

# In Processes, We Believe!

## Marrying Process Algebra and Epistemic Logic

Francien Dechesne and Mohammad Mousavi

TbILLC: Special Session on Logic, Information and Agency

- 1 Introduction: Operational vs Epistemic

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- 2 Bridging the Gap: Specification Framework

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- 5 Conclusions

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(dynamic) epistemic logic



process algebra, formal semantics, verification of distributed processes, security

# The prototypical example: Dining Cryptographers



# Operational vs. Epistemic

## The Gap

approach\spec	<b>Protocol</b>	<b>Goal</b>
<b>Operational</b>	Intuitive	Non-trivial; Difficult with Knowledge Properties
<b>Epistemic</b>	Laborious	Intuitive; combination of epistemic and temporal constructs

## Simple PA: Syntax

$$\begin{aligned} a &::= a[?,!](\vec{k}) \\ p, q &::= a \mid a; p \mid p + q \mid p \parallel q \end{aligned}$$

- 1 receive:  $a?(\vec{k})$ , send:  $a!(\vec{k})$ ,  
individual actions or synchronizations:  $a(\vec{k})$ ;

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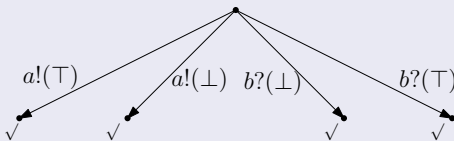
Semantics:

labelled transition system generated by syntactic rules  
(SOS: Structural Operational Semantics)



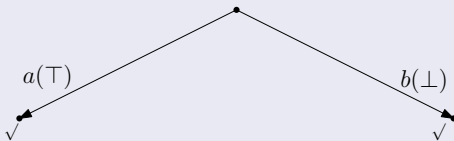
## Simple PA: Intuitive Semantics

$$\sum_{i \in \{\top, \perp\}} (a!(i) + b?(i))$$



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$$\sum_{i \in \{\top, \perp\}} (a!(i) + b?(i)) \parallel (a?( \top ) + b!( \perp ))$$



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Structural Operational Semantics: derived formally from PA-term syntax through set of rules.

(Reference: "Operational and Epistemic approaches to protocol analysis: Bridging the Gap", LPAR 2007. Cf.

<http://www.win.tue.nl/~mousavi/pai.htm>)

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- On these ALTSSs we can check properties in our epistemic temporal language.
- (Spoiler) these ALTSSs are like Interpreted Systems

## Decorated actions:

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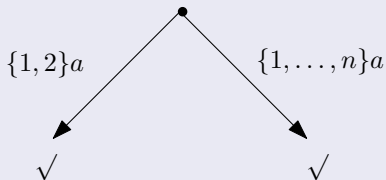
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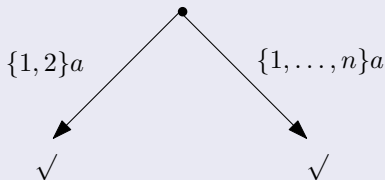
( $\Rightarrow \rho$  is now part of the protocol specification!)

## Simple PA with views: example

$(\{2\}b! + (\{1, \dots, n\})a) \parallel (\{1\})b?$



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Suppose  $\rho(b) = c$ . Then in the left branch,  $\{3, \dots, n\}$  see  $c$ .



## Formal semantics: operational part (Summarized)

$$(a) \frac{}{(d, \pi) \xRightarrow{d} \sqrt{\pi \frown d}}$$

$$(s0) \frac{(x_0, \pi) \xRightarrow{d} (y_0, \pi')}{(x_0; x_1, \pi) \xRightarrow{d} (y_0; x_1, \pi')}$$

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$$(n0) \frac{(x_0, \pi) \xRightarrow{d} (y_0, \pi')}{(x_0 + x_1, \pi) \xRightarrow{d} (y_0, \pi')}$$

$$(p0) \frac{(x_0, \pi) \xRightarrow{d} (y_0, \pi')}{(x_0 \parallel x_1, \pi) \xRightarrow{d} (y_0 \parallel x_1, \pi')}$$

$$(p4) \frac{(x_0, \pi) \xRightarrow{(\mathcal{J})?a} (y_0, \pi') \quad (x_1, \pi) \xRightarrow{(\mathcal{J}')!a} (y_1, \pi'')}{(x_0 \parallel x_1, \pi) \xRightarrow{(\mathcal{J} \cup \mathcal{J}')a} (y_0 \parallel y_1, \pi \frown (\mathcal{J} \cup \mathcal{J}')a)}$$

## Formal semantics: epistemic part (Summarized)

$$\frac{}{\pi \stackrel{i}{=} \pi} \quad \frac{i \in J}{\pi \frown J(a) \stackrel{i}{=} \pi \frown J(a)} \quad \frac{\pi \stackrel{i}{=} \pi' \quad i \notin J \quad \rho(a) = \rho(b)}{\pi \frown J(a) \stackrel{i}{=} \pi' \frown J(b)}$$
$$\frac{\pi \cdot \overset{i}{\dots} \pi' \quad i \notin J \quad \rho(a) = \tau}{\pi \frown J(a) \stackrel{i}{=} \pi} \quad \frac{\pi \cdot \overset{i}{\dots} \pi' \quad i \notin J \quad \rho(a) = \tau}{\pi \stackrel{i}{=} \pi' \frown J(a)}$$

Seminal book:

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Interpreted Systems as semantics for epistemic temporal logic.

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Interpreted Systems as semantics for epistemic temporal logic.

Transition systems with rich states:

- global state is  $n$ -tuple of local states
- indistinguishability relations between global states generated on the basis of local state for each agent.

# Interpreted Systems: framework

- Agents  $\mathcal{I} = \{1, \dots, n\}$
- **Local** states  $L_i$ , **global** states:  $L = \prod_{i=1}^n L_i$
- Run  $r$ : a sequence of global states
- Protocol  $R$ : set of runs
- Valuation function  $\nu : L \rightarrow \Phi$
- Indistinguishability  $\vec{l} \stackrel{i}{\approx} \vec{l}'$  iff  $l_i = l'_i$

Interpreted system:  $(R, \nu)$

Our focus: protocol component (not valuation)

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Work in progress!

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We want to

- Link the process algebra specs to ISs
- Allow to transform results from one to the other:
  - PA as a syntax for generating ISs
  - exploited analysis tools available for IS
  - characterize semantic properties of classes of PA specs



Trace: sequence of decorated actions.

$\llbracket - \rrbracket_{aux}$ : *CCSi* process  $\mapsto$  set of traces.

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$$P \parallel_{tr} \emptyset \doteq \emptyset \parallel_{tr} P \doteq P$$

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$$\{ \langle \rangle \} \uplus P \parallel_{tr} Q \doteq P \parallel_{tr} \{ \langle \rangle \} \uplus Q \doteq (P \parallel_{tr} Q)$$

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$$\begin{aligned}
 \{(\mathcal{J})\alpha \frown tr\} \uplus P \parallel_{tr} \{(\mathcal{J}')\alpha' \frown tr'\} \uplus Q &\doteq \\
 (\mathcal{J})\alpha \frown (\{tr\} \uplus P \parallel_{tr} \{(\mathcal{J}')\alpha' \frown tr'\} \uplus Q) \cup & \\
 (\mathcal{J}')\alpha' \frown (\{(\mathcal{J})\alpha \frown tr'\} \uplus \{tr\} \uplus P \parallel_{tr} Q) \cup & \\
 \bigcup \{(\mathcal{J} \cup \mathcal{J}')a \frown (\{tr\} \uplus P \parallel_{tr} \{tr'\} \cup Q) \mid (\mathcal{J}')a \frown tr' \in Q & \\
 (\alpha = a? \wedge \alpha' = a!) \vee (\alpha = a! \wedge \alpha' = a?)\} &
 \end{aligned}$$

$$\llbracket p \rrbracket_{tr} = \{tr \mid tr \in \llbracket p \rrbracket_{aux} \wedge \mathit{closed}(tr)\}$$

(‘ $\mathit{closed}(tr)$ ’:  $tr$  contains no send or receive actions)



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Consider finite initialized and prefix-closed (fipc) ISs.

Characterizing the class of ISs generated:

- If  $|A| = 1$ : for each fipc interpreted system  $R$ , there is a process algebraic description  $p$  such that  $\llbracket p \rrbracket_{tr} = R$ .
- For  $|A| \geq 2$  and at least 2 agents: there exist fipc ISs that cannot be generated by any process algebraic specification.

Cf. embedding of DEL in ISs (van Benthem et al):

- Perfect Recall: by construction
- Synchronicity: depends on properties  $\rho$
- Uniform No Miracles: ??

Future work: relate different  $\rho$ -types to structural properties of epistemic relations in ISs.

(E.g. with additional parameters distinguishing more groups of agents.)

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Thank You!!!