Relevance semantics: an approach to intensionality *

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Abstract

This paper introduces a semantic system based on relevance logic (hence "relevance semantics"). It is motivated by the inability of a Montague-style semantics based on the modal notion of strict implication to distinguish between mutually entailing expressions, which becomes a problem in "hyperintensional" contexts. Relevance logic isolates implication (rather than the necessity operator) as the locus of the problem. By dropping the structural rule of Weakening, it provides a consequence relation which is not truth-functional, thus avoiding the "paradoxes of strict implication". In the semantics, the absence of Weakening gives rise to partial and inconsistent points of evaluation ("worlds"). Hence, logical truths do not hold at all indices and mutually entailing expressions do not have the same extensions. The paper includes an application of this semantics to attitude reports, which is intended to preserve the insights of existing approaches, while avoiding the problem of hyperintensionality.

1 Introduction

1.1 The background

It is well known that the possible worlds semantics of Montague (1974) fails to distinguish expressions which have the same extension at all indices. This may arise because the expressions are logically equivalent or because they are made equivalent by constraints (meaning postulates).

This problem of "hyperintensionality" has been approached in a variety of ways. Following Lappin and Pollard (2000), I assume that when sufficiently formalized, the only promising approaches boil down to two. One is

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to reverse the Montagovian perspective by taking propositions as primitive and making indices and other extensional objects derivative. Algebraically, this involves declining to equate logically equivalent sentences under the equivalence relation of mutual entailment, thus working with a pre-algebra instead of the familiar Lindenbaum algebra. This approach is developed in Fox, Lappin and Pollard (2001). The other approach is to refine the notion of possible worlds so that logically equivalent propositions no longer have the same denotations. Thus situation semantics (Barwise and Perry (1983)) introduced partial worlds (situations) to model incomplete information. Muskens (1995) extends the idea from partial to impossible worlds (formalized using multiple truth values). Orthogonally to this distinction, situation semantics is based on an independent theory of information termed "ecological realism" rather than being the interpretation of a logical language, while Muskens reverts to a logical perspective closer to Montague's.

The present proposal is located in the second of these two traditions. In particular it is closely related to the programme of Muskens, in that it is both logic-based and invokes a notion of impossible worlds. However, both the logic and its semantics are set up differently. The logic adopted is **relevance logic** (Anderson and Belnap (1975), Dunn (1986)). This provides a non-truth-functional notion of implication, avoiding the paradoxes of both material and strict implication. It is interpreted here using a frame semantics (the Routley-Meyer semantics[1]). This is related to the frame semantics used for modal or intuitionistic logic, but contrasts with them in that logical truths do not hold at all indices and mutually entailing expressions do not have the same extensions. As will be described, this arises in a principled way from the logic. The semantics is more convenient than that of Muskens in a number of respects: first, because of its relation to frames for other logics, which facilitates comparison; and also because of an important connection with channel theory, a development from situation semantics (Barwise (1993), Barwise and Seligman (1997)). This connection has been noted in the literature, and is explored especially in work by Restall (e.g. 1994) and Mares (1997). These writers, however, advocate the use of weaker substructural logics, while relevance logic differs from full intuitionistic logic only in rejecting the single structural rule of Weakening. The perspective of this paper is that this is the heart of the intensionality issue.[2]

[1]Due to Routley and Meyer (1973).

[2]For Contraction, see section 4 below.

1.2 The problem of intensionality

Natural language contains certain intensional or "opaque" contexts, where although A may imply B, it is not the case that an $\Phi(A)$ implies $\Phi(B)$ (where $\Phi(B)$ is identical to $\Phi(A)$ but with B substituted for at least one occurrence of A). Typically, the embedding context Φ represents some predicate of propositional attitude.

Using material implication (1), if A is true then it is implied by any B (**irrelevance**), while if A is false it implies any B (**explosion**). However an agent accepting or rejecting A should not be committed to any such implications, and any report that embeds such an implication inside a propositional attitude will give the wrong truth conditions.

(1) Material implication: $A \supset B = \neg A \vee B$

(2) Strict implication: $A \prec B = [](A \supset B)$

Montague's approach to this problem is to adopt an (S5) modal logic, in which propositions are modelled as sets of possible worlds, and a notion of strict implication (entailment) is obtained by prefixing a necessity operator to a material implication (2). The problem is that the same problems of irrelevance and explosion reappear, whenever (respectively) the consequent is a *necessary* truth or the antecedent a *necessary* falsehood. Traditionally Montague's semantics is called "intensional", and these problems which it fails to solve are described as "hyperintensional".

(3) If John believes [colourless green ideas sleep furiously] then John believes [Mary is pregnant or Mary is not pregnant].

(4) If John believes [colourless green ideas sleep furiously] then John believes [if Mary is pregnant then Mary is pregnant].

(5) If John believes that [Mary is pregnant and Mary is not pregnant] then John believes [the earth is flat].

In (3), the embedded proposition in the consequent is a tautology (the Law of the Excluded Middle, **LEM**, is a tautology in normal modal logic), and is entailed by anything. A partial answer to the problem may lie in taking the content of John's belief to be a situation - a point of evaluation in a partial logic, where not all issues are decided at all points. This would be the approach of situation semantics, or of intuitionistic logic. However, a modified form of the same problem would be (4). Each situation still supports logical truths - it just happens that in situation semantics (or intuitionistic logic) LEM is not among them. Furthermore (5) is a problem for all these approaches. No world or situation can support contradictory information, hence contradictions denote the empty set of situations and anything follows (ex contradictione quodlibet, or **ECQ**).

Thus, neither partial nor modal systems as such seem to be able to fix the problem. I assume that the notion of necessity, as formalized in modal logic, is a useful one in natural language semantics (especially epistemic and doxastic applications). The problem seems to lie not with the notion of necessity, but with the notion of implication. The modal operator merely quantifies over worlds in which formulae with material implication and Boolean or intuitionsistic negation are evaluated, and hence there is no way out of inferences such as (4, 5).

What is needed is a logic in which logical truths do not hold in all situations, and which furthermore allows impossible situations. This requires, first, a notion of implication which is not truth-functional, so that $A \rightarrow A$ does not follow from arbitrary antecedents, and second, a treatment of negation such that contradictions do not entail arbitrary consequents. For the latter we have to "weaken the connection between truth and falsity" (Muskens (1995)) to give a paraconsistent negation. The result in the semantics will be to admit both non-normal worlds where logical truths fail and inconsistent worlds where contradictions can hold. These "impossible worlds" will not (perhaps) be part of the real world, but may correspond to confused epistemic states[3]. In such a model each agent is allowed to be confused in their own way, without all such confused states being identified[4].

Thus we need both an intensional notion of implication and a paraconsistent notion of negation. The following sections will comprise an outline, first, of the way these notions are formalized in relevance logic, and then of the frame semantics for that logic. They are based on Dunn (1986), Gabbay and Olivetti (2000) and Restall (2000). Section 4 makes some suggestions as to how this semantics can be applied to propositional attitudes, using the idea of channels as connections between agents and the situations in which propositions are evaluated.

Note that the implication arrow $\rightarrow$ will henceforth be reserved for relevant implication, and $\sim$ for the corresponding negation (De Morgan nega-

[3]This association of impossible worlds with irrationality may be understating the case for them. Barwise (1997) and Restall (2000) both have interesting discussion of the role of impossibilities in rational investigation.

[4]cf. Landman (1986), who attempts to effect this by different means, and Lappin and Pollard (2000) for criticism.

tion), while the "standard" connectives will be notated $\supset$ and $\neg$.

2 Relevance logic

2.1 Relevant implication

The central concept of relevance logic is a notion of implication which is not reducible to a truth function. It encodes the principle that the premises in a deduction must combine in a non-trivial way to produce the conclusion. The natural deduction rules for relevant implication are given in (6). In $\rightarrow$ elimination (modus ponendo ponens), the conclusion "follows from" precisely the combination X,Y of the premise contexts X and Y from which the major and minor premises follow. Conversely, $\rightarrow$ introduction involves deducting the premise context consisting of the single formula A (equivalent to *non-trivially* discharging an assumption).

(6)
1. ($\rightarrow$ **Elim**): If $X \vdash (A \rightarrow B)$ and $Y \vdash A$, then $X,Y \vdash B$
2. ($\rightarrow$ **Intro**): If $X,A \vdash B$, then $X \vdash (A \rightarrow B)$

This non-triviality condition prevents implication from collapsing into material implication. In terms of structural rules, it amounts to ruling out Weakening. Extra premises which have no role in the deduction cannot be added at will, as in classical or intuitionistic logic where it leads to "irrelevant" deductions such as $A \supset (B \supset A)$, or $\neg A \supset (A \supset B)$.[5]

The implicational fragment of relevance logic rejects Weakening, but retains the other structural rules of implicational intuitionistic logic.[6] Other rules have to be added for extensional connectives, for negation and for modal operators, which will be discussed shortly.

The resulting notion of implication[7] is not truth-functional - it provably depends only on content sharing between antecedent and consequent, not on their truth or falsity - and does not validate the "paradoxes" of material or of strict implication. It is intensional also in that it involves considering more than one premise context; this will be directly reflected in the semantics by evaluation over several worlds using an accessibility relation.

[5] In terms of the correspondence between proof types and lambda terms, a relevant proof must correspond to a lambda term where the variable bound by each lambda actually occurs free in its scope. Weakening corresponds to vacuous abstraction.

[6] Associativity, Permutation and Contraction.

[7] That it is a genuine notion of implication is shown by the existence of a deduction theorem (see Dunn (1986)). The deduction equivalence holds for relevant implication as long as the combination of premises (notated by the comma) is understood in the sense described above, not as conjunction.

2.2 Other connectives

For reasons of space, the logic will be summarized here by a list of axioms[8]. The implicational axioms (7) correspond to structural rules. Connectives other than implication (8-10) are discussed in the following paragraphs. The modal axioms in (11) are simply the familiar axioms for S4.

(7)
1. **Identity**: $A \to A$
2. **Prefixing**: $(A \to B) \to ((C \to A) \to (C \to B))$
3. **Permutation**: $(A \to (B \to C)) \to (B \to (A \to C))$
4. **Contraction**: $(A \to (A \to B)) \to (A \to B)$

(8)
1. **Conjunction elim.**: $A \wedge B \to A$, $A \wedge B \to B$
2. **Conjunction intro.**: $((A \to B) \wedge (A \to C)) \to (A \to B \wedge C)$ (*not* $A \to (B \to A \wedge B)$), which leads to Weakening)
3. **Disjunction intro.**: $A \to A \vee B$, $B \to A \vee B$
4. **Disjunction elim.**: $((A \to C) \wedge (B \to C)) \to (A \vee B \to C)$
5. **Distribution**: $A \wedge (B \vee C) \to (A \wedge B) \vee C$

(9)
1. **Logical truth**: $\mathbf{t} \to (A \to A)$
2. **Trivial truth**: $A \to \top$

(10)
1. **Reductio**: $(A \to {\sim}A) \to {\sim}A$
2. **Contraposition**: $(A \to {\sim}B) \to (B \to {\sim}A)$
3. **Double negation**: ${\sim}{\sim}A \to A$

(11)
1. **$\wedge$-Distribution**: $\Box A \wedge \Box B \to \Box(A \wedge B)$
2. **$\to$-Distribution**: $\Box(A \to B) \to (\Box A \to \Box B)$
3. **Reflexivity**: $\Box A \to A$
4. **Transitivity**: $\Box A \to \Box\Box A$

The connectives $\wedge$ and $\vee$ are "extensional" in the sense that they presuppose a single premise context for both juncts (corresponding to being evaluated in a single world).

This distinction between intensional and extensional extends to nullary connectives (constants). In relevance logic there is an important distinction between the logical truth constant $\mathbf{t}$ and the trivial truth constant $\top$ (top).

[8] With the rules of modus ponens, adjunction and necessitation.

The first represents the conjunction of theorems, and corresponds to the empty premise structure (henceforth 0). $\top$, by contrast, is the trivial truth constant which holds everywhere (follows from all premises)[9]. In standard logic these coincide, but in the absence of Weakening they are crucially distinct[10]. Their negations are written **f** and $\perp$ respectively.

Negation is strict De Morgan negation, characterized by double negation and contraposition, but without supporting either of the complementation laws (Law of the Excluded Middle, LEM, and the Law of Contradiction, which corresponds to ECQ). That is, the logic as set up does not support either of the laws in (12), only the corresponding weaker statements in (13). Thus, LEM holds only at 0; it is a logical truth, but $A \vee \sim A$ is not decided by all situations. (This is as in situation semantics.) Moreover, contradictions do not entail any arbitrary proposition but only **f**; the only thing that follows from them is that somebody is being "illogical".

(12) 1. $\top \vdash A \vee \neg A$ - (Strong) Law of the Excluded Middle, LEM
2. $A \wedge \neg A \vdash \perp$ - Ex Contradictione Quodlibet, ECQ

(13) 1. $\mathbf{t} \vdash A \vee \neg A$
2. $A \wedge \neg A \vdash \mathbf{f}$

3 Frame semantics

The standard semantics for relevance logic is a frame semantics similar to the Kripke frames for modal and intuitionistic logic (of which however the present frames are a proper generalization).

A relevance frame comprises a point set $\langle P, \sqsubseteq \rangle$ ($\sqsubseteq$ being a partial order), together with a *ternary* accessibility relation R on P, and a distinguished point $0 \in P$ (which corresponds to the "empty premise structure" 0, and supports logical truths).

A relevance model for a language $\mathcal{L}$ is a relevance frame equipped with a supports relation $\models$ on $P \times Var(\mathcal{L})$, the variables of the propositional language $\mathcal{L}$. Validity in a model is not computed as truth in all worlds, but as truth at 0.

The point set is understood as the set of *non-trivial prime theories* of $\mathcal{L}$, ordered by inclusion. Because of the way the logic has been set up, such

[9] The disjunction of all propositions, hence the top element of the propositional lattice.

[10] Weakening is used in the proof that they are equivalent: $0 \vdash t$ (by definition of t); but then $0, \top \vdash t$ (by Weakening); therefore $\top \vdash t$. In the other direction, $t \vdash \top$ immediately. (Restall (2000))

theories do not have to be complete or consistent. In logics with Weakening and ECQ, they collapse into consistent theories, and with (strong) LEM into complete ones - the situation in intuitionistic frames and normal modal frames respectively[11]. In relevance logic, which rejects Weakening and ECQ, this is not the case. (Although points may be incomplete and inconsistent, they will often be referred to informally as "worlds".)

Apart from the "extensional" connectives $\wedge$ and $\vee$ (and $\top$ and $\perp$), the evaluation clauses for all connectives involve more than one world, and hence invoke some accessibility relation. There are actually several of these (though they are all expressible in terms of the ternary relation R, the key to the whole system).

The ternary relation Rxyz represents combination of worlds (corresponding to the combination of premise structures in the rules for intensional connectives). If the first argument x supports the implication $A \rightarrow B$, and y supports A, then z supports B.

How does this help? In frames for normal modal logic, an implicational formula is evaluated truth-functionally in a single world. Because of this the paradoxes of material implication surface in a modified form, leading to the problem of hyperintensionality. Logical truths hold everywhere, logical falsities nowhere, and logically equivalent expressions at the same indices. Although the evaluation clause for intuitionistic implication is different, the antecedent and consequent are still evaluated relative to the same worlds. In relevance frames, by contrast, implication is evaluated using separate worlds for the antecedent and consequent. Thus $A \rightarrow A$ is not guaranteed to hold in all worlds. Moreover the bi-implication $A \leftrightarrow B$ does not involve A and B picking out the same set of worlds. In general, there is no entailment from a contradiction to an arbitrary formula, nor from an arbitrary formula to a logical truth. Thus the paradoxes of strict implication and the identification of mutually entailing formulae are avoided.

Rxyz can be written $y \sqsubseteq_x z$, emphasizing the idea of x as a connection between y and z. In the case where $x = 0$, the subscript is omitted and we have $y \sqsubseteq z$, the relation of informational extension.

R is constrained to interact correctly with $\sqsubseteq$, by (14). Note that x and y are downward closed, while z is upward closed. The evaluation condition for implication, using R, is (15).

[11]The non-triviality condition means that every theory contains $\top$ and no theory contains $\perp$. With Weakening, $\mathbf{t} = \top$ and therefore every theory contains $\mathbf{t}$. With ECQ, any inconsistent theory supports $\perp$, and so cannot be non-trivial. Similarly, with strong LEM, any theory containing $\top$ (i.e. every non-trivial theory) must contain either A or $\sim$A.

(14) If Rxyz then for every $x' \sqsubseteq x$, $y' \sqsubseteq y$, $z' \sqsupseteq z$, $Rx'y'z'$

(15) $x \models A \rightarrow B$ iff, if Rxyz and $y \models A$ then $z \models B$

Other constraints on R correspond to structural rules. Those corresponding to the structural rules required for relevance logic are given in the appendix, as they are not transparent at first sight. The rule corresponding to Weakening is simply: R00x (or $0 \sqsubseteq x$) for all x. This makes the point set into a rooted graph (with 0 as root), which corresponds to the situation in intuitionistic logic. 0 supports only logic, while points above 0 add information monotonically.

The other important relations between worlds are the modal accessibility relation used for necessity, and the compatibility relation used for negation. These are both binary. As already mentioned, they can be defined in terms of R, though it will be convenient first to discuss them independently. Their interaction with $\sqsubseteq$ is constrained by (16) and (17). In the first case, x is downward closed and z is upward closed, while in the second case both arguments are downward closed. The evaluation clauses for the modalities which use S and C are given in (18) and (19).

(16) If Sxz then for every $x' \sqsubseteq x$, $z' \sqsupseteq z$, $Sx'z'$.

(17) If Cxy then for every $x' \sqsubseteq x$, $y' \sqsubseteq y$, $Cx'y'$.

(18) $x \models []A$ iff for every z such that Sxz, $z \models A$

(19) $x \models {\sim}A$ iff for every y such that Cxy, $y \not\models A$

The accessibility relation S superimposes a system of modal necessity on relevance logic. S can be constrained further in the ways familiar from normal modal systems. The modal system usually added to relevance logic is S4, where S is reflexive and transitive. Modality will not be discussed in this paper, though there is more detail in Gregory (2001).

The compatibility relation governs negation. Besides the basic constraint in (17), other constraints are added to obtain particular kinds of negation. Those required for De Morgan negation are again given in the appendix[12]. It is assumed that C is symmetric, and also directed (each world has some compatible world). The constraint corresponding to ECQ is that it is also

[12] They amount to the requirement that for each world there is a maximal consistent world (with respect to $\sqsubseteq$), though the formulation is again not immediately transparent. (This corresponds to the "Routley star" operation (Routley and Routley (1972)).) The approach adopted here is due to Dunn (1993).

reflexive. For the De Morgan negation desired here, this condition is of course rejected. Every world must be compatible with some world, but it is not required to be compatible with itself.

4 Channels and attitudes

4.1 Channel theory

The extension of situation semantics to channel theory (Barwise (1993), Israel and Perry (1990)) was intended to give a more satisfactory model for implicational constraints on situations, the basis of the situation-theoretic theory of meaning. A channel bears the same relation to a constraint as a situation does to an infon, namely the $\models$ relation. The relation between the two is given by (20). Thus a relation between situations is set up which mediates a relation between infons (Barwise (1993)).

(20) c $\models \sigma \Rightarrow \tau$ iff, if s $\models \sigma$ then t $\models \tau$

Most constraints are not logical entailments, but hold only given certain background conditions. This leads to the suggestion that they should themselves be situated, perhaps in some kind of ambient situation. However, while infons are generally understood as being upward persistent (with respect to situation inclusion), constraints behave differently; they are *downward* persistent both with respect to the "ambient situation" and to the situation supporting the antecedent (s in the example), though not with respect to t. This behaviour is characteristic of accessibility relations, particularly the ternary relation R in relevance frames. Thus it seems possible to redefine the "channel" c in terms of an accessibility relation. A connection between situations is set up which mediates the relation between infons in a constraint. However, this connection is itself a "part of the world", and may be reasonably be considered a situation in its own right.

As has been observed[13], frames for substructural logics provide a logical interpretation of this conception of a channel. If intuitionistic logic were used, the framework would have essentially the same properties as situation semantics, including its inability to deal with hyperintensionality. The rejection of Weakening leads immediately to the recognition of impossible situations. Dropping other structural rules would have further consequences[14].

[13] cf. Mares (1997) and references therein.

[14] Work in this tradition (Restall (1996), Mares (1997)) has generally advocated very weak substructural logics; in particular, Restall advocates the rejection of Contraction,

4.2 Attitude problems

Channels provide a convenient representation of connections between the situations supporting an embedded proposition and the belief state of an agent. I take the verbs "believe" and "doubt" as examples of positive and negative attitudes, whose interrelated behaviour it is desired to capture. It is well known that positive attitudes show the basic entailment patterns characteristic of necessity operators, while negative attitudes (unsurprisingly) have the basic properties of negation operators. These basic patterns are exemplified below (23ff). At the same time, the further conditions on their behaviour as necessity or negation operators varies between different classes of attitide predicate. The following paragraphs are intended to indicate how the basic entailment patterns can be related to channels, while the "tuning" of frames for different attitudes is deferred to further work.

I make two basic assumptions. The first is that for each agent x there is a world $x \in P$ which supports the set of propositions believed by x. Recall that worlds are theories. They are not in general complete and consistent, but they are required to be closed under modus ponens[15], so that if an agent believes A and $A \to B$ (but not otherwise), he may be reported as believing B. The second is that "believe" and "doubt" are interpreted as operators (written B_x, D_x), parametrized according to agent, on an embedded proposition. The whole attitude report is evaluated using a parametrized accessibility relation defined in (21), using the semantic clauses in (22).

(21)
1. $S_x yz$ iff $Ryxz$ [cf. Syz iff $Ry0z$]
2. $C_x yz$ iff $Ryzx$ [cf. Cyz iff $Ryz0$]

(22)
1. $y \models B_x A$ iff for all z such that $S_x yz$, $z \models A$
2. $y \models D_x A$ iff for all z such that $C_x yz$, $z \not\models A$

The intuition which this notion is trying to capture is that modalities such as belief and doubt are generalizations of necessity and negation, into which they collapse when $x = 0$ (as indicated in the square brackets in (21)). The relationship between their respective accessibility relations and the ternary relation R, and hence with implication, corresponds to the idea that believed statements are those implied by a given belief state, while

as pointed out by a reviewer. In the present application, the presence of Contraction is important to ensure that belief states are closed under modus ponens. Other consequences of this choice, both desirable and undesirable, are discussed in Gregory (2001).

[15] As noted above, this follows from Contraction.

doubted statements are those which imply something incompatible with the particular belief state. The evaluation of an attitude statement involves a channel between the belief state of the agent and the worlds where the embedded proposition is evaluated.

Thus $B_x A$ corresponds to an implication $X \rightarrow A$, where the antecedent $X = \bigwedge\{P: x \models P\}$ - the conjunction of all propositions supported by a given world x associated with the agent x. $B_x A$ is true at y if A is true at every z such that Ryxz. Conversely $D_x A$ corresponds to an implication $A \rightarrow \sim X$, where X this time is in the consequent. $D_x A$ is true at y if A is not true at any z such that Ryzx (otherwise A would be true at x and would be believed not doubted by x). Essentially the meaning postulate linking "believe" and "doubt" (and other such pairs) should consist in fixing x for any given agent.

Much of the behaviour expected from the two verbs then falls out from the proposed semantics. Some of the more important semantic patterns are set out in (23ff). "Positive" attitudes such as belief share important properties with necessity operators, such as conjunction distribution (24)[16]. These properties are not shared by "negative" attitudes such as "doubt", which by contrast have properties characteristic of negation operators: contraposition (28) and the de-Morgan laws (27)[17]. However, these operators do not satisfy ECQ (29) or irrelevance (30).

(23) Mary believes A and Mary believes B $\vdash$ Mary believes A and B
cf. $[]A \wedge []B \vdash [](A \wedge B)$

(24) Mary believes A and B $\vdash$ Mary believes A
cf. $[](A \wedge B) \vdash []A$

(25) Mary doubts A and B $\nvdash$ Mary doubts A
cf. $\sim(A \wedge B) \nvdash \sim A$

(26) Mary believes that if A then B $\vdash$ If Mary believes A then Mary believes B
cf. $[](A \rightarrow B) \vdash []A \rightarrow []B$

(27) Mary doubts A and B $\vdash$ Mary doubts A or Mary doubts B
cf. $\sim(A \wedge B) \vdash \sim A \vee \sim B$

[16] $[]A \wedge []B \vdash [](A \wedge B)$ and distribution over implication (26), which is a property of necessity operators in normal modal systems. The other basic property of such operators, $\top \vdash []\top$, simply means here that every agent must believe some proposition - it seems reasonable to require this of any agent.

[17] This is the most "difficult" of the De Morgan laws (the one that is not immediately true of virtually all forms of negation).

(28) Mary doubts B, Mary believes A implies B $\vdash$ Mary doubts A
cf. $\sim$B, A $\rightarrow$ B $\vdash$ $\sim$A

(29) Mary believes A and not A $\nvdash$ Mary believes B
cf. $[](A \wedge \sim A) \nvdash []B$

(30) Mary believes B $\nvdash$ Mary believes if A then B
cf. $[]B \nvdash [](A \rightarrow B)$

These basic patterns of behaviour rely only on properties of implication where either the antecedent or the consequent (for necessity and negation operators respectively) is kept constant (Restall (2000)).

What does all this mean intuitively? Attitude reports are modelled as constraints on connections between the belief states of cognitive agents and situations where the embedded propositions are evaluated. These constraints are formulated as (relevant) implications. Relevant implication is an intensional operator requiring an accessibility relation for its evaluation, and this is used to give a treatment capturing important semantic affinities between attitude predicates and necessity and negation operators. The correspondence between positive attitude predicates and necessity operators ties in with standard doxastic modal treatments, while at the same time avoiding the paradoxes of strict implication inherent in standard modal systems. The interpretation of negative attitude predicates by means of a compatibility relation is strong enough to capture the similarity of such predicates with negation, while at the same time avoiding the problem of explosion (ECQ).

5 Conclusion and further work

This research attempts to provide a semantics which avoids the problem of hyperintensionality, which has proved intractable on standard approaches. It draws on two traditions of work whose convergence has been the subject of some interest in the literature, though not in connection with the same problem. It adopts the notions of intensional implication and paraconsistent negation offered by relevance logic, and seeks to interpret these in terms of channels. In work in progress, this is being worked out in terms of the more elaborate channel theory of Barwise and Seligman (1997), based on mappings between infon algebras. Other priorities for further work include the extension of the logic to higher order systems, and the treatment of identity of objects in intensional contexts.

6 Appendix

The following extra conditions on accessibility relations are required for relevance logic (as opposed to the "basic" substructural logic discussed in Restall (1995) and Mares (1997)). Only a few of them are mentioned in the text. For ease of comparison, I append the rejected conditions which, if added, would give standard logics.[18]

(31) Conditions on R (for implication)

1. Notational convention: definition of R^2
   (a) $R^2(ab)cd := \exists e(Rabe \wedge Recd)$
   (b) $R^2a(bc)d := \exists e(Raed \wedge Rbce)$
2. (a) $R^2(ab)cd \rightarrow R^2a(bc)d$ $[a\circ(b\circ c) \sqsubseteq (a\circ b)\circ c]$ (Associativity)
   (b) $Rabc \rightarrow R^2(ab)bc$ $[(a\circ b)\circ a \sqsubseteq a\circ b]$ (Contraction)
   (c) for **R**: $Rabc \rightarrow Rbac$ $[a\circ b \sqsubseteq b\circ a]$ (Permutation)
3. For intuitionistic logic: $R00a$ $[0 \sqsubseteq a]$ (Weakening)

(32) Conditions on S (for necessity)

1. Notational conventions:
   (a) $R(xS)yz := \exists w(Sxw \wedge Rwyz)$
   (b) $Rx(yS)z := \exists w(Syw \wedge Rxwz)$
   (c) $Rxy(Sz) := \exists w(Swz \wedge Rxyw)$
2. Conditions for S4 necessity
   (a) Saa (modal axiom **T**)
   (b) $Sab \wedge Sbc \rightarrow Sac$ (axiom **4**)
   (c) $Rcd(Sa) \rightarrow R(cS)(dS)b$ (axiom **K**)
   (d) $S0a \rightarrow a = 0$ (Necessitation)
3. For S5 modal system: $Sab \rightarrow Sba$ (axiom **B**)

(33) Conditions on C (for negation)

1. Extra conditions for De Morgan negation

[18] The conditions on R (31) are more transparent if they are thought of in terms of a fusion operation $\circ$ on worlds, which is like R except of course that R is not a function. (Dunn (1986), Gabbay and Olivetti (2000)). The modal component (32) is based on Read (1988) and Restall (1994), while negation (33) is based on Restall (1995, 2000).

(a) $\forall a \exists b(Cab \wedge \forall c(Cbc \rightarrow c \sqsubseteq b))$ (Double negation)

(b) $\exists a(Rbcd \wedge Cad) \leftrightarrow \exists e(Rbde \wedge Cce)$ (Contraposition)

2. For ECQ: Caa
3. For LEM: $Cab \rightarrow b \sqsubseteq a$

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