Searching for Directions: Epistemic and Deontic Modals in InqS

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01 Introduction

Aims

Aims and structure

1. Show how to lift Aher's modified Andersonian deontic semantics from radical to suppositional inquisitive semantics.

   Apart from the effects of adding suppositional content, we stay as close as possible to Aher's original system.

   We do encode deontic information in a new way, which gives rise to variations of the system, but we don't exploit that here.
2. Show how Veltman's epistemic *might* as consistency check can be lifted to a more general supposability check in InqS, which also gives rise to a sensible notion of epistemic suppositional *must*.
3. Show that the interpretation of implication, suppositional deontic *may* and epistemic *might*, are structurally strongly related in the suppositional inquisitive semantic framework.

Free choice: a puzzle for deontic and epistemic modals

Deontic free choice

(1) a. A country may establish a research center or a laboratory.

b. $\diamond_v (p \vee q)$

Epistemic free choice

(2) a. Estonia might establish a research center or a laboratory.

b. $\diamond (p \vee q)$

02 Deontics in InqS

Basic notions in InqS

Deontic information states

Worlds and rulings

- A world w is a valuation function such that for every atomic sentence p: $w(p) = 1$ (true) or $w(p) = 0$ (false).

  ω refers to the set of all possible worlds.

- A ruling r is a violation function such that for every world $w \in \omega$: $r(w) = 1$ (no violation) or $r(w) = 0$ (violation).

  ρ refers to the set of all possible rulings.

Factive and deontic information

A state determines a set of worlds and a set of rulings:

- the worlds still possible according to factive information;
- the rulings still possible according to deontic information.

Deontic information states

Notation

- Let s be a set of world-ruling pairs: $s \subseteq \omega \times \rho$.
  - $worlds(s) = \{w \in \omega \mid \exists r \in \rho \mid \langle w, r \rangle \in s\}$
  - $rulings(s) = \{r \in \rho \mid \exists w \in \omega \mid \langle w, r \rangle \in s\}$.

Deontic information states

- s is a deontic information state iff

  $s \subseteq \omega \times \rho$ such that $worlds(s) \times rulings(s) = s$.
- A state s is a set of world-ruling pairs such that if a ruling occurs in s, it occurs paired with every world in s.
- This guarantees the independence of deontic and factual information in a state.

Two pictures of deontic states

s_1	w_1	w_2	w_4
r_1	11	10	00
r_2	11	10	00
r_3	11	10	00
r_4	11	10	00

s_2	w_1	w_2	w_4
r_5	11	10	00
r_6	11	10	00
r_7	11	10	00
r_8	11	10	00

Deontic suppositional inquisitive semantics

Ordinary atomic sentences

- $s \models^{+} p$ iff $s \neq \emptyset$ and $\forall w \in worlds(s): w(p) = 1$
- $s \models^{-} p$ iff $s \neq \emptyset$ and $\forall w \in worlds(s): w(p) = 0$
- $s \models^{\circ} p$ iff $s = \emptyset$

The deontic predicate safe

- $s \models^{+} \mathtt{safe}$ iff $s \neq \emptyset$ and $\forall w \in worlds(s)$ and $\forall r \in rulings(s): r(w) = 1$
- $s \models^{-} \mathtt{safe}$ iff $s \neq \emptyset$ and $\forall w \in worlds(s)$ and $\forall r \in rulings(s): r(w) = 0$
- $s \models^{\circ} \mathtt{safe}$ iff $s = \emptyset$

Choosing directions in deontic states

s_1	w_1	w_2	w_4
r_1	11	10	00
r_2	11	10	00
r_3	11	10	00
r_4	11	10	00

s_2	w_1	w_2	w_4
r_5	11	10	00
r_6	11	10	00
r_7	11	10	00
r_8	11	10	00

Choosing directions in deontic states

s_1	w_1	w_2	w_4
r_1	11	10	00
r_2	11	10	00
r_3	11	10	00
r_4	11	10	00

s_2	w_1	w_2	w_4
r_5	11	10	00
r_6	11	10	00
r_7	11	10	00
r_8	11	10	00

Negation, disjunction, conjunction

Negation

- $s \models^{+} \neg\varphi$ iff $s \models^{-} \varphi$
- $s \models^{-} \neg\varphi$ iff $s \models^{+} \varphi$
- $s \models^{\circ} \neg\varphi$ iff $s \models^{\circ} \varphi$

Disjunction

- $s \models^{+} \varphi \vee \psi$ iff $s \models^{+} \varphi$ or $s \models^{+} \psi$
- $s \models^{-} \varphi \vee \psi$ iff $s \models^{-} \varphi$ and $s \models^{-} \psi$
- $s \models^{\circ} \varphi \vee \psi$ iff $s \models^{\circ} \varphi$ or $s \models^{\circ} \psi$

Conjunction

- $s \models^{+} \varphi \wedge \psi$ iff $s \models^{+} \varphi$ and $s \models^{+} \psi$
- $s \models^{-} \varphi \wedge \psi$ iff $s \models^{-} \varphi$ or $s \models^{-} \psi$
- $s \models^{\circ} \varphi \wedge \psi$ iff $s \models^{\circ} \varphi$ or $s \models^{\circ} \psi$

Clauses for Implication

- $s \models^{+} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+}$:
  1. $\forall t$ from u to $u \cap s$: $t \models^{+} \varphi$, and
  2. $u \cap s \models^{+} \psi$
- $s \models^{-} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\exists u \in \textsc{alt}[\varphi]^{+}$:
  1. $\forall t$ from u to $u \cap s$: $t \models^{+} \varphi$, and
  2. $u \cap s \models^{-} \psi$
- $s \models^{\circ} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{+}$:
  1. $\exists t$ from u to $u \cap s$: $t \not\models^{+} \varphi$, or
  2. $u \cap s \models^{\circ} \psi$

Reduction for a non-suppositional antecedent

$s \models^{+} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+}: u \cap s \models^{+} \psi$

$s \models^{-} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\exists u \in \textsc{alt}[\varphi]^{+}: u \cap s \models^{-} \psi$

$s \models^{\circ} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{+}: u \cap s \models^{\circ} \psi$

Deontic *may*

- $s \models^+ \diamondsuit\!\!\!\!\vee\, \varphi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^+$:
  1. $\forall t$ from u to $u \cap s$: $t \models^+ \varphi$, and
  2. $u \cap s \models^+ \texttt{safe}$
- $s \models^- \diamondsuit\!\!\!\!\vee\, \varphi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^+$:
  1. $\forall t$ from u to $u \cap s$: $t \models^+ \varphi$, and
  2. $u \cap s \models^- \texttt{safe}$
- $s \models^\circ \diamondsuit\!\!\!\!\vee\, \varphi$ iff $\textsc{alt}[\varphi]^+ = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^+$:
  1. $\exists t$ from u to $u \cap s$: $t \not\models^+ \varphi$

For non-suppositional φ

$s \models^+ \diamondsuit\!\!\!\!\vee\, \varphi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^+ : u \cap s \models^+ \texttt{safe}$

$s \models^- \diamondsuit\!\!\!\!\vee\, \varphi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^+ : u \cap s \models^- \texttt{safe}$

$s \models^\circ \diamondsuit\!\!\!\!\vee\, \varphi$ iff $\textsc{alt}[\varphi]^+ = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^+ : u \cap s = \emptyset$

03 Comparing implication and may

Comparing support

Comparing deontic *may* and implication

Comparing support clauses

- $s \models^{+} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and
  $\forall u \in \textsc{alt}[\varphi]^{+}$: $\forall t$ from u to $u \cap s$: $t \models^{+} \varphi$
  and $u \cap s \models^{+} \psi$

- $s \models^{+} \diamondsuit\varphi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and
  $\forall u \in \textsc{alt}[\varphi]^{+}$: $\forall t$ from u to $u \cap s$: $t \models^{+} \varphi$
  and $u \cap s \models^{+} \texttt{safe}$

Obvious difference

- The one difference is that the ‘consequent’ of *may* is not an arbitrary formula, but the deontic predicate `safe`.

$$s \models^{+} \diamondsuit\varphi \Longleftrightarrow s \models^{+} \varphi \rightarrow \texttt{safe}$$

Deontic free choice

Free choice

(3) a. A country may establish a research center or a laboratory.
b. $\Diamond\!\!\!\!\!\vee\,(p \vee q)$

Reduced support clause of $\Diamond\!\!\!\!\!\vee\,\varphi$

$s \models^+ \Diamond\!\!\!\!\!\vee\,\varphi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^+ : u \cap s \models^+ \mathtt{safe}$

s_1	w_1	w_2	w_3	w_4
r_1	11	10	01	00
r_2	11	10	01	00

Table: $s_1 \models^+ \Diamond\!\!\!\!\!\vee\,(p \vee q)$

Comparing deontic *may* and implication

Comparing rejection clauses

- $s \models^- \varphi \rightarrow \psi$ iff $\text{ALT}[\varphi]^+ \neq \emptyset$ and
  $\exists u \in \text{ALT}[\varphi]^+ : \forall t$ from u to $u \cap s : t \models^+ \varphi$
  and $u \cap s \models^- \psi$
- $s \models^- \diamondsuit\varphi$ iff $\text{ALT}[\varphi]^+ \neq \emptyset$ and
  $\forall u \in \text{ALT}[\varphi]^+ : \forall t$ from u to $u \cap s : t \models^+ \varphi$
  and $u \cap s \models^- \texttt{safe}$

Crucial difference

- The difference between implication and deontic *may* that is characteristic for the modified Andersionian approach is that, like in the support clause for $\diamondsuit\varphi$, we quantify universally over the support-alternatives for φ in the rejection clause as well.

Comparing deontic *may* and implication

Comparing rejection clauses

- $s \models^- \varphi \to \psi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and
  $\exists u \in \textsc{alt}[\varphi]^+ :$ $\forall t$ from u to $u \cap s : t \models^+ \varphi$
  and $u \cap s \models^- \psi$
- $s \models^- \Diamond\!\!\!\!\!\!\bigcirc\!\!\!\!\!\vee\, \varphi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and
  $\forall u \in \textsc{alt}[\varphi]^+ :$ $\forall t$ from u to $u \cap s : t \models^+ \varphi$
  and $u \cap s \models^- \mathtt{safe}$

Difference disappears, when φ is not support-inquisitive

- If φ is not support-inquisitive:

$$s \models^- \Diamond\!\!\!\!\!\!\bigcirc\!\!\!\!\!\vee\, \varphi \Longleftrightarrow s \models^- \varphi \to \mathtt{safe}$$

Deontic Free choice

Negating free choice

(4) a. A country may not establish a research center or a laboratory.

b. $\neg \Diamond (p \vee q)$

Reduced rejection clause of $\Diamond \varphi$

$s \models^- \Diamond \varphi$ iff $\text{ALT}[\varphi]^+ \neq \emptyset$ and $\forall u \in \text{ALT}[\varphi]^+ : u \cap s \models^- \texttt{safe}$

s_1	w_1	w_2	w_3	w_4
r_1	11	10	01	00
r_2	11	10	01	00

Table: $s_1 \models^+ \neg \Diamond (p \vee q)$

Comparing deontic *may* and implication

Comparing rejection clauses

- $s \models^{-} \varphi \to \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and
  $\exists u \in \textsc{alt}[\varphi]^{+}$: $\forall t$ from u to $u \cap s$: $t \models^{+} \varphi$
  and $u \cap s \models^{-} \psi$
- $s \models^{-} \Diamond_{v} \varphi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and
  $\forall u \in \textsc{alt}[\varphi]^{+}$: $\forall t$ from u to $u \cap s$: $t \models^{+} \varphi$
  and $u \cap s \models^{-} \mathtt{safe}$

Taking the difference into account:

1. $s \models^{-} \Diamond_{v} \varphi \Longleftrightarrow s \models^{+} \varphi \to \neg\mathtt{safe}$
2. $s \models^{+} \neg\Diamond_{v} \varphi \Longleftrightarrow s \models^{+} \varphi \to \neg\mathtt{safe}$
3. $s \models^{+} \Box_{v} \varphi \Longleftrightarrow s \models^{+} \neg\varphi \to \neg\mathtt{safe}$

Comparing deontic *may* and implication

Comparing dismissal clauses

- $s \models^{\circ} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or
  $\exists u \in \textsc{alt}[\varphi]^{+}$: $\exists t$ from u to $u \cap s$: $t \not\models^{+} \varphi$
  or $u \cap s \models^{\circ} \psi$

- $s \models^{\circ} \diamondsuit\varphi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or
  $\exists u \in \textsc{alt}[\varphi]^{+}$: $\exists t$ from u to $u \cap s$: $t \not\models^{+} \varphi$

No significant difference

- The one difference disappears, when the consequent of the implication, like the deontic predicate `safe`, is not suppositional.

$$s \models^{\circ} \diamondsuit\varphi \Longleftrightarrow s \models^{\circ} \varphi \rightarrow \mathtt{safe}$$

Deontic Free choice

Dismissing a free choice prohibition

(5) a. A country may not establish a research center or a laboratory.

b. $\neg\diamondsuit(p \vee q)$

Reduced dismissal clause of $\diamondsuit\varphi$

$s \models^{\circ} \diamondsuit\varphi$ iff $\text{ALT}[\varphi]^{+} = \emptyset$ or $\exists u \in \text{ALT}[\varphi]^{+} : u \cap s = \emptyset$

Dismissal

(6) a. Well, no country will establish a research center.

b. $\neg p$

s_1	w_1	w_2	w_3	w_4
r_1	11	10	01	00
r_2	11	10	01	00

Deontic Free choice

Dismissing a free choice prohibition

(7) a. A country may not establish a research center or a laboratory.
b. $\neg \diamondsuit_{v} (p \vee q)$

Reduced dismissal clause of $\diamondsuit_{v} \varphi$

$s \models^{\circ} \diamondsuit_{v} \varphi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{+} : u \cap s = \emptyset$

Dismissal

(8) a. Well, no country will establish a research center.
b. $\neg p$

s_1	w_1	w_2	w_3	w_4
r_1	11	10	01	00
r_2	11	10	01	00

Conditional permission

Reduction to implication

$s \models^{+} \diamondsuit_{v} \varphi \iff s \models^{+} \varphi \rightarrow \texttt{safe}$

Conditional permission

(9) a. If a country has a laboratory, it **may** establish a research center.

b. $p \rightarrow \diamondsuit_{v} q$

c. $p \rightarrow (q \rightarrow \texttt{safe})$

d. $(p \wedge q) \rightarrow \texttt{safe}$

04 Epistemic modalities

Epistemic might

Suppositional epistemic *might* and *must*

Might as a supposability check

- InqS comes with an epistemic modality $\Diamond\varphi$ that in the most basic cases is a consistency check, like Veltman's *might* in update semantics (US).
- In general, however, $\Diamond\varphi$ in InqS is a supposability check.

Must as a non-supposability check

- InqS -*might* gives rise to a sensible suppositional *must*, standardly defined as $\Box\varphi := \neg\Diamond\neg\varphi$.
- In InqS, $\Box\varphi$ has the same informative content as φ, but it differs from it in suppositional (inquisitive) content.
- Conversationally, $\Box\varphi$ acts as a *non*-supposability check of $\neg\varphi$.

Suppositional epistemic *might* and *must*

Persistence

- For Veltman, $\Diamond\varphi$ is a basic example of a non-persistent update.
- In InqS, both $\Diamond\varphi$ and $\Box\varphi$ are support / reject-persistent modulo suppositional dismissal.

Suppositional *might*: the intuitive idea

$\Diamond\varphi$ is a proposal to check the supposability of φ in s.

- s supports $\Diamond\varphi$ iff
  - (a) $\text{ALT}[\varphi]^+ \neq \emptyset$ and
  - (b) for every $u \in \text{ALT}[\varphi]^+$ it is possible to suppose u in s
- s rejects $\Diamond\varphi$ iff
  - (a) $s \neq \emptyset$ and
  - (b) for every $u \in \text{ALT}[\varphi]^+$: it is impossible to suppose u in s
- s dismisses a supposition of $\Diamond\varphi$ iff
  - (a) $\text{ALT}[\varphi]^+ = \emptyset$ or
  - (b) for some $u \in \text{ALT}[\varphi]^+$: it is impossible to suppose u in s

Clauses for *might*

Suppositional *might*

- $s \models^{+} \Diamond\varphi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and
  $\forall u \in \textsc{alt}[\varphi]^{+}$: $\forall t$ from u to $u \cap s$: $t \models^{+} \varphi$
- $s \models^{-} \Diamond\varphi$ iff $s \neq \emptyset$ and
  $\forall u \in \textsc{alt}[\varphi]^{+}$: $\exists t$ from u to $u \cap s$: $t \not\models^{+} \varphi$
- $s \models^{\circ} \Diamond\varphi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or
  $\exists u \in \textsc{alt}[\varphi]^{+}$: $\exists t$ from u to $u \cap s$: $t \not\models^{+} \varphi$

For a non-suppositional φ

- $s \models^{+} \Diamond\varphi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+}: u \cap s \neq \emptyset$
- $s \models^{-} \Diamond\varphi$ iff $s \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{+}: u \cap s = \emptyset$
- $s \models^{\circ} \Diamond\varphi$ iff $\textsc{alt}[\varphi]^{+} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{+}: u \cap s = \emptyset$

Persistence of suppositional *might*

Two essential features of the clauses for $\Diamond\varphi$

- Support and dismissing a supposition contradict each other
- Rejection implies dismissal

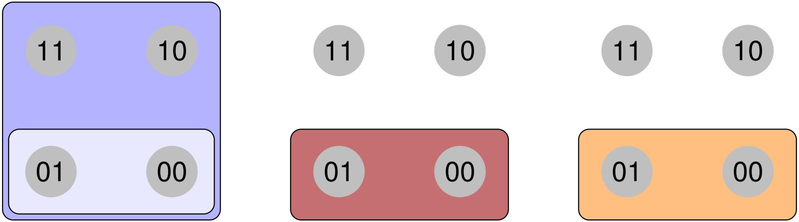
Support of *might* can turn into reject + dismissal

- It can be the case that $s \models^{+} \Diamond\varphi$ and that it holds for some more informed state $t \subset s$ that $t \not\models^{+} \Diamond\varphi$, or even $t \models^{-} \Diamond\varphi$, but then it will also be the case that $t \models^{\circ} \Diamond\varphi$.
- Suppositional *might* is support-persistent, modulo suppositional dismissal.

Picture of meaning *might*

Reduced clauses for *might*

- $s \models^+ \Diamond\varphi$ iff $\textsc{alt}[\varphi]^+ \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^+ : u \cap s \neq \emptyset$
- $s \models^- \Diamond\varphi$ iff $s \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^+ : u \cap s = \emptyset$
- $s \models^\circ \Diamond\varphi$ iff $\textsc{alt}[\varphi]^+ = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^+ : u \cap s = \emptyset$



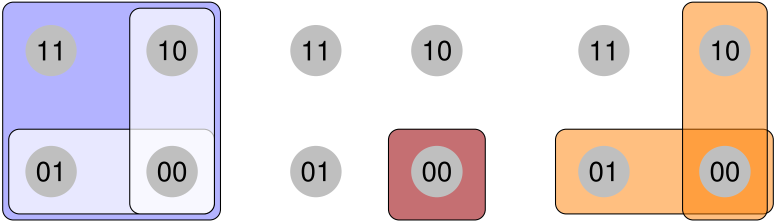
(1) support (2) reject (3) dismissal

Figure: $\Diamond p$

Epistemic free choice

Reduced clauses for *might*

- $s \models^{+} \Diamond\varphi$ iff $\text{ALT}[\varphi]^{+} \neq \emptyset$ and $\forall u \in \text{ALT}[\varphi]^{+} : u \cap s \neq \emptyset$
- $s \models^{-} \Diamond\varphi$ iff $s \neq \emptyset$ and $\forall u \in \text{ALT}[\varphi]^{+} : u \cap s = \emptyset$
- $s \models^{\circ} \Diamond\varphi$ iff $\text{ALT}[\varphi]^{+} = \emptyset$ or $\exists u \in \text{ALT}[\varphi]^{+} : u \cap s = \emptyset$



(1) support (2) reject (3) dismiss

Figure: $\Diamond(p \vee q)$

Derived suppositional *must*

Must as a non-supposability check

- $\Box\varphi$ is defined as $\neg\Diamond\neg\varphi$
- So, $\Box\varphi$ is supported in s, when $\Diamond\neg\varphi$ is rejected in s
- $\Diamond\neg\varphi$ is a proposal to check for supposability of $\neg\varphi$ in s
- When the check for supposability of $\neg\varphi$ fails in s, $\Diamond\neg\varphi$ is rejected in s and $\Box\varphi$ is supported in s.
- Conversationally, a speaker proposing $\Box\varphi$, invites a responder to suppose that $\neg\varphi$, in the hope that in her state $\neg\varphi$ is (also) not supposable.

Suppositional *must*: intuitive idea derived from *may*

$\Box\varphi$ is a proposal to check the non-supposability of $\neg\varphi$ in S

- s supports $\Box\varphi$ iff
  - (a) $s \neq \emptyset$ and
  - (b) for every $u \in \text{ALT}[\varphi]^-$: it is impossible to suppose u in s
- s rejects $\Box\varphi$ iff
  - (a) $\text{ALT}[\varphi]^- \neq \emptyset$ and
  - (b) for every $u \in \text{ALT}[\varphi]^-$: it is possible to suppose u in s.
- s dismisses a supp of $\Box\varphi$ iff
  - (a) $\text{ALT}[\varphi]^- = \emptyset$ or
  - (b) for some $u \in \text{ALT}[\varphi]^-$: it is impossible to suppose u in s

Suppositional epistemic *must*

$s \models^{+} \Box\varphi$ iff $s \neq \emptyset$ and
$\forall u \in \text{ALT}[\varphi]^{-}$: $\exists t$ from u to $u \cap s$: $t \not\models^{-} \varphi$

$s \models^{-} \Box\varphi$ iff $\text{ALT}[\varphi]^{-} \neq \emptyset$ and
$\forall u \in \text{ALT}[\varphi]^{-}$: $\forall t$ from u to $u \cap s$: $t \models^{-} \varphi$

$s \models^{\circ} \Box\varphi$ iff $\text{ALT}[\varphi]^{-} = \emptyset$ or
$\exists u \in \text{ALT}[\varphi]^{-}$: $\exists t$ from u to $u \cap s$: $t \not\models^{-} \varphi$

For non-suppositional φ

$s \models^{+} \Box\varphi$ iff $s \neq \emptyset$ and $\forall u \in \text{ALT}[\varphi]^{-}$: $u \cap s = \emptyset$

$s \models^{-} \Box\varphi$ iff $\text{ALT}[\varphi]^{-} \neq \emptyset$ and $\forall u \in \text{ALT}[\varphi]^{-}$: $u \cap s \neq \emptyset$

$s \models^{\circ} \Box\varphi$ iff $\text{ALT}[\varphi]^{-} = \emptyset$ or $\exists u \in \text{ALT}[\varphi]^{-}$: $u \cap s = \emptyset$

Persistence of suppositional *must*

Two essential features of the clauses for $\Box\varphi$

- Rejection and dismissing a supposition contradict each other
- Support implies dismissal

Rejection of *must* can turn into support + dismissal

- It can be the case that $s \models^- \Box\varphi$ and that it holds for some more informed $t \subset s$ that $t \not\models^- \Box\varphi$, or even $t \models^+ \Box\varphi$, but then it will also be the case that $t \models^\circ \Box\varphi$.
- Suppositional *must* is rejection-persistent, modulo suppositional dismissal.

Picture of meaning *must*

Reduced clauses for *must*

$$s \models^{+} \Box\varphi \quad \text{iff} \quad s \neq \emptyset \quad \text{and} \quad \forall u \in \textsc{alt}[\varphi]^{-}: u \cap s = \emptyset$$

$$s \models^{-} \Box\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{-} \neq \emptyset \quad \text{and} \quad \forall u \in \textsc{alt}[\varphi]^{-}: u \cap s \neq \emptyset$$

$$s \models^{\circ} \Box\varphi \quad \text{iff} \quad \textsc{alt}[\varphi]^{-} = \emptyset \quad \text{or} \quad \exists u \in \textsc{alt}[\varphi]^{-}: u \cap s = \emptyset$$

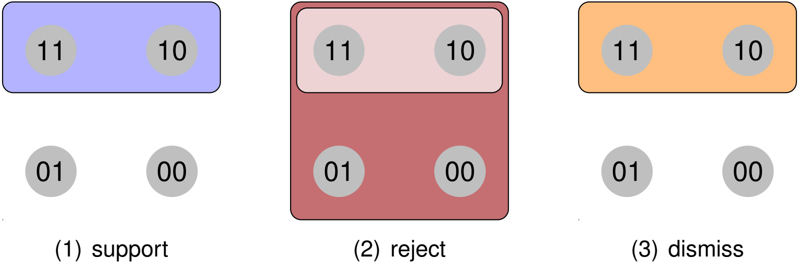


Figure: $\Box p$

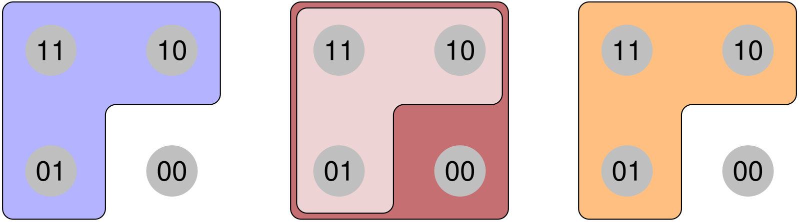
Picture of meaning *must*

Reduced clauses for *must*

$s \models^{+} \Box\varphi$ iff $s \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{-} : u \cap s = \emptyset$

$s \models^{-} \Box\varphi$ iff $\textsc{alt}[\varphi]^{-} \neq \emptyset$ and $\forall u \in \textsc{alt}[\varphi]^{-} : u \cap s \neq \emptyset$

$s \models^{\circ} \Box\varphi$ iff $\textsc{alt}[\varphi]^{-} = \emptyset$ or $\exists u \in \textsc{alt}[\varphi]^{-} : u \cap s = \emptyset$



(1) support (2) reject (3) dismiss

Figure: $\Box(p \vee q)$

Suppositional *must* and non-inquisitive closure

- The reject-informative content of $\Box\varphi$ is nil:

$$\bigcup[\Box\varphi]^{-} = \omega$$

- The support-informative content of $\Box\varphi$ equals that of φ:

$$\bigcup[\Box\varphi]^{+} = \bigcup[\varphi]^{+}$$

- But it does not hold generally that $[\Box\varphi]^{+} = [\varphi]^{+}$.

$$\textsc{alt}[p \vee q]^{+} = \{|p|, |q|\} \quad \neq \quad \textsc{alt}[\Box(p \vee q)]^{+} = \{|p| \cup |q|\}$$

- $p \vee q$ is support-inquisitive, but $\Box(p \vee q)$ is not.
- $\Box(p \vee \neg p)$ is supported in every state, support of $p \vee \neg p$ requires support of p or support of $\neg p$.

Suppositional *might* and non-inquisitive closure

- The support-informative content of $\Diamond\varphi$ is nil:

$$\bigcup[\Diamond\varphi]^+ = \omega$$

- The reject-informative content of $\Diamond\varphi$ equals that of φ:

$$\bigcup[\Diamond\varphi]^- = \bigcup[\varphi]^-$$

- But it does not hold generally that $[\Diamond\varphi]^- = [\varphi]^-$.

$$\textsc{alt}[p \wedge q]^- = \{|\neg p|, |\neg q|\} \quad \neq \quad \textsc{alt}[\Diamond(p \wedge q)]^- = \{|\neg p| \cup |\neg q|\}$$

- $p \wedge q$ is reject-inquisitive, but $\Diamond(p \wedge q)$ is not.
- $\Diamond(p \wedge \neg p)$ is rejected in every state, rejection of $p \wedge \neg p$ requires rejection of p or rejection of $\neg p$.

Suppositional inquisitiveness of *might* and *must*

Suppositional inquisitiveness

- Both $\Diamond\varphi$ and $\Box\varphi$ are never support-inquisitive or rejection-inquisitive.
- But both $\Diamond\varphi$ and $\Box\varphi$ can be inquisitive in their suppositional dismissal.
- $\textsc{alt}[\Diamond(p \vee q)]^\circ = \{|\neg p|, |\neg q|\}$, and $\textsc{alt}[\Diamond(p \vee q)]^- = \{|\neg p| \cap |\neg q|\}$
- $\textsc{alt}[\Box(p \wedge q)]^\circ = \{|p|, |q|\}$, where $\textsc{alt}[\Box(p \wedge q)]^+ = \{|p| \cap |q|\}$

Partial support and rejection

- Dismissing a supposition of $\Diamond(p \vee q)$ amounts to "partially rejecting" $p \vee q$.
- Dismissing a supposition of $\Box(p \wedge q)$ amounts to "partially supporting" $p \wedge q$.

Modal and non-modal implications

Rejecting implication

- In InqS, not just $p \wedge \neg q$, but already $p \rightarrow \neg q$ rejects $p \rightarrow q$.
- Some may feel this is still asking too much, and that $p \rightarrow \Diamond\neg q$ or $\Diamond(p \wedge \neg q)$ would suffice to reject $p \rightarrow q$.
- But both are not support-informative, they are already supported by the ignorant state ω.
- But sheer ignorance about p and q should not suffice to reject the proposal to update the CG with the information that $p \rightarrow q$.
- Responding with $p \rightarrow \Diamond\neg q$ or $\Diamond(p \wedge \neg q)$ to $p \rightarrow q$, signals unwillingness and not unability to accept the proposal.

Modal and non-modal implications

Rejecting implication continued

- Both $p \rightarrow \Diamond\neg q$ and $\Diamond(p \wedge \neg q)$ do suffice to reject $p \rightarrow \Box q$.
- By proposing $p \rightarrow \Box q$ instead of $p \rightarrow q$, one signals that ignorance about p and q suffices to reject the proposal.
- One only intends an update of the CG with $p \rightarrow q$, in case the other participants also already support that $p \rightarrow q$ or $p \rightarrow \Box q$.

05 Comparing might, implication and may

Support

Comparing support clauses

Might

- $s \models^+ \Diamond\varphi$ iff $\text{ALT}[\varphi]^+ \neq \emptyset$ and
  $\forall u \in \text{ALT}[\varphi]^+ :$ $\forall t$ from u to $u \cap s$: $t \models^+ \varphi$

Implication

- $s \models^+ \varphi \rightarrow \psi$ iff $\text{ALT}[\varphi]^+ \neq \emptyset$ and
  $\forall u \in \text{ALT}[\varphi]^+ :$ $\forall t$ from u to $u \cap s$: $t \models^+ \varphi$
  and $u \cap s \models^+ \psi$

May

- $s \models^+ \circledDiamond\varphi$ iff $\text{ALT}[\varphi]^+ \neq \emptyset$ and
  $\forall u \in \text{ALT}[\varphi]^+ :$ $\forall t$ from u to $u \cap s$: $t \models^+ \varphi$
  and $u \cap s \models^+ \texttt{safe}$

Comparing suppositional dismissal clauses

Might

- $s \models^\circ \Diamond\varphi$ iff $\text{ALT}[\varphi]^+ = \emptyset$ or $\exists u \in \text{ALT}[\varphi]^+$:
  $\exists t$ from u to $u \cap s$: $t \not\models^+ \varphi$

Implication

- $s \models^\circ \varphi \to \psi$ iff $\text{ALT}[\varphi]^+ = \emptyset$ or $\exists u \in \text{ALT}[\varphi]^+$:
  $\exists t$ from u to $u \cap s$: $t \not\models^+ \varphi$ or $u \cap s \models^\circ \psi$

May

- $s \models^\circ \Diamond\!\!\!\!\!\vee\,\varphi$ iff $\text{ALT}[\varphi]^+ = \emptyset$ or $\exists u \in \text{ALT}[\varphi]^+$:
  $\exists t$ from u to $u \cap s$: $t \not\models^+ \varphi$

Comparing rejection clauses

Might

- $s \models^{-} \Diamond\varphi$ iff $s \neq \emptyset$ and
  $\forall u \in \textsc{alt}[\varphi]^{+}: \exists t \text{ from } u \text{ to } u \cap s: t \not\models^{+} \varphi$

Implication

- $s \models^{-} \varphi \rightarrow \psi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and
  $\exists u \in \textsc{alt}[\varphi]^{+}: \forall t \text{ from } u \text{ to } u \cap s: t \models^{+} \varphi$
  and $u \cap s \models^{-} \psi$

May

- $s \models^{-} \diamondsuit\!\!\!\!\!\vee\ \varphi$ iff $\textsc{alt}[\varphi]^{+} \neq \emptyset$ and
  $\forall u \in \textsc{alt}[\varphi]^{+}: \forall t \text{ from } u \text{ to } u \cap s: t \models^{+} \varphi$
  and $u \cap s \models^{-} \mathtt{safe}$

The end

Thank you for listening!

Deontic free choice

Ignorance reading

(10) a. A country may establish a research center or a laboratory, but I do not know which.

b. $\Diamond_{v} p \vee \Diamond_{v} q$

s_1	w_1	w_2	w_3	w_4
r_1	11	10	01	00
r_2	11	10	01	00
r_3	11	10	01	00
r_4	11	10	01	00

Table: $\Diamond_{v} p$

s_1	w_1	w_2	w_3	w_4
r_1	11	10	01	00
r_2	11	10	01	00
r_5	11	10	01	00
r_6	11	10	01	00

Table: $\Diamond_{v} q$