Frames in Category Theory

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Abstract

This talk gives a representation of frames in category theory. All of the constituents of a frame — from the underlying universe of objects with thei attributes to the frame itself including constraints on its values — can be expressed in categories. Categories are employed in mathematics to regard structures from a very abstract point of view (MacLaine, 1998). Thus, they can be used to examine the structural properties of frames. In short, categories are given by a class of objects and by the morphisms between all pairs of objects. If a morphism does not uniquely determine its pair of objects we speak of a pre-category.

To represent frames in category theory we employ several interdependent kinds of categories. As a basis, the underlying universe can be described in a pre-category $\mathcal{U}$, with the objects of the universe the objects of the category and the attributes, connecting objects and their values (which, in turn, are objects of the universe), its morphisms. Types can be regarded as a subclass of attributes, relating objects to themselves.

A formal frame, as defined in (Petersen, 2007), is a generalization of a feature structure. It can be represented by a category $\mathcal{G}$ for its frame structure (that is a category of a graph). Here, the nodes are the objects of the category and the paths are the morphisms. On the other hand, the content of a frame is represented by a pre-category, $\mathcal{F}$. Here, objects are sets of things (that is, sets of the objects in $\mathcal{U}$). As in $\mathcal{U}$, morphisms are given by attributes and types, where types denote the identity morphisms. As the same attribute and the same type can occur in a frame more than once, $\mathcal{F}$ is just a pre-category, as the uniqueness requirement for morphisms fails. This makes it straightforward to define a functor from $\mathcal{U}$ to $\mathcal{F}$ that respects attributes and types and thus gives a well-formedness constraint for frame categories. On the other hand, $\mathcal{F}$ is closely connected to $\mathcal{G}$, as types can be interpreted as labels for paths of length zero, again, being just special cases of attributes. Thus for each frame we have a functor connecting its frame category to its graph category to express adequacy constraints. This functor allows to transfer other constraints, as a weak form of uniqueness constraint that is defined on $\mathcal{F}$: Attributes are functional, so if a morphism goes from A to B and from A to B', we can conclude that $B = B'$. This constraint cannot be defined directly on the category of the graph; it is a proper category and thus its morphisms are uniquely assigned.

As in the examples already given, functors prove a useful tool to define constraints on frames. Apart from well-formedness-constraints, there are constraints on values of frames, as Barsalou (1992) proposes them, e.g. monotonicity constraints. We will discuss how to define such constraints in terms of categories.

Functors between categories for different frames can model relations between and operations on frames. For example, we will discuss how frame-subsumption or composition can be defined via functors. As a next step, we regard the category $\mathcal{S}$ of the space of frames. Here, the objects are the frame categories and the morphisms are the functors between them. Some types of frames, like frames for lexicalized concepts or frames for relational concepts, can be captured as subcategories of the space of frames.

References

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