds to a kind of non-monotonicity on which a content may be made true by s but may not be made true by every fact greater than s (Fine 2016c, 9). Interestingly, Fine's non-monotonicity generalizes Yablo's proportionality, according to which a content made true by s is made true by no fact greater than s.

Still, these approaches to exact verification are not enough to tell us what it or truthmaking really is, but on Fine's view that's okay. He finds such elaborations unnecessary,

> We do not attempt to define the notion of exact verification—as a minimal verifier, for example. Its nature is taken to be revealed by the nature of the constraints imposed upon it rather than by a definition in other terms...
>
> Fine 2015a, 3

Here Fine once again calls upon instrumentalism. Don't ask about what exact verification is, he says, look at what it does instead; that should be enough to get an account of truthmaking.

Instrumentalism for truthmakers in the present discussion is as unsatisfying as instrumentalism for facts was in subsection 2.1, and matters only get worse when both instrumentalisms are combined. If truthmaker semantics is the theory of content on which contents are individuated by the facts that make them true, then can one really claim to be offering a truthmaker semantics without saying a little about facts and truthmaking? In my opinion, the answer is "no." More must be said.

3.3 A Decent Proposal

Yablo's and Fine's accounts of truthmakers are both incomplete. Yablo's is because he lacks an account of explanation; Fine's is because he lacks an account of exact verification. Admittedly, the incompleteness here does come from small details. Details, however, can inspire two different sorts of feelings. One might be led to say "small potatoes", or one might follow Flaubert and say

"Le bon Dieu est dans le détail."

"Small potatoes" is a shrug at the details. From this point of view there is little to be gained from details, so why worry about them? The second point of view from Flaubert highlights the importance of details. This perspective differs from the first in that it takes small-scale issues to truly be fundamental.

I happen to agree with Flaubert here. Sure, relative to truthmaking, what it is to be compact or principled or exact is a small matter by some measure. But it is also *fundamental*: Yablo's and Fine's entire theories of content depend on these issues. If they're to say anything at all on truthmakers, they *must* say more.

My plan is to do just that. This subsection provides an account of truthmaking fit for truthmaker semantics. Although I follow Yablo in taking truthmaking to be implication and explanation, the account of explanation I propose here is general enough to incorporate Fine's view of truthmakers as "wholly relevant" to the things they make true. In particular, I argue that all explanations fall into either of two categories. Either they are relevant, reminiscent of Fine, or they are responsible.

Before getting into what I think about explanation, it might be a good idea to say a problematizing word or two on why explanation is under–discussed. My basic point is that explanation is simply not simple. An account of explanation can be lengthy and complicated, surely providing material for an entire book (take, for example, Streven's *Depth* (2008) on scientific explanation). Some disagree, dismissing truthmakers' (and falsemakers') explanatory duties as trivial. To quote,

> Some people might hold that explanation is an unpleasantly murky or controversial concept, best excluded from our philosophical theories. In reply to this, I would claim that the confusion and controversy concern what it is to be an explanation; the notion itself is

> *intuitively clear.* After all, who could doubt that the truth of 'Grass is green' is explained by the colour of grass rather than the flavour of pineapples?
>
> Liggins 2005, 114

where the emphasis is added. To answer the question posed: I could. I would not say that the truth of 'Grass is green' is explained by the color of grass at all. What about grass that has been dyed blue? What about grass that has dried up and turned yellow? Moreover, sometimes the flavor of pineapple does explain why grass is green. Say a pineapple was so delicious that tasting it finally motivated someone to go paint all that peculiar red grass in the yard a more normal green.

This just goes to show that explanation is not clear, not intuitive, and not "intuitively clear". Consider, for instance, the different forms it can take: causal or non-causal, scientific or metaphysical, nomological, teleological, psychological. Whenever it comes up, explanation is always complicated and always contentious. Nonetheless it must be discussed because it plays a central role in truthmaker semantics. In the present context, the complexity of explanation can be bypassed by isolating general, characterizing traits it tends to have. It is argued that explanations are either *relevant* or *responsible.*

Explanation If X explains Y then X is relevant to or responsible for Y.

One does not introduce more complexity by introducing relevance and responsibility, for neither is described in detail and neither needs to be. Relevance and responsibility are intended to form a necessary condition of explanation, i.e. that every explanation must either be relevant or responsible, and necessary conditions can afford to be imprecise.

Keep this notion of explanation in mind, for in Chapter III subsection 2.2 it is argued that there are cases in which falsemakers fail to be explanatory in precisely this way. The argument is particularly strong, for contraposing this forgiving necessary condition of explanatory success produces a forgiving sufficient condition for explanatory failure: if one thing is both irrelevant to and irresponsible for another, then the former has not explained the latter.

Starting in on these two conditions, relevance indicates an intuitive connection. So, for instance, one's diet is intuitively connected to one's health and in this way diet facts can explain health facts. The same goes for the time of day and the temperature, a person's height and their basketball prowess, and so on.

Relevance, whatever it may be, plays a major role in different accounts of explanation throughout the literature. One finds relevance in pragmatic theories of explanation given in, for example, Bas van Fraassen's classic 'The

Pragmatics of Explanation' (1977) or Benjamin Schnieder's more recent 'A Logic for 'Because' (2011). As argued by Kitcher (1989), the unification theory of explanation first promoted by Friedman (1974) can also be seen as prioritizing relevance. Relevance crops up in the metaphysical explanation literature as well (Litland 2013, 21). Yablo even draws from his earlier work on relevance in causation (Yablo 2003) when discussing explanation in §4.10 of *Aboutness*, entitled 'Explaining Truth' (Yablo 2014a, 74–6). In fact, Robert W. Batterman and Collin C. Rice argue that relevance is central to all the major approaches to explanation (Batterman and Rice 2014, esp. §2).

The next requirement is that explanations be responsible. Responsibility may be causal, as the breeze both explains and causes the sound of the leaves. It may be teleological, as someone's ends can be the aim of and an explanation for their means. Responsibility is also to be found in supervenience and grounding, for both the ground and whatever thing is supervened upon are responsible in some way.

One finds responsibility in causal accounts of explanation (Lewis 1986a; Cartwright 2004; Skow 2014), for causes can surely be held responsible for their effects. Leitgeb (2015) in fact uses causal networks as a foundation for an account of explanation that, in turn, is used in a hyperintensional semantics resembling truthmaker semantics. Mechanistic accounts of explanation (Craver 2006; Bechtel and Abrahamsen 2005) also make use of responsibility in that the explanans must represent the mechanism responsible for the explanandum. Kit Fine takes grounding to be a form of metaphysical explanation (Fine 2012a, 2012b).

There are many cases in which relevance and responsibility overlap. Mechanistic explanation, which involves representing the responsible mechanisms, is one example. For another, notice that how someone looks may be both relevant to and responsible for the way one of their blood relatives looks if, for instance, genes are passed from the former to the latter. Relevance and responsibility may also come apart. Say Pat is thirsty and decides to boil water for tea. Pat's thirst is responsible for the temperature of the water, but surely it is not relevant. Pat's thirst shares no intuitive connection to the temperature of a particular amount of water. How could it?

This serves as a sketch of relevance and responsibility. Surely accounts of explanation have been left out and surely relevance and responsibility could be described in more detail. Nonetheless, it would be a challenge to find a reasonable account of explanation on which it is neither relevant nor responsible.

This account of explanation, recall, is given in service of an account of truthmaking: A fact s makes a content S true if and only if s implies and explains S's truth. Before closing the section, I'll say a word or two on how explanation as it has been described fits into the present account of truthmaking.

Given what has been said about explanation, it is the case that if s is a truthmaker for S, then s is relevant to or responsible for S's truth. There is a reason the word 'truth' comes at the end of the previous sentence. If s, a truthmaker for S, need only be relevant to or responsible for S—not S's truth—then relevance runs into some problems.

Consider the following.

(12) the fact that the apple is colorful

(13) the content that the apple is green or self-identical

It seems that (12) is not a truthmaker for (13); in no way does the former make the latter true. However if (12) need only, first, imply (13) and, second, be relevant to or responsible for it—not its truth—then it would stand to reason that (12) is a good candidate for being a truthmaker for (13). (12) does, after all, imply (13) and, further, (12) is relevant to (13) since both involve the same apple and, in part, its color.

How is one to avoid (12)'s being a truthmaker for (13)? I propose that something in the area of truth is the missing piece: (12) may be relevant to (13) but it is not relevant to (13)'s truth. Officially, any truthmaker s for the content S is either relevant to or responsible for S's truth. This, then, amounts to a sort of definition of truthmaking.

Truthmaking s is a truthmaker for S iff s implies and explains S's truth.

Explanation s explains S's truth by being relevant to or responsible for it.

for any fact s and content S.

I do not intend to get into issues concerning inflationism or deflationism about truth and, bluntly, I fear questions like "Is S the same as the truth of S?" To wiggle out of these issues, one may divert attention away from truth and towards explanation. The distinction between s's explaining S and s's explaining S's truth, then, depends on the difference between explaining and explaining truth, not on the difference between S and S's truth. Think, for instance, of a history of science class in which the teacher is explaining phlogiston theory. The teacher is not explaining the truth of phlogiston theory, which is more famous for being false than anything else, but instead explaining what the theory itself is.

When one hears that truthmakers imply and explain truth, therefore, one should have some idea of what explaining truth is: if a fact s is a truthmaker for a content S, then s is either relevant to or responsible for S's truth.

4. Conclusion: Progress So Far

This chapter opened by mentioning two principles shared by accounts of truthmaker semantics: pertinence, that contents are individuated by whichever facts are pertinent, and truthiness, that contents are individuated by *truthmakers.* Despite the central roles played by facts and truthmakers, the literature has said too little about them. The chapter proceeded with discussions facts and truthmakers and here, in the conclusion, I summarize the points made.

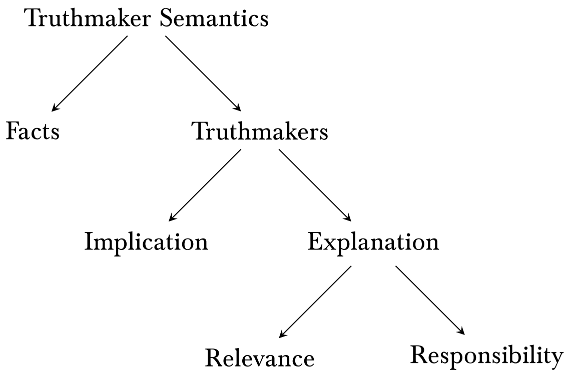
Subsection 2.1 provided a few ideas about the kinds of things facts might be and it was said that facts, for the purpose of illustration, are viewed here as on demands on reality. Subsections 2.2 and 2.3 provided a way of formalizing the facts, which is summarized in the following remark.

Remark 2 (Formalization of Facts).
The quadruple $(\mathfrak{F}, \mathfrak{P}, \leq, \perp)$ obeys the following definition.

- $\mathfrak{F}$ is non-empty and has the facts as elements.
- $\perp \subseteq \mathfrak{F} \times \mathfrak{F}$ is the contrariety relation on $\mathfrak{F}$.
- A fact with contrary parts is inconsistent, otherwise it's consistent.
- $\leq \subseteq \mathfrak{F} \times \mathfrak{F}$ is fact parthood, a partial order on $\mathfrak{F}$.
- $\bigwedge, \bigvee, \max_{\leq}, \min_{\leq}$ are defined in the usual way with $\leq$.
- $\mathfrak{P} \subseteq \mathfrak{F}$ is non-empty and has the possible facts as elements.
- $\mathfrak{P}$ is downward-closed with respect to $\leq$.
- $\mathfrak{W}$, the set of possible worlds, is defined to be $\max_{\leq}(\mathfrak{P})$.

Facts additionally can be truthmakers for contents. What, then, is truthmaking? On the view I adopt from Yablo, a fact makes a content true just in case the fact implies and explains the content's truth (where, once again, it is one thing for a fact to explain a content and another thing all together for a fact to explain the truth of a content). While implication has already been discussed in section 2.1 of Chapter I, section 3.3 provided a necessary condition for explanation: all explanations are either relevant or responsible. An explanation is relevant if the explanans is intuitively related to the explanandum, just as height is intuitively related to basketball skill or snowy weather is intuitively related to winter. An explanation is responsible if the explanans itself bears some responsibility for the explanandum in the way of cause or grounding or supervenience. For example, Pat's thirst both explains the temperature of the water Pat boiled for tea and also bears responsibility for it.

This chapter has provided some idea of what truthmaker semantics looks like, and the following figure illustrates the connections made between the various notions involved.



What remains to be settled is the notion of pertinence, i.e. the issue of which facts individuate contents. This is settled in the next chapter: the consistent ones.

III. Truthmaker Semantics: Contents

A theory of content is strongly hyperintensional if it lets necessarily false contents be distinct. To date, there are two strategies for making an account of truthmaker semantics strongly hyperintensional: falsism and fusionism. I argue here that neither strategy is successful. I propose my own strategy, consistentism, and show its advantages over its competitors.

1. Introduction

The naïve account of truthmaker semantics has the following two tenets.

Naïveté Contents are individuated by the possible facts.

Truthiness Contents are individuated by their truthmakers.

Contents, then, are individuated by the possible facts that make them true. Same facts, same content; different facts, different contents.

The naïve account has the attractive feature of being *hyperintensional*.

Hyperintensionality Necessarily equivalent contents may be distinct.

To see how, note that two contents may be necessarily equivalent by being implied by the same possible facts, but nonetheless distinct by having different truthmakers. The naïve account also has an unappealing feature: *weak hyperintensionality*.

Weak Hyperintensionality Necessarily false contents are the same.

To see how, recall Definition 5 from Chapter I above, which says that no possible fact implies a necessarily false content. Since truthmaking requires implication, it follows that no possible fact is a truthmaker for a necessarily false content. Therefore all necessarily false contents are made true by the same possible facts—none—and so, on the naïve account, are all the same.

Naïveté and truthiness allow too few facts to individuate contents. In order to build the naïve account into a more appealing strongly hyperintensional truthmaker semantics, contents must be individuated more finely by more content individuators. There are two strategies to do so: supplement truthiness or supplement naïveté. The first strategy, which I call ‘falsism’, includes falsemakers.

Falsism Contents are individuated by their *falsemakers.*

The second strategy, which I call ‘impossibilism’, adds impossible facts to the mix.

Impossibilism Contents are individuated by *impossible facts.*

Given that truthmaking is the implication and explanation of truth, falsism straightforwardly takes contents to be individuated by the impliers and explainers of falsity. Impossibilism is not as straightforward, for different accounts may opt to add in different impossible facts. To date, all impossibilist strategies have been the same: add the impossible fusions of possible facts to the mix, e.g. the fact that the candle is both bent and straight.[1] I call this strategy ‘fusionism’.

Fusionism Contents are individuated by impossible fusions of possible facts.

None of the existing strategies for achieving strong hyperintensionality do the trick. Falsism runs into problems with contradictions, i.e. any content of the form $S \mathbin{\&} \neg S$, and fusionism leaves out useful impossible facts that should qualify as truthmakers.

I propose a third alternative, consistentism, a version of impossibilism that takes the consistent facts to individuate contents.

Consistentism Contents are individuated by the *consistent* facts.

Consistentism is not only strongly hyperintensional, it also avoids the problems posed to falsism and fusionism. In the end, it emerges as the best among the available options. Before getting to consistentism in section 4, falsism and fusionism are presented and assessed in section 2 and section 3 respectively.

1. In the parlance of Chapter II subsection 2.3, the impossible fusions of possible facts are the fusions of non-co-possible facts.

2. Falsism

Falsism is intended to help the naïve account become strongly hyperintensional, but how is this supposed to work? Taking naïveté, truthiness, and falsism together, contents are individuated by the possible facts that make them either true or false. Although necessarily false contents are made true by all the same possible facts—none—they may yet be made false by distinct ones. Falsism, it is supposed, achieves strong hyperintensionality in this way.

Not so fast. What exactly are falsemakers? Given that truthmakers imply and explain truth, falsemakers must imply and explain falsity. Since this is the only account of falsemaking in town, it's the one falsism is stuck with. However, as argued in subsection 2.2 below, this notion of falsemaking is problematic. Not only is it to be abandoned, but falsism—which depends on falsemaking—is to be left behind as well.

One might protest that this is an argument by throwing the baby away with the bath water. After all, one might not need to throw away falsemaking to throw away falsism. This comment is well-received. However, generality should be a virtue, not a vice, of an otherwise correct argument.

In particular, falsemaking is problematic because of its treatment of contradictions, i.e. contents of the form $S \,\&\, \neg S$. Facts, e.g. the fact that ruminant mammals eat hearty grains, cannot explain the falsity of contradictions because they do not meet the necessary condition above in Chapter II subsection 3.3: they are neither relevant to nor responsible for contradictions' falsity.

Before getting into any of that, a little must be said on contents and connectives. After all, this section makes use of contradictions, which are defined in terms of the connectives "and", &, and "not", ¬. In subsection 2.1, I present the two treatments of connectives employed in existing accounts of truthmaker semantics: the common account and the alternative account. Both definitions of connectives, however, run into the same problems with contradictions. Having gained some idea of how the connectives work in section 2.1, subsection 2.2 presents the problematic cases for falsism.

2.1 Contents and Connectives

This subsection presents the two ways falsist approaches to truthmaker semantics define the connectives "and", &, "or", v, and "not", ¬. Two different definitions given here: one common and one alternative. The common definition of connectives is found in the most foundational works of truthmaker semantics, offered not only by Yablo (2014a, 58) and Fine (2015a, 35) but also by van Fraassen (1969, 484). The common definition is also simpler than the alternative definition, which found in a separate incarnation of Fine (2015c) and in Mark Jago's joint work with Fine (2015).

What are connectives for contents? They are functions from contents, or from pairs of contents, to contents. These functions affect the truthmakers and, as the case may be, falsemakers of their arguments in specified ways. Conjunction on the common account, for example, is a function $(\cdot)\ \&\ (\cdot) : \mathfrak{C} \times \mathfrak{C} \rightarrow \mathfrak{C}$ such that if s is a truthmaker for S and t is a truthmaker for T then $s \vee t$, if it exists, is a truthmaker for $S\ \&\ T$. This makes sense, fusion having been described as a kind of conjunction for facts in Chapter II subsection 2.2.

Because different definitions of connectives affect truthmakers and falsemakers in different ways, one's conception of truthmaking changes with one's treatment of connectives. So, for instance, one may take any truthmaker for $\neg\neg S$ to be a truthmaker for S or one may not. The treatments of connectives surely differ, but so too do the treatments of truthmaking. For this reason, I use different symbols for truthmaking (and for falsemaking) when giving the different definitions of the connectives.

Definition 17 (Truthmaking and Falsemaking for Defining Connectives).
Let $\mathfrak{F}$ be the set of facts and $\mathfrak{C}$ be the set of contents.

a. $|||\!\!-, -\!\!||| \subseteq \mathfrak{F} \times \mathfrak{C}$ are the common truthmaking and falsemaking relations used in the common definition of connectives.

b. $|||\!\!-^*, -\!\!|||^* \subseteq \mathfrak{F} \times \mathfrak{C}$ are the alternative truthmaking and falsemaking relations used in the alternative definition of connectives.

I start with the common definition of connectives.

Definition 18 (Common Definition of Connectives).
Let $\mathfrak{F}$ be the set of facts and $\mathfrak{C}$ be the set of contents. The common definitions of the connectives not ($\neg$), or (v), and and (&) are as follows.

a. Negation, $\neg(\cdot) : \mathfrak{C} \rightarrow \mathfrak{C}$

(i) $s\ |||\!\!-\ \neg S$ iff $s\ -\!\!|||\ S$

(ii) $s\ -\!\!|||\ \neg S$ iff $s\ |||\!\!-\ S$

b. Disjunction, $(\cdot) \text{ v } (\cdot) : \mathfrak{C} \times \mathfrak{C} \rightarrow \mathfrak{C}$

(i) $s\ |||\!\!-\ S \text{ v } T$ iff $s\ |||\!\!-\ S$ or $s\ |||\!\!-\ T$

(ii) $u\ -\!\!|||\ S \text{ v } T$ iff $s\ -\!\!|||\ S$ and $t\ -\!\!|||\ T$ and $u = s \vee t$

c. Conjunction, $(\cdot)\ \&\ (\cdot) : \mathfrak{C} \times \mathfrak{C} \rightarrow \mathfrak{C}$

(i) $u\ |||\!\!-\ S\ \&\ T$ iff $s\ |||\!\!-\ S$ and $t\ |||\!\!-\ T$ and $u = s \vee t$

(ii) $s \dashv||| S \mathbin{\&} T$ iff $s \dashv|| S$ or $s \dashv||| T$

for all $s, t, u \in \mathfrak{F}$ and $S, T \in \mathfrak{C}$.[2]

The rationale behind these definitions is rather simple, and I'll go through a few clauses. Definition 18a(i), for instance, says that any truthmaker for $\neg S$—i.e. any fact that implies and explains the truth of $\neg S$—also implies and explains the falsity of S and vice versa. The falsemakers for a disjunction, according to Definition 18b(ii), are exactly the fusions of falsemakers for the disjuncts.

Definition 18 allows for two sorts of cases, among others, that one might have trouble with. First, a fact may be a truthmaker for $\neg S \mathbin{\&} \neg T$ but not a falsemaker for $S \mathbin{\&} T$; second, a fact may be a truthmaker for $S \mathbin{\&} T$ but not a truthmaker for the corresponding disjunction $S \mathbin{\mathrm{v}} T$. The alternative definition of the connectives only makes two slight changes to the common definition, specifically to clauses 18b(i) and 18c(ii), to address these cases.

Definition 19 (Alternative Definition of Connectives).
Let $\mathfrak{F}$ be the set of facts and $\mathfrak{C}$ be the set of contents. The alternative definitions of the connectives not ($\neg$), or (v), and and (&) are the same as the common definition above, except for the following clauses.

b. Disjunction, $(\cdot) \mathbin{\mathrm{v}} (\cdot) : \mathfrak{C} \times \mathfrak{C} \to \mathfrak{C}$

(i) $s \,|||\!\vdash^* S \mathbin{\mathrm{v}} T$ iff $s \,|||\!\vdash^* S$ or $s \,|||\!\vdash^* T$ or $s \,|||\!\vdash^* S \mathbin{\&} T$

c. Conjunction, $(\cdot) \mathbin{\&} (\cdot) : \mathfrak{C} \times \mathfrak{C} \to \mathfrak{C}$

(ii) $s \dashv|||^* S \mathbin{\&} T$ iff $s \dashv|||^* S$ or $s \dashv|||^* T$ or $s \,|||\!\vdash^* \neg S \mathbin{\&} \neg T$

for all $s \in \mathfrak{F}$ and $S, T \in \mathfrak{C}$.

The two definitions of connectives found in truthmaker semantics have now been put into play: the common and the alternative. The next section shows that both approaches face serious problems when it comes to contradictions.

2.2 Against Falsism: Problems with Falsemaking

Although there are different forms of falsism, all existing approaches agree on the following three principles (Yablo 2014a, 58; 2014b, §8; 2016, 20; Fine 2015a, 35; 2016a, 21; 2015c, 8; van Fraassen 1969, 484; Jago 2015b).

2. Although the above looks a little like a definition of sentential connectives, it is in fact a definition of connectives for contents. For this reason, I have not included an atomic case. If, as Bealer (1998) contends, contents are *sui generis* entities then would there really be *atomic* contents? Some might say no, others might say yes. To be accommodating to everybody, I've simply left them out.

Remark 3 (Principles of Falsism).
The present forms of falsism agree on the following.

a. Double negation introduction
If s is a truthmaker for S then s is a truthmaker for $\neg\neg S$.

b. Truthmakers of negations are falsemakers
If s is a truthmaker for $\neg S$ then s is a falsemaker for S.

c. Falsemakers of a conjunct are falsemakers
If s is a falsemaker for S then s is a falsemaker for $S \mathbin{\&} T$.

for all facts s and contents S and T.

These three principles are enough to create a problems for falsemaking. If one wanted to protect falsemaking from the resulting problems, if indeed their results are as problematic as I claim, it is not possible to simply tweak the principles. Once falsism opens the door to falsemaking, Remark 3a, 3b, and 3c fall out.

As casual evidence that the principles in Remark 3 are unavoidable, let's go through each case intuitively. While double negation elimination has been called into question by intuitionists, among others, double negation introduction is nearly universally agreed upon. Given that truthmaking and falsemaking amount to implication and explanation of truth and, respectively, falsity, the second and third principles are also hard to dispute. Starting with Remark 3b, if a fact s implies (explains) the truth of $\neg S$, it surely implies (explains) the falsity of S. One gets Remark 3c from falsemaking trickling up from conjuncts to conjunctions. If a fact implies and explains the falsity of S, the reasoning goes, it also implies and explains the falsity of any content $S \mathbin{\&} T$.

To see how falsism itself inevitably leads to Remark 3, notice that they follow from all the existing accounts of connectives: both from the common definition and the alternative one.

Proposition 4 (Common & Alternative Connectives Prove Remark 3).
The common connectives, as found in Definition 18, and the alternative connectives, as found in Definition 19, obey the principles of falsism, as found in Remark 3.

Proof. Take each of Remark 3a–3c in turn.

- Double negation introduction
  If $s \mathrel{|||\!\!-} S$ then, by 18a(ii), $s \mathrel{-\!\!|||} \neg S$ and, by 18a(i), $s \mathrel{|||\!\!-} \neg\neg S$.

- Truthmakers of negations are falsemakers
  If $s \mathrel{|||\!\!-} \neg S$ then, by 18a(i), $s \mathrel{-\!\!|||} S$.

- Falsemakers of a conjunct are falsemakers
  If $s \dashv\!||| \ S$ then, by 18c(ii), $s \dashv\!||| \ S \,\&\, T$.
  If $s \dashv\!|||^* \ S$ then, by 19c(ii), $s \dashv\!|||^* \ S \,\&\, T$.

where s is any fact and S is any content.

With the principles in Remark 3 firmly in place, let's move to the actual problem they pose to falsemaking.

Proposition 5 (Problem for Falsism).
Let S be a content. Any truthmaker for S is a falsemaker for $S \,\&\, \neg S$.

Proof. If s is a truthmaker for S then, by Remark 3a, it is a truthmaker for $\neg\neg S$ and, by Remark 3b, it is also a falsemaker for $\neg S$. By Remark 3c, it follows that s is a falsemaker for $S \,\&\, \neg S$.

Proposition 5 is enough to give rise to falsemakers that fail to be explanatory. To see this, consider the following concrete example.

(1) the fact that ruminant mammals eat hearty grains

(2) the content that does eat oats

(3) the content that does do and don't eat oats

(1) is a good candidate for a truthmaker for (2). Surely (1) implies (2), seeing as does are ruminant mammals and oats are hearty grains. (1) also explains (2)'s truth by being responsible for it: the fact that ruminant mammals eat hearty grains is a reason for it being true.

Whether (1) is relevant to the truth of (2) is up in the air. From one angle, (1) is irrelevant to (2)'s truth because ruminant mammals have nothing to do with the truth-values of contents. This is pretty convincing. (1) really does have to explain (2)'s truth—not (2) itself—as discussed when distinguishing explanation from explanation of truth in Chapter II subsection 3.3. From another angle, (1) is relevant to (2) seeing as both involve animals' diets, and it also has bearing on (2)'s truth. Perhaps this is enough for (1) to be relevant to (2)'s truth.

The problem posed by Proposition 5 makes itself clear when looking closer at (3). Given that (1) is a truthmaker for (2), Proposition 5 says that (1) is a falsemaker for (3). However, (1) cannot be a falsemaker for (3). Although (1) does *imply* (3)'s falsity—after all, every fact implies the falsity of a contradiction—(1) fails to *explain* it. This is because (1) is neither *relevant to* nor *responsible for* (3)'s falsity.

The fact that (1) fails to be responsible is easy to see. Whatever is responsible for the falsity of (3), it surely is not the eating habits of ruminant mammals. No matter whether ruminant mammals eat hearty grains or not, (3) is going to be false and (1) is not to be held responsible.

The fact that (1) fails to be relevant is not as straightforward. It is hard to imagine how the fact that ruminant mammals eat hearty grains is intuitively connected to the falsity of a content. When weighing whether (1) was relevant to the truth of (2), matters were a little murky as well. On the one hand (1) is not intuitively connected to (2)'s truth or the truth of any content, at least not in the way someone's height is intuitively connected to their basketball skill. On the other (1) is concerns the same thing as (2) and does bear on its truth; maybe that's enough for relevance.

Matters for (1) and (3) are not parallel to those for (1) and (2). While (1) has some bearing on (2)'s truth, (1) has no bearing on the falsity of (3) whatsoever. The truth value of (3), being a contradiction, is unaffected by facts concerning animals' diets. It is just hard to find a way in which (1) is relevant to (3)'s falsity. It is fair, then, to conclude that (1) does not explain (3)'s falsity and therefore that (1) is not a falsemaker for (3). This is a worrisome since Proposition 5 says that it must be. (1) is, after all, a truthmaker for (2).[3] Having reached a contradiction, something must be given up. But what?

Retracing the steps made in this section, the choice is clear. Accepting falsism means commitment to falsemaking. Having opened the door to falsemaking, the principles of falsism follow. From the principles of falsism, one reaches a contradiction. Falsemaking—and therefore falsism—must be left behind.

The argument is valid to all appearances, but what are its assumptions? First it assumes that relevance and responsibility form a necessary condition for explanation, as argued subsection 3.3 of Chapter II. Second it assumes on the principles of falsism listed in Remark 3. The attempt to preserve falsism at the cost of either assumption would only be in vain. There are good reasons to think explanation does bring either relevance or responsibility with it. As argued in Chapter II, it is a general characterization of explanation that accommodates both Yablo's and Fine's views on truthmaking. The principles of falsism are even harder to drop given their intuitiveness and given the long list of people who accept them.

Falsism—the doctrine that contents are individuated by falsemakers—is untenable because falsemaking is incoherent. Moreover, there is no apparent way of escaping this conclusion. Other options for achieving strong hyperintensionality are to be considered.[4]

3. Sometimes Yablo speaks of truthmakers and falsemakers as "ways of being" true or false (Yablo 2014a, §2.5; Yablo 2015, §5). Other times he says they "tell us why" (Yablo 2014b, §8) something is true or false. These notions of falsemaking, however, do not provide an escape from the present argument. Is the fact that does eat oats a *way* in which the sentence 'Does eat oats and does do not eat oats' is false? Does it tell us why? To answer both questions: surely not.

4. There is formal precedent for falsism in "bilateral" accounts of content that treat truth and falsity

3. Fusionism

Maybe falsism errs in piling falsemakers on top of truthmakers. Maybe, just maybe, there are better ways to add to the naïve account in order to achieve strong hyperintensionality. Aside from supplementing truthiness, which did not work for falsism, the only other option is to supplement naïveté and have contents be individuated by more than just the possible facts. This is the strategy behind impossibilism: become strongly hyperintensional by individuating necessarily false contents with distinct impossible facts.

I myself advocate this impossibilism, but what I advocate is distinct from the versions of impossibilism found in the truthmaker semantics literature. At present, there is only one available option, which I call 'fusionism',

Fusionism Impossible facts are fusions of possible facts.

Fusionism has two existing incarnations and Yablo (2014a, §4.2) offers the first. In describing how distinct contradictions may be made true by distinct impossible facts on his account (and, therefore, how his account is strongly hyperintensional) Yablo betrays his fusionism. If s and s' are respectively truthmakers of S and $\neg S$ and if t and t' are respectively truthmakers of T and $\neg T$ then, says Yablo, $s \vee s'$ and $t \vee t'$ are respectively truthmakers of $S \,\&\, \neg S$ and $T \,\&\, \neg T$. Yablo makes a point of emphasizing that the fusions $s \vee s'$ and $t \vee t'$ are impossible,[5] providing no other way for impossible facts to enter his framework. Yablo is therefore a fusionist: his impossible facts are the fusions of possible ones.[6]

The second approach to impossibilism, from Fine (2016a), is not all that different. He too is a kind of fusionist, getting at the impossible by fusing possible facts. However Fine does things a little differently. While Yablo takes

separately. Restall (1997) is one such example, and he cites Anderson and Belnap (1975) and Dunn (1976) as predecessors, however the precedent goes back at least ten years earlier to (Dunn 1966, ch. IX). There are recent advocates of bilateral approaches as well, including Restall himself (Restall 2005, 2013) and David Ripley (2013, forthcoming). A bilateral approach to meaning has also been attempted in inquisitive semantics, but was proven incapable of defining implication by Katsuhiko Sano (2012). The present results only apply to these other bilateral accounts if, firstly, they depend upon explanation in a way that resembles truthmaker semantics and, secondly, if bring the result that facts like (1) explain the falsity of contradictions. Since this is not apparently the case in any of the accounts mentioned in this note, none of these authors need worry.

5. To quote, "If $p \,\&\, \neg p$ were to be true, that would be because of the fact that $\{\mathsf{p}, \bar{\mathsf{p}}\}$. A different fact, $\{\mathsf{q}, \bar{\mathsf{q}}\}$, makes, or would make, $q \,\&\, \neg q$ true. (It is because these facts can't obtain that the sentences can't be true.)" (Yablo 2014a, 58).

6. As an interesting side note, Yablo's impossibilist side is incompatible with his facts-as-possible worlds side. Fusion, recall, is a kind of conjunction and so in the context of possible worlds fusion amounts to set-theoretic intersection. However, $s \vee s' = \varnothing = t \vee t'$ if s, s', t, t' are all sets of worlds.

impossible content-individuators like $s \vee s'$ or $t \vee t'$ to be facts like any other, Fine does not. On Fine's account, the impossible is captured by *virtual* facts, which are different from facts proper.[7] While facts are simply the elements of the partial order $(\mathfrak{F}, \leq)$ mentioned above, virtual facts are sets of possible facts obeying certain constraints, i.e. particular subsets of $\mathfrak{P}$.

Fine's and Yablo's versions of fusionism share a lot in common, but are quite distinct. Even so, I do not find there to be strong reasons why either version is superior. Yablo's has the advantage of simplicity but Fine's, which is much more intricate, has many bells and whistles. I do not have space to recount all the many applications and motivations Fine gives for his account, which occupy a large portion of (Fine 2016a), but I mention one to say more than nothing on the matter.

Note that Yablo's fusionism is committed to impossible facts. In the above example, $s \vee s'$ and $t \vee t'$, although impossible, are facts like any other. Fine's, on the other hand, is not so-committed. Instead of positing impossible facts *à la* Yablo, Fine simulates impossible facts with virtual facts. Virtual facts, importantly, are just sets of possible facts and do not carry commitment to anything impossible. Fine's fusionism allows for flexible commitments, which is an advantage over Yablo's.

This section presents two criticisms of fusionism, the first aimed at Fine's account and the second aimed at both. Although this section touches on both accounts, Fine's takes center stage simply because the first criticism, which focuses on his account alone, is longer and more complicated than the second. This first criticism takes issue with the impossible "facts" (really, impossible *virtual* facts) that Fine's account includes. In particular, after looking closely at the assumptions of fusionism, it becomes clear that some possible facts are simulated as having impossible parts which, given the downward closure of the set of possible facts, should not be accepted.

The second criticism, on the other hand, takes issue with the impossible facts that Yablo's and Fine's accounts leave out. In particular, there are many impossible facts that should be accounted for (since it seems they act as truth-makers) but Yablo and Fine both fail to do so. Why? Because no fusionist can: they are impossible but not the fusion of possible facts.

Subsection 3.1 describes how Fine builds virtual facts, while the first and second criticisms are given in subsections 3.2 and 3.3. All attempts at achieving strong hyperintensionality failed, consistentism saves the day in section 4.

7. Strictly speaking, Fine does not use the term 'virtual fact' but 'virtual fusion' (Fine 2016a, 6–7).

3.1 Constructing Virtual Facts by Ideal Completion

Fine's fusionist truthmaker semantics in (Fine 2016a) has contents be individuated by virtual facts allowing, as mentioned, for strong hyperintensionality. It is still left open what these virtual facts are and what they are supposed to do. There are, admittedly, interesting questions about the status of virtual facts. Do they exist? Are they real? How are they related to content? While these are legitimate questions that may even lead to strong arguments against impossibilism, they are not pursued here.

As for what virtual facts are supposed to do, they serve to furnish Fine's variety of fusionism: virtual facts are supposed to help Fine supplement the possible facts with their impossible fusions. The virtual facts, whatever they may be, must therefore accomplish two feats: preserving the structure of the possible facts and supplementing that structure with the impossible fusions of possible facts. The word I use here is 'simulate': Fine's virtual facts are to *simulate* both the possible facts and their impossible fusions the way virtual reality simulates reality.

How is one to find virtual facts that fit this description? Notice that the possible facts are partially ordered by fact parthood, i.e. that $(\mathfrak{P}, \leq)$ is a poset, and notice further that fusion $\bigvee : 2^{\mathfrak{P}} \to \mathfrak{P}$ is simply a supremum. The desired virtual facts, then, preserve the structure of the possible facts, i.e. preserve the structure of the poset $(\mathfrak{P}, \leq)$, while adding in all the impossible fusions of possible facts, i.e. while adding to $(\mathfrak{P}, \leq)$ all the suprema it lacks.

Lattice and order theory provide an elegant option for doing so: the completion of a poset. This, it turns out, provides exactly the virtual facts the fusionist desires.

Definition 20 (Completion of a Poset).
Let $(U, \leq)$ and $(U', \leq')$ be posets, and let $\bigvee : 2^U \to U$ and $\bigvee' : 2^{U'} \to U'$ be their respective suprema. $(U', \leq')$ is a completion of $(U, \leq)$ if and only if

a. There is an order–embedding $c : U \to U'$.
For all $s, t \in U$, $s \leq t$ iff $c(s) \leq' c(t)$.

b. U' is supremum complete.
For all $X \subseteq U'$, $\bigvee' X \in U'$.

To check that a completion does the trick, let $(\mathfrak{P}, \leq)$ be the possible facts ordered by fact parthood and let $(\mathfrak{P}', \leq')$ be its completion. Does the completion simulate the possible facts? Indeed so: Definition 20a guarantees that that $(\mathfrak{P}, \leq)$ is isomorphic to a subset of $\mathfrak{P}'$ in providing the embedding $c : \mathfrak{P} \to \mathfrak{P}'$:

$(\mathfrak{P}, \leq)$ is isomorphic to $(c(\mathfrak{P}), \leq' \restriction c(\mathfrak{P}))$. The elements of $c(\mathfrak{P})$, then, are well-qualified to simulate the possible facts in $\mathfrak{P}$.

Does the completion simulate the impossible fusions of possible facts? Once again, the answer is 'yes'. Let $s, t \in \mathfrak{P}$ be the possible facts that Pat is smart and that Pat is not smart, respectively. s and t are each simulated in $\mathfrak{P}'$ by $c(s), c(t) \in \mathfrak{P}'$ and, since $\mathfrak{P}'$ is fusion complete by Definition 20b, $c(s) \vee' c(t)$ is in $\mathfrak{P}'$ as well. $c(s) \vee' c(t)$, then, is the virtual fact that Pat is both smart and not smart.

A completion of the possible facts does the trick and results in virtual facts that meet the desired criteria. However, there are many different completions (cf. Harding 2008) and so there remains a question of which completion to use. Fine suggests the arbitrary-ideal completion, which I simply call the 'ideal completion', and motivates it with an example (Fine 2016a, 6–7). I devote some space here to restating Fine's example in order to present his account.

The idea behind Fine's example is quite subtle. The goal is to find a function $f : 2^{\mathfrak{P}} \to 2^{\mathfrak{P}}$ that, when given a set of possible facts $X \subseteq \mathfrak{P}$, returns the virtual fact $f(X) \subseteq \mathfrak{P}$ simulating X's fusion. If X contains the fact that Pat is smart and the fact that Pat is not smart, for example, then $f(X)$ is the virtual fact simulating their impossible fusion, the virtual that Pat is both smart and not smart.

Fine moves pretty quickly here, taking a few stairs in one stride. It is not immediately clear how a function like f leads one to a completion of $(\mathfrak{P}, \leq)$, so I'll go step-by-step. A completion of the possible facts must contain two different sorts of virtual facts. First, it must have virtual facts simulating the possible facts themselves; second, it must have virtual facts simulating the impossible fusions of virtual facts. f helps to achieve both tasks. If $s \in \mathfrak{P}$ is any possible fact, then $f(\{s\})$ is the virtual fact simulating the fusion of s with itself—i.e., the virtual fact simulating s. Generally, if $X \subseteq \mathfrak{P}$ then $f(X)$ is the virtual fact simulating X's fusion. This holds for all sets of possible facts, even the ones with impossible fusions. The image of $f : 2^{\mathfrak{P}} \to 2^{\mathfrak{P}}$, $f(2^{\mathfrak{P}}) \subseteq 2^{\mathfrak{P}}$, contains virtual facts simulating all the possible facts and all their fusions, both possible and impossible. $f(2^{\mathfrak{P}})$, then, contains exactly the sought-after virtual facts.

There is still some work to be done, for there are many such functions. The focus now is on finding an f that assigns the right virtual facts, and this is where Fine's example enters. The example considers four sets of possible facts, X_1 through X_4, none of which have possible fusions. The same virtual fact, however, should simulate all their impossible fusions.

$$X_1 = \{s \vee s', t\} \quad X_2 = \{s, s \vee s', t\} \quad X_3 = \{s, s', t\} \quad X_4 = \{s, s', s \vee s', t\}$$

$$\begin{aligned} s &= \text{ the fact that the ball is red} \\ s' &= \text{ the fact that the ball is round} \\ s \vee s' &= \text{ the fact that the ball is red and round} \\ t &= \text{ the fact that the ball is blue} \end{aligned}$$

The impossible fusions of all these sets should all be simulated by the same virtual fact: the virtual fact that the ball is red, blue, and round.

What does this example have to do with f? Instead of asking that the same virtual fact simulate the fusions of X_1 through X_4 one may equivalently ask that f assign the same value to all four sets, and this is what Fine sets out to do. Fine has a two-step plan for finding such an f. Step one: find an f that assigns the same value to both X_1 and X_2; step two: find an f that does the same for X_3 and X_4. Combining both steps in the right way, one gets an f that assigns the same value to all the sets.

Fine's proposed answer to step one is *downward closure.* Recall the definition of downward closure from Definition 8 above: $X^{\downarrow} \subseteq \mathfrak{P}$, the downward closure of $X \subseteq \mathfrak{P}$, contains every possible fact smaller than some element of X. Note that the downsets of X_1 and X_2 are the same,

$$(X_1)^{\downarrow} = (X_2)^{\downarrow}$$

and Fine's first step is solved.

Downward closure does not solve step two, however, because the downsets of X_3 and X_4 are distinct. While $s \vee s' \in (X_4)^{\downarrow}$ since, after all, it is trivially smaller than itself, $s \vee s'$ is not in the downset of $(X_3)^{\downarrow}$. Step two, therefore, has a different solution, and Fine proposes the *fusion closure.*

Definition 21 (Fusion Closure).
Let $(U, \leq)$ be a poset and let $X \subseteq U$ be non-empty. The fusion closure of X, written $X^{\vee}$, obeys the following.

$$X^{\vee} = \left\{ \bigvee Y \in \mathfrak{P} \mid Y \subseteq X \right\}$$

If $X^{\vee} = X$, then X is said to be fusion-closed.

Fusion closure helps with the second step, for X_3 and X_4 have the same fusion closure,

$$(X_3)^{\vee} = (X_4)^{\vee}$$

and so the second step is also solved.

Putting these two steps together, one gets that the right f assigns to any set of possible facts $X \subseteq \mathfrak{P}$ its fusion-cum-downward closure $(X^{\vee})^{\downarrow}$.[8] The virtual facts Fine is after are these fusion- and downward-closed sets of possible facts.

In any poset $(U, \leq)$, the fusion- and downward-closed subsets are called 'arbitrary ideals', or just 'ideals' for short.

Definition 22 (Ideal of a Poset).
Let $(U, \leq)$ be a poset and let $X \subseteq U$ be non-empty. The ideal-closure of X, written X°, obeys the following.

$$X^{\circ} = (X^{\vee})^{\downarrow}$$

If $X = X^{\circ}$, then X is said to be an ideal.

Notice that $f(2^{\mathfrak{P}})$ contains exactly the ideals of $(\mathfrak{P}, \leq)$. These play an important role in what's to come and therefore merit their own definition.

Definition 23 (Ideals of Possible Facts).
Let $(\mathfrak{P}, \leq)$ be the possible facts ordered by fact parthood. The set of ideals of possible facts, written $\mathfrak{P}^{\circ}$, obeys the following.

$$\mathfrak{P}^{\circ} = \{X \in 2^{\mathfrak{P}} \mid X = X^{\circ}\}$$

Every $X \in \mathfrak{P}^{\circ}$ is said to be an ideal of possible facts.

It has not been made explicit how one gets a completion from $\mathfrak{P}^{\circ}$, however. In particular, there must be an order on $\mathfrak{P}^{\circ}$ that, first, allows one to embed $(\mathfrak{P}, \leq)$ into it and, second, defines a supremum that makes $\mathfrak{P}^{\circ}$ fusion complete. The way to find this embedding has already been hinted at: to find the virtual fact simulating some possible fact $s \in \mathfrak{P}$, apply f to its singleton—i.e. apply $(\cdot)^{\circ}$ to $\{s\}$,

$$(\{s\})^{\circ} = (\{s\}^{\vee})^{\downarrow} = \{s\}^{\downarrow}$$

and also note that for any $t, u \in \mathfrak{P}$,

$$t \leq u \iff \{t\}^{\downarrow} \subseteq \{u\}^{\downarrow}.$$

Therefore $\{\cdot\}^{\downarrow} : \mathfrak{P} \to \mathfrak{P}^{\circ}$ embeds $(\mathfrak{P}, \leq)$ into $(\mathfrak{P}^{\circ}, \subseteq)$. So far so good, but there's one last thing to check: does the order $\subseteq$ give rise to a complete on $\mathfrak{P}^{\circ}$? The answer happens to be 'yes': $\bigvee^{\circ} : 2^{\mathfrak{P}^{\circ}} \to \mathfrak{P}^{\circ}$, defined to be

$$\bigvee^{\circ} Y = \left(\bigcup Y\right)^{\circ}$$

8. I'm playing it a bit fast and loose with my discussion of these closures. One might worry that fusion closures are not always fusion-closed, downward closures are not downward closed, and, even having settled these two worries, that fusion-cum-downward closures like $(X^{\vee})^{\downarrow}$ are no longer fusion closed. One can prove that there is no reason to worry by proving that all of the above are closure operators in the sense of Definition 10, given in Chapter II subsection 2.2.

for any non-empty ideal of possible facts $Y \subseteq \mathfrak{P}^\circ$, is a complete supremum.

This way of completing the possible facts is an instance of sometimes called the ideal completion of a poset, which has the following characterization.

Definition 24 (Ideal Completion of a Poset).
Let $(U, \leq)$ and $(U', \leq')$ be posets, and let $\bigvee : 2^U \to U$ and $\bigvee' : 2^{U'} \to U'$ be their respective suprema. $(U', \leq')$ is the ideal completion of $(U, \leq)$ if and only if

a. The elements of U' are the ideals of U.
$X \in U'$ iff $X = X^\circ$, for any $X \subseteq U$.

b. $\leq'$ is the subset relation.
$X \leq' Y$ iff $X \subseteq Y$, for all $X, Y \in U'$.

c. $\bigvee' : 2^{U'} \to U'$ is the ideal closure of union.
$\bigvee' x = (\bigcup x)^\circ$, for any non-empty $x \subseteq U'$.

d. $\{\cdot\}^\downarrow : U \to U'$ order-embeds $(U, \leq)$ into $(U', \leq')$.
$s \leq t$ iff $\{s\}^\downarrow \leq' \{t\}^\downarrow$, for all $s, t \in U$.

It remains to show that the ideal completion is indeed a *completion* in the sense of Definition 20. The proof of this is omitted because it is already well-known.

The ideal completion is the method of constructing virtual facts Fine proposes. The following remark shows how the virtual facts constructed with the ideal completion simulate the possible facts and their fusions.

Remark 4 (Virtual Facts from an Ideal Completion).
Let $(\mathfrak{P}, \leq)$ be the possible facts ordered by fact parthood and let $\bigvee : 2^{\mathfrak{P}} \to \mathfrak{P}$ be its fusion. Let $(\mathfrak{P}^\circ, \subseteq)$ be the ideal completion of $(\mathfrak{P}, \leq)$ and let $\bigvee^\circ : 2^{\mathfrak{P}^\circ} \to \mathfrak{P}^\circ$ be its supremum. For any possible fact $s \in \mathfrak{P}$ and any set of possible facts $X \subseteq \mathfrak{P}$, note the following.

- The fusion of X is simulated by $X^\circ \in \mathfrak{P}^\circ$.
- s is simulated by the virtual fact $\{s\}^\downarrow \in \mathfrak{P}^\circ$.
- X is simulated by the set of virtual facts $\left\{\{s\}^\downarrow \mid s \in X\right\} \subseteq \mathfrak{P}^\circ$.
- The fusion of X is also simulated by the virtual fact

$$\bigvee\nolimits^\circ \left\{\{s\}^\downarrow \mid s \in X\right\},$$

which one can show to be the same as X°.

3.2 Against Fusionism: a Criticism from Possibility

The ideal completion is not an appropriate method of constructing virtual facts. This section presents a reason for this involving the possible facts and the way they are simulated by the ideal completion's virtual facts. Given that the set of possible facts is downward closed , the set of virtual facts simulating them should follow in suit. Under reasonable assumptions about fusion, the virtual facts delivered by the ideal completion simulate some possible facts having impossible parts.

Without argument, Fine places a restriction on the set of possible facts: whenever $X \subseteq \mathfrak{P}$ has an upper bound, it has a least upper bound. This is also called 'bounded completeness'.

Definition 25 (Bounded Complete Poset).
Let $(U, \leq)$ be a poset. $(U, \leq)$ is bounded complete if and only if,

$$\text{if there is a } u \in U \text{ such that } u \leq x \text{ for all } x \in X, \text{ then } \bigvee X \in U$$

for any $X \subseteq U$.

There are good reasons to question whether the set of possible facts is bounded complete in the way Fine assumes. Perhaps there are pairs of possible facts $s, t \in \mathfrak{P}$ that share some upper bounds with no *least* upper bound among them. Maybe, for instance, s and t's upper bounds descend infinitely. If this were the case, $s \vee t \notin \mathfrak{P}$ and the possible facts would not be bounded complete.

I have a controversial example of facts like these. However, I acknowledge that some might not accept it as a counterexample. Even if the example is unconvincing my argument still holds *some* water; no one has presented an argument for the bounded completeness of the possible facts as of yet and so there is no reason to assume them to be.

(4) the fact that it is hot outside

(5) the fact that it is cold outside

(4) and (5) are, individually, possible facts. Their fusion—the fact that the weather is both hot and cold—appears, to me at the very least, impossible.

Nonetheless, there are arguably possible facts that bound both (4) and (5) above; perhaps, for instance, the following.[9]

(6) the fact that it is 80°F outside

9. For reference: 80°F is 26°C.

(6) is not only possible, it also concerns the temperature outside as the facts (4) and (5) did before it. (6), however, provides considerably more detail. I claim that both (4) and (5) are parts of (6). The fact that it is 80°F outside, as far as I can tell, demands more of reality then either the fact that it is warm or the fact that it is cool and is capable of tightening the demands of each. Although (4) and (5) do not have a possible fusion, they do share a possible upper bound: (6)

This particular example touches on at least one highly contentious area in philosophy (namely, vagueness), and perhaps touches on others as well. The basic idea behind it is that some facts that are non-co-possible become co-possible when detail is added. Increasing precision, as numerical degrees are more precise than loose concepts like heat or coolness, may reveal commonalities between facts that were not at first apparent.

Now that the example is in place, we may proceed to ask why it poses a problem for Fine's account. I claim that it results in a simulation of the possible facts that is not downward-closed, and I'll demonstrate this casually. Inspect the virtual fact simulating the fusion of (4) and (5). Using the ideal completion, one first closes the set containing the facts (4) and (5) under fusion. Since (4) and (5) have no possible fusion, the result of the fusion closure is the set containing (4) and (5) once again. The second step is downward-closing this set, which simply produces the union of the individual downward closures of (5) and of (4). This, then, is the virtual fact simulating the impossible fusion of (5) and (4).

What virtual fact simulates (6)? As has been shown, simply its downward-closure. Notice that the downward-closure of (6) has both (4) and (5) as elements since (6) has both facts as parts. Notice, moreover, that every fact less than either (4) or (5) is also in (6)'s downward-closure. This is enough to show that the virtual fact simulating (6) is a superset of the virtual fact simulating the impossible fusion of (4) and (5). In the ideal completion superset indicates parthood, and so (6) is simulated as having the fusion of (4) and (5) as a part. But (6) is possible and this fusion is impossible, and no possible fact has impossible parts.

Clearly, the possible facts should not be simulated as having impossible parts for all agree that the possible facts are downward closed. This to me is sufficient reason to question whether the ideal closure provides a suitable simulation of the possible facts. It seems to me that it is not.

3.3 Against Fusionism: a Criticism from Impossibility

The ideal completion is not a good method of constructing virtual facts because it misrepresents the possible facts, simulating them as having features they do not. Taking down the ideal completion takes down only one version of fusionism. Others still lurk about, and the present subsection gives a reason why they all fall short: fusionism inevitably omits facts that are well-qualified to be truthmakers. This, I claim, is an error. Because these omitted facts meet all the right standards, they should be accepted as truthmakers and allowed to individuate contents. Instead of fusionism, one should opt for a different form of impossibilism that does not make these sorts of omissions.

Consider an example. Alice has very peculiar curly eyebrows that she has seen on no one else. She learned from the doctor that these curly eyebrows are a symptom of a rare genetic condition passed down from mother to daughter. Alice does not know anything about her biological parents, however. She has never met them or seen them. One day, Alice sees a movie starring an actor named Beatrice. Alice was shocked: Beatrice had curly eyebrows as well!

(7) the fact that Beatrice is Alice's biological mother

(8) the content that Alice has curly eyebrows

(7), I claim is well-suited to be a truthmaker for (8). (7) implies (8)'s truth for, as the doctor told her, her curly eyebrows are inherited from her mother. (7) also explains (8)'s truth in assuming responsibility for it. The fact that Beatrice is Alice's mother is a reason for (8) to be true.

(7), then, is well-qualified to be a truthmaker for (8). The above paragraph argued for this without considering whether or not (7) is true. If it's true or if it's false, (7) still implies (8)'s truth and also still explains it; truth has nothing to do with the matter. Assuming, *pace* Kripke, that it is impossible to have different biological parents, (7) is impossible if it's false. Even so, being impossible makes (7) no less qualified to make (8) true. I claim that (7), if false, is a truthmaker for (8) and an impossible fact all at the same time.

Even though (7) is a well-qualified truthmaker, it is not accounted for by fusionism. Why? Although it is impossible, (7) is not the impossible fusion of possible facts. To see this, try to find possible facts that fuse into (7). One might venture a guess along the lines of the fact that Beatrice is the biological parent of *somebody* and the fact that *somebody* is the biological parent of Alice. These facts do not have the desired fusion, for it is still left open who each of these somebodies are.

A better guess is the fusion of a set of possible facts that lead Beatrice to become a biological parent. Call this set X. One need not be too specific about the contents of X, but it suffices to say that if the fact that Beatrice is Alice's biological mother already lives in X, then X is not a set of possible facts. However, in order for the fusion of X to be (7), it must already contain something along those lines by containing possible facts enough to specify that Beatrice is the biological parent of somebody and that this somebody is Alice.

Fusionism simply leaves out important truthmakers. One might resist the argument by challenging Kripke's thoughts about biological parentage and modality. Challenge away, Kripke's help is not needed. One can find other examples that work just as well, examples involving biological parents are easier to explain.

Say that Ryan and Sam are identical twins that share all the same friends. However none of their friends have seen both Ryan and Sam at the same time. As the years go on, suspicions start to grow.

(9) the fact that Ryan and Sam are the same person

(10) the content that none of Ryan and Sam's friends have seen them both at the same time

One can run the same argument on this example as well. (9) is a well-qualified truthmaker for (10), impossible, and not the fusion of possible facts. One can craft other good examples, many including identity (9).

Having seen this, one cannot unsee it. Facts like (7) and (9) show that fusionism inexplicably omits a large number of well-qualified truthmakers. It is my contention that this is an error. All well-qualified facts should be given the same opportunity to act as truthmakers and it is wrong for fusionism to rob facts like (7) and (9) of the opportunity.

4. Consistentism

The goal of this section has been to find a way for truthmaker semantics to be strongly hyperintensional. The naïve account is not an option since it is only weakly hyperintensional, not differentiating between necessarily false contents. To become strongly hyperintensional, one must supplement the naïve account with more content-individuators.

Falsism and fusionism, each attempted by both Yablo and Fine, encounter problems. Falsism gives contradictions peculiar falsemakers that, although required to be explanatory, do not meet the necessary condition for explanation: they are neither relevant nor responsible. Fusionism has difficulties with both possible and impossible facts, misrepresenting the former and omitting important instances of the latter.

This section presents consistentism, the view that contents are individuated by the consistent facts. Consistentism is new, distinct from the naïve account, from falsism, and from fusionism. This is argued in subsection 4.1, and in subsection 4.2 it is argued that consistentism avoids the problems faced by the other accounts. In the end, consistentism emerges as the best way for truthmaker semantics to achieve strong hyperintensionality.

4.1 Consistentism is New

So far, three versions of truthmaker semantics have been presented: the naïve view, falsism, and fusionism. Is consistentism distinct from these other views? The present subsection argues that it is indeed so.

It is easy to show that consistentism is distinct from falsism: falsism allows falsemakers to individuate contents while consistentism does not. It is also easy to show that consistentism is distinct from fusionism: fusionism allows inconsistent facts, e.g. the fact that Pat is both smart and not smart, to individuate contents but consistentism does not. The work is in showing consistentism to be distinct from the naïve account.

Since consistentism and the naïve account take contents to be individuated by *consistent* facts and *possible* facts respectively, one can distinguish the two views by showing there to be facts that are simultaneously *consistent and impossible.*[10] This subsection argues that consistentism is indeed new by providing examples of such facts.

Before getting into these examples, it would be good to recall what possibility and consistency are. A fact's possibility depends on reality for facts can be thought of as demands on reality and, it has been said, a fact is possible if the demands it makes can be met. The fact that Pat is eating ice cream, for instance, is possible because reality can meet the demand it makes, Pat really can be eating ice cream.

A fact's consistency depends on its parts, not on reality. Consistency depends in particular upon contrariety, a relation between facts indicating a natural opposition. The fact that the apple is red, for example, is contrary to the fact that it's green. A fact is consistent just if none of its parts stand in this sort of opposition, just if none of its parts are contrary.

Looking back on possibility and consistency might already lead one to believe that some facts are both consistent and impossible. Notice that possibility is global: whether or not a fact is possible depends upon the way all of reality can be. Consistency, on the other hand, is local, not global. A fact's consistency depends only upon its parts, not upon whether reality can meet its demands. The gap between globality and locality provides an opportunity for some facts to fall between the cracks, meeting local constraints by being consistent while failing global ones by being impossible.

What are some examples consistent and impossible facts? The facts that presented problems for fusionism in subsection 3.3, I claim, are perfectly good examples,

10. One also must show the such facts are capable of individuating contents, but this is a trivial task and so it is ignored.

(7) the fact that Beatrice is Alice's biological mother

(9) the fact that Ryan and Sam are the same person

where it is assumed that Beatrice is not Alice's mother and Ryan and Sam are not the same person. As above, the present argument focuses on (7) for explanatory ease but can be replicated just as well for (9).

While the impossibility of (7) can be assumed without argument, there is no reason as of yet to think that it is consistent. Subsection 3.3 above argues that (7) is not the result of fusing possible facts, but this is of no help in the present discussion. What needs to be shown is that (7) has no contrary parts.

One argument is by exhaustion: try to find contrary parts of (7) and, after inevitably growing exhausted, give up and admit there are none. This isn't a good choice. Not only is argument by exhaustion unfair to the reader, it is also not all that convincing.

Luckily, I have a more convincing argument at hand. Thinking of facts as demands on the reality, recall what fact parthood is: a fact s is part of another t if s loosens the demands t makes of reality. From this point of view, (7) is inconsistent if there are different ways of loosening the demands (7) makes of reality two of which result in contrary facts; otherwise (7) is consistent.

Take the example of the fact that the apple is both red and green. One can loosen this fact's demands to get the fact that the apple is red and to get the fact that the apple is green. The results of these loosenings are contraries, the fact that the apple is both red and green is inconsistent.

Can one find ways of loosening (7)'s demands on reality that result in contrary facts? It is my contention that (7) is consistent and one cannot get contrary facts by loosening its demands in certain ways. To see why, assume that Mary, not Beatrice, is Alice's biological mother and consider the following fact.

(11) the fact that Mary is Alice's biological mother

This fact, which is true, is also presumably consistent.

Locally speaking, (7) and (11) share a lot in common. Both concern Alice's biological mother, both share many of the same parts (e.g. the fact that Alice has a relative, the fact that somebody has a biological mother). Because of how much they share, if there are ways of loosening the demands made by (7) to reach contraries, one should be able to reach contraries by loosen (11)'s demands in the very same ways. However, (11) is consistent and there are no such ways of loosening its demands. It follows that (7), too, must be consistent.

One might try to resist this argument by attempting to mark a difference between the fact that Beatrice is Alice's biological mother and the fact that Mary is, but resistance of this sort is futile. There is nothing that distinguishes (7) from (11) except that the former is false and the latter is true, but this has no bearing on consistency. Truth, like possibility, depends on a fact's relation to reality. This was mentioned at the beginning of Chapter II subsection 2.2. The fact that (11) just so happens to be true gives it no advantage over (7) when it comes to consistency, which does not depend upon reality at all.

Admittedly, this argument does not fill in every blank. Importantly, not much as been said on what exactly it is to loosen a fact's demands on reality. Nonetheless the points here are plausible. If the fact that Beatrice is Alice's biological parent is inconsistent then what are its contrary parts? What's more, how could the fact that Mary is Alice's biological parent, which is presumably consistent, avoid having contrary parts if (7) were to have them? Although the above does not answer these questions completely, it does provide reasons to think that the correct answers come out in consistentism's favor.

(7) and (9) are examples of consistent yet impossible facts.[11] Consistentism and the naïve account are therefore distinct: the former individuates contents with more facts than the latter. Consistentism is therefore new: it is not naïve, it is not falsism, and it is not fusionism. It is different from all the existing accounts of truthmaker semantics on the market.

4.2 Consistentism Avoids the Above Problems

So far, the alternatives to consistentism have encountered a total of four problems.

1. (Naïve Account, 1) Being weakly hyperintensional.

2. (Falsism, 2.2) Employing a problematic account of falsemaking.

3. (Fusionism, 3.2) Using an inapt means of constructing virtual facts.

4. (Fusionism, 3.3) Barring well-qualified facts from differentiating contents.

Out of hand consistentism avoids the first three problems. While consistentism plans to become strongly hyperintensional by permitting *some* impossible facts (the consistent ones) to differentiate between contents, it avoids the second problem by not resorting to falsemaking, and avoids the third by not constructing virtual facts.

Consistentism also avoids the fourth problem and the remainder of the section contains two short arguments showing this to be the case. The first argument is a bottom-up, and simply points out that the problematic cases that

11. And, in reference to note 10 both facts also individuate contents. Section 3.3 showed this to be the case by arguing that they are well-qualified truthmakers.

plagued fusionism have already been shown to be dealt with by consistentism. The previous section showed that both (7) and (9) are themselves consistent and, therefore, accounted for.

The second, top-down argument claims that consistentism allows all well-qualified facts to be content differentiators. Recall what it is to be well-qualified: to be a good candidate truthmaker in having the ability to *explain* truth. All well-qualified facts are therefore explanatory. For consistentism, which takes consistent facts to individuate contents, it is enough to show that all explanatory facts are consistent in order to argue that it has all well-qualified facts individuate contents.

The argument argues contrapositively, that inconsistent facts are not explanatory. This seems to hold true. Consider one of the running examples of an inconsistent fact: the fact that Pat both is and is not smart. Does this explain anything? Going back to the two traits of explanation, it's hard to see how it could be relevant to or responsible for the truth of any content. This fact, then, is explanatorily inert and the same goes for other inconsistent facts as well.

Thanks to these top-down and bottom-up arguments, one can confidently say that consistentism has a strong advantage over its competitors in not encountering the problems they face. For this reason, I recommend that the full power of consistent facts be taken advantage of in truthmaker semantics.

4.3 A Consistentist Treatment of Connectives

The new definition of connectives, which I stand behind, differs from the common and alternative definitions in two ways. First, it does not employ falsemakers to speak of. As a result, the truthmakers for a negation are given by *contrariety*, not by falsemaking. A fact is a truthmaker for $\neg S$ on my view just in case it is contrary to all the truthmakers for S. On this view, truthmakers for any negated content $\neg S$ must have a natural opposition the truthmakers for the unnegated content S and, in a sense, rob S of the ability to be made true.

The second departure the new definition makes from the others involves disjunction. On the other views, conjunction and disjunction are asymmetrical. While conjunction's truthmakers come from fusion, disjunction's involves no such operations on facts. So the truthmakers for a conjunction $S \mathbin{\&} T$ are truthmakers for S fused with truthmakers for T. Truthmakers for a disjunction $S \text{ v } T$, on the other hand, are truthmakers for S or—not overlap, but or—truthmakers for T.

Why is this the case? Admittedly, there is some formal neatness involved in using only fusion instead of fusion and overlap, but this should count as no real motivation. In fact, there is good motivation for thinking that truthmakers for a disjunction are the overlap of truthmakers for each disjunct.

When dealing with implication alone and not with explanation, implying a disjunct is certainly enough to imply a disjunction. Is the same true for explanation of truth? I think not. Say for instance that a fact s explains the truth of a content S. One might say that s also explains the truth of any disjunction $S \text{ v } T$, but what argument does one have for this?

The only arguments I can think of are no good. Take, for example, the following.

> "s explains the truth of $S \text{ v } T$ because (i) it explains the truth of S and (ii) whenever a disjunct is true then so is the disjunction."

It seems that (i) and (ii) are both true, but they do not allow one to reach the conclusion that s explains the truth of $S \text{ v } T$. In order to do so, (ii) would have to be altered to (ii′): that explaining the truth of a disjunct is enough to explain the truth of a disjunction. One cannot assume (ii′) to be true, however, because it is the very assumption being questioned.

Let's try another angle. A disjunction is true when some of its disjuncts are true. Explaining the truth of $S \text{ v } T$, then, means explaining the following: reality is such a way that either S is the case or T is the case. Truthmakers for disjunctions, therefore, must allow for all disjuncts to be true. A truthmaker, say, that explains the truth of S does not always explain the truth of $S \text{ v } T$ because it does not always allow for the truth of T. Take for example the fact that the apple is red and the content that either the apple is a warm color or my aunt is visiting. Does the fact that the apple is red really explain the truth of this disjunctive content? I think not. It explains the truth of the content that the apple is a warm color, surely, but not the truth of the disjunction as a whole.

Insofar as disjuncts are non-identical with disjunctions, explaining the truth of some disjuncts is not the same as explaining the truth of a disjunction. The treatment of disjunction, then, should be parallel to that of conjunction on the existing accounts (cf. Definition 18c(i)): for a fact to make a disjunction true, it must be the overlap of facts that make each disjunct true. The new definition of connectives is then the following.

Definition 26 (Consistentist Definition of Connectives).
Let $\mathfrak{F}$ be the set of facts and $\mathfrak{C}$ be the set of contents. From Definition 14 in Chapter II, let $\perp \subseteq \mathfrak{F} \times \mathfrak{F}$ be the contrariety relation. The new definitions of the connectives not ($\neg$), or (v), and and (&) are as follows.

a. Negation, $\neg(\cdot) : \mathfrak{C} \to \mathfrak{C}$
$s \mid\!\mid\!\!- \neg S$ iff $t \mid\!\mid\!\!- S$ implies that $s \perp t$

b. Disjunction, $(\cdot) \mathrm{v} (\cdot) : \mathfrak{C} \times \mathfrak{C} \to \mathfrak{C}$
$u \mathrel{||}\!\!- S \mathrm{v} T$ iff $s \mathrel{||}\!\!- S$ and $t \mathrel{||}\!\!- T$ and $u = s \wedge t$

c. Conjunction, $(\cdot) \,\&\, (\cdot) : \mathfrak{C} \times \mathfrak{C} \to \mathfrak{C}$
$u \mathrel{||}\!\!- S \,\&\, T$ iff $s \mathrel{||}\!\!- S$ and $t \mathrel{||}\!\!- T$ and $u = s \vee t$

for all $s, t, u \in \mathfrak{F}$ and $S, T \in \mathfrak{C}$.

5. Conclusion

The naïve account of truthmaker semantics consists of two tenets: truthiness, that contents be individuated by truthmakers, and naïveté, that contents be individuated by possible facts. Together these tenets do not achieve strong hyperintensionality for, when combined, there are too few facts to individuate the contents of necessarily false contents. The desired account of truthmaker semantics is strongly hyperintensional and must supplement the tenets to allow for more content-individuators.

There are two options: falsism, which supplements truthiness by individuating contents with *more than just truthmakers*, and impossibilism, which supplements naïveté by individuating contents with *more than just possible* facts. Falsism, then, has contents be individuated by truthmakers *and falsemakers*; impossibilism, on the other hand, has contents be individuated by possible facts *and impossible* facts.

Falsism is the wrong way to go. Opening the door to falsemakers opens the door to too many problems, as section 2.2 discussed. Impossibilism is therefore the right way to achieve strong hyperintensionality, but in which of its incarnations? The only existing incarnation of impossibilism in truthmaker semantics is fusionism: the impossible fusions of possible facts individuate contents, e.g. the impossible fact that Pat both is and is not smart.

Fusionism is not right either. As argued in section 3.2, it misrepresents the possible facts, and, as argued in section 3.3, it prevents well-qualified facts from being truthmakers. Although fusionism does not help get us to strong hyperintensionality, not all hope is lost. Other forms of impossibilism that individuate contents with different impossible facts are still out there.

My proposal, consistentism, is a form of impossibilism that avoids the problems faced by the other accounts. Consistentism does not fall prey to the problems of falsism because it does not resort to falsemaking. Consistentism is not vulnerable to the problems of fusionism because it does not simulates the possible facts and it accounts for the important impossible facts fusionism left out. Consistentism, additionally, is strongly hyperintensional. If one is after an account of truthmaker semantics, consistentism seems to be the best option on the table.

Conclusion

I close with a brief recap of the areas covered. Chapter I used hyperintensionality to motivate truthmaker semantics. The truth conditional theory of content, which is not hyperintensional, was presented and criticized. The problems it faced all stemmed from a common source: its lack of hyperintensionality. After spending some time defining hyperintensionality and extolling its virtues, two types of hyperintensionality were distinguished: the strong, on which necessarily false contents may be distinct, and the weak, on which they may not; it is argued that strong hyperintensionality, in particular, is needed to address some of the problems faced by the truth conditional account. The first chapter ends by tying hyperintensionality to truthmaker semantics and showing how one jumps from the former motivates the latter.

Chapter II was largely devoted to filling in gaps in the truthmaker semantics literature. In particular, the notions of fact, of explanation, and of truthmaking needed elaboration. With the ultimate aim of furnishing an account of truthmaker semantics, I provide my own view on these matters. While facts can be one of the many options chosen by related semantic theories, it was argued that explanation, whatever it may be, must bring either relevance or responsibility with it. Truthmaking is just implication, a truth theoretic relation, in concert with explanation.

Chapter III attempted a novel contribution to truthmaker semantics with an argument for consistentism, the view that contents are individuated by their consistent truthmakers. The argument started by referring back to the first chapter, which motivated strong hyperintensionality. The existing accounts of truthmaker semantics have tried to achieve strong hyperintensionality with recourse to either falsism or fusionism. Both views, however, encounter problems. Falsism runs into problems with falsemaking, which assigns unintuitive truthmakers to contradictions. Fusionism, as found in Fine's 'Constructing the Impossible' (2016), misrepresents the possible facts and, as found in any form, omits facts that are well-qualified to be truthmakers. Consistentism is shown to be distinct from the earlier views and additionally to avoid their problems.

In the end, I argue, consistentism is the view to be adopted.

Bibliography

Anderson, Alan R., and Nuel D. Belnap. 1975. *Entailment: The Logic of Relevance and Necessity, Vol. I.* Princeton University Press.

Armstrong, D.M. 2004. *Truth and Truthmakers.* Cambridge University Press.

Bader, Ralf M. 2013. "Towards a Hyperintensional Theory of Intrinsicality." *Journal of Philosophy* 110:525–563.

Barwise, Jon. 1989. "Logic and Information." In *The Situation in Logic.* CSLI Lecture Notes Number 17. Center for the Study of Language.

Barwise, Jon, and John Perry. 1983. *Situations and Attitudes.* MIT Press.

Batterman, Robert W, and Collin C Rice. 2014. "Minimal Model Explanations." *Philosophy of Science* 81 (3): 349–376.

Bealer, George. 1998. "Propositions." *Mind* 107:1–32.

Bechtel, William, and Adele Abrahamsen. 2005. "Explanation: A Mechanist Alternative." *Studies in History and Philosophy of Science: Studies in History and Philosophy of Biological and Biomedical Sciences* 36 (2): 421–441.

Berto, Francesco. 2015. "A Modality Called 'Negation'." *Mind* 124 (495): 761–793.

Bezhanishvili, Nick. 2006. "Lattices of Intermediate and Cylindric Modal Logics." Doctoral dissertation, University of Amsterdam.

Birkhoff, George. 1940. *Lattice Theory.* American Mathematical Society.

Brandom, Robert. 2008. *Between Saying and Doing: Towards an Analytic Pragmatism.* Oxford University Press.

Cartwright, Nancy. 2004. "From Causation to Explanation and Back." In *The Future for Philosophy*, edited by Brian Leiter. Clarendon Press.

Ciardelli, Ivano. 2016. "Questions in Logic." Doctoral dissertation, University of Amsterdam.

Ciardelli, Ivano, Jeroen Groenendijk, and Floris Roelofsen. 2013. "Inquisitive Semantics: A New Notion of Meaning." *Language and Linguistics Compass* 7:459–476.

— . 2015. *Inquisitive Semantics.* ESSLLI Lecture Notes.

Craver, Carl F. 2006. "When mechanistic models explain." *Synthese* 153 (3): 355–376.

Cresswell, M.J. 1975. "Hyperintensional Logic." *Studia Logica* 34:25–38.

Davidson, Donald. 1967. "Truth and Meaning." *Synthese* 17:304–23.

Devlin, Keith. 2008. "Situation Theory and Situation Semantics." In *Handbook of the History of Logic*, edited by Dov Gabbay and John Woods, 7:601–664. Elsevier.

Dunn, J. Michael. 1966. "The Algebra of Intensional Logics." Doctoral dissertation, University of Pittsburgh.

— . 1976. "Intuitive Semantics for First-Degree Entailments and 'Coupled Trees'." *Philosophical Studies* 29 (3): 149–168.

— . 1993. "Star and Perp: Two Treatments of Negation." *Philosophical Perspectives* 7:331–357.

— . 1996. "Generalized Ortho Negation." In *Negation: A Notion in Focus*, edited by H. Wansing, 3–26. W. De Gruyter.

Eddon, Maya. 2011. "Intrinsicality and Hyperintensionality." *Philosophy and Phenomenological Research* 82:314–336.

Fine, Kit. 2012a. "Guide to Ground." In *Metaphysical Grounding*, edited by Fabrice Correia and Benjamin Schnieder, 37–80. Cambridge University Press.

— . 2012b. "The Pure Logic of Ground." *Review of Symbolic Logic* 5 (1): 1–25.

— . 2014. "Truth-Maker Semantics for Intuitionistic Logic." *Journal of Philosophical Logic* 43 (2-3): 549–577.

— . 2015a. "A Theory of Truth-Conditional Content I: Conjunction, Disjunction and Negation." Retrieved from academia.edu.

— . 2015b. "A Theory of Truth-conditional Content II: Subject-matter, Common Content, Remainder and Ground." Retrieved from academia.edu.

— . 2015c. "Angellic Content." *Journal of Philosophical Logic.*

— . 2016a. "Constructing the Impossible." Retrieved from academia.edu.

— . 2016b. "The World of Truthmaking." Retrived from academia.edu, Draft.

— . 2016c. "Truthmaker Semantics." In *Blackwell Philosophy of Language Handbook.* Blackwell.

Foulis, D.J., and C.H. Randall. 1969. "Lexicographic Orthogonality." *Journal of Combinatorial Theory, Series A* 11:157–162.

Friedman, Michael. 1974. "Explanation and Scientific Understanding." *The Journal of Philosophy* 71 (1): 5–19.

Girard, Jean-Yves. 1987. "Linear Logic." *Theoretical Computer Science* 50:1–102.

Goldblatt, Robert. 1974. "Semantic Analysis of Orthologic." *Journal of Philosophical Logic* 3:19–35.

— . 2011. *Quantifiers, Propositions and Identity.* Cambridge University Press.

Goodman, Nelson. 1961. "About." *Mind* 70:1–24.

Harding, John. 2008. "Completions of Ordered Algebraic Structures: a Survey." In *Interval/Probabilistic Uncertainty and Non-classical Logics*, 231–244. Springer.

Holliday, Wesley. 2014. "Partiality and Adjointness in Modal Logic." In *Advances in Modal Logic*, edited by Rajeev Goré, Barteld Kooi, and Agi Kurucz, 10:312–332. College Publications.

— . 2015. *Possibility Frames and Forcing for Modal Logic.* UC Berkeley Working Paper in Logic and the Methodology of Science.

Humberstone, I. L. 1981. "From Worlds to Possibilities." *Journal of Philosophical Logic* 10 (3): 313–339.

— . 2013. "Logical Relations." *Philosophical Perspectives* 27:175–230.

Jago, Mark. 2015a. "Hyperintensional Propositions." *Synthese* 192:585–601.

— . 2015b. "Truthmaking, Content and Same-Saying." Visited on 09/05/2015. `http://www.markjago.net/uploads/1/5/4/5/15456840/truthmaking_content_samesaying.pdf`.

Jespersen, Bjørn, and Marie Duži. 2015. "Introduction." *Synthese* 192:525–534.

Kaplan, David. 1968. "Quantifying In." *Synthese* 19:178–214.

Kemp, Gary. 1998. "Meaning and Truth-Conditions." *The Philosophical Quarterly* 48:483–493.

King, Jeffrey C. 2007. *The Nature and Structure of Content.* Oxford University Press.

Kitcher, Philip. 1989. "Explanatary Unification and the Causal Structure of the World." In *Scientific Explanation*, edited by Philip Kitcher and Wesley Salmon, 8:410–505. Minneapolis: University of Minnesota Press.

Kratzer, Angelika. 1989. "An Investigation of the Lumps of Thought." *Linguistics and Philosophy* 12 (5): 607–653.

— . 2002. "Facts: Particulars or Information Units?" *Linguistics and Philosophy* 25 (5): 655–670.

Leitgeb, Hannes. 2015. "A Hyperintensional Logic for the Causal 'Because'?" In *The Inquisitive Turn, Workshop 2: Questions in Logic and Semantics.* `www.illc.uva.nl/inquisitivesemantics/workshops/inqturn-closing-event-2015`.

Lewis, David. 1986a. "Causal Explanation." In *Philosophical Papers*, volume II. Oxford University Press.

— . 1986b. *On the Plurality of Worlds.* Blackwell.

— . 1988a. "Relevant Implication." *Theoria* 54 (3): 161–174.

— . 1988b. "Statements Partly About Observation." *Philosophical Papers* 17:1–31.

Liggins, David. 2005. "Truthmakers and Explanation." In *Truthmakers: The Contemporary Debate*, edited by Helen Beebee and Julian Dodd, 105–115. Clarendon Press.

Litland, Jon Erling. 2013. "On Some Counterexamples to the Transitivity of Grounding." *Essays in Philosophy* 14 (1): 3.

Merricks, Trenton. 2007. *Truth and Ontology.* Clarendon Press.

Nolan, Daniel. 2014. "Hyperintensional Metaphysics." *Philosophical Studies* 171:149–160.

Perry, John. 1986. "From Worlds to Situations." *Journal of Philosophical Logic* 15 (1): 83–107.

Pollard, Carl. 2008. "Hyperintensions." *Journal of Logic and Computation* 18:257–282.

Restall, Greg. 1997. "Ways Things Can't Be." *Notre Dame Journal of Formal Logic* 38 (4): 583–596.

— . 2005. "Multiple Conclusions." In *Logic, Methodology and Philosophy of Science*, edited by Petr Hájek, Luis Valdés-Villanueva, and Dag Westerståhl. College Publications.

— . 2013. "Assertion, Denial and Non-Classical Theories." In *Paraconsistency: Logic and Applications*, edited by Francesco Berto et al., 81–99. Springer.

Reyes, Gonzalo E., and Houman Zolfaghari. 1996. "Bi–Heyting Algebras, Toposes and Modalities." *Journal of Philosophical Logic* 25:25–43.

Ripley, David. 2013. "Paradoxes and Failures of Cut." *Australasian Journal of Philosophy* 91 (1): 139–164.

— . Forthcoming. "Bilateralism, coherence, warrant." In *Act-Based Conceptions of Propositional Content. Contemporary and Historical Perspectives*, edited by Friederike Moltmann and Mark Textor. Oxford University Press.

Rodriguez-Pereyra, Gonzalo. 2002. *Resemblance Nominalism.* Clarendon Press.

Roelofsen, Floris. 2013. "Algebraic Foundations for the Semantic Treatment of Inquisitive Content." *Synthese* 190:79–102.

Rumfitt, Ian. 2012. "On a Neglected Path to Intuitionism." *Topoi* 31:101–109.

Ryle, Gilbert. 1933. "About." *Analysis* 1:10–12.

Sano, Katsuhiko. 2012. "An Impossibility Theorem in Radical Inquisitive Semantics." In Workshop on Relating Particles to Evidence and Inference, Göttigen 2012.

Schiffer, Stephen. 2012. "Propositions, What Are They Good For?" In *Prospects for Meaning*, edited by R. Schantz, 3:259–285. Current Issues in Theoretical Philosophy. De Gruyter.

Schnieder, Benjamin. 2011. "A Logic for 'Because'." *The Review of Symbolic Logic* 4 (3): 445–465.

Skow, Bradford. 2014. "Are There Non-Causal Explanations (of Particular Events)?" *The British Journal for the Philosophy of Science* 65:445–467.

Soames, Scott. 1987. "Direct Reference, Propositional Attitudes, and Semantic Content." *Philosophical Topics* 15:47–87.

— . 2008. "Why Propositions Cannot be Sets of Truth-Supporting Circumstances." *Journal of Philosophical Logic* 37:267–276.

— . 2014. "Why the Traditional Conceptions of Propositions Can't be Correct." Chapter 3 in *New Thinking about Propositions*, 25–44. Oxford University Press.

Sorensen, Ray. 2002. "The Art of the Impossible." Chapter 9 in *Conceivability and Possibility*, edited by Tamar Szabó Gendler and John Hawthorne, 337–368. Oxford University Press.

Stalnaker, Robert. 1975. "Indicative Conditionals." *Philosophia* 5.

— . 1986. "Possible Worlds and Situations." *Journal of Philosophical Logic* 15 (1): 109–123.

Strevens, Michael. 2008. *Depth: An account of scientific explanation.* Harvard University Press.

van Fraassen, Bas. 1969. "Facts and Tautological Entailments." *The Journal of Philosophy* 66 (15): 477–487.

— . 1977. "The Pragmatics of Explanation." *American Philosophical Quarterly* 14 (2): 143–150.

Wittgenstein, Ludwig. 2014. *Tractatus Logico-Philosophicus.* Edited by C.K. Ogden. Routledge.

Yablo, Stephen. 2003. "Causal Relevance." *Philosophical Issues* 13 (1): 316–329.

— . 2014a. *Aboutness.* Princeton University Press.

— . 2014b. "Aboutness Theory." Supplement to *Aboutness.* `www.mit.edu/~yablo/home/Papers_files/aboutness%20theory.pdf`.

— . 2016. "Ifs, Ands, and Buts: An Incremental Truthmaker Semantics for Conditionals." *Analytic Philosophy* ().