The Dynamics of Adaptive Proofs
A Case for Open Worlds

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Adaptive Logic: Some Background

Adaptive logics are logics for defeasible inference that are based on

1. A formula-preferential semantics, and
2. a dynamic proof-theory (nonmonotonic inference procedure).

Adaptive logic and modal logic

1. Adaptive consequence relations can be reformulated in a Kripke-style semantics.
2. This type of reformulation does not properly account for the dynamics of the proof-theory of adaptive logic.

A Question

Is the dynamics of the dynamic proof-theory something new, or is it a standard type of inferential dynamics?

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The Argument in Brief

Getting Rid of Logical and Deductive Omniscience

1. Dilute the space of possibilities.
2. Add a syntactic *awareness* function.
3. Use a model for *fragmented* beliefs.

A Widely Spread View

What can be done with non-logical worlds can equally well be done with an awareness function (and fragmented beliefs).

Challenging Orthodoxy!

Models for "realistic" defeasible inference do require non-logical worlds because a *revision without new information / tentative application of a default rule* presupposes a notion of explicit belief that does not imply implicit belief.

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An Inconsistency Adaptive Logic

Language $\mathcal{L}_\mathbf{P}$

$$\phi := p \mid \sim \phi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \phi_1 \rightarrow \phi_2$$

The Paraconsistent Logic CluNs

Full positive classical propositional logic, excluded middle, and all *De Morgan* equivalences.

The Adaptive Logic

- The logic **CluNs**,
- the set of abnormalities $\Omega = \{\phi \wedge \sim \phi \mid \phi \in \text{Prop}\}$, and
- the permission to derive ϕ conditionally whenever $\phi \vee \psi$ can be derived unconditionally, and $\psi \in \Omega$ has not yet been derived (as a disjunct of a minimal disjunction of members of Ω).

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(2)	$\sim p \vee r$	Prem.	$\emptyset$	
(3)	$\sim q \vee (p \rightarrow \sim q)$	Prem.	$\emptyset$	
(4)	$\sim p \vee \sim q$	Prem.	$\emptyset$	
(5)	$q \rightarrow \sim p$	Prem.	$\emptyset$	
(6)	p	Simpl. (1)	$\emptyset$	
(7)	q	Simpl. (1)	$\emptyset$	
(8)	r	(2), (6)	$\{p\}$	$\surd^{11}$
(9)	$p \rightarrow \sim q$	(3), (7)	$\{q\}$	$\surd^{11}$
(10)	$\sim q$	MP (6), (9)	$\{q\}$	$\surd^{11}$
(11)	$(p \wedge \sim p) \vee (q \wedge \sim q)$	PbC. (4), (6), (7)	$\emptyset$	
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Preference Models based on CluNs

$\mathfrak{M} = (S, \preceq, \Omega, Ab_{\mathfrak{M}}, \|\cdot\|_{\mathfrak{M}})$ WITH

$(S, \preceq)$ is a preference frame, $\Omega = \{\phi \wedge \sim \phi \mid \phi \in \mathsf{Prop}\}$, $Ab_{\mathfrak{M}}$ a map: $S \mapsto \mathcal{P}(\Omega)$, and $\|\cdot\|_{\mathfrak{M}}$ a **CluNs**-valuation-function such that:

1. $Ab_{\mathfrak{M}}(s) = \{\omega \in \Omega : s \in \|\omega\|_{\mathfrak{M}}\}$
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ϕ IS AN MA-CONSEQUENCE OF Γ IN $\mathfrak{M}$ IFF

$\|\Gamma\|^m_{\mathfrak{M}} \subseteq \|\phi\|_{\mathfrak{M}}$, with:

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This is the consequence-relation we've used!

Adaptive Consequence

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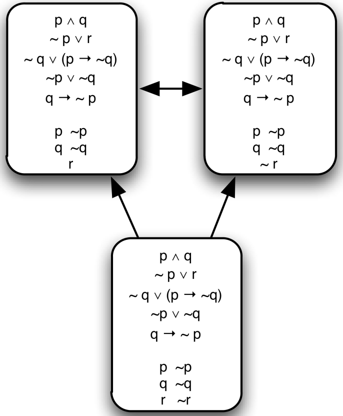
No trace of the dynamics!



How to account for proof-dynamics?

Deductive inferences yield insight in premises

- Deducing r from $p \wedge q$ and $(p \wedge q) \to r$ requires one to recognise the immediate sub-formulae of $(p \wedge q) \to r$, but not the sub-formulae of $p \wedge q$.
- We use a dedicated block-language $\mathcal{L}^{\mathrm{bl}}_{\mathbf{CLuNs}}$ to reflect this; e.g. $[\![(p \wedge q) \to r]\!]$ versus $[\![(p \wedge q)]\!] \to [\![r]\!]$.

(Open) Worlds and Block-Formulae

- Arbitrary worlds where there need not be a logical connection between arbitrary ϕ and ψ.
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Extended Adaptive Preference Model

$\mathfrak{M} = (O, S, \preceq, \Omega, Ab_{\mathfrak{M}}, \|\cdot\|_{\mathfrak{M}}, \mathsf{A}, \mathsf{R}, R)$ WITH

$(O \cup S, \preceq, \Omega, Ab_{\mathfrak{M}}, \|\cdot\|_{\mathfrak{M}})$ an adaptive preference model such that:

1. The restriction of $\|\cdot\|_{\mathfrak{M}}$ to S is a **CluNs**-valuation, and
2. the restriction of $\|\cdot\|_{\mathfrak{M}}$ to O is arbitrary;

and with $\mathsf{A} : S \mapsto \mathcal{P}(\mathcal{L}^{\mathrm{bl}}_{\mathbf{CLuNs}})$ the *access-set* function, $\mathsf{R} : S \mapsto \mathcal{P}(\mathcal{R})$ the *rule-set* function, and $R \subseteq S \times (O \cup S)$ a binary relation such that:

3. A is a constant function,
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Main notions of belief

- ϕ is implicitly believed iff it is true in all states in $R[s] = \{t \in S : Rst\}$.
- ϕ is explicitly believed iff it is true in all minimal/reliable states in $R[s]$.

Further refinements

- ϕ is an explicitly derived indefeasible belief iff $\phi \in \mathrm{A}$.
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(4)	$q \vee (p \wedge \sim p)$	PbC. , 1, 3	$[\![[\![q]\!] \vee [\![([\![p]\!] \wedge [\![\sim p]\!])]\!]]\!]$
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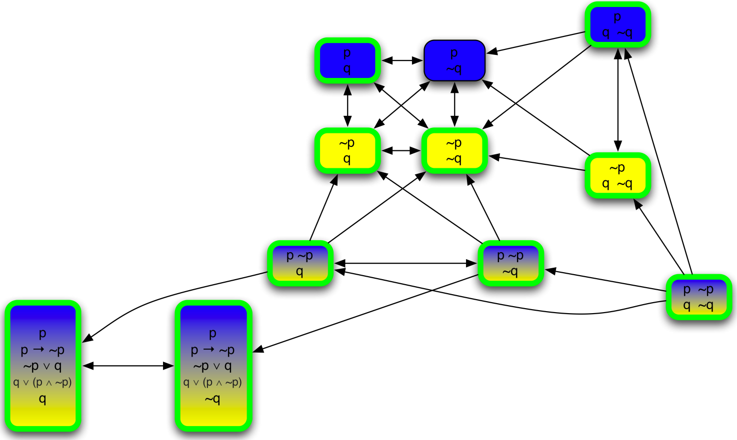

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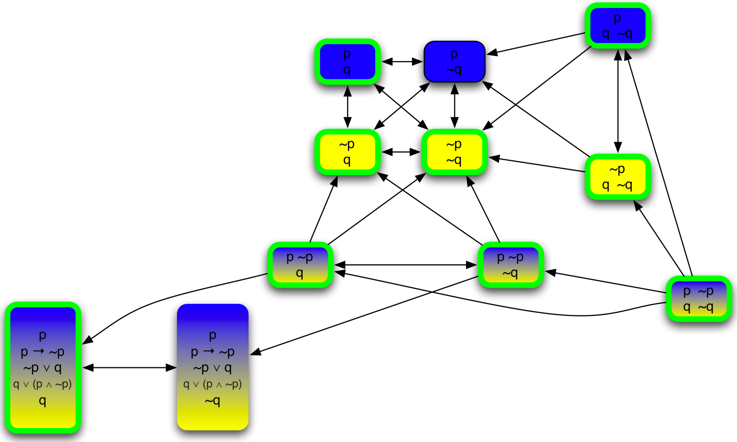

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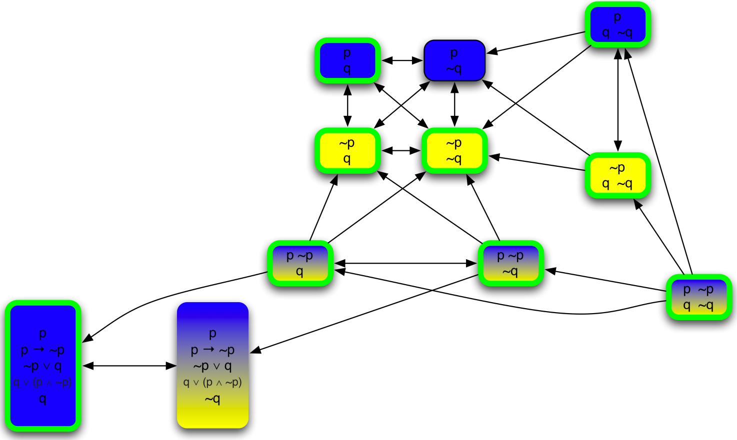

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