XII
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Centre for Language, Logic and Speech
at the Tbilisi State University

Georgian Academy of Sciences

Institute for Logic, Language and Computation
University of Amsterdam

Contents

Tbilisi Symposium on Language, Logic and Computation

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The Tbilisi Symposium on Language, Logic and Computation is an interdisciplinary conference at the interface of logic, linguistics and computer science with the goal of sharing new results and developing mutually beneficial relationship between these fields. The Symposium is held biennially in different parts of Georgia. It is organized by the Institute for Logic, Language and Computation (ILLC) of the University of Amsterdam in conjunction with the Centre for Language, Logic and Speech and Razmadze Mathematical Institute of the Tbilisi State University.

Lagodekhi

The Twelfth International Tbilisi Symposium on Language, Logic and Computation will be held on 18-22 September 2017 in Lagodekhi in the Kakheti region in Georgia. It will feature 3 tutorials on Logic, Language and Computation, respectively, 6 invited lectures and 38 contributed talks. Lagodekhi is a pretty little town in the eastern part of Georgia, in the Kakheti region famous for its wine. The town is also an entry place for the Lagodekhi Nature Reserve, one of the most well-preserved natural habitats in Georgia, with hiking trails to forests, waterfalls and alpine lakes.

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Program

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10.00 - 10.20	BREAK	
10.20 - 10.45	*Philippe Balbiani and Jie Fan* **A complete axiomatization of Euclidean strong non-contingency logic**	*Carla Umbach and Stefan Hinterwimmer* **German "wie"-complements: manners, methods and events in progress**
10.45 - 11.10	*Philippe Balbiani and Mikhail Rybakov* **An axiomatization of iteration-free PDL with loop**	*Kerstin Schwabe* **The uniform representation of German embedded polar interrogatives, a typology of their embedding predicates and adaptors**
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11.35 - 12.00	*Gianluca Grilletti and Ivano Ciardelli* **An Ehrenfeucht-Fraïssé Game for Inquisitive First-Order Logic**	*Curt Anderson and Sebastian Löbner* **Roles and the lexical semantics of role-denoting relational adjectives**
12.00 - 14.00	LUNCH	
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11.10 - 11.35	*Dick de Jongh and Fateme Shirmohammadzadeh Maleki* **Two Neighborhood Semantics for Subintuitionistic Logics**	*Ralf Naumann and Wiebke Petersen* **Bridging inferences in a dynamic, probabilistic frame theory**
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15.30 - 16.30	*Jakub Szymanik* **Language Tutorial: Generalized Quantifiers. Logical, Computational, and Cognitive Approaches**	
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WEDNESDAY

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10.45 - 11.10	*Diego Valota* **Computing Spectra of Finite Goedel Algebras through Finite Forests**	*Nino Javashvili* **Derivation Models According to Otar Tchiladze Text Corpus**
11.10 - 11.35	*Dick de Jongh and Fateme Shirmohammadzadeh Maleki* **Two Neighborhood Semantics for Subintuitionistic Logics**	*Irina Lobzhanidze* **Computational Model of Modern Georgian Language and Searching Patterns for On-line Dictionary of Idioms**
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10.45 - 11.10	*Ondrej Majer* **Belief Aggregation for Non-standard Reasoners**	*Kurt Erbach, Hana Filip and Peter Sutton* **Hungarian measure phrases, bare singular nouns, and mass/count distinction**
11.10 - 11.35	*Gary Mar* **A Brief History of the Logic of Time**	*Giuli Shabashvili* **Syntactic Realization of Present Perfect Forms in Georgian**
11.35 - 12.00	*Tobias Heindel* **The Chomsky-Schützenberger Theorem with Circuit Diagrams in the Role of Words**	*Katherine Fraser* **Exploring the multi-dimensional meaning of extra arguments in English**
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FRIDAY AFTERNOON

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15.30 - 16.30	*Jakub Szymanik* **Language Tutorial: Generalized Quantifiers. Logical, Computational, and Cognitive Approaches**	
16.30 - 16.45	SHORT BREAK	
16.45 - 17.10	*Yulia Zinova* **Explaining meaning: The interplay of syntax, semantics, and pragmatics**	*Dan Zeman* **Some Solutions to the Perspectival Plurality Problem for Relativism**
17.10 - 17.35	*Sumiyo Nishiguchi* **Liberalism and Bouletic/Deontic Modality**	*Yael Greenberg and Lavi Wolf* **Assertions as degree relations**
17.35 - 18.00	*Andres Soria Ruiz* **Comparative Evaluative Judgments**	*Thomas Gamerschlag, Jens Fleischhauer and Wiebke Petersen* **Why event structure templates are not enough - A frame account of bleeding and droning**

Logic Workshop:

LOGIC, ALGEBRA, CATEGORIES AND QUANTITATIVE MODELS

Organizers: *Alexander Kurz, Alex Simpson*

14.00 - 14.45	*Radu Mardare* **Stone dualities for Markov processes**
14.45 - 15.30	*Matteo Mio* **Riesz Modal Logic for Markov Processes**
15.30 - 16.00	BREAK
16.00 - 16.45	*Sam Staton* **Semantic models of higher-order probability theory**
16.45 - 17.30	*Alex Simpson* **Programming with correlated random variables**

Language Workshop:

TRANSMODAL PERSPECTIVES ON SECONDARY MEANING

Organizers: *Daniel Hole, Fabian Bross, Daniel Gutzmann, Katharina Turgay*

11.20 - 12.00	*Fabian Bross, Daniel Hole* **Scope-taking strategies in German Sign Language and the at-issue/not-at-issue divide**
12.00 - 14.00	LUNCH
14.00 - 14.45	*Daniel Gutzmann, Katharina Turgay* **Expectedness exclamations and unexpected common ground states**
14.45 - 15.30	*Henk Zeevat* **Non truth conditional attributes: affordance and danger**
15.30 - 16.00	BREAK
16.00 - 16.45	*Katherine Fraser, Daniel Hole* **Secondary meanings in argument alternations**
16.45 - 17.30	*Stefan Hinterwimmer* **A comparison of the modal particles "fei" and "aber"**
17.30 - 18.15	*Lavi Wolf* **Secondary meanings of epistemic modality**

Workshops

Logic workshop:
Logic, Algebra, Categories and Quantitative Models

This workshop will focus on the application of mathematical techniques from the fields of logic, algebra and category theory in quantitative areas such as probability theory and economics. A characteristic feature of research in this direction is the combination of "structural" techniques from abstract mathematics with concrete numerical methods. Such research is also motivated by applications to areas of computer science such as probabilistic programming, verification of probabilistic systems, and distributed systems.

Organizers: Alexander Kurz, Alex Simpson

Radu Mardare
Stone dualities for Markov processes

In this talk I will present two Stone-type dualities that can be proven for Markov processes (MPs). The first duality is for a restricted class of MPs and the dual category is of the countably generated Aumann algebras. The second duality, which relies on Sikorski's duality between sigma-perfect sigma-fields and sigma-spatial Boolean algebras, involves a larger class of MPs and the dual category is of an alternative, less restrictive type of Aumann algebra. The two dualities are independent of each other.

This is a joint work with Dexter Kozen, Prakash Panangaden, Robert Furber and Kim Larsen (LICS'13 and LICS'17).

Matteo Mio
Riesz Modal Logic for Markov Processes

We investigate a modal logic for expressing properties of Markov processes whose semantics is real-valued, rather than Boolean, and based on the mathematical theory of Riesz spaces (lattice ordered vector spaces). We use the duality theory of Riesz spaces to provide a connection between Markov processes and the logic. This takes the form of a duality between the category of coalgebras of the Radon monad (modeling Markov processes) and the category of a new class of algebras (algebraizing the logic) which we call modal Riesz spaces. As a result, we obtain a sound and complete axiomatization of the Riesz Modal logic.

Sam Staton
Semantic models of higher-order probability theory

Bayesian statistics is full of interesting structure, particularly in its 'non-parametric' aspects. We have been working on semantic models for this structure from the perspective of probabilistic programming. For example, regression problems involve random functions, and we have built a new version of probability theory that allows probability measures over function spaces.

The talk will be based around recent papers in Proc. LICS and Proc. ESOP, partly in collaboration with Chris Heunen, Ohad Kammar, Frank Wood, Hongseok Yang and others.

Alex Simpson

Programming with correlated random variables

I shall describe a natural style of programming with correlated random variables as first-class citizens, using a simple typed functional language to illustrate it. The language primitives include data types of real numbers and infinite streams, and constructs for creating and sampling from continuous random variables. Semantically, this style of programming is naturally modelled in a presheaf category over probability spaces, which I shall briefly outline.

Language workshop: *Transmodal perspectives on secondary meaning*

Recent years have seen a growing or renewed interest in components of secondary meaning, meaning that is not entailed, that is: not-at-issue meaning, conventional implicature, evaluative and expressive meaning, use-conditional meaning etc. At the same time, formal accounts of sign language and speech-accompanying gesture have made substantial progress. Some of these transmodal approaches focus on secondary meaning, too. The aim of this workshop is to bring together researchers with expertise in the realm of secondary meaning in natural language with researchers specializing in sign language and gesture research with a (partial) emphasis on secondary meaning.

Organizers: Daniel Hole, Fabian Bross, Daniel Gutzmann, Katharina Turgay

Fabian Bross, Daniel Hole
Scope-taking strategies in German Sign Language and the at-issue/not-at-issue divide

The scope order of clausal categories has been claimed to be universal. In this talk we adopt a universalist cartographic approach to clausal syntax. By discussing the categories of speech acts, evaluation, epistemic modality, scalarity, volition and deontic, as well as other kinds of modality, we illustrate a striking regularity in strategies of scope-taking in German Sign Language (DGS): The wider/higher the scope of a clausal operator is, the more likely its expression will occur with a high body part by way of layering. Namely, descending from the eyebrows to the lower face, tentatively to the shoulders, and finally switching to manual expressions. For intermediate operators a left-to-right concatenation strategy is employed, and low categories are expressed by way of a manual right-to-left concatenation strategy. Hence, we propose a highly regular natural mapping of the scope-order of clausal categories onto the body. This sort of mapping can also be observed in other sign languages and may turn out to be universal.

Daniel Gutzmann, Katharina Turgay
Expectedness exclamations and unexpected common ground states

In this talk, investigate a previously unstudied construction in German which we dub "expectedness exclamations". These are utterances in which a possibly reduced declarative is preceded or followed by the unintegrated adjective normal "normal/usual".

A: Was für Sport machst du? 'What kind of sports do you do?'

B: (Ich gehe zu) McFit – normal! '(I'm going to) McFit, of course!'

What is crucial about this construction is that the adjective normal does not target the propositional content of the utterance, but rather expresses the use-conditional speaker attitude that the assertion itself has to be considered as normal and expectable. We discuss the syntactic, semantic, and pragmatic properties of these expectedness exclamatives and compare them to other construction like ordinary declaratives with the adverb normalerweise "usually" and standard unexpectedness exclamations. Crucially – and on contrast to the other constructions – expectedness exclamations are licensed if there is (unexpected) discrepancy in the interlocutors' beliefs about what is common ground. In order to model the discourse effects that expectedness exclamations have, we extend the common ground based model of assertion to represent the beliefs of the interlocutors about what is in the common ground, which enables the modeling of discrepancies in their perception of the common ground. The effect of normal exclamations is then to align the beliefs of speaker and hearer regarding the common ground, thereby removing the discrepancies in the state of the common ground.

Henk Zeevat

Non truth conditional attributes: affordance and danger

Frame semantics is or should be interpreted as an attempt to define concepts in terms of natural attributes that give values in some range for the objects that fall under the concept. Attributes like weight, size and colour can be linked to the appearance of the objects in visual perception and are estimated within visual perception though their values can be determined with more precision with other measuring techniques. Features of this kind, also for other kinds of perception and for generalisations of perception (e.g. administrative procedures for determining the proper age of a person or medical procedures to determine parenthood) produce truth-conditional content.

But it is also clear that conceptual content involves other attributes. If one lives in the jungle, one has mastered the concept of a tiger or a snake insufficiently if one is unaware of the considerable danger that these animals represent to oneself. Similarly, one misses out in one's concept of a chair if it does not tell one that it typically affords sitting. Or, the concept of an olive would be incomplete if it does not represent one's private evaluation of its taste. Knowing an object by a concept is part of determining one's action with respect to its bearer and danger, affordance and one's evaluation of olives as a food stuff are directly relevant.

If this is so, it should be unsurprising that expressive meaning can be part of lexically coded meaning, as well as judgments about taste and judgments about possible use. The representation of such information is not different from that of truth-conditional information. The talk aims to make it clear that non-truth-conditional content needs to be assumed for NLI by examples in discourse processing, and to discuss the projective properties of such content. It will also argue for agents as information states for a dynamic semantic account of such content.

Katherine Fraser, Daniel Hole

Secondary meanings in argument alternations

Although there is already extensive work on the semantics and syntax of argument alternations (see, e.g., Levin 2015 for an overview), the literature lacks a systematic investigation of the not-at-issue meaning that the non-base variants contribute. For example, the emotional valency of prepositional passives (1b) has been noted before (Davison 1980), but not accounted for. We take a selection of syntactically high, not-at-issue categories from the literature (Cinque 1999, Hole 2015) and observe their various, hitherto overlooked, not-at-issue meanings in non-base variants of argument alternations; (1)-(3) present some of our observations. At a more general level, our work contributes to the growing body of literature that describes morphosyntactic communication between inflectional/argument-structural categories on the one hand, and C-level categories on the other (Kratzer 2009, Wiltschko 2014, Hole 2015).

1. **Mirativity in prepositional passives**

a. *normal passive* That bridge was built in a day ... but this is not surprising.

b. *prepositional* That bridge was flown under ... #but this is not surprising.

2. **Scalarity in the pseudo-conative alternation**

a. The mouse ate the cheese, ... and it was not a small amount.

b. The mouse ate at the cheese, ... #and it was not a small amount.

3. **Evidentiality in there-insertion alternations**

a. Basques marched through the square, ... but i didn't see them.

b. There marched Basques through the square, ... #but i didn't see them.

The not-at-issue categories attested above may only be targeted by the metalinguistic *Hey, wait a minute*-refutations: *Hey, wait a minute, {that's not surprising/that's not a little/you didn't witness it*; they cannot be targeted by an entailment-cancelling refutation (cf. Simons et al. 2011).

We use diagnostics from the syntactic literature to tease out the functional hierarchy these alternations. For example, higher, speaker-anchored, categories (e.g., evaluativity/evidentiality) are known to be unable to combine with unanchored conditionals, but can easily combine with reason clauses (cf. Haegeman 2010, a.o.); cf. the behaviour of (4b) vs. (4b').

4. **Conditional clause: no evidential projection**

a. Madrid would have sent the police if Basques marched in the square.

b.* Madrid would have sent the police *if* **there** marched Basques in the square.

b. Madrid sent the police *because* **there** marched Basques in the square.

Following Wiltschko (2014), Ritter & Wiltschko (2014), we assume that there is morphosyntactic communication between C level and INFL level categories. In this case, *there* at INFL level is a pronounced locative element that will try to communicate with the evidential projection of the C domain.

In a nutshell, argument alternations are a highly useful empirical domain to render visible high not-at-issue categories that are frequently overlooked.

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Stefan Hinterwimmer
A comparison of the modal particles "fei" and "aber"

This paper deals with the interpretation of two modal particles that intuitively express some form of contrast and correction – the Bavarian modal particle fei (Schlieben-Lange 1979, Thoma 2009), which does not have an equivalent in standard German, and the modal particle aber (not to be confused with the conjuction aber, 'but'), which exists in standard German as well as in Bavarian. We will show that both are special among discourse particles in the following sense: They not only make a contribution that is interpreted at a level distinct from the level where at issue content (Potts 2005) is interpreted – as is standard for modal particles (see Gutzmann 2015 and the references therein). Rather, they also exclusively relate to propositions that do not have entered the Common Ground via being the at-issue content of an assertion made by the addressee. Intuitively, fei is used by the speaker in order to direct the addressee's attention to a conflict between her own beliefs and the addressee's beliefs that is not salient at the point where the sentence containing fei is uttered: The proposition p believed by the addressee that contradicts the proposition q believed by the speaker has not been made a topic of the ongoing conversation. At the same time, by asserting q the speaker implicitly proposes to resolve the conflict as follows: The addressee ceases to believe p and believes q instead. The modal particle aber, in contrast, requires there to be proposition p contradicting the proposition q asserted by the speaker that is, on the hand, salient at the point where q is asserted. On the other hand, just as in the case of fei, p may not be the at-issue content of assertion made by the addressee.

Lavi Wolf
Secondary meanings of epistemic modality

Epistemic modality is classically (cf. Kratzer 1981, 1991, 2012 and many others) represented as truth conditional quantification over possible worlds. However, in addition to this primary meaning, epistemic modals exhibit a wide range of secondary meanings, e.g.:

1. **A:** John might be upset.
   B: No, he's just tired.
2. **A:** You are late.
   B: I might be late but I brought cookies.
3. John might possibly be upset.

In (1), the first conversational participant utters an epistemically modalized content but the second participant replies to the prejacent (non-modal content), i.e. the modal does not seem to be at issue. In (2), the first conversational participant asserts a non-modal claim, which is true, yet the second participant uses an epistemic modal in the first conjunct and again the modal does not seem to be at issue. And (3) is a modal concord (cf. inter alia Geurts & Huitink, 2006; Zeijlstra, 2007; Huitink, 2012) sentence, in which one epistemic modal seems to be semantically inert, again not taking part in the propositional content. This talk discusses these and other linguistic phenomena in which epistemic modals manifest secondary meanings, and presents a new formal account based on Greenberg & Wolf (2017), in which assertion is a gradable speech act, i.e. represented as a degree relation between truth-conditional content and the use-conditional degree of speaker's belief in the asserted proposition. The secondary meanings of epistemic modals are then represented as degree modification over gradable assertions.

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Tutorials

Ana Sokolova
Semantics for Probabilistic Systems

The tutorial covers both basic and new results on coalgebraic modelling and semantics of probabilistic systems.

The material is divided in three parts:

1. Branching-time semantics;

2. Linear-time semantics;

3. Belief-state semantics

Each of these parts will motivate a generic treatment and introduce all necessary category and coalgebra notions on-the-fly, when needed. In Part (1) we will discuss standard modelling of various types of probabilistic systems as coalgebras on Sets, which suffices for branching-time semantics like bisimilarity and behavioural equivalence. I will also present a comparison hierarchy for these various system types. Part (2) is all about traces: generic trace theory, trace semantics via determinisation, and sound and complete axiomatisation of trace semantics for probabilistic systems. We will see that for this we need to move to more complex categories like the Kleisli and Eilenberg-Moore categories of a monad. This is where convexity and convex algebras start to play an important role. In Part (3) we will discover the foundations of probabilistic systems as belief-state transformers, used in AI and machine learning, together with their natural distribution semantics. Also here we work in Eilenberg-Moore categories and convexity theory provides the tools.

Jakub Szymanik
Generalized Quantifiers. Logical, Computational, and Cognitive Approaches

The course gives an introduction to the generalized quantifier theory, overviewing some crucial notions of formal semantics and logic. We survey how mathematical methods may be rigorously applied in linguistics to study the possible meanings, the inferential power, and computational properties of quantifier expressions. The course novelty lies mostly in combining classical generalized quantifier themes with a computational perspective and explicitly connecting the formal theory with psycholinguistic research. During the course I will use as lecture notes fragments of my book "Quantifiers & Cognition. Logical and Computational Approaches", Studies in Linguistics and Philosophy, Springer, 2016.

Sam van Gool
Machines, Models, Monoids, and Modal logic

Automata are abstract machines that yield a simple model of computation. It is fairly easy to understand what automata are, as we will see in the first part of this tutorial. Still, automata encode deep, difficult, and often unsolved problems about the power of computation and its limits.

In the second part of the tutorial, we will focus on two powerful mathematical techniques, which can be used to study these problems in automata theory: algebra, in the form of monoids, and logic, in the form of models. We will see how duality theory forms a bridge between the two techniques, in a similar way to the syntax/semantics duality of modal logic.

After introducing this mathematical core, in the last part we will look at applications of this theory to problems in the computer science study of automata and formal languages; for example, the problem of separating two regular languages by a language from a specified class of first-order-definable languages.

The tutorial will naturally fall into three parts:

1. what are automata and the problems associated to them?

2. what algebraic and logical techniques are used for studying automata theory?

3. how do the techniques help to make progress on these problems?

Along the way, there will be opportunity for questions, exercises, (partial) solutions, and discussion.

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    - H. Straubing, Finite Automata, Formal Logic, and Circuit Complexity, Birkhauser, 1994. A highly readable introduction to the theory of automata and formal languages.
    - J.-É. Pin, Mathematical Foundations of Automata Theory[1], Lecture notes from one of the leading researchers in automata theory. The introduction gives a historical overview. Also see Pin's recent survey papers[2].

2. On duality theory:

    - M. Gehrke, Duality, inaugural lecture[3], 2009. A gentle introduction to duality theory, written for a general audience.
    - M. Gehrke, Stone duality, topological algebra, and recognition[4], Journal of Pure and Applied Algebra, Volume 220, Issue 7, July 2016, Pages 2711–2747. The introduction to this paper gives an overview of the applications of Stone duality in formal language theory and profinite algebra.

3. My own research on duality and algebra in automata theory:

    - S. J. v. Gool and B. Steinberg, Pro-aperiodic monoids via saturated models[5], submitted, 2017. Extended version of my STACS 2017 paper on using model theory for studying pro-aperiodic monoids and logic on words.
    - S. J. v. Gool and B. Steinberg, Merge decompositions, two-sided Krohn-Rhodes, and aperiodic pointlikes[6], submitted, 2017. A preprint from last month, going more deeply into the algebraic theory.

[1]https://www.irif.fr/ jep/PDF/MPRI/MPRI.pdf
[2]https://www.irif.fr/ jep/publications.html
[3]https://www.irif.fr/ mgehrke/oratie.pdf
[4]https://hal.archives-ouvertes.fr/hal-00859717v4/document
[5]https://arxiv.org/abs/1609.07736
[6]https://arxiv.org/abs/1708.08118

Invited Talks

Gemma Boleda
Distributional semantics in linguistic research

This talk, which reports on a position paper in progress with Louise McNally, Josep M. Fontana and Alessandro Lenci, is about the potential of distributional semantics for linguistic theory. Distributional semantics is an approach to meaning representation which has come to dominate computational semantics. It has its modern roots in structuralist approaches to Linguistics, work on concepts in Cognitive Science, and "vector space" models in Information Retrieval. Distributional representations are usage-based, non-discrete, not limited to truth-conditionally relevant components of meaning, integratable with multi-modal sources of information (e.g. image, sound), and amenable not only to compositional but also to "decompositional" views of meaning. Despite the existence for some time of notable and very varied efforts within computational semantics to connect distributional semantics to the logical semantics tradition (a representative sample includes Coecke et al. 2010, Garrette et al. 2011, Copestake and Herbelot 2012, Lewis and Steedman 2013, Baroni et al. 2014 and the papers in Boleda and Herbelot 2016), the method has had virtually no impact on formally-oriented theoretical linguistic research. We want to change this by highlighting the potential of distributional methods, when properly combined with formal methods, not only to afford insight into the semantic problems for which they are already well known, such as the analysis of polysemy, but also to illuminate certain claims in the syntax literature that have lacked any meaningful connection to semantic theory (arguably to the detriment of both subfields), as well as patterns of semantic change.

Ruth Kempson
Language as a Tool for Interaction: An Evolutionary Tale

In this talk I bring together the Dynamic Syntax (DS) of Kempson et al (2001), Kempson et al (2016) in which natural-language syntax as core of the grammar formalism is defined as conditional actions for growing semantic constructs reflecting real time dynamics, the Andy Clark theory of cognition (Predictive Processing (PP) Clark 2016) in which cognition and all its effects are grounded in anticipatory action processes, selections driven by probabilistic weighting of context-relative assumptions, and the Multi-Level Selection Hypothesis (MLS, Sober & Sloan Wilson 1988) in which groups can be taken nonreductively as explanatory adaptive units in their own right. From this cross-disciplinary marriage, I shall argue, a gradualist account of language evolution, following language acquisition and change, can be seen to emerge.

DS is grounded in the Logic of Finite Trees (Blackburn & Meyer-Viol 1994) from which a system of tree growth defines, as core, underspecification and incremental update of structure, content, context, and procedures, reflecting the dynamics of real-time processing. These mechanisms apply in tandem in parsing and production, a stance from which the interactivity of dialogue exchanges emerges in virtue of both parties building up representations of content (Gregoromichelaki et al 2011). Clark (2016) argues in like manner but from a distinct starting point, that perception and action are essentially the same, with all cognitive processing involving forward top-down expectations about the immediate future based on one's given knowledge and the immediate and evolving context, input stimuli serving merely as corrective filters eliminating otherwise potential choices. With DS and PP stances displaying parallelism, the DS perspective can be directly nested within the PP model, both subject to probabilistically driven choice mechanisms, adding to the PP model the contribution which human potential for social interaction brings to the PP dynamics.

With DS/PP integration, languages can be seen as systems adaptively promoting group evolutionary success, capacity for language emerging gradually from prior capacity for making interaction manifest, following first-language acquisition. Apparent altruistic/group effects become consequences of how the system-internal mechanisms interact, with no need to invoke over-arching superimposed inference systems, rich innate structure, mind-reading, or normativity assumptions as in other frameworks, in either acquisition or evolution. Adopting the MLS methodology, which establishes adaptiveness of groups by investigating tensions between group and individual adaptivity trait by trait, languages can be characterised as adaptive systems, with language acquisition, change and evolution subject to analysis as involving gradual transitions, matching informal intuition.

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Alexander Kurz

Lawvere's Generalized Logic Revisited

In his influential paper "Metric Spaces, Generalized Logic, and Closed Categories", Lawvere developed the insight that the logical rules of implication and the definition of a metric space are both instances of a generalized logic that arises from working with a lattice of 'truth values' considered as a category. But, thinking of the relationship of Boolean algebra and classical logic, what then is the algebra of this generalized logic? In this talk, we will discuss this question from the point of view of Stone-type dualities.

Based on joint work with Octavian Babus, Adriana Balan and Jiri Velebil.

Eric Pacuit

The Logic of Decisive Sets

Beginning with Arrow's classic Social Choice and Individual Values, the literature on Arrow's Impossibility Theorem has shown the importance for social choice theory of reasoning about decisive coalitions of voters. This talk will discuss a fine-grained analysis of reasoning about decisive coalitions, formalizing how the concept of a decisive coalition gives rise to a social choice theoretic language and logic all of its own. It will be shown that Arrow's axioms of the Independence of Irrelevant Alternatives and Universal Domain (independently of the Pareto principle or weakenings thereof) correspond to strong axioms about decisive coalitions. This correspondence will be demonstrated with results of a kind familiar in economics—representation theorems—as well as results of a kind coming from mathematical logic—completeness theorems. The talk will present a complete logic for reasoning about decisive coalitions, along with formal proofs of Arrow's and Wilson's theorems. The talk will also discuss other logical analysis of Arrow's Theorem and related results.

This is joint work with Wes Holliday.

Dexter Kozen

On Free ω-Continuous and Regular Ordered Algebras

We study varieties of certain ordered Σ-algebras with restricted completeness and continuity properties. We give a general characterization of their free algebras in terms of submonads of the monad of Σ-coterms. Varieties of this form are called quasi-regular. For example, we show that if E is a set of inequalities between finite Σ-terms, and if V_ω and V_r denote the varieties of all ω-continuous ordered Σ-algebras and regular ordered Σ-algebras satisfying E, respectively, then the free V_r-algebra $R(X)$ on generators X is the subalgebra of the corresponding free V_ω-algebra $F_\omega(X)$ determined by those elements of $F_\omega(X)$ denoted by the regular Σ-coterms. This is a special case of a more general construction that applies to any quasi-regular family. Examples include the $*$-continuous Kleene algebras, context-free languages, ω-continuous semirings and ω-continuous idempotent semirings, OI-macro languages, and iteration theories.

Alex Simpson

Modalities of effectful computation

Most real-world computation is "effectful"; that is, it has an effect on and/or is affected by the environment within which it is computed. In the talk, I shall discuss how behavioural properties of effectful computation can be naturally captured using suitably tailored modalities. Different kinds of effect are distinguished through having different modalities associated with them. This leads to a family of modal logics for effectful computation. Such logics provide a framework for reasoning about computation with effects, and also underline the naturalness of "applicative bisimilarity" as a general notion of behavioural equivalence between effectful programs.

This is joint work with Niels Voorneveld.

Contributed Talks

Nino Amiridze, Rusudan Asatiani and Zurab Baratashvili

Loan verb typology and Georgian-English language contact

Based on the typology of verbal borrowings by Wohlgemuth [Woh09] and on the spoken Georgian data coming from various social networks, blogs, discussion forums as well as printed media of the last decade, we overview the ways English items are accommodated in Georgian.

Literature on verb borrowing knows several strategies of loan verb accommodation [Woh09]: direct insertion, indirect insertion, light verb strategy and paradigm insertion. Direct insertion means adapting a form from the donor language into the recipient language without any morphological modification (1a), while indirect insertion implies such a modification tool, usually a verbalizer (1b). Light verb strategy is the use of a native inflected light verb in combination with a borrowed item (1c). Paradigm insertion is a rare case of borrowing of an entire paradigm, like finite verbal paradigm borrowing from Russian in the mixed language Mednyi Aleut [Tho97].

We identify a special accommodation technique for English items as the root of Georgian synthetic verbs, namely, the Georgian preverb *da-*. Apart from widely used forms (2), (3), there are individual uses of all kinds of English material as a root, depending on the creativity of the utterer. For instance, in figurative speech, in marked contexts even English phrases can serve as roots (4a), (4b).

It seems that the preverb *da-* does not fit neatly into the typology of accommodation types above. It cannot be considered as an indirect strategy, since it does not match the cases that define this category in [Woh09]: *da-* is neither a verbalizer (its function is not to accommodate a borrowed verb into a verb of the recipient language), nor a factitive/causative, nor is it special for loan verbs only since it can be used with native ones too. Neither it is a direct insertion, as loans do not get directly inserted in the native Georgian verb frame with just any preverb but require the use of a special one, the preverb *da-*. By default, it can be neither a light verb strategy nor a paradigm insertion.

By form, Georgian preverbs are of two types, simple and complex. The latter is formed via adding the element *mo-* to the former, to refer to the speaker-oriented movement. This particular preverb *da-* is different from any other simple preverb by lacking its complex counterpart (*damo-*) in Modern Georgian [Sha73] and by being more grammaticalized, that is by having acquired an extra grammatical function of expressing distributivity. The use of *da-* in loans could be argued to illustrate a further extension of its grammatical function to a loan verb marker.

(1) a. German, [Woh09, p. 88]
download-en
(Eng.)download-INF
'to download'

b. Modern Greek, [Woh09, p. 96]
tsek-ar-i
(Eng.)check-VBLZ-3SG
'(It) checks.'

c. Korlai Portuguese, [CL15, p. 237]
tray hedze
(Eng.)try do
'try, give a try.'

(2) Georgian, http://forum.tsu.ge/lofiversion/index.php?t9774.html

da-m-serč-e.
PV-O1.SG-search-TAM

'You made an online search on me.'

(3) Georgian, about a movie, from a private conversation

ert-i-or-i siṭq̇v-it moq̇evi... ḳi ar da-a-sp̣oil-o.
one-NOM-two-NOM word-INST you.tell.it PART not PV-PRV-spoil-TAM

'Tell in a word or two... don't spoil it.'

(4) Georgian

a. https://forum.ge/?showtopic=33957881&view=findpost&p=13547826
da-v-sit+daun-d-e-t. (Ironic)
PV-S1.SG-sit+down-INTR-TAM-PL
'Let's sit down.'

b. http://forum.ge/?showtopic=33558967&view=findpost&p=163204

rame eget-s tu naxav xolme,
something like.that-DAT if you.will.see PART
da-gv-i-poṣṭ+linḳ-e. (Individual use)
PV-o1.PL-PRV-post+link-TAM

'If you see something like that, post the link.'

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Curt Anderson and Sebastian Löbner

Roles and the lexical semantics of role-denoting relational adjectives

Introduction Property adjectives such as *green*, *round*, or *old* define a property of their argument; when combined with a noun, the property is attributed to the referent of the noun. Adjectives of this sort can be thought to define classes in their own right, such as *the green ones* or *the round ones*. Relational or classificatory adjectives, such as *nuclear*, *dental*, or *musical*, in contrast, do not form classes in their own right, but are used to define subclasses of the class denoted by a noun alone: *dental surgery* specifies a subclass of surgeries, while *dental assistant* specifies a subclass of assistants according to who they assist or in what area of medicine they assist in. We examine the lexical semantics of one class of relational adjectives, what we call role-denoting relational adjectives (RAs). In examining these adjectives, we advance a model of the semantics where lexical information is available for compositional operations, and argue against recent theories that claim RAs denote properties of kinds.

Examples of role-denoting RAs include but are not limited to *presidential*, *papal*, *senatorial*, *mayoral*, and *royal*. The examples we consider are often derived from nominals. When used attributively, these adjectives have a meaning that is similar but not identical to that of a possessor in a Saxon genitive. To illustrate, (1a) and (2a) have the implication that the referent of the nominal has a relation to the president with respect to their official duties and responsibilities while in office. Although the referents in (1b) and (2b) may be used by the president during their time in office, they do not have an implication that they have any necessary connection to the office itself; the president's desk may be a favorite desk brought to use in a private study, and the president's advisor may refer to an advisor in non-official matters, such as personal finances. In contrast, the presidential desk is the desk used by the president for their official duties while in office, and a presidential advisor is an advisor to the president in the president's official capacity.

(1) a. the presidential desk
 b. the president's desk (i.e., his personal desk)

(2) a. a presidential advisor
 b. the president's advisor (i.e., a personal finance advisor)

This observation holds for event nominals like *visit* as well. The use of the RA implicates that the visit is an official duty of the president. This relation does not obtain when the agent (the president) is represented as a possessor (compare (3a) and (3b)), or with a verbal predication (see (4)).

(3) a. the president's visit (to his mother)
 b. a presidential visit (#to the president's mother)

(4) The president visited his mother $\not\rightsquigarrow$ There was a presidential visit to the president's mother.

Criticism of Previous Theories Recent semantic accounts of relational adjectives have claimed that Carlsonian kinds play a role in their predication. McNally & Boleda [5] propose that RAs are properties of kinds. They assume that a common noun such as *arquitecte* 'architect' (Catalan) has a kind argument (x_k) in addition to an argument for an ordinary individual (y_o). A Carlsonian R relation asserts that the ordinary individual is an instantiation of the kind. Relational adjectives such as *tècnic* 'technical' are interpreted intersectively and predicated of the kind.

(5) a. $[\![arquitecte]\!] = \lambda x_k \lambda y_o [R(y_o, x_k) \wedge \textbf{architect}(x_k)]$
b. $[\![t\grave{e}cnic]\!] = \lambda x_k [\textbf{technical}(x_k)]$

(6) $[\![arquitecte\ t\grave{e}cnic]\!] = \lambda x_k \lambda y_o [R(y_o, x_k) \wedge \textbf{architect}(x_k) \wedge \textbf{technical}(x_k)]$

Arsenijevic et al. [2] extend this account to ethnic adjectives (EAs) such as *French*. These adjectives are also assumed to be predicates of kinds, but encode an additional **Origin** relation that asserts that the kind arises in the nation denoted by the adjective.

(7) $[\![French\ wine]\!] = \lambda y_o \exists x_k [\textbf{wine}(x_k) \wedge \textbf{Origin}(x_k, \textbf{France}) \wedge R(y, x_k)]$

McNally & Boleda and Arsenijevic et al. predict that RAs should be able to be used predicatively when their argument is a kind. But some RAs (such as *medical*) cannot be used predicatively even with kind-referring expressions (such as *doctors/a doctor*) (as in (8b)). Moreover, some RAs (such as *public*) can be used predicatively even when their argument is not a kind (as in (9)). These inconsistencies weaken the case for RAs simply being properties of kinds.

(8) a. Why would someone choose not to become a medical doctor? (Google)
b. *Doctors/*A doctor can be medical.

(9) **This university is public**, but there are also some private universities and colleges on the island.

If relational adjectives are properties of kinds, we might also expect paraphrases using the noun *kind* to be generally available. However, with the *presidential*+N combinations in the table below, paraphrases with *kind* (e.g., *a presidential kind of desk*) are inapplicable even when paraphrases constructed in other ways are possible. This against suggests that *presidential* is not predicating of kinds.

		="presidential kind of N"
presidential election	election of the president (THEME)	(inapplicable paraphrase)
presidential office	office [room] used by the president	(inapplicable paraphrase)
presidential desk	desk used by the president	(inapplicable paraphrase)
presidential visit (1)	visit by the president	(inapplicable paraphrase)
presidential visit (2)	visit to the president	(inapplicable paraphrase)
presidential motorcade	motorcade escorting the president	(inapplicable paraphrase)

Table 1: Paraphrases for *presidential*+N combinations

As demonstrated in (10) and also via the available paraphrases for *presidential*+N in the previous table, RAs can show different relations between the adjective and the modified noun. This calls into question the strategy of specifying a relation internally to the adjective, such as with EAs and **Origin**. And although an **Origin** relation is intuitive for the EA subclass, what relation to use for other classes of RA is not clear, making the strategy difficult to generalize.

(10) a. gynecological clinic (clinic for gynecological problems)
b. gynecological education (education about gynecology)
c. gynecological conference (conference for gynecologists)

Last, we worry about the proliferation of kinds, especially about considering too many utterances to be kind-related and trivializing the notion of what counts as a kind.

What's in a role? Crucial for understanding these types of adjectives exemplified by *presidential* is the concept of a role, and in particular social roles. We consider social roles to belong to a level of social reality distinct from physical reality (cf. Searle's (1995) institutional facts and brute facts), where social roles are defined by certain duties and responsibilities for the role, and with special rights carved out for individuals acting within the context of this role. The individuals that act within the context of a role must not be confused for the role itself; the role is distinct from the individual occupying the role, and the individual need not occupy the role at all moments of their existence.

Individuals may act within a role as well as acting outside of a role. To illustrate this, consider again the example of a president; presidents act within a role when presiding over the affairs of a nation, giving orders to other agents in the government, meeting foreign dignitaries, and so on, but not all actions undertaken individual who is president are actions taken in the context of the role of president. A nap during the day or a lunchtime meal are actions a president often performs as a private person. Accordingly, an action is action within a role only if it is part of the implementation of that role (e.g. if the action is performed with respect to the official duties and responsibilities of the role). Role-related actions must be realized by lower-level physical action (although the two are not identical). A president may enact legislation by signing a document, but the movement of the hand and pen across the paper is only its physical manifestation. This basic philosophical outlook is adopted in our representation.

Proposal The core of the adjective *presidential* is the noun *president*. We represent the concept for *president* as a frame, a structured representation consisting of functional attributes and their values. In our analysis, we lexically decompose the concept for *president* and model *president* as making reference to an event of leading or presiding over an institution or organization (what we label in our representation as preside) where a president is the agent of this event. Evidence for events in the semantics of roles comes from pairs such as *president/presidency*, where *presidency* denotes the event of being president. Examples such as those in (11) support *presidency* denoting an event.

(11) a. Barack Obama's presidency lasted eight years.
b. Because his presidency occurred between those of Grover Cleveland and Theodore Roosevelt, McKinley's accomplishments have often been overlooked. (Google)

This event of presiding is extended over time and has as its subparts the events that a president participates in during the course of their presidency. We assume that these events represent actions that take place as part of and are derived from the responsibilities of leading an organization or nation. Additionally, as the officeholder for the presidency does not stay the same over time, the role of the president as agent is separate from the person implementing the office at some particular time. IMPL maps agents of presiding events to the person implementing that role at some time *i*.

(12) $$\textit{president (of the United States)} \rightsquigarrow \iota x \exists e \begin{bmatrix} \textsc{Impl}_i(\textsc{Agent}(e)) = x \wedge \\ \mathsf{preside}(e) \wedge \textsc{Theme}(e) = \mathbf{USA} \end{bmatrix}$$

An event nominal such as *visit* is analyzed as a predicate of events. A *presidential visit* is modeled as a visit that occurs as part of the preside event from *president*. The event participant role that *president* has in the *visit* event is left unspecified. Rather, we argue that the particular role is inferred from the president's duties and responsibilities while in office. This predicts roles other than agent (such as theme) should be available for examples such as *presidential visit*, contra other accounts of RAs, such as Alexiadou & Stavrou [1], which predict only agents should be possible. (14) shows an example that confirms this prediction, where the theme of the event of visiting (rather than the agent) is the US president.

(13) $$\textit{presidential visit} \rightsquigarrow \lambda e' \exists x \exists e \begin{bmatrix} \textsc{Impl}_i(\textsc{Agent}(e)) = x \wedge \\ \mathsf{preside}(e) \wedge \mathsf{visit}(e') \wedge \underline{e' \sqsubseteq e} \end{bmatrix}$$

(14) Will NBA champions continue to visit the White House under Donald Trump? One of the first players to make **the presidential visit** gives his opinion. (Google)

A non-event nominal is in a relation with the agent of the preside event rather than the event itself. In the case of *presidential desk*, the agent of preside is equated to the be possessor of the desk. This avoids incorrectly attributing possession to the officeholder. We surmise the uniqueness of *presidential desk* is a consequence of the uniqueness of *president*, following observations that possessors determine uniqueness for the noun phrase if the head noun is a functional concept, as is the case here [4].

(15) *(the) presidential desk* $\rightsquigarrow \iota y \exists x \exists e \begin{bmatrix} \textsc{Impl}_i(\textsc{Agent}(e)) = x \wedge \mathsf{preside}(e) \wedge \\ \mathsf{desk}(y) \wedge \underline{\textsc{Possessor}(y) = \textsc{Agent}(e)} \end{bmatrix}$

Lastly, *presidential advisor* is considered to encode an event of advising. The agent of the presiding event from *president* is asserted to be the theme of the advising, again distinguishing assertions about the role of the president from the officeholder at a particular time.

(16) *presidential advisor* $\rightsquigarrow \lambda y \lambda x \exists e \exists e' \begin{bmatrix} \textsc{Impl}_i(\textsc{Agent}(e)) = x \wedge \mathsf{preside}(e) \wedge \mathsf{advise}(e') \wedge \\ \textsc{Agent}(e') = y \wedge \underline{\textsc{Theme}(e') = \textsc{Agent}(e)} \end{bmatrix}$

Our proposal models the observation that an individual may stop inhabiting a role (e.g., a president often stops being president at some point in time) but cannot so easily cease to be an instantiation of a natural kind (cf. Sowa [7], Guarino [3]). This is captured through the IMPL attribute. Additionally, the representation we propose differs crucially from the kind-based analyses we criticize in explicitly analyzing role-denoting RAs as involving reference to an event. Linguistic evidence, such as reference shifts induced by derivational morphology (cf. *president/presidency*), provide support for events at some level of representation. Putting these events into the lexical semantic model allows us to be explicit about how the semantics of the modified nominal interact with these events.

Conclusion In our analysis we distinguish the agent of presiding over an institution/nation from the its implementor. In this way, we can model why the adjective *presidential* predicates of the role corresponding to the president rather than an ordinary individual. This shows that lexical information is vital to understanding attributions with RAs; analyses that expose the lexical semantics of modifiers and modifiees offer a better chance of correctly capturing the fine-grained and manifold meanings found with RAs and how they interface with world knowledge. Our results are discussed in the context of a decompositional theory of lexical meaning that allows for subcompositional processes. And, although we focus on *presidential*, we argue that our results are generalizable to other role-adjectives such as *senatorial*, *papal*, and *royal*, providing additional insight into how natural language represents roles.

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Philippe Balbiani and Jie Fan

A complete axiomatization of Euclidean strong non-contingency logic

1 Introduction

Past decades have witnessed a variety of research on logics with a sole primitive modality that is essentially a combination of another modality and boolean connectives. For instance, in non-contingency logic [10,1,7,8,2,4,5], a formula is noncontingent iff it is either necessarily true or necessarily false, whereas a formula is contingent iff it is possibly true and also possibly false, in symbol, $\Delta\varphi = \Box\varphi \vee \Box\neg\varphi$, $\nabla\varphi = \Diamond\varphi \wedge \Diamond\neg\varphi$; in the logic of essence and accident [9,11], a formula is essential iff once it is true, it is necessarily true, while a formula is accident iff it is true but possibly false, in symbol, $\circ\varphi = \varphi \to \Box\varphi$, $\bullet\varphi = \varphi \wedge \Diamond\neg\varphi$; in the logic for false belief [12], φ is a false belief iff φ is false but believed, in symbol, $W\varphi = \neg\varphi \wedge B\varphi$. Despite being definable with known modalities such as necessity/belief, these modalities have philosophical interests in their own right, and deserve to be studied independently.

Recently, Fan [3] has introduced the notion of *strong non-contingency* by saying that a formula is strongly non-contingent iff it is necessarily true when it is true and it is necessarily false when it is false. This notion is related to Hintikka's treatment of question embedding verbs like 'know', 'remember' in [6]. According to his treatment, "Mary knows (remembers) whether it is raining" is equivalent to "if it is raining, then Mary knows (remembers) it is raining, and if it is not raining, then Mary knows (remembers) it is not raining". Just as necessity means (propositional) knowledge in the setting of epistemic logic, strong non-contingency means knowledge whether in the sense of Hintikka's aforementioned treatment.

As shown in [3], the logic with strong non-contingency as a sole primitive modality, a non-normal modal logic, is less expressive than standard modal logic on various classes of models, and cannot define many usual frame properties including Euclideanity. This may invite technical difficulties and novelties in completely axiomatizing this new logic over various frames. [3] has completely axiomatized different modal logics of strong non-contingency, and leave open the question of the complete axiomatization of the modal logic of strong non-contingency determined by the class of all Euclidean frames. In this note, we answer the open question.

2 Formal definitions

We adopt the notation from [3].

Syntax Let ATO be a countable set of atoms (p, q, etc). The set FOR of all formulas (φ, ψ, etc) is inductively defined as follows:

– $\varphi ::= p \mid \bot \mid \neg\varphi \mid (\varphi \vee \psi) \mid \blacktriangle\varphi$.

The formula $\blacktriangledown\varphi$ is obtained as an abbreviation for $\blacktriangledown\varphi ::= \neg\blacktriangle\varphi$. The formulas φ^0, φ^1 and φ^2 are respectively obtained as abbreviations for $\blacktriangledown\varphi \wedge \blacktriangle\blacktriangle\varphi$, $\varphi \wedge \blacktriangle\varphi$ and $\neg\varphi \wedge \blacktriangle\varphi$.

Semantics A frame is a structure of the form $\mathcal{F} = (W, R)$ where W is a nonempty set of states (s, t, etc) and R is a binary relation on W. A model based on a frame $\mathcal{F} = (W, R)$ is a structure of the form $\mathcal{M} = (W, R, V)$ where V is a valuation on W, i.e. a function associating to each atom p a set $V(p)$ of states. Atoms and Boolean connectives being classically interpreted, we inductively define the truth of $\varphi \in FOR$ in model $\mathcal{M} = (W, R, V)$ at state $s \in W$ ($\mathcal{M}, s \models \varphi$) as follows:

– $\mathcal{M}, s \models \blacktriangle\varphi$ iff if $\mathcal{M}, s \models \varphi$ then $\mathcal{M}, t \models \varphi$ for all states $t \in W$ such that sRt and if $\mathcal{M}, s \not\models \varphi$ then $\mathcal{M}, t \not\models \varphi$ for all states $t \in W$ such that sRt.

Let $\mathcal{F} = (W, R)$ be a frame. We shall say $\varphi \in FOR$ is $\mathcal{F}$-valid ($\mathcal{F} \models \varphi$) iff for all models $\mathcal{M}$ based on $\mathcal{F}$ and for all states $s \in W$, $\mathcal{M}, s \models \varphi$. Let $\mathcal{C}$ be a class of frames. We shall say $\varphi \in FOR$ is $\mathcal{C}$-valid ($\mathcal{C} \models \varphi$) iff for all frames $\mathcal{F}$ in $\mathcal{C}$, $\mathcal{F} \models \varphi$. The logic determined by a class $\mathcal{C}$ of frames ($Log(\mathcal{C})$) is the set of $\varphi \in FOR$ such that $\mathcal{C} \models \varphi$.

Generated subframes The subframe of a frame $\mathcal{F} = (W, R)$ generated by a state $s \in W$ is the restriction of $\mathcal{F}$ to the states in $R^\star(s)$ where $R^\star$ is the reflexive transitive closure of R; in other words, $R^\star = \bigcup_{n \in \mathbb{N}} R^n$. We shall say that a frame $\mathcal{F} = (W, R)$ is generated iff there exists a state $s \in W$ such that $R^\star(s) = W$.

Proposition 1 (Generated Subframe Lemma). *Let $\mathcal{F} = (W, R)$ and $\mathcal{F}' = (W', R')$ be frames. If $\mathcal{F}'$ is a subframe of $\mathcal{F}$ generated by a state $s \in W$ then for all $\varphi \in FOR$, if $\mathcal{F} \models \varphi$ then $\mathcal{F}' \models \varphi$.*

Bounded morphisms Let $\mathcal{F} = (W, R)$ and $\mathcal{F}' = (W', R')$ be frames. A function μ associating to each state $s \in W$ a state $\mu(s) \in W'$ is said to be a bounded morphism from $\mathcal{F}$ to $\mathcal{F}'$ iff the following conditions hold:

– If sRt and $s \neq t$ then $\mu(s)R'\mu(t)$.
– If $\mu(s)R't'$ and $\mu(s) \neq t'$ then there exists a state $t \in W$ such that sRt and $\mu(t) = t'$.

We shall say that $\mathcal{F}'$ is a bounded morphic image of $\mathcal{F}$ iff there exists a surjective bounded morphism from from $\mathcal{F}$ to $\mathcal{F}'$.

Proposition 2 (Bounded Morphism Lemma). *Let $\mathcal{F} = (W, R)$ and $\mathcal{F}' = (W', R')$ be frames. If $\mathcal{F}'$ is a bounded morphic image of $\mathcal{F}$ then for all $\varphi \in FOR$, if $\mathcal{F} \models \varphi$ then $\mathcal{F}' \models \varphi$.*

Euclidean frames A frame $\mathcal{F} = (W, R)$ is said to be Euclidean iff for all states $s, t, u \in W$, if sRt and sRu then tRu and uRt. Let $\mathcal{C}_{euc}$ be the class of all Euclidean frames.

Proposition 3. *Let $\mathcal{F} = (W, R)$ be an Euclidean frame. For all states $s \in W$, $R^\star(s) = \{s\} \cup R(s) \cup R(R(s))$.*

Proof. By the definition of $R^\star$, we need only show

$$(*) \quad \text{for all } n \geq 3, R^n(s) \subseteq R(R(s)).$$

We prove this by induction on $n \geq 3$.

- $n = 3$. Suppose $t \in R^3(s)$, then there exist u_1, u_2 such that sRu_1Ru_2Rt. Since R is Euclidean, u_1Ru_1, and then u_2Ru_1. From this and u_2Rt, by using Euclideanity of R, we obtain u_1Rt, then $t \in R(R(s))$.
- Inductively hypothesize (IH) that $(*)$ holds for $n = k$, we show it also holds for $n = k+1$. Assume $t \in R^{k+1}(s)$, then there is a u such that sR^kuRt. By $u \in R^k(s)$ and IH, we derive $u \in R(R(s))$, and thus $t \in R^3(s)$. By a similar argument as the case $n = 3$, we conclude that $t \in R(R(s))$.

Let A, B and C be pairwise disjoint sets. Let $\mathcal{F}^A_{B,C} = (W^A_{B,C}, R^A_{B,C})$ be the frame such that $W^A_{B,C} = A \cup B \cup C$ and $R^A_{B,C} = \{(s, t)$: either $s \notin A$ and $t \in C$, or $t \in B\}$.

Proposition 4. *Let A, B and C be pairwise disjoint sets. Then $\mathcal{F}^A_{B,C}$ is in $\mathcal{C}_{euc}$.*

Proof. Suppose $\mathcal{F}^A_{B,C}$ is not in $\mathcal{C}_{euc}$. Let $s, t, u \in W^A_{B,C}$ be states such that $sR^A_{B,C}t$, $sR^A_{B,C}u$ and either not $tR^A_{B,C}u$, or not $uR^A_{B,C}t$. Without loss of generality, suppose not $tR^A_{B,C}u$. Since $sR^A_{B,C}t$, therefore either $s \notin A$ and $t \in C$, or $t \in B$. Hence, either $t \in C$, or $t \in B$. Since $sR^A_{B,C}u$, therefore either $s \notin A$ and $u \in C$, or $u \in B$. Thus, either $u \in C$, or $u \in B$. Since not $tR^A_{B,C}u$, therefore either $t \in A$, or $u \notin C$. Moreover, $u \notin B$. Since either $t \in C$, or $t \in B$, therefore $u \notin C$: a contradiction.

Proposition 5. *Let $\mathcal{F} = (W, R)$ be a generated Euclidean frame. There exists pairwise disjoint sets A, B and C such that $\mathcal{F}$ is a bounded morphic image of $\mathcal{F}^A_{B,C}$.*

Proof. Let $s \in W$ be a state such that $R^\star(s) = W$. Since $\mathcal{F}$ is Euclidean, therefore $W = \{s\} \cup R(s) \cup R(R(s))$. Let A, B and C be the pairwise disjoint sets defined as follows:

- $A = \{(s, 0)\}$.
- $B = \{(t, 1) : t \in R(s)\}$.
- $C = \{(u, 2) : u \in R(R(s))\}$.

Let μ be the function associating to each state $x \in W^A_{B,C}$ a state $\mu(x) \in W$ defined as follows:

- $\mu(s, 0) = s$.
- For all $t \in R(s)$, $\mu(t, 1) = t$.
- For all $u \in R(R(s))$, $\mu(u, 2) = u$.

Obviously, μ is a surjective bounded morphism from $\mathcal{F}^A_{B,C}$ to $\mathcal{F}$.

3 Axiomatization

Let $L^{\blacktriangle}_{euc}$ be the least set of formulas closed under the inference rules of modus ponens and uniform substitution, closed under the inference rule $\dfrac{\varphi \to \psi}{\varphi \wedge \blacktriangle\varphi \to \psi \wedge \blacktriangle\psi}$, containing all propositional tautologies and containing the following formulas:

$(K1)$ $\blacktriangle\top$,
$(K2)$ $\blacktriangle\neg p \leftrightarrow \blacktriangle p$,
$(K3)$ $\blacktriangle p \wedge \blacktriangle q \to \blacktriangle(p \wedge q)$,
$(A1)$ $\blacktriangledown(p \wedge q^1) \wedge \blacktriangledown(p \wedge \neg q^1) \to p$,
$(A2)$ $\blacktriangle(p \wedge q^0) \wedge \blacktriangle(p \wedge q^1) \wedge \blacktriangle(p \wedge q^2) \to p \vee \blacktriangle p$.

The inference rule $\dfrac{\varphi \to \psi}{\varphi \wedge \blacktriangle\varphi \to \psi \wedge \blacktriangle\psi}$ and the formulas $(K1)$, $(K2)$ and $(K3)$ have already been considered in [3].

Proposition 6 (Soundness). *For all $\varphi \in FOR$, if $\varphi \in L^{\blacktriangle}_{euc}$ then $\varphi \in Log(\mathcal{C}_{euc})$.*

Proof. Suppose $(A1)$ is not in $Log(\mathcal{C}_{euc})$. Let $\mathcal{F} = (W, R)$ be an Euclidean frame, $\mathcal{M} = (W, R, V)$ be a model based on $\mathcal{F}$ and $s \in W$ be a state such that $\mathcal{M}, s \not\models \blacktriangledown(p \wedge q^1) \wedge \blacktriangledown(p \wedge \neg q^1) \to p$. Hence, $\mathcal{M}, s \models \blacktriangledown(p \wedge q^1)$, $\mathcal{M}, s \models \blacktriangledown(p \wedge \neg q^1)$ and $\mathcal{M}, s \not\models p$. Let $t, u \in W$ be states such that sRt, sRu, $\mathcal{M}, t \models q^1$ and $\mathcal{M}, u \not\models q^1$. Thus, $\mathcal{M}, t \models q$, $\mathcal{M}, t \models \blacktriangle q$ and either $\mathcal{M}, u \not\models q$, or $\mathcal{M}, u \not\models \blacktriangle q$. Since $\mathcal{F}$ is Euclidean, sRt and sRu, therefore tRu. Since $\mathcal{M}, t \models q$ and $\mathcal{M}, t \models \blacktriangle q$, therefore $\mathcal{M}, u \models q$. Since either $\mathcal{M}, u \not\models q$, or $\mathcal{M}, u \not\models \blacktriangle q$, therefore $\mathcal{M}, u \not\models \blacktriangle q$. Since $\mathcal{M}, u \models q$, therefore let $v \in W$ be a state such that uRv and $\mathcal{M}, v \not\models q$. Since $\mathcal{F}$ is Euclidean, sRt, sRu and uRv, therefore tRv. Since $\mathcal{M}, t \models q$ and $\mathcal{M}, t \models \blacktriangle q$, therefore $\mathcal{M}, v \models q$: a contradiction.
Suppose $(A2)$ is not in $Log(\mathcal{C}_{euc})$. Let $\mathcal{F} = (W, R)$ be an Euclidean frame, $\mathcal{M} = (W, R, V)$ be a model based on $\mathcal{F}$ and $s \in W$ be a state such that $\mathcal{M}, s \not\models \blacktriangle(p \wedge q^0) \wedge \blacktriangle(p \wedge q^1) \wedge \blacktriangle(p \wedge q^2) \to p \vee \blacktriangle p$. Hence, $\mathcal{M}, s \models \blacktriangle(p \wedge q^0)$, $\mathcal{M}, s \models \blacktriangle(p \wedge q^1)$, $\mathcal{M}, s \models \blacktriangle(p \wedge q^2)$, $\mathcal{M}, s \not\models p$ and $\mathcal{M}, s \not\models \blacktriangle p$. Let $t \in W$ be a state such that sRt and $\mathcal{M}, t \models p$. Since $\mathcal{M}, s \models \blacktriangle(p \wedge q^0)$, $\mathcal{M}, s \models \blacktriangle(p \wedge q^1)$, $\mathcal{M}, s \models \blacktriangle(p \wedge q^2)$ and $\mathcal{M}, s \not\models p$, therefore $\mathcal{M}, t \not\models p \wedge q^0$, $\mathcal{M}, t \not\models p \wedge q^1$ and $\mathcal{M}, t \not\models p \wedge q^2$. Since $\mathcal{M}, t \models p$, therefore $\mathcal{M}, t \not\models q^0$, $\mathcal{M}, t \not\models q^1$ and $\mathcal{M}, t \not\models q^2$. Thus, $\mathcal{M}, t \not\models \blacktriangle q$ and $\mathcal{M}, t \not\models \blacktriangle\blacktriangle q$. Without loss of generality, suppose $\mathcal{M}, t \models q$. Let $u, v \in W$ be a state such that tRu, tRv, $\mathcal{M}, u \not\models q$ and $\mathcal{M}, v \models \blacktriangle q$. Since $\mathcal{F}$ is Euclidean, sRt, tRu and tRv, therefore vRt and vRu. Since $\mathcal{M}, v \models \blacktriangle q$ and $\mathcal{M}, t \models q$, therefore $\mathcal{M}, v \models q$. Since $\mathcal{M}, v \models \blacktriangle q$, $\mathcal{M}, u \not\models q$ and vRu, therefore $\mathcal{M}, v \not\models q$: a contradiction.

We end this section by giving examples of derivable formulas. Let us consider the following formulas:

$(A3)$ $\blacktriangledown(p \wedge q^0) \wedge \blacktriangledown(p \wedge \neg q^0) \to p$,
$(A4)$ $\blacktriangledown(p \wedge q^2) \wedge \blacktriangledown(p \wedge \neg q^2) \to p$,
$(B1)$ $\blacktriangledown(p \wedge \neg q^0) \to p \vee \blacktriangledown(p \wedge q^1) \vee \blacktriangledown(p \wedge q^2)$,
$(B2)$ $\blacktriangledown(p \wedge \neg q^1) \to p \vee \blacktriangledown(p \wedge q^0) \vee \blacktriangledown(p \wedge q^2)$,

$(B3)$ $\blacktriangledown(p \wedge \neg q^2) \to p \vee \blacktriangledown(p \wedge q^0) \vee \blacktriangledown(p \wedge q^1)$,
(E) $p \to \blacktriangle(p \vee q^0 \vee q^1 \vee q^2)$.

Proposition 7. *$(A3)$, $(A4)$, $(B1)$, $(B2)$, $(B3)$ and (E) are in $L^{\blacktriangle}_{euc}$.*

For all $n \geq 0$ and for all $\alpha : i \in \{1, \ldots, n\} \mapsto \alpha(i) \in \{0, 1, 2\}$, let us consider the following formula:

(C^{α}_n) $\bigwedge\{p_i \wedge \blacktriangledown(\neg p_i \wedge \neg q_i^{\alpha(i)}) : 1 \leq i \leq n\} \to \bigwedge\{\blacktriangledown(\neg p_i \wedge \bigwedge\{\neg q_j^{\alpha(j)} : 1 \leq j \leq n\}) : 1 \leq i \leq n\}$.

Proposition 8. *For all $n \geq 0$ and for all $\alpha : i \in \{1, \ldots, n\} \mapsto \alpha(i) \in \{0, 1, 2\}$, (C^{α}_n) is in $L^{\blacktriangle}_{euc}$.*

4 Completeness

Our proof of the completeness of $L^{\blacktriangle}_{euc}$ is based on maximal consistent sets of formulas where "consistency" means "$L^{\blacktriangle}_{euc}$-consistency". If Γ is a set of formulas then let $\Box\Gamma = \{\varphi : \varphi \wedge \blacktriangle\varphi \in \Gamma\}$. Let Γ_0 be a maximal consistent set of formulas. We consider the following two cases: **(i)** for all $\varphi \in FOR$, $\blacktriangle\varphi \in \Gamma_0$, **(ii)** there exists $\varphi \in FOR$ such that $\blacktriangledown\varphi \in \Gamma_0$. In the former case, let $A = \{(\Gamma_0, 0)\}$, $B = \emptyset$ and $C = \emptyset$. Let $\mathcal{M} = (W^A_{B,C}, R^A_{B,C}, V)$ be the model based on $\mathcal{F}^A_{B,C}$ where V is the valuation on $W^A_{B,C}$ such that for all atoms p, $(\Gamma_0, 0) \in V(p)$ iff $p \in \Gamma_0$.

Proposition 9 (Former case: Truth Lemma). *For all $\varphi \in FOR$, $\mathcal{M}, (\Gamma_0, 0) \models \varphi$ iff $\varphi \in \Gamma_0$.*

Proof. By induction on $\varphi \in FOR$. We only treat the case $\blacktriangle\varphi$. As mentioned, in this case $\blacktriangle\varphi \in \Gamma_0$. Moreover, it is easy to show that $R^A_{B,C} = \emptyset$, from which and the semantics of $\blacktriangle$, it follows immediately that $\mathcal{M}, (\Gamma_0, 0) \models \blacktriangle\varphi$.

In the latter case, let $(\varphi_1, \varphi_2, \ldots)$ be an enumeration, possibly with repetitions, of the set of all $\varphi \in FOR$ such that $\varphi \wedge \blacktriangledown\varphi \in \Gamma_0$. For all $i \geq 1$, let $\Delta_i = \Box\Gamma_0 \cup \{\neg\varphi_i\}$. Let $(\psi_1, \psi_2, \ldots)$ be an enumeration, possibly with repetitions, of FOR. Let $(a_1, a_2, \ldots) \in \{0, 1, 2\}^{\omega}$ be such that for all $n \geq 0$ and for all $i \geq 1$, $\Delta_i \cup \{\psi_1^{a_1} \wedge \cdots \wedge \psi_n^{a_n}\}$ is consistent. For all $i \geq 1$, let Δ'_i be a maximal consistent set of formulas such that for all $n \geq 0$, $\Delta_i \cup \{\psi_1^{a_1} \wedge \ldots \wedge \psi_n^{a_n}\} \subseteq \Delta'_i$. Let $\Delta' = \Delta'_1 \cup \Delta'_2 \cup \ldots$. Let $(\chi_1, \chi_2, \ldots)$ be an enumeration, possibly with repetitions, of the set of all $\chi \in FOR$ such that $\chi \wedge \blacktriangledown\chi \in \Delta'$. For all $i \geq 1$, let Λ'_i be a maximal consistent set of formulas such that $\Box\Delta' \cup \{\neg\chi_i\} \subseteq \Lambda'_i$. Let $A = \{(\Gamma_0, 0)\}$, $B = \{(\Delta'_i, 1) : i \geq 1\}$ and $C = \{(\Lambda'_i, 2) : i \geq 1\}$. Let $\mathcal{M} = (W^A_{B,C}, R^A_{B,C}, V)$ be the model based on $\mathcal{F}^A_{B,C}$ where V is the valuation on $W^A_{B,C}$ such that for all atoms p, $(\Gamma_0, 0) \in V(p)$ iff $p \in \Gamma_0$ and for all $i \geq 1$, $(\Delta'_i, 1) \in V(p)$ iff $p \in \Delta'_i$ and $(\Lambda'_i, 2) \in V(p)$ iff $p \in \Lambda'_i$.

Proposition 10 (Latter case: Truth Lemma). *For all $\varphi \in FOR$, $\mathcal{M}, (\Gamma_0, 0) \models \varphi$ iff $\varphi \in \Gamma_0$ and for all $i \geq 1$, $\mathcal{M}, (\Delta'_i, 1) \models \varphi$ iff $\varphi \in \Delta'_i$ and $\mathcal{M}, (\Lambda'_i, 2) \models \varphi$ iff $\varphi \in \Lambda'_i$.*

Propositions 9 and 10 immediately yield the following result:

Proposition 11 (Completeness). *For all $\varphi \in FOR$, if $\varphi \in Log(\mathcal{C}_{euc})$ then $\varphi \in L^{\blacktriangle}_{euc}$.*

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An axiomatization of iteration-free PDL with loop

1 Introduction

Propositional dynamic logic (PDL) is based on the idea of associating with each program γ a modality $[\gamma]$, $[\gamma]\varphi$ being read "whenever γ terminates it must do so in a state satisfying φ" [6]. Hence, PDL is a modal logic with an algebraic structure in the set of modalities: composition $(\gamma;\delta)$, test $\varphi?$, union $(\gamma\cup\delta)$ and iteration $\gamma^\star$. Additional topics include results about axiomatization and decidability of PDL variants. An interesting variant of PDL is PDL with loop [4]. Its chief feature is that loop of programs is not modally definable in the ordinary language of PDL [9]. In this paper, we present the deductive system of iteration-free PDL with loop.

2 Syntax and semantics

Syntax p ranging over a countable set of propositional variables and π ranging over a countable set of program variables, the set FOR of all formulas (φ, ψ, etc) and the set PRO of all programs (γ, δ, etc) are defined as follows

$$\varphi ::= p \mid \bot \mid [\gamma]\varphi \mid \gamma!,$$

$$\gamma ::= \pi \mid (\gamma;\delta) \mid \varphi?.$$

The other constructs are defined as usual. In particular,

$$\neg\varphi ::= [\varphi?]\bot,$$

$$(\varphi\rightarrow\psi) ::= [\varphi?]\psi,$$

$$\langle\gamma\rangle\varphi ::= [[\gamma][\varphi?]\bot?]\bot.$$

We follow the standard rules for omission of the parentheses.

Semantics A model is a triple (W,R,V) where $W\neq\emptyset$, $R:\ PRO\mapsto 2^{W\times W}$ and $V:\ W\mapsto 2^{FOR}$ are such that
(i) $\bot\not\in V(x)$,
(ii) $[\gamma]\varphi\in V(x)$ iff for all $y\in W$, if $xR(\gamma)y$ then $\varphi\in V(y)$,
(iii) $\gamma!\in V(x)$ iff $xR(\gamma)x$,
(iv) $xR(\gamma;\delta)y$ iff there is $z\in W$ such that $xR(\gamma)z$ and $zR(\delta)y$,
(v) $xR(\varphi?)y$ iff $x=y$ and $\varphi\in V(y)$.
We say φ is m-valid iff for all models (W,R,V) and for all $x\in W$, $\varphi\in V(x)$.

3 Axiomatization

Let $f : PRO \mapsto PRO$ be defined by
(i) $f(\pi) = \pi$,
(ii) $f(\gamma;\delta) = f(\gamma); (\top?; f(\delta))$,
(iii) $f(\varphi?) = \varphi?$.
Let $\dim : PRO \mapsto \mathbb{N}$ be defined by
(i) $\dim(\pi) = 1$,
(ii) $\dim(\gamma;\delta) = \dim(\gamma) + \dim(\delta)$,
(iii) $\dim(\varphi?) = 0$.
Let $\equiv$ be the least equivalence relation on PRO compatible with ; and such that $\gamma; (\delta; \lambda) \equiv (\gamma;\delta); \lambda$. Let $\preceq$ be the least reflexive transitive relation on PRO containing $\equiv$, compatible with ; and such that
(i) if $\dim(\gamma) = 0$ then $\gamma;\delta \preceq \delta$,
(ii) if $\dim(\delta) = 0$ then $\gamma;\delta \preceq \gamma$,
(iii) $\gamma \preceq \gamma; \top?$,
(iv) $\delta \preceq \top?; \delta$,
(v) $\gamma!? \preceq \gamma$,
(vi) if $\dim(\gamma) = 0$ then $\gamma \preceq \gamma!?$,
(vii) $(\varphi \wedge \psi)? \preceq \varphi?; \psi?$.
Let PDL_0^{loop} be the least normal logic that contains the axioms
$(\mathbf{A_1})$ $\langle\gamma;\delta\rangle\varphi \leftrightarrow \langle\gamma\rangle\langle\delta\rangle\varphi$,
$(\mathbf{A_2})$ if $\gamma \equiv \delta$ then $\langle\gamma\rangle\varphi \leftrightarrow \langle\delta\rangle\varphi$,
$(\mathbf{A_3})$ if $\dim(\gamma) = 0$ then $\langle\gamma\rangle\varphi \to \varphi$,
$(\mathbf{A_4})$ if $\gamma \preceq \delta$ then $\langle\gamma\rangle\varphi \to \langle\delta\rangle\varphi$,
$(\mathbf{A_5})$ $\langle\gamma(\varphi?)\rangle\chi \to \langle\gamma((\varphi \wedge \psi)?)\rangle\chi \vee \langle\gamma((\varphi \wedge \neg\psi)?)\rangle\chi$,
$(\mathbf{A_6})$ $\langle\gamma((\varphi \vee \psi)?)\rangle\chi \to \langle\gamma(\varphi?)\rangle\chi \vee \langle\gamma(\psi?)\rangle\chi$,
$(\mathbf{A_7})$ $\varphi \to \neg(\gamma; \neg(\delta; \varphi?; \gamma)!?; \delta)!$.
Obviously, every axiom is m-valid. Hence,

Proposition 1. *Let $\varphi \in FOR$. If $\varphi \in PDL_0^{loop}$ then φ is m-valid.*

A special case of axiom $(\mathbf{A_4})$ is given by the formulas
$(\mathbf{A_4^k})$ $\langle\varphi_1?;\ldots;\varphi_k?\rangle\psi \to \langle(\varphi_1?;\ldots;\varphi_k?)!?\rangle\psi$
where $k \geq 1$. For all $n \in \mathbb{N}$, let $PDL_0^{loop|n}$ be the least normal logic that contains all axioms of PDL_0^{loop} but the formulas $(\mathbf{A_4^k})$ where $k > n$. Obviously, $\bigcup\{PDL_0^{loop|n} : n \in \mathbb{N}\} = PDL_0^{loop}$. Moreover, for all $n \in \mathbb{N}$, one can find a $PDL_0^{loop|n}$-model in which $PDL_0^{loop|n+1}$ does not hold. Hence,

Proposition 2. *Axiom $(\mathbf{A_4})$ cannot be replaced by finitely many formulas.*

4 Theories, large programs and large systems

Theories A theory is any set of formulas containing PDL_0^{loop} and closed under modus ponens. We say a theory S is consistent iff $\bot \notin S$. We say a theory S is maximal iff for all $\varphi \in FOR$, either $\varphi \in S$, or $\neg\varphi \in S$. Let MAX be the set of all maximal theories.

By Lindenbaum's Lemma, for all $\varphi \in FOR$ and for all theories S, if $\varphi \notin S$ then there is $T \in MAX$ such that $S \subseteq T$ and $\varphi \notin T$. If γ is a program and S is a theory then let $[\gamma]S = \{\varphi\text{: } [\gamma]\varphi \in S\}$. The canonical model for PDL_0^{loop} possesses all properties characterizing models but the third one, seeing that loop of programs is not modally definable in the ordinary language of PDL [9]. Hence, following the line of reasoning suggested in [1, 2], the concept of large programs will be used.

Large programs For all theories S, let $S?$ be a new symbol. The set LAR of all large programs (Γ, Δ, etc) is defined by

$$\Gamma ::= \pi \mid (\Gamma; \Delta) \mid S?.$$

We say large program $\Gamma(S_1?, \ldots, S_n?)$ is maximal iff $S_1, \ldots, S_n \in MAX$. Let $\ker : LAR \mapsto 2^{PRO}$ be defined by
(i) $\ker(\pi) = \{\pi\}$,
(ii) $\ker(\Gamma; \Delta) = \{\gamma; \delta\text{: } \gamma \in \ker(\Gamma) \text{ and } \delta \in \ker(\Delta)\}$,
(iii) $\ker(S?) = \{\varphi?\text{: } \varphi \in S\} \cup \{\gamma\text{: } \gamma! \in S\}$.
Let $\dim : LAR \mapsto \mathbb{N}$ be defined by
(i) $\dim(\pi) = 1$,
(ii) $\dim(\Gamma; \Delta) = \dim(\Gamma) + \dim(\Delta)$,
(iii) $\dim(S?) = 0$.
If $\Gamma \in LAR$ and S is a theory then let $[\Gamma]S = \{\varphi : \gamma \in \ker(\Gamma) \text{ and } [\gamma]\varphi \in S\}$. Let $\equiv$ be the binary relation on LAR such that $\Gamma \equiv \Delta$ iff

- for all $\gamma \in \ker(\Gamma)$, if $\dim(\gamma) = \dim(\Gamma)$ then there is $\delta \in \ker(\Delta)$ such that $\dim(\delta) = \dim(\Delta)$ and $\gamma \equiv \delta$,
- for all $\delta \in \ker(\Delta)$, if $\dim(\delta) = \dim(\Delta)$ then there is $\gamma \in \ker(\Gamma)$ such that $\dim(\gamma) = \dim(\Gamma)$ and $\gamma \equiv \delta$.

Let $\preceq$ be the binary relation on LAR such that $\Gamma \preceq \Delta$ iff

- for all $\delta \in \ker(\Delta)$, if $\dim(\delta) = \dim(\Delta)$ then there is $\gamma \in \ker(\Gamma)$ such that $\dim(\gamma) = \dim(\Gamma)$ and $\gamma \preceq \delta$.

Large systems A large system is a triple (W, R, V) where $W \neq \emptyset$ and $R : LAR \mapsto 2^{W \times W}$ and $V : W \mapsto MAX$ are such that
(i) $\bot \notin V(x)$,
(ii) $[\gamma]\varphi \in V(x)$ iff for all $y \in W$, if there is a maximal $\Gamma \in LAR$ such that $f(\gamma) \in \ker(\Gamma)$ and $xR(\Gamma)y$ then $\varphi \in V(y)$,
(iii) $\gamma! \in V(x)$ iff there is a maximal $\Gamma \in LAR$ such that $f(\gamma) \in \ker(\Gamma)$ and $xR(\Gamma)x$,
(iv) $R(\Gamma; S?; \Delta) = \{(x, y)\text{: there is } z \in W \text{ such that } xR(\Gamma)z,\ S \subseteq V(z) \text{ and } zR(\Delta)y\}$,
(v) $R(\Gamma; S?) = \{(x, y)\text{: } xR(\Gamma)y \text{ and } S \subseteq V(y)\}$,
(vi) $R(S?; \Delta) = \{(x, y)\text{: } S \subseteq V(x) \text{ and } xR(\Delta)y\}$,
(vii) $R(S?) = \{(x, y)\text{: } x = y \text{ and } S \subseteq V(y)\}$,
(viii) if $\Gamma \preceq \Delta$ then $R(\Gamma) \subseteq R(\Delta)$.
We say φ is ls-valid iff for all large systems (W, R, V) and for all $x \in W$, $\varphi \in V(x)$.
Obviously, every large system corresponds to a model. Hence,

Proposition 3. *Let $\varphi \in FOR$. If φ is m-valid then φ is ls-valid.*

5 Subordination models

A subordination model is a triple (W, R, V) where $W \neq \emptyset$ and $R :\ LAR \mapsto 2^{W \times W}$ and $V :\ W \mapsto MAX$ are such that
(i) $\perp \notin V(x)$,
(ii) if $[\gamma]\varphi \in V(x)$ then for all $y \in W$, if there is a maximal $\Gamma \in LAR$ such that $f(\gamma) \in \ker(\Gamma)$ and $xR(\Gamma)y$ then $\varphi \in V(y)$,
(iii) $\gamma! \in V(x)$ iff there is a maximal $\Gamma \in LAR$ such that $f(\gamma) \in \ker(\Gamma)$ and $xR(\Gamma)x$,
(iv) $R(\Gamma; S?; \Delta) \supseteq \{(x, y)$: there is a $z \in W$ such that $xR(\Gamma)z$, $S \subseteq V(z)$ and $zR(\Delta)y\}$,
(v) $R(\Gamma; S?) = \{(x, y)$: $xR(\Gamma)y$ and $S \subseteq V(y)\}$,
(vi) $R(S?; \Delta) = \{(x, y)$: $S \subseteq V(x)$ and $xR(\Delta)y\}$,
(vii) $R(S?) = \{(x, y)$: $x = y$ and $S \subseteq V(y)\}$,
(viii) if $\Gamma \preceq \Delta$ then $R(\Gamma) \subseteq R(\Delta)$.
We say φ is sm-valid iff for all subordination models (W, R, V) and for all $x \in W$, $\varphi \in V(x)$. Obviously, for all consistent $S \in MAX$, the triple (W, R, V) where $W = \{S\}$, $SR(\Gamma)S$ iff $S? \preceq \Gamma$ and $V(S) = S$ is a subordination model. Hence,

Proposition 4. *Let $\varphi \in FOR$. If φ is sm-valid then $\vdash_{PDL_0^{loop}} \varphi$.*

Given a subordination model (W, R, V), it may contain imperfections:
(i) triples (γ, φ, x) where $\gamma \in PRO$, $\varphi \in FOR$ and $x \in W$ are such that $[\gamma]\varphi \notin V(x)$ and for all $y \in W$, if there is a maximal $\Gamma \in LAR$ such that $f(\gamma) \in \ker(\Gamma)$ and $xR(\Gamma)y$ then $\varphi \in V(y)$,
(ii) 5-tuples $(\Gamma, S, \Delta, x, y)$ where $\Gamma, \Delta \in LAR$ are maximal, $S \in MAX$ and $x, y \in W$ are such that $xR(\Gamma; S?; \Delta)y$ and for all $z \in W$, either $x\overline{R(\Gamma)}z$, or $S \not\subseteq V(z)$, or $z\overline{R(\Delta)}y$.
An imperfection (γ, φ, x) can be repaired by adding a new element y to W and by extending the functions R and V in such a way that y will be γ-reachable from x and y will not satisfy φ whereas an imperfection $(\Gamma, S, \Delta, x, y)$ can be repaired by adding a new element z to W and by extending the functions R and V in such a way that z will be Γ-reachable from x, y will be Δ-reachable from z and z will satisfy S. The heart of our method consists in step-by-step repairing all these imperfections, therefore transforming every subordination model into an equivalent large system. Hence,

Proposition 5. *Let $\varphi \in FOR$. If φ is ls-valid then φ is sm-valid.*

From Proposition 1 and Propositions 3–5, we obtain the following

Theorem 1. *Let $\varphi \in FOR$. The following conditions are equivalent:*
(i) $\vdash_{PDL_0^{loop}} \varphi$,
(ii) *φ is m-valid,*
(iii) *φ is ls-valid,*
(iv) *φ is sm-valid.*

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A lattice of subminimal logics of negation

Minimal propositional calculus (MPC) is the system obtained from the positive fragment of intuitionistic propositional calculus (equivalently, positive logic [12]) by adding a unary negation operator satisfying the so-called principle of contradiction. This system was introduced by Johansson in 1937 [10] (even before, by Kolmogorov [11]) by discarding *ex falso quodlibet* from the standard axioms for intuitionistic logic.

The aim of this work is to focus on the bounded lattice of propositional logical systems arising from the language of minimal logic and obtained by weakening the requirements for the negation operator in a 'maximal way'. More precisely, the bottom element of this lattice of logics is a system where the unary operator $\neg$ has no properties at all, except the property of being functional; the top element is minimal logic. We use the term *N-logic* to denote an arbitrary logical system in this lattice. The setting is paraconsistent, in the sense that contradictory theories do not necessarily contain all formulas.

Some of these subsystems of intuitionistic logic have been studied in [7], with focus on their syntax as well as on the corresponding relational structures (e.g., their Kripke semantics). In this abstract we take the first steps towards a uniform treatment of this family of logical systems by developing their algebraic semantics. We also introduce descriptive frames for these systems and prove that every logic in this lattice is complete with respect to these descriptive frames. These results allow us to export the techniques of [8, 9, 3] to our setting and, in particular, to prove the existence of continuum many N-logics.

Given the language of positive logic (equivalently, the language of intuitionistic logic with neither negation nor $\bot$) over countably many propositional variables, we consider the axioms of positive logic and a unary operator $\neg$ satisfying the additional axiom $(p \leftrightarrow q) \to (\neg p \leftrightarrow \neg q)$. We call the resulting system N. We keep a fixed positive logical fragment, and we strengthen the negation operator up to reaching minimal propositional logic, which can be seen in this language as the system obtained by adding the axiom $(p \to q) \wedge (p \to \neg q) \to \neg p$ to positive logic [12]. Note that another axiomatization of minimal logic is obtained by extending N with the axiom $(p \to \neg p) \to \neg p$ [6, Proposition 1.2.5].

From an algebraic point of view, we deal with relatively pseudo-complemented lattices (which algebraically characterize positive logic [12]) equipped with a unary operation $\neg$ satisfying the equation $(x \leftrightarrow y) \to (\neg x \leftrightarrow \neg y) \approx 1$. Observe that the latter can be equivalently formulated as

$$x \wedge \neg y \approx x \wedge \neg(x \wedge y).$$

We denote the variety of these algebras as $\mathcal{NA}$ and we call these structures *N-algebras*. Using the standard argument we can show that every N-logic L is complete with respect to a variety of N-algebras in which all the theorems of L are valid. The least variety among the ones we are considering, corresponding to minimal logic, is the one of contrapositionally complemented lattices [12].

Next we discuss a uniform frame-based completeness result for every N-logic. In order to do this, we introduce the notion of *top descriptive frame*: a top descriptive frame is a quadruple

$\mathfrak{F} = \langle W, R, \mathcal{P}, N \rangle$, where $\langle W, R \rangle$ is a partial order with a top node t, the set $\mathcal{P}$ is a family of admissible upsets as in the intuitionistic case [5, 3] with the difference that the top element t must be contained in every admissible upset, and $N : \mathcal{P} \to \mathcal{P}$ is a map satisfying, for all $U, V \in \mathcal{P}$,

$$U \cap N(V) = U \cap N(U \cap V). \tag{1}$$

Observe that the notion of admissible upset in this setting excludes the empty set. The positive reducts of these frame structures are presented topologically in [2] as *pointed Esakia spaces.*

It can be proved [6] that for every N-algebra there is a corresponding dual top descriptive frame, and vice versa. More precisely, given a top descriptive frame $\mathfrak{F}$, the structure

$$\mathfrak{F}^* = \langle \mathcal{P}, \cap, \cup, \to, W, N \rangle,$$

where $\to$ is the Heyting implication, is the N-algebra dual to $\mathfrak{F}$. On the other hand, the set of prime filters of any N-algebra $\mathfrak{A} = \langle A, \wedge, \vee, \to, 1, \neg \rangle$ induces a dual top descriptive frame $\mathfrak{A}_*$ defining the map N as $N(\hat{a}) = \widehat{(\neg a)}$ for every element a of $\mathfrak{A}$, where $\hat{a}$ is the upset of all prime filters containing a. Observe that the notion of prime filter in this context does not require the filter to be proper, i.e., the whole algebra A is always a prime filter, and this ensures the corresponding frame structure to have a top element.

Every N-logic L is complete with respect to the corresponding class of N-algebras. Given the afore-sketched duality between N-algebras and top descriptive frames, completeness of every N-logic with respect to the corresponding class of top descriptive frames (i.e., the ones dual to the corresponding class of N-algebras) follows easily. As in the case of Heyting algebras, for N-algebras there exists a one-to-one correspondence between congruences and filters. We can therefore characterize subdirectly irreducible N-algebras as those N-algebras containing a second greatest element, thereby obtaining a completeness result of L with respect to the class of finitely generated rooted top descriptive frames.

We conclude by proving the existence of continuum many N-logics in the interval $[\mathsf{N}, \mathsf{MPC}]$. We consider a countable family of formulas without negation that can be used to define independent systems enhancing the basic logic N, and we adapt them to ensure the logics we obtain to be subsystems of minimal propositional logic. Given a top descriptive frame and a persistent valuation map on admissible upsets, truth of a formula $\neg\varphi$ at a node w is equivalent to w being an element of the upset $N(V(\varphi))$. In what follows, we call a *top frame* a partial order with a top node; following [4], we call *top model* an intuitionistic Kripke model whose underlying frame is a top frame and such that every propositional variable is true at the top node.

Let $\Delta = \{\mathfrak{F}_n : n \in \omega\}$ be an infinite set of finite rooted top frames (see Figure 1), which forms an antichain with respect to the frame order [3] $\leq$ defined by: $\mathfrak{F} \leq \mathfrak{G}$ if and only if $\mathfrak{F}$ is an order-preserving image of a generated subframe of $\mathfrak{G}$ ([1, Lemma 6.12]; in fact, the antichain of [1] does not contain top frames, but it is easy to see that adding the top nodes still makes the sequence Δ a $\leq$-antichain). Recall (e.g., [5, 4]) that a positive morphism is a partial p-morphism f such that $\mathrm{dom}(f)$ is a downset, and consider the relation $\preceq$ defined by: $\mathfrak{F} \preceq \mathfrak{G}$ if and only if $\mathfrak{F}$ is an image, via a positive morphism, of a generated subframe of $\mathfrak{G}$. Now observe that, if $\mathfrak{F} \preceq \mathfrak{G}$ and $\mathfrak{F}$ is a top frame, then $\mathfrak{F} \leq \mathfrak{G}$. Indeed, assume $\mathfrak{F} \preceq \mathfrak{G}$ and let f be a partial p-morphism from a generated subframe $\mathfrak{G}'$ of $\mathfrak{G}$ onto $\mathfrak{F}$. Extending f by mapping all the points of $\mathfrak{G}' \setminus \mathrm{dom}(f)$ to the top node of $\mathfrak{F}$, we obtain a total order-preserving map, yielding $\mathfrak{F} \leq \mathfrak{G}$. Thus, every $\leq$-antichain is a $\preceq$-antichain, and so Δ is a $\preceq$-antichain.

Having constructed the desired $\preceq$-antichain, we will now proceed by adjusting the technique of Jankov-de Jongh formulas to obtain a continuum of logics in between N and MPC. Every finite rooted top frame $\mathfrak{F}_n$ equipped with an appropriate valuation V_n can be mapped as a top model

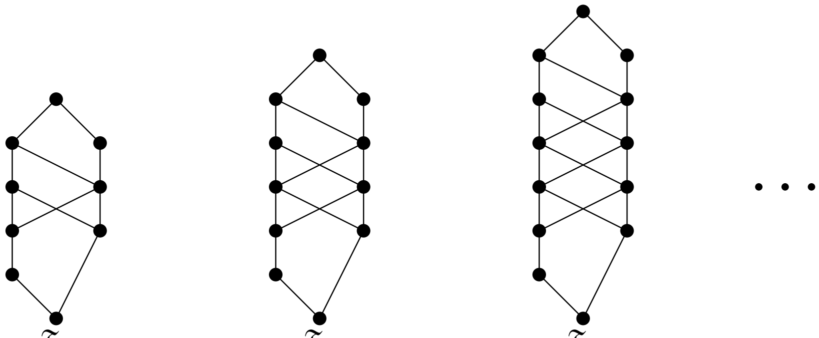

Figure 1: The antichain Δ

onto a generated submodel of $(\mathcal{U}^\star(2))^+$ via a p-morphism [4]. Consider the world $w_n \in (\mathcal{U}^\star(2))^+$ in the positive universal model corresponding to the root of $\mathfrak{F}_n$ in the sense described above. We can assume without losing generality the positive Jankov-de Jongh formula of $\mathfrak{F}_n$ to be defined as follows [4]:

$$\chi^\star(\mathfrak{F}_n) = \psi^\star_{w_n} := \varphi^\star_{w_n} \to \bigvee_{i=1}^{r} \varphi^\star_{w_{n_i}} ,$$

where $\varphi^\star_{w_n}, \varphi^\star_{w_{n_i}}$ are defined as in [4] and $w_n \prec \{w_{n_1}, \ldots, w_{n_r}\}$. So, a descriptive frame $\mathfrak{G}$ refutes $\chi^\star(\mathfrak{F}_n)$ if and only if $\mathfrak{F}_n \preceq \mathfrak{G}$. As Δ is a $\preceq$-antichain, this means that, for every $n, m \in \omega$, the formula $\chi^\star(\mathfrak{F}_m)$ is valid on the frame $\mathfrak{F}_n$ if and only if $n \neq m$. In fact, it is easy to see that $\varphi^\star_{w_n}$ is satisfied at the root w_n in $(\mathcal{U}^\star(2))^+$, while none of the formulas $\varphi^\star_{w_{n_i}}$ are.

Now, we equip each frame $\mathfrak{F}_n$ with an appropriate function N_n to make it a top descriptive frame such that $N_n(\{t\}) = \{t\}$, where t is the top node of $\mathfrak{F}_n$. We denote the new family of top descriptive frames $\langle \mathfrak{F}_n, N_n \rangle$ by Δ_N. We consider a new propositional variable p and let

$$\theta(\mathfrak{F}_n) = (p \to \neg p) \wedge \varphi^\star_{w_n} \to \neg p \vee \bigvee_{i=1}^{r} \varphi^\star_{w_{n_i}} .$$

It is easy to see that, if $n \neq m$, the formula $\theta(\mathfrak{F}_n)$ is valid on the frame $\langle \mathfrak{F}_m, N_m \rangle$. On the other hand, for checking that $\langle \mathfrak{F}_n, N_n \rangle \not\models \theta(\mathfrak{F}_n)$ it is enough to consider a valuation $\tilde{V}_n$ enhancing V_n in such a way that $\tilde{V}_n(p) = \{t\}$. In this way, the root of $\mathfrak{F}_n$ under the considered valuation makes the whole antecedent of $\theta(\mathfrak{F}_n)$ true, while the consequent is not true at w_n. We note that the formulas $\theta(\mathfrak{F}_n)$ are not the Jankov-de Jongh formulas for the considered signature; in fact, $\theta(\mathfrak{F}_n)$ has the defining property of the Jankov-de Jongh formulas for the signature of positive logic with an extra addition that $\theta(\mathfrak{F}_n)$ is a theorem of MPC. The latter ensures that for each subset $\Gamma \subseteq \Delta_N$, the logic $L(\Gamma) = \mathsf{N} + \{\theta(\mathfrak{F}) : \mathfrak{F} \in \Gamma\}$ belongs to the interval $[\mathsf{N}, \mathsf{MPC}]$ i.e., $\mathsf{N} \subseteq L(\Gamma) \subseteq \mathsf{MPC}$. In particular, the logics $L(\Gamma)$ share the same positive fragment.

Finally, observe that for each pair of different subsets $\Gamma_1 \neq \Gamma_2$ of Δ_N, we have $L(\Gamma_1) \neq L(\Gamma_2)$. Indeed, without loss of generality we may assume that there is $\mathfrak{F} \in \Gamma_1$ such that $\mathfrak{F} \notin \Gamma_2$. Moreover, we have $\mathfrak{F} \not\models \theta(\mathfrak{F})$ and $\mathfrak{F} \models \theta(\mathfrak{G})$, for each $\mathfrak{G}$ in Γ_2. Therefore, there is a top descriptive frame $\mathfrak{F}$ which is an $L(\Gamma_2)$-frame and not an $L(\Gamma_1)$-frame. Since every N-logic is complete with respect to top descriptive frames, the latter entails that $L(\Gamma_1) \neq L(\Gamma_2)$. As a consequence, we obtain uncountably many distinct N-logics.

Theorem 1. *There are continuum many logics in the interval* $[\mathsf{N}, \mathsf{MPC}]$.

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Two Neighborhood Semantics for Subintuitionistic Logics

In [7] we defined two neighborhood semantics for subintuitionistic logics. NB-semantics, our main semantics, is for most purposes best suitable to study the basic logic and its extensions. The N-semantics is closer to the usual neighborhood semantics for modal logics, and is thereby more suitable to study Gödel-type translations into modal logics. The relationship between the two semantics remained unclear. Our basic logic WF is sound and complete for NB-semantics and sound for N-semantics but completeness remained an open issue. Here we clear up their relationship. We introduce a new rule N, which added to WF gives a system $\mathsf{WF_N}$ complete for N-semantics. Two new axioms, falsifiable in NB-semantics, can be derived from it. Gödel-type translations into modal logic can now be realized properly.

Definition 1. *$\mathfrak{F} = \langle W, g, NB, \mathcal{X}\rangle$ is called an **NB-Neighborhood Frame** of subintuitionistic logic if $W \neq \emptyset$ and $\mathcal{X}$ is a non-empty collection of subsets of W such that $\emptyset$ and W belong to $\mathcal{X}$, and $\mathcal{X}$ is closed under $\cup$, $\cap$ and $\to$ defined by*

$$U \to V := \{w \in W \mid (U, V) \in NB(w)\},$$

where NB is a function from W into $\mathcal{P}(\mathcal{X}^2)$ such that:

1. $\forall w \in W,\ \forall X, Y \in \mathcal{X},\ (X \subseteq Y \ \Rightarrow\ (X, Y) \in NB(w))$,
2. $NB(g) = \big\{(X, Y) \in \mathcal{X}^2 \mid X \subseteq Y\big\}$ *(g is called **omniscient**).*

*In an **NB-Neighborhood Model** $\mathfrak{M} = \langle W, g, NB, \mathcal{X}, V\rangle$, $V\colon At \to \mathcal{X}$ is a valuation function on the set of propositional variables At.*

***Truth** of A in w, $w \Vdash A$ is defined as usual except for: $\mathfrak{M}, w \Vdash A \to B \ \Leftrightarrow\ (A^{\mathfrak{M}}, B^{\mathfrak{M}}) \in NB(w)$, where $A^{\mathfrak{M}} := \{w \in W \mid \mathfrak{M}, w \Vdash A\}$.*

Definition 2. *$\mathfrak{F} = \langle W, g, N, \mathcal{X}\rangle$ is an **N-Neighborhood Frame** if W is a non-empty set and $\mathcal{X}$ is a non-empty collection of subsets of W such that $\emptyset$ and W belong to $\mathcal{X}$ and $\mathcal{X}$ is closed under $\cup$, $\cap$ and $\to$ defined by*

$$U \to V := \big\{w \in W \mid \overline{U} \cup V \in N(w)\big\},$$

*where N is a function from W into $\mathcal{P}(\mathcal{X})$, $g \in W$, for each $w \in W$, $W \in N(w)$, $N(g) = \{W\}$ (g is called **omniscient**). Valuation $V : At \to \mathcal{X}$ makes $\mathfrak{M} = \langle W, g, N, \mathcal{X}, V\rangle$ an **N-Neighborhood Model** with the clause:*

$$\mathfrak{M}, w \Vdash A \to B \Leftrightarrow \{v \mid v \Vdash A \Rightarrow v \Vdash B\} = \overline{A^{\mathfrak{M}}} \cup B^{\mathfrak{M}} \in N(w).$$

Definition 3. WF *is the logic given by the following axioms and rules,*

1. $A \to A \vee B$	2. $B \to A \vee B$	3. $A \to A$
4. $A \wedge B \to A$	5. $A \wedge B \to B$	6. $\frac{A \quad A\to B}{B}$
7. $\frac{A\to B \quad A\to C}{A\to B\wedge C}$	8. $\frac{A\to C \quad B\to C}{A\vee B\to C}$	9. $\frac{A\to B \quad B\to C}{A\to C}$
10. $\frac{A}{B\to A}$	11. $\frac{A\leftrightarrow B \quad C\leftrightarrow D}{(A\to C)\leftrightarrow(B\to D)}$	12. $\frac{A \quad B}{A\wedge B}$
13. $A \wedge (B \vee C) \to (A \wedge B) \vee (A \wedge C)$		14. $\bot \to A$

To the system WF *we add the rule* N *to obtain the logic* $\mathsf{WF_N}$*:*

$$\frac{C \to A \vee D \qquad A \wedge C \wedge B \to D}{(A \to B) \to (C \to D)} \qquad (\mathsf{N})$$

A rule like N is considered to be valid on a frame $\mathfrak{F}$ if, on each $\mathfrak{M}$ on which the premises of the rule are valid, the conclusion is valid as well.

Lemma 1. *(Soundness of* $\mathsf{WF_N}$*)* N *is valid on* N*-neighborhood frames.*

Proof. Recall that, by Theorem 2.13(1) of [7], for all E, F, $\mathfrak{M} \Vdash E \to F$ iff $E^{\mathfrak{M}} \subseteq F^{\mathfrak{M}}$.

Assume, (1) $\mathfrak{M} \Vdash C \to A \vee D$, i.e. $C^{\mathfrak{M}} \subseteq A^{\mathfrak{M}} \cup D^{\mathfrak{M}}$, and (2) $\mathfrak{M} \Vdash A \wedge C \wedge B \to D$, i.e. $A^{\mathfrak{M}} \cap C^{\mathfrak{M}} \cap B^{\mathfrak{M}} \subseteq D^{\mathfrak{M}}$. It will suffice to prove that $\overline{A^{\mathfrak{M}}} \cup B^{\mathfrak{M}} \subseteq \overline{C^{\mathfrak{M}}} \cup D^{\mathfrak{M}}$.

Let $w \in \overline{A^{\mathfrak{M}}} \cup B^{\mathfrak{M}}$. Then $w \in \overline{A^{\mathfrak{M}}}$ or ($w \in A^{\mathfrak{M}}$ and $w \in B^{\mathfrak{M}}$). If $w \in \overline{A^{\mathfrak{M}}}$, we distinguish the cases $w \in D^{\mathfrak{M}}$ and $w \in \overline{D^{\mathfrak{M}}}$. In the first case we are done directly. In the second case, we can conclude from (1) that $w \in \overline{C^{\mathfrak{M}}}$ and we are done as well. If $w \in A^{\mathfrak{M}}$ and $w \in B^{\mathfrak{M}}$, we distinguish the cases $w \in \overline{C^{\mathfrak{M}}}$ and $w \in C^{\mathfrak{M}}$. In the first case we are done directly. In the second case, we can conclude from (2) that $w \in D^{\mathfrak{M}}$ and we are done as well.

□

Definition 4. *A set of sentences* Δ *is a* ***prime theory*** *if and only if*

- $A, B \in \Delta \;\Rightarrow\; A \wedge B \in \Delta$,
- $\vdash A \to B$ and $A \in \Delta \;\Rightarrow\; B \in \Delta$,
- $\vdash A \;\Rightarrow\; A \in \Delta$,
- $A \vee B \in \Delta \;\Rightarrow\; A \in \Delta$ or $B \in \Delta$.

Lemma 2. $\mathsf{WF_N}$ *is a prime theory (has the disjunction property).*

Proof. Using Kleene's $|$ ([6]) as in [7], Theorem 2.12. □

Definition 5. *Let* $W_{\mathsf{WF_N}}$ *be the set of all consistent prime theories of* $\mathsf{WF_N}$*. Given a formula* A*, we define* $\llbracket A \rrbracket = \{\Delta \mid \Delta \in W_{\mathsf{WF_N}},\ A \in \Delta\}$*. The* ***N-Canonical model*** $\mathfrak{M}_{\mathsf{WF_N}} = \langle W, g, N, \mathcal{X}, V\rangle$ *is defined by:*

- $W = W_{\mathsf{WF_N}}$,
- $g = \mathsf{WF_N}$,
- *For each* $\Gamma \in W$, $N(\Gamma) = \{\overline{\llbracket A \rrbracket} \cup \llbracket B \rrbracket \mid A \to B \in \Gamma\}$,
- $\mathcal{X}$ *is the set of all* $\llbracket A \rrbracket$,
- *If* $p \in At$*, then* $V(p) = \llbracket p \rrbracket = \{\Gamma \mid \Gamma \in W$ *and* $p \in \Gamma\}$.

Lemma 3. (Truth Lemma) *In the N-canonical Model* $\mathfrak{M}_{\mathsf{WF_N}}$*,* $A \in \Gamma$ *iff* $\Gamma \Vdash A$.

Proof. The crucial part of the proof is showing that, if $\overline{\llbracket A \rrbracket} \cup \llbracket B \rrbracket = \overline{\llbracket C \rrbracket} \cup \llbracket D \rrbracket$, then $\mathsf{WF_N} \vdash (A \to B) \leftrightarrow (C \to D)$. So, assume $\overline{\llbracket A \rrbracket} \cup \llbracket B \rrbracket = \overline{\llbracket C \rrbracket} \cup \llbracket D \rrbracket$. It suffices to show (1) $\mathsf{WF_N} \vdash A \to B \vee C$, $\mathsf{WF_N} \vdash A \wedge C \wedge D \to B$ and (2) $\mathsf{WF_N} \vdash C \to A \vee D$, $\mathsf{WF_N} \vdash A \wedge C \wedge B \to D$. We will show (1); (2) is analogous.

From $\overline{\llbracket A \rrbracket} \cup \llbracket B \rrbracket = \overline{\llbracket C \rrbracket} \cup \llbracket D \rrbracket$ we get $\llbracket A \rrbracket \cap \overline{\llbracket B \rrbracket} = \llbracket C \rrbracket \cap \overline{\llbracket D \rrbracket}$. We have $\llbracket A \rrbracket \subseteq \llbracket B \rrbracket \cup \llbracket A \rrbracket$, so also, $\llbracket A \rrbracket \subseteq \llbracket B \rrbracket \cup (\llbracket A \rrbracket \cap \overline{\llbracket B \rrbracket})$, This means that $\llbracket A \rrbracket \subseteq \llbracket B \rrbracket \cup (\llbracket C \rrbracket \cap \overline{\llbracket D \rrbracket})$, so $\llbracket A \rrbracket \subseteq \llbracket B \rrbracket \cup \llbracket C \rrbracket$. Therefore, $A \to B \vee C \in g$, so $\mathsf{WF_N} \vdash A \to B \vee C$.

Again using $\llbracket A \rrbracket \cap \overline{\llbracket B \rrbracket} = \llbracket C \rrbracket \cap \overline{\llbracket D \rrbracket}$, we get $\llbracket A \rrbracket \cap \llbracket C \rrbracket \cap \llbracket D \rrbracket \cap \overline{\llbracket B \rrbracket} = \llbracket A \rrbracket \cap \overline{\llbracket B \rrbracket} \cap \llbracket C \rrbracket \cap \llbracket D \rrbracket = \llbracket C \rrbracket \cap \overline{\llbracket D \rrbracket} \cap \llbracket C \rrbracket \cap \llbracket D \rrbracket = \emptyset$. So, $\llbracket A \rrbracket \cap \llbracket C \rrbracket \cap \llbracket D \rrbracket \subseteq \llbracket B \rrbracket$, and, reasoning as above, $\mathsf{WF_N} \vdash A \wedge C \wedge D \to B$.

□

Theorem 1. (Completeness of $\mathsf{WF_N}$) $\Sigma \vdash_{\mathsf{WF_N}} A$ *iff for all* $w \in \mathfrak{M}_{\mathsf{WF_N}}$*, if* $w \Vdash \Sigma$*, then* $w \Vdash A$.

Lemma 4. $\mathsf{WF_N} \vdash (A \to B) \leftrightarrow (A \vee B \to B)$, $\quad \mathsf{WF} \nvdash (A \to B) \leftrightarrow (A \vee B \to B)$.
$\mathsf{WF_N} \vdash (A \to B) \leftrightarrow (A \to A \wedge B)$, $\quad \mathsf{WF} \nvdash (A \to B) \leftrightarrow (A \to A \wedge B)$.

The exact relationship between the axioms of Lemma 4 and rule N is unclear. We can derive the axioms of Lemma 4 from WF+N, but the other direction seems unlikely, probably N is not derivable from WF + the axioms of Lemma 4.

We can now extend the translation results of Corsi [3] and others [4, 1] on subintuitionistic logics into modal logics to weaker logics. We consider the translation $\Box$ from $\mathcal{L}$, the language of IPC, to $\mathcal{L}_\Box$, the language of modal propositional logic. It is given by:

1. $p^{\Box} = p$;
2. $(A \wedge B)^{\Box} = A^{\Box} \wedge B^{\Box}$;
3. $(A \vee B)^{\Box} = A^{\Box} \vee B^{\Box}$;
4. $(A \to B)^{\Box} = \Box(A^{\Box} \to B^{\Box})$.

Theorem 2. *For all formulas A, $\mathsf{WF_N} \vdash A$ iff $\mathsf{EN} \vdash A^{\Box}$.*
For all formulas A, $\mathsf{WF_N I_R I_L} \vdash A$ iff $\mathsf{M} \vdash A^{\Box}$.

Here *classical modal logic* E, based on $\frac{A \leftrightarrow B}{\Box A \leftrightarrow \Box B}$, is the smallest non-normal modal logic, and EN extends E by adding necessitation. Also a system of modal logic is *monotonic* iff it is closed under RM ($\frac{A \to B}{\Box A \to \Box B}$), and M is the smallest monotonic modal logic [2, 5]. In [7], $\mathsf{I_L}$ is the rule $\frac{A \to B}{(C \to A) \to (C \to B)}$ and $\mathsf{I_R}$ is the rule $\frac{A \to B}{(B \to C) \to (A \to C)}$. In the meantime we have been able to show that rule $\mathsf{I_L}$ is equivalent to the axiom $\check{\mathsf{C}}$: $(A \to B \wedge C) \to (A \to B) \wedge (A \to C)$, and rule $\mathsf{I_R}$ to the axiom $\check{\mathsf{D}}$: $(A \vee B \to C) \to (A \to C) \wedge (B \to C)$.

In [7] the relationship between the logic WF and the non-normal modal logic EN was already indicated. But because of the difference of the models we were able to prove only the direction $\vdash_{\mathsf{WF}} A \Rightarrow \vdash_{\mathsf{EN}} A$. A similar situation arose between the basic monotonic modal logic M and our system $\mathsf{WFI_R I_L}$.

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Finite Identification with positive and with complete data

1 Introduction

The groundbreaking work of Gold (3) from 1967 started a new era for developing mathematical and computational frameworks for studying the formal process of *learning*. Gold's model, *identification in the limit*, has been studied for learning recursive functions, recursively enumerable languages, and recursive languages with *positive data* and with *complete data*. The learning task consists of identifying languages as members of a family of languages, the learning function can output infinitely many conjectures but they need to stabilize in one permanent conjecture. In Gold's model, a huge difference in power between learning with positive data and with complete data (i.e. positive plus negative data) is exposed. With positive data no family of languages containing all finite languages and at least one infinite one can be learnable. With complete data the learning task becomes almost trivial.
Based on Gold's model and results, Angluin's work (1) focuses on indexed families of recursive languages, i.e., families of languages with a uniform decision procedure for membership. Such families are of interest because of their naturalness for languages generated by types of grammars. In particular, Angluin (1) gave a characterization when Gold's learning task can be executed. Her work shows that many non-trivial families of recursive languages can be learned by means of positive data only.

A few years later, Mukouchi (6) (and simultaneously Lange and Zeugmann (5)) introduced the framework of *finite identification* "in Angluin's style" for both positive and complete data. The learning task is as in Gold's model with the difference that the learning function can only guess once. Mukouchi presents an Angluin style characterization theorem for positive and complete finite identification. As expected, finite identification with complete data is more powerful than with positive data only. However, the distinction is much less marked than in Gold's framework. His work didn't draw much attention until recently Gierasimczuk and de Jongh (2) further developed the theory of finite identification.

It is often believed that children do not use negative data when they learn their native language. In opposition to that, a large amount of theoretical and experimental work in computational linguistics has been conducted to analyze and test the intuition in the powerful contribution of "negative" data for improving and speeding up children's language acquisition (see Hiller and Fernandez (4)).

In this work, we focus on a more fine grained theoretical analysis of the distinction between finite identification with positive and with complete data in Angluin-style. Our aim is to formally study the concrete difference: what can we do more with complete information for families of recursive languages than with only positive information.
We start with finite identification of finite families, in which the distinction between positive and complete data comes out very clearly: the difference is exactly described by the fact that with positive data families can only be identified if they are antichains w.r.t. $\subseteq$. Then, we question whether any finitely identifiable family is contained in a maximal finitely identifiable one. First we address this in the positive data setting. Maximal learnable families are of special interest because a learner for a maximal learnable family is a learner for all of its subfamilies. We provide a mildly positive result for families concerning any number of finite languages and give some hints about the obstacles to a more general result. Then, surprisingly, we provide a negative result concerning maximal learnable families for finite identification

with complete data: any finitely identifiable family can be extended to a larger one which is also finitely identifiable and is therefore not maximal.

We then return to families which are antichains. We show that infinite antichains of infinite languages exist which can be identified with complete information but not with positive information only. For infinite antichains of finite languages we show that such an example cannot exist if the indexing of the languages is by canonical indexes. The case of arbitrary indexing is investigated but not fully solved.

2 Preliminaries

We use standard notions from recursion theory and learning theory (see e.g., Osherson and Weinstien (7)), and for "Angluin style" identification in the limit (see (1), (6)).

Since we can represent strings of symbols by natural numbers, we will always refer to $\mathbb{N}$ as our universal set. Thus *languages* are sets of natural numbers, i.e. $L \subseteq \mathbb{N}$. A *family* $\mathcal{L} = \{L_i | i \in \mathbb{N}\}$ will be *an indexed family of recursive languages*, i.e. the two-place predicate $y \in L_i$ is recursive. In case all languages are finite and there is a recursive function F such that for each i, $F(i)$ is a canonical index for L_i, then we call $\mathcal{L}$ a *canonical family*. In finite identification a *learner* will be a total recursive function that takes its values in $\mathbb{N} \cup \{\uparrow\}$ where $\uparrow$ stands for *undefined*.

A *positive data presentation* of a language L is an infinite sequence $\sigma^+ := x_1, x_2, \ldots$ of elements of $\mathbb{N}$ such that $\{x_1, x_2, \ldots\} = L$. A *complete data presentation* of a language L is an infinite sequence of pairs $\sigma := (x_1, t_1), (x_2, t_2), \ldots$ of $\mathbb{N} \times \{0, 1\}$ such that $\{x_n | t_n = 1, n \geq 0\} = L$ and $\{x_m | t_m = 0, m \geq 0\} = \mathbb{N} \setminus L$. An initial segment of length n of σ is indicated by $\sigma[n]$. A family $\mathcal{L}$ of languages is said to be *finitely identifiable from positive data (p.f.i.)*, or *finitely identifiable from complete data (c.f.i.)*, if there exists a recursive learner φ which satisfies the following: for any language L_i of $\mathcal{L}$ and for any positive data sequence σ^+ (or complete data sequence σ) of L_i as input to φ, φ produces on exactly one initial segment $\sigma^+[n]$ a conjecture $\varphi(\sigma^+[n]) = j$ such that $L_j = L_i$, and stops.

Let $\mathcal{L}$ be a family of languages, and let L be a language in $\mathcal{L}$. A finite set D_L is a *definite tell-tale set* (DFTT) for L if $D_L \subseteq L$ and $\forall L' \in \mathcal{L}, (D_L \subseteq L' \rightarrow L' = L)$.

A language L' is said to be *consistent* with a pair of finite sets (B, C) if $B \subseteq L'$ and $C \subseteq \mathbb{N} \setminus L'$. A pair of finite sets D_L, $\overline{D}_L$ is a *definite,co-definite pair of tell-tale sets* (DFTT, co-DFTT) for L if L is consistent with $(D_L, \overline{D}_L)$, and $\forall L' \in \mathcal{L}$, if L' is consistent with $(D_L, \overline{D}_L)$ then, $L' = L$.

Theorem 1. (Mukouchi's Characterization Theorem)(6)(5)

A family $\mathcal{L}$ of languages is finitely identifiable from positive data (p.f.i.) iff for every $L \in \mathcal{L}$ there is a uniformly computable DFTT set D_L, that is, there exists an effective procedure that on input i, index of L, produces the canonical index of some definite finite tell-tale of L and then halts.

A family $\mathcal{L}$ of languages is finitely identifiable from complete data (c.f.i.) iff there is, by an effective procedure, for every $L \in \mathcal{L}$ a uniformly computable pair of DFTT,co-DFTT sets (D_L,$\overline{D}_L$).

Corollary 1. (6) *If a family $\mathcal{L}$ has two languages such that $L_i \subset L_j$, then $\mathcal{L}$ is not p.f.i..*

3 Finite families of languages

This section is dedicated to finite families of languages. A pair of simple but striking results already provides a good insight on a feature underlying the difference between finite identification on positive and on complete data.

Theorem 2. *A finite family of languages $\mathcal{L}$ is finitely identifiable from positive data iff no language $L \in \mathcal{L}$ is a proper subset of another $L' \in \mathcal{L}$.*

Theorem 3. *Any finite collection of languages $\mathcal{L} = \{L_1, \ldots, L_n\}$ is finitely identifiable with complete data.*

4 Looking for maximal learnable families

4.1 Finding maximal p.f.i. families

In this section we study maximal p.f.i. families. We address the follow up question: Is each p.f.i. family contained in a maximal p.f.i. family?

Theorem 4. *Every recursive family of finite languages which is p.f.i. is contained in a maximal family of languages which is (non-effectively) p.f.i..*

Proving theorem 4 is by a classical Zorn lemma construction. If infinite languages are present in the family, such a Zorn lemma construction cannot be applied since not every family of incomparable languages is non-effectively p.f.i..

Conjecture 1: Every p.f.i. family can be effectively extended into a maximal effective p.f.i. family.

How many maximal extensions can a p.f.i. family have? Consider the following example: Let $\mathcal{L}^s$ be the family of all singletons. Clearly it is maximal with respect to p.f.i.. However if we take out one of the singletons, say $\{0\}$, we obtain a p.f.i. subfamily $\mathcal{L}^s_0$ which is no longer maximal and its only p.f.i. extension is $\mathcal{L}^s$. If we remove $\{1\}$ from $\mathcal{L}^s_0$, we can maximally extend this family in two different ways, either adding $\{0,1\}$ or adding $\{0\}$ and $\{1\}$. Thus we have two independent maximal p.f.i. extensions for $\mathcal{L}^s_1$. We can repeat this effective deletion-procedure finitely many times and still obtain finitely many extensions. For regaining maximality, we are indeed "restricted" in the structural sense. The following lemma illustrates this.

Lemma 1. *Let $\mathcal{L}$ be a maximal p.f.i. family and $\mathcal{L} \setminus \{x\}$ where $x \in \mathbb{N}$ and $\{x\} \in \mathcal{L}$. If $\mathcal{L}'$ is a maximal p.f.i. extension of $\mathcal{L} \setminus \{x\}$, then for all $L \in \mathcal{L}'$ which is not in $\mathcal{L} \setminus \{x\}$ we have that L is of the form $\{x\} \cup A$ for some $A \subseteq L_i \in \mathcal{L} \setminus \{x\}$.*

In the following example we see that even when the languages are all finite, we can still regain uncountably many maximal p.f.i. extensions. Let $\mathcal{L} = \{\{0\} \cup \mathcal{L}_3\}$ where $\mathcal{L}_3 = \{\{i,j,k\} : i,j,k \in \mathbb{N} \setminus \{0\}\}$. Clearly $\mathcal{L}$ is maximal p.f.i. family. Consider $\mathcal{L}_3 = \mathcal{L} \setminus \{0\}$, by lemma 1 in order to regain maximality, the languages to add must be of the form $\{0\} \cup A$ for some $A \subseteq L_i$ for some $L_i \in \mathcal{L}_3\}$. Therefore we have the following procedure for achieving maximal p.f.i. extensions of $\mathcal{L}_3$: For each $B \subseteq \mathbb{N} \setminus \{0\}$ add the triplets of the form $\{0,n,m\}$ with $n \neq m$ and $n,m \in B$ and all the pairs of the form $\{0,c\}$ with $c \notin B$. This construction applies to all $B \subseteq \mathbb{N} \setminus \{0\}$, thus $\mathcal{L}_3$ has uncountable many maximal p.f.i. extensions.

Conjecture 2: Every p.f.i. family has either finitely many maximal p.f.i. extensions or uncountably many.

4.2 Do maximal c.f.i. families exist?

In this section we address the question whether every c.f.i. family is contained in a maximal one. Or in other words, if we can always find c.f.i. extensions for c.f.i. families. Surprisingly, we show that the latter is indeed always possible, the question whether maximal c.f.i. families exist is answered negatively.

Theorem 5. *Take $\mathcal{L}$ an indexed c.f.i. family and $L \in \mathcal{L}$. For any co-DFTT $\overline{D}_L$ of L, if $D_L \cup \{n\}$ is such that $n \notin \overline{D}_L \cup L$ then $\mathcal{L} \cup \{D_L \cup \{n\}\}$ is c.f.i..*

Corollary 2. *Maximal c.f.i. extensions do not exist for any c.f.i. family $\mathcal{L}$.*

There may be other ways of extending a c.f.i. family than the one described in Theorem 5 as the following example shows.

Example 1. *Take the family $\mathcal{L} = \{\{0\}, \{0,1\}, \{0,1,2\}, \dots, \{0,1,2,3,\dots,n\}, \dots\}$. This family is c.f.i.. Note that for $L = \{0\}$ we can extend $\mathcal{L}$ with $L \cup \{2\}$ and preserve c.f.i. even though a co-DFTT is $\{1,2\}$. Moreover we can extend it with $L \cup \{3\}$, $L \cup \{4\}$ and so on, and preserve c.f.i..*

5 Infinite antichains

Contrary to the results of Section 3, c.f.i. identification is on infinite families more powerful on antichains than p.f.i. identification. The class of all co-singletons, $\{\mathbb{N} \setminus \{i\} \,|\, i \in \mathbb{N}\}$, is easily seen to be c.f.i. but not p.f.i. The case of infinite families of finite languages is less clear. It is a trivial fact that canonical families which are antichains are always p.f.i. By following a diagonalization strategy, we can construct a non-canonical family which is an antichain but not p.f.i.

Theorem 6. *There is a family $\mathcal{L}$ of finite languages which is an antichain and for which there is no canonically indexed $\{D_{f(n)} : n \in \omega\}$ such that $D_i \subseteq L_i$ for all $i \in \mathbb{N}$ and $D_i \nsubseteq L_j$ for all $j \neq i$, i.e, this family is not p.f.i.*

This example happens to be not c.f.i. either. The question remains open, whether there exists such a family which is c.f.i. but not p.f.i.

Conjecture 3: If $\mathcal{L}$ is an antichain of finite languages which is c.f.i., then it is p.f.i.

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Hungarian measure phrases, bare singular nouns, and mass/count distinction

1 Introduction

In this talk, we argue that Hungarian notionally singular count nouns like *könyv* ('book'), *toll* ('pen'), and *ház* ('house') are semantically number neutral (see also [3]). This is in opposition to the view that such nouns are dual-life such as *cake* or *stone* in English, as recently argued by [13] and [15]. Number neutral nouns denote a join-semilattice and are therefore compatible with singular and plural interpretations, while dual-life nouns have both mass and count denotations (the nature of these denotations depend on which theory of the mass/count distinction is chosen). The dual-life analysis of the majority of Hungarian nouns rests on (i) the observation that Hungarian notionally singular count nouns are felicitous in numerical constructions with count cardinal quantifiers, but also in measure NPs (pseudo-partitives); and on (ii) the (not uncontroversial) claim that measure NPs require their constituting nouns to have a mass interpretation. One major implication of their analysis is that Hungarian has an unusual distribution of nouns across countability classes, which makes Hungarian (more) like Brazilian Portuguese [12] for instance rather than English. According to [15], Hungarian has few count nouns (only *fej* 'head' and *csepp* 'drop'), a larger number of mass nouns (i.e. 'stuff' and substance denoting nouns like *víz* 'water' and *kosz* 'dirt'), and a many dual-life nouns (e.g. *könyv* 'book', *toll* 'pen', and *ház* 'house')

However, assuming a different and standardly accepted analysis of measure NPs, we arrive at a more adequate analysis of notionally count nouns in Hungarian, and consequently also a different distribution of nouns across Hungarian countability classes. As is standardly assumed (see the original proposal by [8], also [5], [6], [11], [16], [10], i.a.), measure NPs sanction *cumulative* predicates (expressed by mass or bare plural count nouns), but disallow quantized predicates (expressed by singular count nouns). Based on this analysis of measure NPs and the compatibility of the alleged dual-life nouns in Hungarian with a variety of expressions of quantity and quantification, we conclude that they are best interpreted as number neutral predicates as was done on alternative grounds by [3] (see a similar analysis for Catalan and Spanish in [4]). Our proposed analysis of Hungarian notionally singular count nouns, which partially builds on [10], leads us to the conclusion that the Hungarian count/mass distinction and the distribution of nouns across countability classes in Hungarian are aligned with languages like English, rather than Brazilian Portuguese, *pace* previous proposals, including [13] and [15].

2 Previous proposal: Dual-life nouns in Hungarian

[13] and [15] have argued that Hungarian has a clear mass/count distinction, but unlike English, many nouns are dual-life in sor far as they freely occur in count or mass syntax. Furthermore, in mass syntax contexts, Hungarian dual-life nouns behave like object ('fake') mass nouns, such as *furniture* in English [15]. For instance *könyv* ('book') may be used in questions with the WH-quantifier *mennyi* ('what quantity of'), as in (1-a) (taken from [15]) Felicitous answers to

(1-a) may either measure referents in terms of, for example, weight (1-b) or count its referents in terms of their cardinality (1-c). [15] argue that the availability of (1-b) and (1-c) as answers to (1-a) shows that nouns such as *könyv* ('book') are dual-life nouns because (1-b) indicates the availability of a mass interpretation whereas (1-c) indicates the availability of a count interpretation.

(1) a. Mennyi könyvet tudsz cipelni?
what.quantity.of book-ACC able.you to.carry
'What quantity of book can you carry?'

b. Három kiló-t.
three kilo-ACC
'Three kilos.'

c. Hármat.
three.ACC
'Three.'

Their analysis of measure NPs builds on [14] who independently argues that measure NPs only admit mass noun denotations, which she supports by examples like (2)

(2) #Twenty kilos of books are lying on top of each other on the floor.

According to [14], (2) is infelicitous because the individual books are not semantically accessible by the reciprocal operator *on top of each other*, and so *each* has no grammatical antecedent. This is precisely because the plural count noun *books* must first shift into a mass interpretation in order to intersectively combine with the measure phrase *twenty kilos of*, which is mass in the intersective analysis of measure NPs in [14], and supported by data like (3) taken from [14] (p. 23, ex. 41b,c).

(3) a. #I have read many of the twenty kilos of books that we sent.
b. I have(n't) read much of the twenty boxes/kilos of books in our house.

Given this analysis, a singular noun like *könyv* ('book') is both mass and count (i.e. dual-life). As a mass noun, *könyv* ('book') denotes a root noun, a plural subset of the mass domain equal to the upward closure of a vague set of atoms (N_{root} = *A where *X = m $\in$ M: $\exists$ Y $\subseteq$ X: m = $\sqcup_M$Y). As a count noun, *könyv* ('book') denotes a set of objects in a context k, which is a set of objects in M and which are countable atoms. Count nouns are derived from the root via the $COUNT_k$ operation, which picks out the set of atoms in context k, the ordered pairs $\langle$d,k$\rangle$: d $\in$ k [14]. Each dual-life noun in Hungarian therefore has two denotations, one mass and one count.

3 Counterarguments

The claim that notionally count nouns in Hungarian are dual-life ([13], [15]) heavily relies on the assumption that nouns in measure NPs are mass denoting. We argue against this assumption on empirical and theoretical grounds. We provide four main arguments. First, many native English speakers find (2) acceptable (pace [14]) and straightforwardly interpret it as meaning that the books are stacked one on top of the other, and their cumulative weight is twenty kilos, i.e., the individual books are accessible by *on top of each other*. This weakens the claim in [14] that plural count nouns shift into a mass interpretation when combined wiht measure phrases like *twenty kilos (of)*. Second and related to the first empirical counterargument, on the standardly accepted view ([8], [5], [6], [11], [16], [10], i.a.), measure phrases like *twenty kilos (of)* select for cumulative predicates, which are expressed either by mass (e.g. *flour* or plural count nouns (e.g. *books, apples*), and are built with extensive measure functions (e.g., KILO) which can only apply to cumulative Ps (4) to yield quantized predicates (e.g. *twenty kilos of flour/books*), defined in (5) [9]. Crucially, measure phrases (e.g. *twenty kilos (of)*) cannot apply to singular count nouns, because they are already quantized. (Singular count nouns like *fence, wall* fail to be quantized, but this is outside the scope of this talk.) In other words, if Hungarian singular nouns are shown to have cumulative (and thus not-quantized) reference, then there is an alternative answer for the felicity of (1-b) and (1-c).

(4) $\forall$ P[CUM(P) $\leftrightarrow \forall$ x $\forall$ y[P(x) $\wedge$ P(y) $\rightarrow$ P(x $\sqcup$ y)]]

(5) $\forall$ P[QUA(P) $\leftrightarrow \forall$ x $\forall$ y[P(x) $\wedge$ P(y) $\rightarrow \neg$y$\sqsubseteq$x)]]

The third argument against the assumption that nouns in measure NPs are mass denoting is that plural nouns retain their atomicity when used in measure NPs (see e.g. [10]). This can be shown by the observation that it is posisble to anaphorically refer to atomic individuals in the denotation of a plural count noun in a measure NP (6-a). Such an anaphoric reference is excluded with a mass noun in the same context (6-b), despite its denoting stuff that consists of perceptually and conceptually salient entities (e.g. individual pieces of furniture). However, on the view that nouns in measure NPs uniformly have a mass interpretation [14]—i.e. lack denotations with an accessible atomic structure—which cannot explain the difference between (6-a) and (6-b).

(6) a. I bought 500 grams of bonbons and gave each one to a different person.
b. I bought 500 grams of furniture # and gave each one to a different person.[1]

The fourth argument is that the same anaphoric accessibility obtains for alleged dual-life nouns like *könyv* ('book') in Hungarian, as in the measure NP *könyv mennyiséget* ('book quantity') in (7). Contrary to the claim of [13] and [15] that *könyv* ('book') must have a mass interpretation in constructions such as (7), it is nonetheless possible to anaphorically refer to the individual books in the measure NP.

(7) Egy életbe telne hogy elolvassam a **önyv-mennyiség-et** amit te a
a life would.take that read.1S the book-quantity-ACC that you the
nyáron olvastál. Még ha azok rövid-ek is.
summer read.2S still if those short-PL too
'It would take me a lifetime to read the quantity of books that you read this summer. Even if they are short!' [7]

4 Proposal

Having invalidated the main arguments of [13] and [15] for the dual-life status of notionally count nouns like *könyv* ('book') or *alma* ('apple') in Hungarian, we follow [3] in assuming that such nouns are best analyzed as semantically number neutral. In addition to the data presented in [3], one of our key arguments comes from the observation that in their singular form, nouns

[1]While a reviewer noted that *livestock* would be fine in this sort of construction and thus is a counterexample, [1] has shown that such nouns like *cattle* belong in a class of their own, separate from other mass nouns, and we argue that *livestock* belongs in this separate class.

denote both singularities and pluralities, and in this sense they are number neutral. For example, one can use the bare singular as in (8) to announce that books have arrived, and then follow up with a specific number, which in this case is four.[2]

(8) könyv érkezett. Négy.
Book arrived.3SG four
'(A) Book(s) arrived. Four' [7]

The English version, *A book arrived. Four.* would be infelicitous because a plurality with the cardinality *four* is not in the denotation of the singular *a book.*

Our formal analysis of the mass/count distinction in Hungarian build on [10] in so far as we treat lexical nominal predicates as ordered pairs $\langle \mathbf{body}(X), \mathbf{base}(X) \rangle$, where **body** and **base** are both subsets of the Boolean interpretation domain B: $\mathbf{body}(X)$, $\mathbf{base}(X) \subseteq B$. The **base** is a set of individuals, which via the sum operation $\sqcup$ (9) is used to generate the **body**—i.e. the standard denotation of a noun. The body is therefore a subset of the base (10)—i.e. the body is grounded in the base.

(9) $^*X = b \in B$: $\exists Y \subseteq X$: $b = \sqcup Y$ (closure under sum: the set of all sums of elements of X)

(10) $\mathbf{body}(X) \subseteq {}^*\mathbf{base}(X)$

Count nouns have a disjoint base that is used to generate countable sums in the body. Mass nouns do not have a disjoint base, and therefore do not generate countable sums in the body. A set is disjoint if no two members overlap (11).

(11) disjointness $\forall$ x $\forall$ y$[P(x) \wedge P(y) \rightarrow x \sqcap y = 0]$

The representation of a number neutral noun like *könyv* ('book') is given in (12), where both its singular and plural interpretations are counted in terms of the same disjoint base BOOK.

(12) $[\![$ *könyv* $]\!] = \langle {}^*\text{BOOK}, \text{BOOK} \rangle$

Measure NPs (pseudo-partitives) are also represented as $\langle \mathbf{body}(X), \mathbf{base}(X) \rangle$ pairs. For instance, *three kilos of books* has a body consisting of sums of whole, disjoint, countable books that measure up to the appropriate measure value (13). The base is the parts of the set of books that measures three kilos and weighs less than the contextually given measure value **m** kilos (14), where $\downarrow$ is an operation used to access entities from entity, measure-value pairs.

(13) $\mathbf{body}$: $\lambda x.\mathbf{kilo}(x) = 3 \wedge {}^*\text{BOOK}(x)$

(14) $\downarrow\mathbf{base}$: $\lambda y.y \sqsubseteq(\lambda x.{}^*\text{BOOK}(x) \wedge \mathbf{kilo}(x) = 3) \wedge \mathbf{kilo}(y) \leq \mathbf{m}_{kilo}$

This representation allows us to capture the following insights: (i) bare plural count nons (which are semantically cumulative) retain their atomicity when used in measure NPs ([8], [10] *pace* [14]) (see e.g. (6-a) and (7)) on the assumption that measure phrases select for cumulative predicates; (ii) measure NPs pattern with count (quantized) nouns under their classifier interpretation (including portion interpretation) [8], [10], [14]; (iii) measure NPs pattern with mass nouns under their measure interpretation [10] [14], which is evident in the possibility of singular subject-verb agreement (see e.g. *Twenty kilos/boxes of books was/were put through the shredder last night*, example taken from [14]). Most importantly, analyzed in this way, measure NPs straightforwardly can admin Hungarian nouns like *könyv* ('book') as long as they are cumulative

[2]The plural *könyvek érkeztek* ('Books arrived') could also be used, but would entail exclusive reference (only to sums), while the singular makes no commitment to the reference of sums.

predicates—i.e. denoting a semi-lattice either mass or count. Moreover, it can be shown that such nouns as *könyv* ('book') fail to behave like mass nouns in a number of other syntactic environments, apart from measure NPs, contrary to [13], [15]. We folllow [3] in proposing that the meaning of lexical nouns like *könyv* ('book') in Hungarian corresponds to the number neutral property, whose denotation is built from the set of book atoms via closure under sum $\sqcup$ (***body** in our Landman inspired lexical representations), which, as is commonly assumed (e.g. [2] and references therein), includes singularities in its extension. But this also means that the denotation of *könyv*-like ('book') nouns in Hungarian can be assimilated to that of count nouns, meaning the nominal system has less ambiguity and is therefore simpler. We follow [3] in assuming that plural nouns denote either a semi-lattice or a semi-lattice minus its atoms—i.e. is either inclusive or exclusive–depending on a set of pragmatic constraints on the context in which the plural occurs.

The claim that singular nouns are number neutral explains why they are singular in measure phrases, with numericals, and in other environments in which languages like English use plurals. One major implication of our proposal is that Hungarian patterns with English, rather than with Brazilian Portuguese (as analyzed by [12]), when it comes to the distribution of nouns across countability classes—namely, a substantial number of mass and count nouns, but few dual life nouns—and therefore shifts the typological classification of Hungarian.

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Exploring the multi-dimensional meaning of extra arguments in English

INTRODUCTION The behaviour of verbs in argument alternations provide essential clues not just about their respective argument structure, but also about their meaning (Levin, 2015, a.o.). More interesting, however, is when a verb does not follow the expected alternation patterns of its class. For example, with an unaccusative construction like in (1-a), one could expect that *tear* behaves in a transitivity alternation either like *bounce*, an anticausative, optionally assigning an AGENT theta role, or like *appear*, a pure unaccusative, unable to take an AGENT or CAUSER subject (Schäfer, 2009). (1-b) demonstrates the anticipated anti-causative pattern, showing that *tear* can have both agentive and non-agentive causers as subjects. (1-c), on the other hand, shows an additional possibility for the subject, one lacking intentional or causative properties: the LOCATION of the event, boldfaced in (1-c). Constructions like (1-c) are the focus of the present investigation.

(1) a. A sail tore.
 b. Claire/the wind tore a sail.
 c. **The ship** (#intentionally) tore a sail (#with its mast).

More specifically, this paper looks at a special subclass of unaccusative verbs: a set of English change-of-state verbs which exhibit unexpected linguistic behaviour (examples of eligible verbs in (2)-(5)). The construction is called "extra argument" (here, ExArg) by Hole (2006), because the surface subject is not the theme, as typical for unaccusatives, nor is it an external argument. Instead, the extra argument subject has a non-canonical thematic role: the event's LOCATION. There is also a part-of relation between this extra argument and the THEME; if there is no part-of relation, the interpretation becomes one of causation, as in (1b) above.

Additionally, there is a locative alternation which tells us more about the meaning of ExArg. This location is when the LOCATION is located in a PP and the THEME is the subject, henceforth called LocAlt; this alternation is presented in (2-5b), with the locative PP bracketed and the LOCATION boldfaced. Both variants entail that the change-of-state event is localised on the boldfaced DP. As such, one might think the two alternations describe the same event.

(2) a. **The ship** tore a sail.
 b. A sail tore [**on the ship**].

(3) a. **The skater** chipped a tooth.
 b. A tooth chipped [**in her mouth**].

(4) a. **The lizard** grew a tail.
 b. A tail grew [**on the lizard**].

(5) a. **The bucket** spilled water.
 b. Water spilled [**from the bucket**].

However, there is actually a subtle difference: LocAlt has a non-defeasible implication that the maximum change-of-state has been reached, whereas ExArg's change-of-state (COS) can also be partial if the respective verb's lexical semantics allow it. (From now on, we will only be looking at these predicates in the past tense, to exclude ambiguously non-maximal events.) This paper argues that LocAlt contributes projective meaning: an implicature of 'maximality'. (6) and (7) display the difference in not-at-issue meaning.

(6) *No 'maximality' implicature in ExArg*

a. The ship tore a sail, ... but the damage was minor.

b. The lizard grew a tail, ... but not an entire tail.

(7) *'Maximality' implicature in LocAlt*

a. A sail tore on the ship, ... #but the damage was minor.

b. A tail grew on the lizard, ... #but not an entire tail.

This implicature of 'maximality' is not a conversational one, as it does not come soley from pragmatic context, but rather a "conventional" one, as the meaning is from the construction itself and is non-cancelable (Horn, 2007; Potts, 2015). Of course, the lexical semantics of the eligible verbs can play a role in the COS interpretation (although the localisation entailment remains constant). For example, the nature of a verb like *burst* in *the car burst a tire/a tire burst on the car* blocks a partial reading in the LocAlt variant. However, this would be a further argument against a conversational implicature classification.

This paper argues that both alternations have a localisation entailment, but that they differ in their not-at-issue contribution. The paper's focus is the semantic-pragmatic interface, but syntactic reasons for the licensing of an extra argument subject will be discussed in the talk.

PREVIOUS WORK Rohdenburg (1974) described Germanic "secondary subjects" in his dissertation, but without a formal analysis. Hole (2006) examines ExArg cross-linguistically, addressing the localisation entailment with a binding analysis. To my knowledge, that is the extent of theoretical research concerning the ExArg construction. As such, there currently lacks a multi-dimensional analysis accounting for the behaviour exhibited in (6) and (7) above. This study tackles that gap and, more generally, contributes to the discussion on non-canonical subjects and to the growing research on not-at-issue meaning of constructions (and not simply of lexical items).

ANALYSIS PREREQUISITES The localisation entailment, of the tear-event being on the ship, is at-issue in both alternations. In ExArg, this localisation is realised in the surface subject, as it is the LOCATION; while in LocAlt, it is more transparent, being the PP object. The diagnostic is presented with ExArg, as its localisation entailment is less transparent than LocAlt's. In the relevant construction (i.e., unintentional causer), it is infelicitous to deny that the location of the event is the LOCATION (8); as at-issue information is the descriptive content entailed in the utterance, it cannot be denied.

(8) *At-issue meaning cannot be denied*

a. {The lizard}$_i$ grew a tail **#but it was not on {the lizard}$_i$**

b. {The ship}$_i$ tore a sail **#but it was not on {the ship}$_i$**

Similar to at-issue entailments, CIs cannot be denied; cf. (7) above. However, not-at-issue material is independent of the at-issue dimension and is able to project (Potts, 2005; Simons et al., 2010). The 'maximality' implicature behaves as a CI would, as is seen in the following two examples.

(9) *No projective meaning*

a. Did the lizard grow a tail?
$\not\rightsquigarrow$ 'if the COS event had occurred, it would have been a maximal COS'

b. Did the ship tear a sail?
$\not\rightsquigarrow$ 'if the COS event had occurred, it would have been a maximal COS'

(10) *Projective meaning*

a. Did a tail grow on the lizard?
$\rightsquigarrow$ 'if the COS event had occurred, it would have been a maximal COS'

b. Did a sail tear on the ship?
$\rightsquigarrow$ 'if the COS event had occurred, it would have been a maximal COS'

There does exist a further multi-dimensional type to consider: presuppositions. The 'maximality' implication of LocAlt is, however, no presupposition. Presuppositions, being backgrounded, contribute old information, whereas CIs contribute new information (Potts, 2005, a.o.). In the test of (11), the (prospective) content of the presupposition is underlined and the trigger is boldfaced. The presupposed content of the possessive pronoun in (a) can be repeated without sounding redundant (example after Potts 2015, 178). For the CI in (b), this repetition is odd.

(11) a. <u>Sam has a dog</u>. **Her** dog is sick.
b. A tail grew on the lizard. <u>#It was an entire tail.</u>

<u>Analysis</u> To formalise both at-issue and not-at-issue meanings, both alternation variants are analysed as having two dimensions, following proposals by, e.g., Potts (2005) and Gutzmann (2015). Along the first dimension, both include the localisation entailment by virtue of the LOCATION DP. The additional dimension contains the CI for LocAlt.

First, the at-issue meaning of both ExArg and LocAlt. The verbs *tear* and *grow* are change-of-state verbs, with a lexicalised multi-value scale, bound at the upper end by the maximally-possible COS (Kennedy and Levin, 2008). To model this, we need to define a measure function (12-a). To operationalise the measure function, we need a degree morpheme; with verbs, it is a null morpheme (12-b). The LOCATION and past are added per conjunction in (13).

(12) a. For any measure function m, $\mathrm{m}_\Delta = \lambda x.\lambda e.\mathrm{m}\uparrow m(x)(init(e))(x)(fin(e))$
b. $pos = \lambda m_\Delta.\lambda x.\lambda e.\mathrm{m}_\Delta(x, e) \geq \; stnd(m_\Delta)$

In (12-a), the measure of change function m_Δ outputs the degree amount of change that x undergoes in e, within the interval represented by *init* and *fin*. In (12-b), *stnd* is the standard of comparision for the measure of change. The following combines *pos* with the verb, *tear*, resulting in a relation between entities and events. Also, in (b), the predicates *sail* and *boat* are inputted.

(13) a. $pos([\![\text{ tear }]\!]) = \lambda x.\lambda y.\lambda e.\mathbf{tear}(x, e) \geq \; stnd(\mathbf{tear}) \wedge \textsc{location}(\mathrm{y, e}) \wedge fin(\mathrm{e}) < t_{\mathrm{now}}$
b. $pos([\![\text{ tear }]\!]\text{ (sail)}) = \lambda y.\exists e.\mathbf{tear}(\mathbf{sail}, e) \geq \; stnd(\mathbf{tear}) \wedge \textsc{location}(\mathrm{y, e}) \wedge fin(\mathrm{e}) < t_{\mathrm{now}}$
c. $pos([\![\text{ tear}]\!]\text{(sail)(boat)}) = \exists e.\mathbf{tear}(\mathbf{sail}, e) \geq \; stnd(\mathbf{tear}) \wedge \textsc{location}(\mathbf{boat}, \mathrm{e}) \wedge fin(\mathrm{e}) < t_{\mathrm{now}}$

In order to account for the not-at-issue meaning, this paper follows Kennedy 2012, via Spalek 2012, in incorporating an incremental *part-of* function (14). More precisely, the variability in defining d allows for either a maximality CI or not, depending on the syntactic difference.

(14) part-of$_{\text{inc}}$ = $\lambda x.\lambda d.\lambda p.\lambda e.$**part-of**$\Delta$(x,p,e) $\geq d$

a. d >0 : For each part that undergoes tearing, the tearing of the part is not a gradable event, but the whole VP can be gradable

b. d = 1 : achievement-like interpretation; maximal tearing reached

In (14), p is a portion of x; the output of *part-of* is the degree to which the consitutive parts p of x changes in the event e. The definition of the degree d would be for ExArg as in (a), as the event can be gradable, that is, does not have to reach the maximal threshold, just be greater than zero. For LocAlt, (b) represents the definition of d, which accounts for the CI of 'maximality'.

(15) *part-of* $[\![$ sail $]\!]$ =

a. <u>ExArg</u>
The boat **partially/completely** tore a sail : $\lambda d.\lambda p.\lambda e.$**part-of**$\Delta(\mathbf{s},p,e)>0$

b. <u>LocAlt</u>
A sail **completely/#partially** tore on the boat : $\lambda d.\lambda p.\lambda e.$**part-of**$\Delta(\mathbf{s},p,e) = 1$

This *part-of* function is on a different meaning dimension than the at-issue material of (13). Depending on which syntactic variant is in use, either (a) or (b) will define the necessary condition on felicitous use. For those change-of-state verbs, such as *burst*, which necessarily have a maximal COS only the binary-valued degree d (as in (b)) is compatible.

<u>Conclusion</u> This short paper discussed the meaning an understudied construction alongside an argument alternation. The puzzle of why a location can be subject may be able to be explained by a possessor-raising account (Deal 2013), given the part-of relationship of the location and theme. Alternatively, a binding account of interparticipant relations and affecteehood could be a possibility (Hole, 2006). To further explore the meaning, a study with an empirical emphasis is in order.

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An Axiomatization of the d-Logic of Planar Polygons

Introduction. Topological semantics of modal logic starts with McKinsey and Tarski paper [7] where semantic treatment of modal diamond is provided by closure operator of a space. Every topological model generates a Closure algebra (Boolean algebra with closure operator) which is a subalgebra of closure algebra of all subsets of a model. It turns out that the set of all polygons, in particular the planar polygons, also forms a closure algebra. This idea has been explored and developed in [6]. In C-semantics based on planar polygons, modal formulas are evaluated on a subalgebra of the closure algebra generated by all planar polygons.

An alternative topological semantics also suggested in [7] and later studied in detail by number of authors [4], [5], [8] etc., is provided by derivative operator interpretation of the diamond modality. More precisely, the truth set of $\Diamond p$ in a topological model is a set of all limit points of the truth set of p. A derivative algebra is a Boolean algebra with operator which satisfies algebraic properties of topological derivative operator. The set of planar polygons generates a derivative algebra. In this paper, we give an axiomatization of the modal logic generated by the derivative algebra of planar polygons. The main result (Theorem 2) states that the logic $\mathbf{PL}_d^2$ described in the next paragraph is sound and complete d-logic of the polygonal plane.

Syntax. Fix a signature consisting of countable set $\mathbf{Prop}$ of symbols for propositions. The *propositional modal language* consists of formulas φ that are built up inductively according to the grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Diamond\varphi,$$

where p ranges over proposition symbols in $\mathbf{Prop}$. The logical symbols '$\top$' and '$\bot$', and the additional connectives such as '$\vee$', '$\rightarrow$' and '$\leftrightarrow$' and the dual modalities '$\Box$' are defined as usual.

Let $\mathbf{PL}_d^2$ be a classical normal modal logic containing the following axiom schemes:

$$\begin{array}{ll}
(I) & \Diamond\Diamond p \leftrightarrow \Diamond p \\
(II) & \Diamond\top \\
(III) & (\Diamond p \wedge \Diamond\neg p) \rightarrow \Diamond\big((p \wedge \Diamond\neg p) \vee (\neg p \wedge \Diamond p)\big) \\
(IV) & \Box\big(p \rightarrow \Box(\neg p \rightarrow \Box\neg p)\big) \\
(V) & \Box(r \rightarrow \gamma) \rightarrow \big(r \rightarrow \Diamond(\neg r \wedge \Diamond\Box p \wedge \Diamond\Box\neg p)\big)
\end{array}$$

where γ is the formula $\Diamond\Box(p \wedge q) \wedge \Diamond\Box(p \wedge \neg q) \wedge \Diamond\Box(\neg p \wedge q)$

Kripke Semantics. An adequate Kripke semantics for the logic $\mathbf{PL}_d^2$ is provided by the class of finite frames which we call crown frames with irreflexive root. Below we give the definition of these frames.

Definition 1. *A* crown frame with irreflexive root $\mathfrak{S}_n$ *is a frame* (S_n, Q_n) *such that* $S_n = \{r, s_1, \cdots, s_{2n}\}$ *and* Q_n *is defined as follows:*

$$\begin{array}{ll} (r, r) \notin Q_n; & \\ (r, s_i) \in Q_n & \textit{for all } s_i \in S_n; \\ (s_i, s_i) \in Q_n & \textit{for all } s_i \in S_n; \\ (s_i, s_j) \in Q_n & \textit{when } i < 2n \textit{ is even and } j = i-1, i+1; \\ (s_{2n}, s_1) \in Q_n; & \\ (s_{2n}, s_{2n-1}) \in Q_n. & \end{array}$$

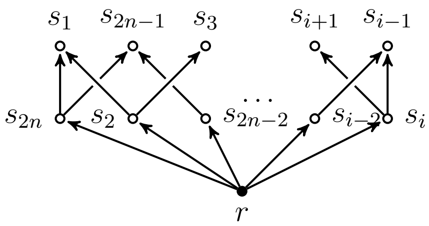

Figure 1: Crown frame

Figure 1 represents the structure of the crown frame with irreflexive root. The black bullet represents an irreflexive point and the white bullets represent reflexive points. Let **CROWN** denote the class of all crown frames with irreflexive root. We only consider Kripke models which are based on crown frames with irreflexive root. We omit the definition of satisfaction and validity of modal formula in Kripke structures. These are standard definitions and can be found in any modal logic textbook. Crown frames form the main link towards proving the topological completeness theorem essential part of which is provided by the following Kripke completeness result.

Theorem 1. $\mathbf{PL}^2_d$ *is sound and complete w.r.t. the class* ***CROWN***.

Topological Semantics. Now we define the topological semantics for our modal language. The main object of our study is the *polygonal plane* which we now proceed to define. Consider the regions of the plane obtained by the intersections of finitely many half-planes and generate the Boolean algebra using the set-theoretic operations from these regions. An arbitrary member of the obtained boolean algebra is called a *polygon* and the collection of all polygons is denoted by P_2. The structure $\mathfrak{P}_2 = (\mathbb{R}^2, P_2)$ is called the *polygonal plane*. A typical bounded member of P_2 is a finite union of (open) n-gons, line segments and points. In other words, we consider as entities not only the 2-dimensional n-gons, but also their boundaries, i.e. 'polygons' of lower dimension. It is readily checked that P_2 forms the derivative algebra.

To interpret the modal language over the polygonal plane, we allow for valuations ν : $\textsc{Prop} \to P_2$ to range over polygons only. The valuations are extended to arbitrary modal formulas using the set-theoretic counterparts for the propositional connectives, interpreting $\Diamond$ as the topological derivative operator, and $\Box$ as its dual in the following way:

$$\begin{array}{lll}
x \models p & \text{iff} & x \in \nu(p); \\
x \models \neg\varphi & \text{iff} & x \not\models \varphi; \\
x \models \varphi \vee \psi & \text{iff} & x \models \varphi \text{ or } x \models \psi; \\
x \models \Diamond\varphi & \text{iff} & x \in d(|\varphi|);
\end{array}$$

Where $|p| = \{x \in \mathbb{R}^2 \mid x \models p\}$. The regions denoted by the propositional letters are specified in advance by means of a valuation, and $\vee$, $\neg$ and $\Diamond$ are interpreted as union, complement and derivative operator respectively. We skip the standard notions from general topology and definitions of satisfaction at a point, validity in the topological model etc. The reader is referred to [3], [1] for these definitions. To give the proof idea of the main theorem we need a definition of maps which preserve modal formulas i.e. maps which are alike to p-morphisms, but when one structure is a topological space and the other one—a Kripke frame.

Definition 2 ([2]). *A map $f : X \to W$ where (X, τ) is a topological space and (W, R) is a transitive Kripke frame, is called a d-morphism if the following properties are satisfied:*

(i) For each open $U \in \tau$ it holds that $R\big(f(U)\big) \subseteq f(U)$;

(ii) For each $V \subseteq W$ such that $R(V) \subseteq V$ it holds that $f^{-1}(V) \in \tau$;

(iii) For each irreflexive point $w \in W$ it holds that $f^{-1}(w)$ is discrete space w.r.t. subspace topology;

(iv) For each reflexive point $w \in W$ it holds that $f^{-1}(w) \subseteq d(f^{-1}(w))$.

It is a well known fact that d-morphisms preserve validity of modal formulas.

Proposition 1. *Every rooted crown frame $\mathfrak{F}$ is a d-morphic image of a polygonal plane $\mathfrak{P}_2$ in such a way that the preimage of an arbitrary point $w \in \mathfrak{F}$ belongs to the algebra of planar polygons P_2.*

Now we are ready to state the main result of our paper.

Theorem 2. *$\mathbf{PL}_d^2$ is sound and complete logic of polygonal plane $\mathfrak{P}_2$.*

Proof. (Sketch of Completeness). By Theorem 1, for an arbitrary formula φ which is not a theorem of $\mathbf{PL}_d^2$, there exists a rooted crown frame $\mathfrak{F}$ such that $\mathfrak{F} \not\models \varphi$. By Proposition 1, φ is falsified on $\mathfrak{P}_2$. □

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Thomas Gamerschlag, Jens Fleischhauer and Wiebke Petersen

Why event structure templates are not enough – A frame account of bleeding and droning

The common assumption of most decompositional approaches to natural language semantics is that event structure templates as in (1) represent the grammatically relevant meaning components of verbs.

(1) Event structure templates (Rappaport Hovav and Levin 1998, p.108, based on Dowty 1979)
State: $[x\langle\text{PRED}\rangle]$
Activity: $[x\ \text{ACT}_{\langle\text{PRED}\rangle}]$
Achievement: $\text{BECOME}[x\langle\text{PRED}\rangle]$
Accomplishment: $[x\ \text{CAUSE}[\text{BECOME}[y\langle\text{PRED}\rangle]]$

As these approaches are confined to representing event structural properties, the idiosyncratic lexical content is often reduced to an unanalyzed atomic root. In our talk we will demonstrate by the example of *verbs of emission* that a more fine-grained analysis is necessary in order to account for the semantics of these verbs. Following the traditional approach in Rappaport Hovav and Levin (2010), verbs of emission would be represented as in (2), in which the specific type of emission appears as a subscripted modifier root of the primitive predicate ACT.

(2) a. bleed: $[x\ \text{ACT}_{\langle\text{BLEED}\rangle}]$
b. drone: $[x\ \text{ACT}_{\langle\text{DRONE}\rangle}]$

Representations in this fashion, however, neglect the semantic differences that exist between verbs of substance emission like *bleed* in (2-a) and verbs of sound emission like *drone* in (2-b): while the ACT-predicate indicates that both verbs denote activities, it does not express that they fundamentally differ with respect to the relation between the properties of the emission and the progression of the event. In the case of (2-a) the emission of substance is monotonically related to the progression of the event, i.e., the quantity of emitted substance increases in the course of the event (event-dependent emission). By contrast, there is no relation between the progression of the event and the emission of a sound in (2-b) such that any property (quantity, intensity or whatever) necessarily increases in the progress of the event (event-independent emission). This difference is evident in the context of verbal degree gradation: *sehr* 'very' specifies the quantity of emitted blood in (3). If the verb is used in a progressive construction as in (3-a), the quantity of blood at a certain stage of the event is specified whereas the perfective-like construction in (3-b) refers to the total amount of emitted blood:

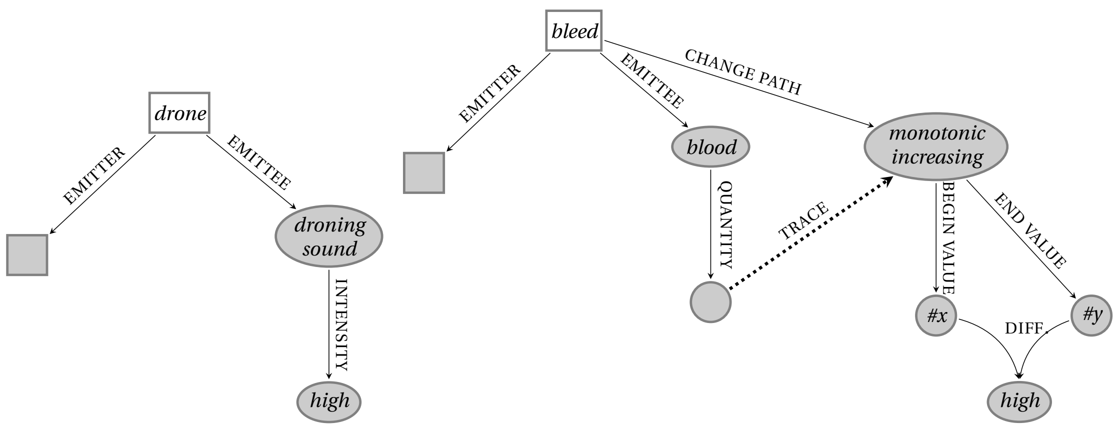
(3) a. Die Wunde war sehr am Bluten.
the wound was very at.the bleeding
'The wound was bleeding a lot.'
b. Die Wunde hat sehr geblutet.
the wound has very bled
'The wound bled a lot.'

By contrast, grammatical aspect does not affect the interpretation of degree gradation in case of verbs of sound emission. In both examples in (4), *sehr* indicates the intensity (= loudness) of the emitted sound.

(4) a. Der Motor ist sehr am Dröhnen.
the engine is very at.the droning
'The engine was droning a lot.'
b. Der Motor hat sehr gedröhnt.
the engine has very droned
'The engine droned a lot.'

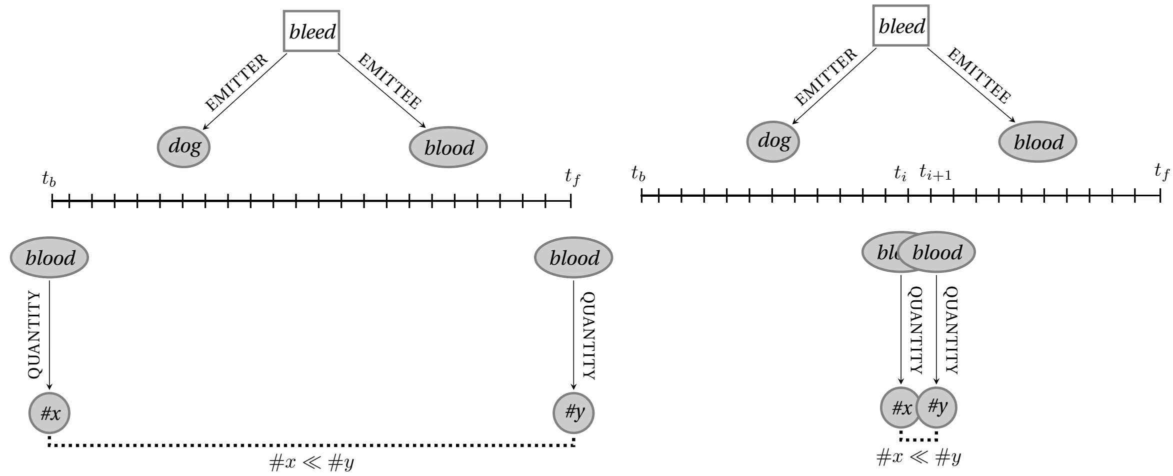
Decompositional representations like those in (2) are not able to capture this difference between verbs of substance emission and verbs of sound emission as they do not represent the relation that holds between the event and the emitted stimulus.

A promising framework for the analysis of emission verbs is frame theory which is based on Barsalou's ideas about frames as the fundamental structures of cognitive representation (Barsalou, 1992). Frames are recursive attribute-value structures that allow one to zoom into conceptual structures to any desired degree and to access meaning components by attribute paths (cf. Petersen, 2007). The static event frame of *sehr dröhnen* as in (4) is given in (5)(a). It models the static dimensions of the event (cf. Fillmore, 1982), that is the relations to the two participating objects, i.e. the *emitter* and the *emittee*, of the event (note that the emittee is an implicit argument while the emitter is an open argument). Additionally, the frame represents the result of applying the intensifier *sehr* 'very': it restricts the value of the INTENSITY-attribute of the emitted sound to 'high' (which is a context-dependent subinterval of the intensity scale).

(5)


(a) (b)

The case of the event-dependent degree gradation in (3) is more complex. In order to model the dependency relation that the more the event progresses, the higher the degree on the quantity scale is, the level of static event frames is not sufficient. In our analysis we follow the three-level event decomposition model proposed in Naumann (2013) and further developed and exemplified in Gamerschlag et al. (2014). Figure (6) shows the three level model for the examples in (3) ((3-b) is depicted in (6)(a) and (3-a) in (6)(b)). At the top, the *static event frame level* represents the relation of the event to the participating objects (emitter and emittee). In the middle, on the *event decomposition level* the event is decomposed into single subevents. This level represents the temporal structure of the event and links it to the *situation frame level* at the bottom that represents the participating objects and the changes they undergo at the different time points of the event, here the amount of emitted blood.

(6) 

(a) (b)

The three levels can be merged into the single frame in Figure (5)(b) by establishing the dynamic attribute TRACE that is projected from the event decomposition frame in (6) and maps the QUANTITY value of the emmitted blood to the record of its trace in the time span of the event. The recorded trace is of type ‘path’ and hence a static spatial object with a begin and an end value. The intensifier *sehr* restricts the difference of these two values (here indicated by the 2-place attribute DIFF) to the value range ‘high’. Thus, in our frame account the attested asymmetry between substance and sound emission verbs illustrated in (3) and (4) results from the structural difference between the representation of intensity scales (as used in sound emission frames) and quantity specifications in frames. In particular, the accumulation of the quantity of a substance over the course of the event is made explicit at a level of the frame representation which captures the temporal change of the participants’ properties.

In our talk we will further demonstrate that frame theory allows for an adequate analysis of a second class of grammatical asymmetries which are not predicted by the representations in (2) – this time within the class of sound emission verbs. In German, motion verbs can be derived from verbs of sound emission such as *jaulen* ‘whine’ as in (7):

(7) Kaufmann (1995, p.91)

a. Der Welpe jault.
the Puppy whines
‘The puppy yowls.’ ‘

b. § Das Motorrad jault.
the Motorbike whines
‘The motorbike yowls.’

c. Das Motorrad jault über die Kreuzung.
the Motorbike whines over the crossing
‘The motorbike whines over the crossing.’

d. § Der Welpe jault unter das Bett.
the Puppy whines under the bed
‘The puppy whines under (dir) the bed.’

As already observed by Kaufmann (1995) and Levin and Hovav (1995) among others, the motion verb use of sound emission verbs is accessible only if the specific sound can be interpreted as a side-effect of motion as in (7-c) whereas this use is not licensed if such a relation does not hold as in (7-d). At the same time, the sortal restrictions of the verb in the basic use and the derived use are reversed as illustrated by the contrast between (7-a)/(7-b) and (7-c)/(7-d). Neither the accessibility of the motion verb use nor the change in sortal restrictions is adequately captured by representations as in (2). What is needed instead is a representational framework which allows for making reference to the co-occurrence of sound and motion.

We will demonstrate how this grammatical asymmetries can be analysed in frame theory. Here, the strength of frame representations is that we can model the detailed relations between an event, its participants and the sound produced either independently by the actor ('puppy' in (7-a), (7-d)) or by the theme in dependence of the event ('motorbike' in (7-b), (7-c)). The constructional constraints can be formulated by making reference to specific frame components. In particular, the frame of the base verb referring to the emission of a particular sound licenses the activation of a movement frame in which the theme argument is embedded. Thereby, it introduces an additional argument, namely the directed path PP. We will show that frame representations show a flexible degree of complexity (zooming in and out by expanding/not expanding nodes) which allows for easy access to the details of verb and noun meaning needed for an analysis of the different uses of emission verbs.

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Kristina Gogoladze and Alexandru Baltag

Evidence-Based Belief Revision for Non-Omniscient Agents

It has long been recognized that inconsistencies may easily occur in people's beliefs in real life. Even if one is rational, one may hold inconsistent beliefs due to receiving conflicting information along with the fact that our limited capacity for information processing (or limited memory) may make it hard to spot the inconsistency. A rational agent would, of course, like to revise his beliefs when he becomes aware of an inconsistency. However, the usual discussion in the Belief Revision literature on solving the contradiction involving old evidence and new evidence assumes that the agent is always aware of this contradiction (because of his logical omniscience).

An important outstanding problem in epistemic and doxastic logics is the problem of logical omniscience, unrealistic assumptions with regard to the reasoning power of the agents. It would be nice, of course, to have perfect reasoners, but even in powerful computers the resources for reasoning are limited. Epistemic logicians usually consider the following features as different issues involving logical omniscience: Knowledge of all logical validities; Monotonicity; Closure under known implication, logical equivalence or conjunction; Introspection. Each of these principles is a feature of idealized perfect reasoners that may not exist in a rational agent in real life.

Assuming that the agent is rational, the reasons he may be non-omniscient are typically limited computational power, time constraints and insufficient memory. These restrictions may also cause the agent to believe some contradictory facts (in this case, he simply may not have noticed the contradiction yet). We are not aware of any previous work that deals with inconsistent beliefs and that has a framework that would allow agents to fix inconsistent beliefs later. There are so-called paraconsistent logics [Priest, 2002] that allow reasoning about inconsistencies, but the underlying philosophy of these logics is that believing a contradiction may be rational and that, in principle, there is no need to resolve logical contradictions. So, if we want to be able to explain why agents can hold inconsistent beliefs, we need to think of something different.

We introduce and investigate a model of belief formation that is closer to real-life reasoning than existing models. In particular, we want to propose a model that enables agents to reason about inconsistent beliefs when they are not aware of the inconsistency due to some limitations by introducing more natural definition of beliefs. Even rational agents may happen to believe irrational things either because they read/were told something or have false evidence from other sources. An agent will never believe an explicit contradiction $\perp$. If he notices such inconsistency, he will have to revise his current beliefs to keep them consistent.

Since one of the main reasons why people hold inconsistent beliefs is limited computational resources, as a possible solution, we, firstly, restrict agents to the usage of only finite amount of sentences at every given period of time. These are going to be (the agent's) *explicit* beliefs — a finite set of syntactically given formulas. *Implicit* beliefs will not have all the restrictions we impose on the explicit beliefs, but agents can reason only with their explicit belief sets. The explicit belief sets are not required to be closed under any of the logical operations, the only restriction will be that they do not contain an *explicit inconsistency* which we denote by $\perp$. Then we go one level up and start with *explicit evidence pieces* instead of explicit beliefs by borrowing some ideas from van Benthem and Pacuit's work [2011] on evidence-based beliefs. The explicit beliefs of an agent will then be computed using his explicit evidence pieces. This "computation" is defined in such a way that it does not allow an explicit inconsistency in the agent's explicit belief set even when his evidence set does contain $\bot$.

Interestingly, our proposed models of explicit beliefs naturally validate axiom schemes that correspond to nice properties of knowledge and belief that one may want to have.

Definition 1 (Explicit Evidence Language)**.** Let At be a set of atomic propositions. Formulas ϕ of language $\mathcal{L}$ are given by

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid B^e\phi \mid B^i\phi \mid K^e\phi \mid K^i\phi$$

with $p \in \mathsf{At}$. ◁

We use the B^e and B^i modalities for explicit and implicit belief respectively, and, similarly, K^e and K^i for explicit and implicit knowledge.

Definition 2 (Semantic Model of Explicit Evidence)**.** An *explicit evidence model* (EE-model) is a tuple $\mathfrak{M} = (W, W_0, \mathcal{E}_s, \mathcal{E}_h, V)$ where

W is a set of possible worlds.

$W_0 \subseteq W$ is a set of worlds that represents the agent's background beliefs or "biases".

$\mathcal{E}_s \subseteq \mathscr{P}(\mathcal{L})$ is a set of formulas that represent agent's (soft) evidence pieces.

$\mathcal{E}_h \subseteq \mathcal{E}_s$ is a set of hard evidence pieces.

$V : \mathsf{At} \to \mathscr{P}(W)$ is a valuation function.

The following condition is imposed on the models:

$$\top \in \mathcal{E}_h(w)$$

◁

The above condition means that the agent has some knowledge to start with. Note that here the soft evidence set may contain $\bot$ (but the explicit beliefs will not). We will use $\bot$ to "mark" a contradiction, but this is only a convention because we did not want to restrict ourselves — it could have been any formula.

The idea is that we can now think of the explicit knowledge set also as of an evidence set — it is hard evidence set $\mathcal{E}_h$ that is infallibly true, whereas $\mathcal{E}_s$ is a soft evidence set: an agent is not absolutely certain about those evidence pieces and they may even be inconsistent with each other.

Definition 3 (Quasi-consistency)**.** Let U be a set of formulas. We say that U is *quasi-consistent* if $\bot \notin U$. ◁

Definition 4 (Closed Evidence)**.** A set $F \subseteq \mathcal{E}_s$ of (soft) evidence pieces is said to be *closed* if, and only if, it includes all the hard evidence (i.e. $\mathcal{E}_h \subseteq F$) and it is closed under Modus Ponens within $\mathcal{E}_s$ (i.e. if ϕ and $\phi \to \psi$ belong to F and ψ belongs to $\mathcal{E}_s$, then ψ belongs to F). ◁

Definition 5 (Q-max Evidence)**.** A set $F \subseteq \mathcal{E}_s$ of (soft) evidence pieces is said to be *maximal closed quasi-consistent set* (or *q-max*, for short) if it is (1) closed (in the above sense), (2) quasi-consistent, and (3) maximal with respect to properties (1) and (2) (i.e. for every other closed quasi-consistent set F', if $F \subseteq F' \subseteq \mathcal{E}_s$, then $F' = F$). ◁

Since we do not have explicit beliefs in the model, we have to define them. Soft evidences are on the more abstract level than beliefs, the evidence pieces play a role of the derivations an agent made so far, and beliefs are encoded there. We say that the agent *explicitly believes* a formula at some world if, and only if, that formula belongs to the intersection of all maximal closed quasi-consistent sets:

$$\mathcal{B} := \bigcap\{F : F \text{ is q-max}\}$$

Let us use this abbreviation for the explicit belief set from now on.

The choice of such definition naturally arises from our line of research — since we assume the agent has the fast "working" memory where he can easily compute even exponential things. According to this definition, the agent stays safe and cautious and sticks with what is included to every maximal closed quasi-consistent set of evidence pieces. This can be seen as the appropriate syntactic counterpart of the van Benthem and Pacuit's definition of Maximal Consistent Evidence [2011]. Required $\bot \notin \mathcal{B}$ will hold automatically by construction.

In our models, implicit belief is a defined notion: it is defined as a closure of agent's explicit beliefs together with his prior background biases. From this it follows that implicit beliefs may be inconsistent (in the usual sense). We think, that defining implicit beliefs via explicit ones is more natural than treating them as an independent notion, and it makes perfect sense that the implicit beliefs of an agent may happen to be inconsistent at some point in time. Of course, these beliefs can become consistent if the agent manages to resolve the inconsistencies in his explicit beliefs.

Working with the assumption that the agent remains rational, and that he does not find it rational to believe in explicit inconsistencies, we provided a model that in a sense corrects the explicit inconsistencies itself.

Our proposed solution addresses this problem by providing a basis for agent's beliefs — syntactic pieces of evidence that an agent uses to justify his beliefs. Then, the explicit beliefs of an agent are computed using his explicit evidence set. The explicit belief set is purely syntactic as well, which allows an agent to hold any kind of sentences without identifying them with an inconsistency, unless it is indeed an explicit inconsistency.

Since we allow our agents to operate only with (finite) syntactic explicit information, our proposed explicit evidence models happen to resolve all the omniscience problems that epistemic logicians are usually concerned with. Closure properties need not hold at all for the explicit sets of knowledge and belief, as well as explicit introspection.

We prove the following theorem.

Theorem. *The logic is completely axiomatized by the following system of axioms and rules:*

$S5$ *axioms and rules for* K^i	(1)	$B^e\phi \to B^i\phi$	(2)
$K45$ *axioms and rules for* B^i	(3)	$K^e\phi \to K^iK^e\phi$	(4)
$\neg B^e\bot$	(5)	$\neg K^e\phi \to K^i\neg K^e\phi$	(6)
$K^e\top$	(7)	$B^e\phi \to K^iB^e\phi$	(8)
$K^i\phi \to B^i\phi$	(9)	$\neg B^e\phi \to K^i\neg B^e\phi$	(10)
$K^e\phi \to B^e\phi$	(11)	$B^i\phi \to K^iB^i\phi$	(12)
$K^e\phi \to K^i\phi$	(13)	$\neg B^i\phi \to K^i\neg B^i\phi$	(14)

Here, reflexivity of knowledge represents its *veracity*, transitivity — *positive introspection*, and Euclideanness — *negative introspection*; as for belief, transitivity means positive introspection, Euclideanness — negative introspection, and seriality would mean *consistency*; there is also *strong introspection* of belief (axioms 8,10,12 and 14), and the obvious requirement that knowledge should imply belief (axioms 9 and 11).

It is worthwhile to mention that neither B^e nor B^i satisfy the standard $KD45$ axioms for belief. Implicit beliefs do not satisfy the seriality axiom, as explained above, whereas explicit beliefs do not satisfy the K-axiom (The axiom K means that the set of formulas that the agent knows is deductively closed. It would imply logical omniscience of the agent.). Interestingly, B^e does satisfy the D-axiom in the sense that $\neg B^e \bot$ (but not $B^e\phi \rightarrow \neg B^e \neg \phi$). Consequently, one could argue that the standard notion of belief is a mixture of these two.

One of our main intentions in this work was to allow agents to resolve the inconsistency once they become aware of it. To model this, we have to express some *actions* that describe how the agent becomes aware of new information. We focus on some of the possible evidence dynamics which are also called *updates*. The updates are informational actions that *change* the original model. Some of these changes may remove the possible worlds of the agent, another — will just modify the explicit information of the agent. First of all, one could look at the usual DEL [van Ditmarsch *et al.*, 2007] updates. For example, the operation of update models the situation when the agent receives a piece of evidence from an infallible source. Another, more natural for our models, scenario is when the agent learns new pieces of evidence. They may be consistent or inconsistent with the previously learned information. It can be even explicit inconsistency $\bot$. There are two possibilities: either the agent adds ϕ to his explicit knowledge, or he just accepts ϕ as a piece of evidence. We would like to model belief revision of realistic agents, and realistic agents cannot hold all the information they learn forever. In real life, agents do forget some things from time to time. It, therefore, makes perfect sense to consider evidence removal operation as well. With these dynamic operations, the agent can become aware of the inconsistency and is able to fix it (this happens automatically). This means that both explicit and implicit beliefs of the agent can become consistent (if they were not).

We have given an axiomatization for the (static) logic of explicit evidence which is complete. Next, we presented the dynamic actions that describe change of models due to modifications in the evidence pieces. We saw various examples that illustrate how our models work. In the key part of this work, we showed how the problem of inconsistent belief revision is solved with the help of our models. Lastly, we discussed some possible extensions of the language of explicit evidence in order to provide sound and complete system for the extended dynamic language.

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Assertions as degree relations

Introduction: This paper builds on two main ideas in the literature. First, that some epistemic modal expressions are gradable (*similarly to tall/clean*), specifically that they are not quantifiers over possible worlds (Kratzer 1981, 1991, 2012) but rather denote relations between propositions and degrees of subjective probability / belief, *aka* credence. This has been claimed, for (some) modal adjectives *(e.g. possible/likely)* (Yalcin 2005, 2007, Lassiter 2010, 2015, to appear) for particles like the German *eh-* (Herburger & Rubinstein 2014, 2017, Goncharov & Irimia 2017), and motivated by the ability of such expressions to be compared (*more likely/ eher*), and / or to be modified by e.g. degree modifiers / questions (*How likely?*).Second, that Speech Acts (SA) can participate in the compositional interpretation and be embedded (e.g. Krifka 2014, 2015, 2017, Cohen & Krifka 2014, Thomas 2014, Beck 2016)[1]. We focus on assertions and on the speech act operator *ASSERT.*

Our proposal is to examine a way to integrate these two ideas, and move them one step forward so that (bare) assertion speech acts are modeled as gradable, and are compositionally modifiable by (overt and covert) degree modifiers.

The starting point motivation for our proposal relies on existing claims concerning Modal adverbs: Piñón 2006, Wolf & Cohen 2009, Wolf 2015 observe that, unlike modal adjectives (MADJs), modal adverbs (MADVs) act as modifiers of assertion speech acts. E.g.

(A) MADVs, but not MADJs can only be embedded in the consequent but not the antecedent of conditionals (cf. Bellert 1977, Nilsen 2004, Piñón 2006, Ernst 2009):

(1) a. #If John possibly/probably arrived at the office early, I will call the office.
b. If it's possible/probable that John arrived at the office early, I will call the office.
(2) a. If John is in the office, it is possible / probable that he arrived there early.
b. If John is in the office, he possibly / probably arrived there early.

We support such contrasts by data from COCA (Davies, 2008), as seen in e.g. (4):

*(3)*a. *If it is/it's possible* (243) vs. *If it is/it's/he is/he's/she is/she's possibly* (0)
b. *If possible* (1725) vs. *If possibly* (14; 12 out of these are non-conditional *ifs* as *whether*)

(B) Only MADVs are speaker-oriented (Nuyts, 2001, Ernst 2009, Nilsen 2004):

(4) A: It is probable that they have run out of fuel. B: Whose opinion is this?
(5) A: They have probably run out of fuel. B: #Whose opinion is this?

Following Piñón 2006, Wolf & Cohen 2009 and Wolf 2015 conclude that MADVs combine with *ASSERT* and lower/raise the speaker's credence degree regarding the propositional content she asserts.

Analysis: We adopt Wolf's 2015 conclusion, and suggest that if MADVs indeed lower / raise the degree of credence in assertions, then assertions, crucially, even those containing no modal expression, should involve credence degrees to begin with. There are several ways to implement this idea, depending on the specific entry for *ASSERT* one favors. Suppose, for example, we follow Thomas' 2014 and Beck's 2016 implementation of Krifka 2014, where *ASSERT* is type <<s,t>, <c,c>> as in (6), (c is the type of contexts, including a speaker, hearer, time of utterance and Common Ground (c_{sp}, c_h, c_t, C_w)):

(6) $[[ASSERT]] = \lambda p.\lambda c.\ \iota c': c'=\langle c_{sp}, c_h, c_t, C_w \cap \{w: assert\ (p)(c)\}\rangle$ *Where assert (p)(c) is true iff in w* c_{sp} *is committed to behave as though she believes that p at* c_t

We now proceed by making two moves. First, we take bare assertions to denote degree

[1] But cf. Han 1998, Palmer 1986, Platzack and Rosengren 1997, Rivero and Terzi 1995, Sadock and Zwicky 1985, Condoravdi and Lauer 2012, Lauer 2015 for a non / extra compositional view of speech acts.

relations, by adding a credence degree argument to the denotation of *ASSERT*. Adopting, for example, the entry for *ASSERT* as in (6), this will result in (7), with *ASSERT* being now type <<s,t>, <d, <c,c>>>:

(7) [[ASSERT]]: λp. λd.λc. ιc': c'=<c_{sp}, c_h, c_t, C_w ∩{w: Assert (p) (d)(c)}>, Where assert (p)(d)(c) is true iff in w the speaker of c, c_{sp}, is committed to behave as though she believes that p to a degree d, at the time c_t, and the hearer c_h is a witness to this commitment.

Second, we propose that similarly to degree modifiers over adjectives (e.g. *completely*,), MADVs are degree modifiers over gradable speech acts, G. Within the framework in (7), for example, we will end up with (8)-(10):

(8) [[Probably]]: λG. λp. λd. λc. ιc': c'=< c_{sp}, c_h, c_t, C_w∩{w: ∃d d>0.5 ∧G(p)(d)(c)}>

[[Possibly]]: λ G. λp.λd. λc. ιc': c'=< c_{sp}, c_h, c_t, C_w∩{w:∃d d> 0 ∧ G(p)(d)(c)}>

(9)(a) *John is probably a thief* b. *[Probably(Assert)] (John is a thief) (c)*

(10) ιc': c'=< c_{sp}, c_h, c_t, C_w∩{w: ∃d d >0.5 ∧ Assert (John is a thief)(d)(c)}>

I.e. (9b) combines with a context c and yields a context c' which is just like c except that the CG is updated with the information that the speaker, c_s, in c is committed at the time c_t, to behave as though her credence in "John is a thief" is greater than 0.5.

Predictions: We discuss several predictions of our proposal:

a. MADVs and degree questions. Our proposal predicts that unlike gradable MADJs, which have been shown to be modifiable by degree questions (11), MADVs will not be felicitous with such questions. This is because unlike gradable MADJs (analyzed in the literature as denoting degree relations, and modifiable by degree modifiers), under our analysis MADVs, are themselves degree modifiers (of *ASSERT*) and hence should not be modified by other degree questions due to type mismatch. Indeed, as seen in (12), this prediction is borne out:

(11) How probable is it that John left? (12) #How (much) probably is it that John left?

We discuss the better status of MADVs with e.g. *very* (as in *very possibly*) and following Kennedy & McNally (K&M) 2005, and Lassiter (to appear) who suggest that *very* is not a 'true' degree modifier. Rather, it can apply to [*possibly ASSERT]*.

b. MADVs and (some) epistemic comparatives: Goncharov & Irimia 2017 propose that some cases of epistemic comparatives in e.g. Rumanian, Bulgarian and Russian are instantiations of the comparative morpheme *–er* in the left periphery of the sentence, operating over a high epistemic covert operator, EPIST, expressing degree of speaker's credence of the proposition (cf. Rubinstein & Herburger 2014, 2017 on German *eher*). Taking this epistemic operator to be, in fact, *ASSERT*, our analysis predicts that such epistemic comparatives, being degree modifiers, will be compatible with propositional, 'low', modal expressions (expressing degree relations), but not with MADVs, which are themselves degree modifiers. This prediction seems to be borne out, at least for Russian, as seen in the contrast between (13b) with the 'low' modals and (14b) with MADVs (Goncharov, p.c.):

(13) a. Ivan mozhet byt' na rabote.
Ivan may be at work - "Ivan may be at work"
b. Ivan mozhet byt' skoree na rabote chem doma.
Ivan may be sooner at work than home "It is more plausible that Ivan may be at work than that he is at home"

(14) a. Vozmozhno Ivan na rabote. (Modal adverb)
Maybe-adv Ivan at work – "Maybe / perhaps Ivan is at work"
*b. */?? Vozmozhno Ivan skoree na rabote chem doma.*
maybe Ivan sooner at work than home

Intended: *"It is more plausible that maybe/perhaps Ivan is at work than that he is at home"*

c. The contextual variability of apparently unmodified assertions. If *ASSERT* denotes a degree relation, and is modifiable by MADVs (and some epistemic comparatives), what happens when assertions appear 'bare', i.e. when they do not seem to be modified by any overt degree modifiers?

Our analysis predicts that in such cases apparently unmodified assertions cannot stay unmodified. Instead, they will be modified by a **covert** degree modifier, which will help set the value for the degree argument of *ASSERT*. We suggest that this is indeed the case, and that such a covert degree modifier behaves in a similar way to *POS* with apparently unmodified (upper closed) adjectives.

This prediction is supported by existing observations about the contextual variability of assertions. Following Lewis 1976 Potts 2006 and Davis et al. 2007 propose that pragmatically, Grice's maxim of quality should be relaxed, as speakers do not always assert propositions with complete certainty, i.e. with subjective probability of 1. Moreover, they suggest that subjective probability varies with context. We make a similar observation i.e. that the probability that the speaker takes assertions wuch as *John is a crook* to have may be higher when this proposition is asserted, for example, as part of a testimony in court than in a casual conversation in a bar.

We now propose that the (apparent) variability of $C\tau$ with assertions is strikingly similar to the (apparent) variability found with upper-closed gradable adjectives in their 'positive form'. In general, contextual variability with adjectives is often captured by taking apparently unmodified adjectives to be modified by a covert *POS*, setting the standard of comparison, as in (15):

(15) $||POS|| = \lambda G.\lambda x.\ \exists d\ d \geq \textbf{standard (G,C)} \wedge G(x,d)]$ (e.g. von Stechow 1984, K&M 2005)

We propose that apparently unmodified assertions are also modified by a covert *POS,* identical in type to MADVs. For example, using the framework for *ASSERT* in (7) above, such a covert *POS* operator will have the denotation in (16), as illustrated in (17)-(18):

(16) $[[POS]]: \lambda G.\ \lambda p.\ \lambda c.\ \iota c': c' = < c_{sp}, c_h, c_t, C_w \cap \{w: \exists d\ d \geq \textbf{standard (G,C)} \wedge G(p)(d)(c)\}>$

(17) a. *John is a thief* b. *[POS (Assert)] (John is a thief) (c)*

(18) $\iota c': c' = < c_{sp}, c_h, c_t, C_w \cap \{w: \exists d\ d \geq \textbf{stand (ASSERT,C)} \wedge \text{Assert (John is a thief)}(d)(c)\}>$

In words, (17b) combines with a context c and yields a new context c' which is just like c except that the common ground is updated with the information that the speaker, c_s, in c is committed at the time c_t, to behave as though her credence in "John is a thief" is at least as high as the standard of credence for assertions in the context.

A potential problem with this suggestion is how the contextual variability of assertions, observed in Davis et al and Potts, is compatible with the total closeness of the credence scale, given K&M's 2005 claim that with upper closed adjectives (like *clean*) the standard of comparison is always at the maximal point. Notice, though, that K&M themselves point out cases where the positive form with such adjectives is used with an (apparently) non-maximal standard (e.g. *The theatre is empty today* when several people are present), and that contextual variability is found there too (compare *The glass is clean* when uttered by a pedant lab worker vs. by a child). This has been either accounted for by insisting on the maximal endpoint standard and deriving apparently lower standards in the positive form from imprecision, using e.g. pragmatic halos (Lasersohn 1995) as in K&M 2005, (cf. Burnett 2014 for an elaborated view), or by dissociating the standard from scale structure, allowing the former to be contextually supplied after all. In the latter direction the standard can be restricted to the upper interval of the scale, but is still allowed to vary and be lower than the maximum (as in McNally 2011. cf. also Lassiter (forthcoming) on modal adjectives with probability scales, cf. Klecha 2012).

The crucial point for us is that the contextual variability found with apparently unmodified assertions is indeed similar to the one found with Upper closed adjectives. Thus, no matter which strategy is chosen for capturing contextual variability with apparently unmodified upper-closed gradable adjectives, we suggest that the same choice can be made for apparently unmodified assertions, with the upper closed credence scale.

To conclude: In this paper we are not committed toward any specific entry for *ASSERT*, but rather suggest **a general recipe: Take your favorite entry for *ASSERT,* supplement it with a credence degree argument, and allow degree modifiers to operate over it and manipulate this degree in direct and indirect ways.**

A more general point, though, concerns the fact that our proposal that assertions are gradable and that they are modifiable by (overt and covert) degree modifiers, is to a large extent inspired by similarities with well-studied propositional constructions involving modified and (apparently) unmodified gradable predicates, which are part of the compositional process. A general take home message of our proposal, then, is that such similarities lend support to the view that speech acts should be part of the compositional process as well (e.g. Krifka 2014, 2015, 2017, Cohen & Krifka 2014, Thomas 2014, Beck 2016).

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An Ehrenfeucht-Fraïssé Game for Inquisitive First-Order Logic

Background: inquisitive first-order logic

Inquisitive first-order logic, `InqBQ`, ([2], [6], [4]) generalizes classical first-order logic to interpret not only formulas that stand for statements, but also formulas expressing questions and dependencies. Technically, `InqBQ` fits within the family of logics based on *team semantics*, being closely related to Dependence Logic ([7], [1]; for a discussion of this connection, see [3], [8]). A model for `InqBQ` represents a variety of states of affairs, or *worlds*, where each world corresponds to a first-order structure. While standard first-order formulas only express local requirements, which have to be satisfied at each world in the model, inquisitive formulas (aka questions) allow us to express global requirements, having to do with the way the worlds are related to each another. Thus, for instance, in `InqBQ` we can express that there is one individual that, uniformly across all the worlds in our model, has property P; or that the extension of property P is the same in all worlds in the model; or that, within the model, the extension of property Q is functionally determined by the extension of property P.

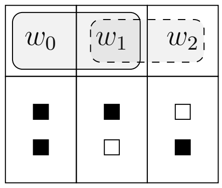
Let us recall here the essential definitions. In this paper, we assume that the given signature is finite and relational, that is, contains no function symbols. While the finiteness requirement is essential for our result to hold, the relationality requirement can be dropped. The syntax of `InqBQ` is given by the following inductive definition.

$$\varphi \quad ::= \quad R(x_1, \ldots, x_n) \mid (x = y) \mid \bot \mid \varphi \wedge \varphi \mid \varphi \to \varphi \mid \forall x.\varphi \mid \varphi \mathbin{\rlap{\vee}\,\vee} \varphi \mid \exists\!\!\!\exists x.\varphi$$

Classical first-order logic can be identified with the $\{\mathbin{\rlap{\vee}\,\vee}, \exists\!\!\!\exists\}$-free fragment of the language (the operators $\neg, \vee, \exists$ can be defined in the usual way). A model for `InqBQ` is a tuple $\mathcal{M} = \langle W, D, I, \sim\rangle$ where W is a set (the worlds), D is a non-empty set (the individuals), and:

$\mathcal{M}, s \vDash_g R(\overline{x})$	$\iff$	$\forall w \in s.\ g(\overline{x}) \in I_w(R)$
$\mathcal{M}, s \vDash_g [x = y]$	$\iff$	$\forall w \in s.\ g(x) \sim_w g(y)$
$\mathcal{M}, s \vDash_g \bot$	$\iff$	$s = \emptyset$
$\mathcal{M}, s \vDash_g \varphi \wedge \psi$	$\iff$	$\mathcal{M}, s \vDash_g \varphi$ and $\mathcal{M}, s \vDash_g \psi$
$\mathcal{M}, s \vDash_g \varphi \to \psi$	$\iff$	$\forall t \subseteq s. [\mathcal{M}, t \vDash_g \varphi \Rightarrow \mathcal{M}, t \vDash_g \psi]$
$\mathcal{M}, s \vDash_g \forall x.\varphi$	$\iff$	$\forall d \in D^{\mathcal{M}}.\ \mathcal{M}, s \vDash_{g[x \mapsto d]} \varphi$
$\mathcal{M}, s \vDash_g \varphi \mathbin{\rlap{\vee}\,\vee} \psi$	$\iff$	$\mathcal{M}, s \vDash_g \varphi$ or $\mathcal{M}, s \vDash_g \psi$
$\mathcal{M}, s \vDash_g \exists\!\!\!\exists x.\varphi$	$\iff$	$\exists d \in D^{\mathcal{M}}.\ \mathcal{M}, s \vDash_{g[x \mapsto d]} \varphi$

• I assigns to each world $w \in W$ a first-order structure I_w over the domain D; we will denote by $I_w(R)$ the interpretation of the relation symbol R in I_w.
• $\sim$ assigns to each world $w \in W$ an equivalence relation $\sim_w$ over D (the identity relation at w), with the requirement that the relations and functions respect the classes of $\sim_w$.



We refer to a set $s \subseteq W$ as an *information state*. Intuitively, we can regard s as encoding the information that the actual state of affairs corresponds to one of the worlds in s. An assignment is a map $g : \texttt{Var} \to D$. The semantics of `InqBQ` is then given by a recursive definition of the relation of *support* relative to an information state $s \subseteq W$ and an assignment g as in the table in page 1. We write $\mathcal{M} \models_g \varphi$ for $\mathcal{M}, W \models_g \varphi$. In the picture, a representation of a simple model $\mathcal{M}$ in the signature $\{P^{(1)}\}$ is depicted: it has three worlds (w_0, w_1, w_2) and two elements; under each w_i, a copy of the domain is shown, together with the interpretation of property P (black squares); $\sim$ is the actual identity relation at each world. We have $\{w_1, w_2\} \vDash \exists x.P(x)$ but $\{w_1, w_2\} \nvDash \overline{\exists} x.P(x)$: although in state $\{w_1, w_2\}$ we know that some individual with property P exists, we do not know of any individual that has the property P. By contrast, we have $\{w_0, w_1\} \models \overline{\exists} x.P(x)$, since in the state $\{w_0, w_1\}$ it is known that the upper individual has property P.

An Ehrenfeucht-Fraïssé game for `InqBQ`

Ehrenfeucht-Fraïssé games ([5]) have established themselves as one of the most useful model-theoretic tools to check if two models for a given logic are distinguishable by sentences of the logic itself. While these games were first developed for first-order logic, they have since been extended to a number of settings, including monadic second-order logic and modal logic. In this paper, we propose an Ehrenfeucht-Fraïssé game for `InqBQ`, which allows us to provide a game-theoretic characterization of equivalence with respect to `InqBQ` formulas.

Fix $\mathcal{M}$ and $\mathcal{N}$. The game can be thought as a dialog between two players, **I** (the *spoiler*) and **II** (the duplicator). **I** tries to show that $\mathcal{M}$ supports some information that $\mathcal{N}$ does not support by picking every turn an element or an info state that witness this fact. The role of **II** is to respond accordingly, simulating the choices of **I** and thus showing that such an info cannot exist.

Definition 1 (The game)**.** Consider two tuples $\langle \mathcal{M}_1, s_1, \overline{a}_1 \rangle$ and $\langle \mathcal{M}_2, s_2, \overline{a}_2 \rangle$ where $\mathcal{M}_1, \mathcal{M}_2$ are information models, s_1, s_2 are info states in the corresponding models, and $\overline{a}_1, \overline{a}_2$ are two tuples of individuals from the corresponding domains of the same length. For $m, n \in \mathbb{N}$ we define a zero-sum game $\mathrm{EF}_{m,n}(M_1, s_1, \overline{a}_1; M_2, s_2, \overline{a}_2)$, played between two players, **I** (the *spoiler*) and **II** (the *duplicator*). The game is defined inductively on the pair $\langle m, n \rangle$ as follows:
- Base case: $\langle m, n \rangle = \langle 0, 0 \rangle$. No move is performed and the game ends. Player **II** wins iff for every atomic $\varphi(\overline{x})$ with $\texttt{length}(\overline{x}) = \texttt{length}(\overline{a}_i)$: $\mathcal{M}_1, s_1 \vDash \varphi(\overline{a}_1) \Rightarrow \mathcal{M}_2, s_2 \vDash \varphi(\overline{a}_2)$
- Inductive case: **I** moves and **II** must respond accordingly. The following three options are allowed:

$\forall$ **move:** this move is allowed only if $n > 0$. **I** chooses $d_2 \in D^{\mathcal{M}_2}$ and **II** chooses $d_1 \in D^{\mathcal{M}_1}$. The game $\mathrm{EF}_{m,n-1}(\mathcal{M}_1, s_1, \overline{a}_1 d_1; \mathcal{M}_2, s_2, \overline{a}_2 d_2)$ starts. To win, **II** must win this game.

$\overline{\exists}$ **move:** this move is allowed only if $n > 0$. **I** chooses $d_1 \in D^{\mathcal{M}_1}$ and **II** chooses $d_2 \in D^{\mathcal{M}_2}$. The game $\mathrm{EF}_{m,n-1}(\mathcal{M}_1, s_1, \overline{a}_1 d_1; \mathcal{M}_2, s_2, \overline{a}_2 d_2)$ starts. To win, **II** must win this game.

$\to$ **move:** this move is allowed only if $m > 0$. **I** chooses $s_2' \subseteq s_2$ and **II** chooses $s_1' \subseteq s_1$. Then the games $\mathrm{EF}_{m-1,n}(\mathcal{M}_1, s_1', \overline{a}_1; \mathcal{M}_2, s_2', \overline{a}_2)$ and $\mathrm{EF}_{m-1,n}(\mathcal{M}_2, s_2', \overline{a}_2; \mathcal{M}_1, s_1', \overline{a}_1)$ start. In order to win, **II** must win both these games.

We indicate with $\mathcal{M}_1, s_1, \overline{a}_1 \preceq_{m,n} \mathcal{M}_2, s_2, \overline{a}_2$ that player **II** has a **winning strategy** in the game $\mathrm{EF}_{m,n}(M_1, s_1, \overline{a}_1; M_2, s_2, \overline{a}_2)$.

To connect the game to the expressivity of `InqBQ`-formulas, we introduce a notion of complexity which takes into account not only nesting of quantifiers, but also nesting of implications.

Definition 2 ($\mathtt{IQ}$ complexity)**.** The $\mathtt{I}$ and $\mathtt{Q}$ complexities of a formula φ are defined in the table in page 2. Moreover, we define $\mathtt{IQ}(\varphi) = \langle \mathtt{I}(\varphi), \mathtt{Q}(\varphi)\rangle$ and $\mathcal{L}^l_{m,n}$ the set of formulas of $\mathtt{IQ}$ complexity at most $\langle m, n\rangle$ (with respect to the component-wise order) with free variables among $\{x_1, \ldots, x_l\}$.

$\mathtt{I}(A) = 0$	$\mathtt{Q}(A) = 0$
$\mathtt{I}(\psi \circ \chi) = \max(\mathtt{I}(\psi), \mathtt{I}(\chi))$	$\mathtt{Q}(\psi \circ \chi) = \max(\mathtt{Q}(\psi), \mathtt{Q}(\chi))$
$\mathtt{I}(\psi \to \chi) = \max(\mathtt{I}(\psi), \mathtt{I}(\chi)) + 1$	$\mathtt{Q}(\psi \to \chi) = \max(\mathtt{Q}(\psi), \mathtt{Q}(\chi))$
$\mathtt{I}(\Pi x.\psi) = \mathtt{I}(\psi)$	$\mathtt{Q}(\Pi x.\psi) = \mathtt{Q}(\psi) + 1$

For A atomic, $\circ \in \{\wedge, \vee\}$ and $\Pi \in \{\overline{\exists}, \forall\}$

Theorem 3 (Ehrenfeucht-Fraïssé theorem for $\mathtt{InqBQ}$)**.** *For $l = \mathtt{length}(\overline{a}_i)$, we have:*

$$\mathcal{M}_1, s_1, \overline{a}_1 \preceq_{m,n} \mathcal{M}_2, s_2, \overline{a}_2 \iff \forall \varphi(\overline{x}) \in \mathcal{L}^l_{m,n}. [\mathcal{M}_1, s_1 \vDash \varphi(\overline{a}_1) \Rightarrow \mathcal{M}_2, s_2 \vDash \varphi(\overline{a}_2)]$$

Proof sketch. ($\Rightarrow$) By contraposition: let $\varphi \in \mathcal{L}^l_{m,n}$ be such that $\mathcal{M}_1, s_1 \vDash \varphi(\overline{a}_1)$ but $\mathcal{M}_2, s_2 \nvDash \varphi(\overline{a}_2)$. We will indicate this condition as $\mathfrak{I}(\varphi)$. We define recursively a winning strategy for **I** by maintaining the invariant $\mathfrak{I}$ for a subformula ψ of φ of suitable $\mathtt{IQ}$ complexity during a run of the game. In particular, if the subgame has index $\langle m, n\rangle$ then $\mathtt{IQ}(\psi) \leq \langle m, n\rangle$ (componentwise), assuring that ψ is atomic at the end of the game. The non-trivial cases are the following:
<u>If $\varphi \equiv \forall x.\psi$</u>, consider $d_2 \in D^{\mathcal{M}_2}$ such that $\mathcal{M}_2, s_2 \nvDash \psi(\overline{a}_2, d_2)$. Then by performing a $\forall$ move and by choosing d_2, **I** ensures that the condition $\mathfrak{I}(\psi)$ will hold in the next turn. With a similar argument we also obtain the result for $\varphi \equiv \overline{\exists} x.\psi$.
<u>If $\varphi \equiv \psi \to \chi$</u>, there exists an info state $s_2' \subseteq s_2$ s.t. $\mathcal{M}_2, s_2' \vDash \psi(\overline{a}_2)$ but $\mathcal{M}_2, s_2' \nvDash \chi(\overline{a}_2)$. Note that for each $s_1' \subseteq s_1$, one of the two corresponding relations fails. Then by performing a $\to$ move and by choosing s_2', **I** ensures that one of the conditions $\mathfrak{I}(\psi)$ and $\mathfrak{I}(\chi)$ holds for one of the subgames.
($\Leftarrow$) Again by contraposition: if **II** doesn't have a winning strategy, then by Gale-Stewart theorem, **I** does. By well-founded induction on $\langle m, n\rangle$ we can define φ for which condition $\mathfrak{I}(\varphi)$ holds.
<u>Suppose the strategy of **I** starts with a $\forall$ move</u> by choosing $d_2 \in D^{\mathcal{M}_2}$. Then by inductive hypothesis, for every $d_1 \in D^{\mathcal{M}_1}$ a formula $\psi_{d_1} \in \mathcal{L}^{l+1}_{m,n-1}$ can be found for which $\mathfrak{I}(\psi_{d_1})$ holds. From this we obtain $\mathfrak{I}(\varphi)$ for the following φ. For an $\overline{\exists}$ move, the argument is analogous.

$$\varphi :\equiv \forall y. \bigvee\!\!\!\bigvee \left\{\psi(\overline{x}, y) \in \mathcal{L}^{l+1}_{m,n-1} | \mathcal{M}_2, s_2 \nvDash \psi(\overline{a}_2, d_2)\right\}$$

<u>If the strategy of **I** starts with a $\to$ move</u>, by reasoning as in the previous case we can find a state $s_2' \subseteq s_2$ such that $\mathfrak{I}(\varphi)$ holds for the following φ:

$$X := \{\psi \in \mathcal{L}^l_{m-1,n} | \mathcal{M}_2, s_2' \vDash \psi(\overline{a}_2)\} \qquad \varphi :\equiv \bigwedge X \to \bigvee\!\!\!\bigvee(\mathcal{L}^l_{m-1,n} \setminus X)$$

Note that, since our signature is finite, the class $\mathcal{L}^l_{m-1,n}$ contains only finitely many non-equivalent formulas, and so φ can be expressed by choosing appropriate representatives. $\square$

An application: number of individuals

Ehrenfeucht-Fraïssé games are a very useful tool to investigate the expressive power of a logic. For instance, consider the following question. In a given world w, the actual individuals are given by the equivalence classes $D/{\sim_w}$. Let us denote by $c(w)$ the number of these individuals, so that world w represents a state of affairs where there are exactly $c(w)$ individuals. Now, can $\mathtt{InqBQ}$ express the question of how many individuals there actually are? In more formal terms, is there a sentence φ of $\mathtt{InqBQ}$ with the following semantics?

$$M \vDash \varphi \iff \forall w, w' \in W : c(w) = c(w')$$

Using the game, we can show that a formula with this property does not exist, and this provides an excellent example of the sort of open questions that our result allows us to answer.

Proof sketch. For simplicity we consider the empty signature, but the proof easily generalizes to an arbitrary signature Σ. By contradiction: suppose a formula φ as above exists and define $n = \mathtt{Q}(\varphi)$. Toward a contradiction, we will present two models that entail the same formulas up to $\mathtt{Q}$-complexity n, but that disagree on the property $\forall w, w' \in W : c(w) = c(w')$.

For h, k positive natural numbers, define $\mathcal{M}_{\langle h,k\rangle}$ as the model with set of worlds $\{w_0, w_1\}$, with domain $[h] \times [k] = \{\langle a, b\rangle \,|1 \leq a \leq h, 1 \leq b \leq k\}$ and with $\sim$ defined by

$$\langle a, b\rangle \sim_{w_0} \langle a', b'\rangle \iff a = a' \qquad\qquad \langle a, b\rangle \sim_{w_1} \langle a', b'\rangle \iff b = b'$$

Clearly the property $c(w_0) = c(w_1)$ holds if and only if $h = k$. Moreover, given a sequence of elements $\overline{U} = \langle\langle a_1, b_1\rangle, \ldots, \langle a_p, b_p\rangle\rangle$, the set of atomic formulas with parameters in $\overline{U}$ supported at an info state is determined by the sets $E_1^{\overline{U}} = \{\langle i, j\rangle \,|a_i = a_j\}$ and $E_2^{\overline{U}} = \{\langle i, j\rangle \,|b_i = b_j\}$ and is independent from h and k.

Using the EF game we can show that $\mathcal{M}_{\langle n,n\rangle}$ and $\mathcal{M}_{\langle n,n+1\rangle}$ satisfy the same formulas of $\mathtt{Q}$ complexity up to n. To show this, we describe a winning strategy for **II** in the game $\mathrm{EF}_{m,n}(M_{\langle n,n\rangle}; M_{\langle n,n+1\rangle})$ for an arbitrary m. Suppose that the current position of the game is $\left\langle \mathcal{M}_{\langle n,n\rangle}, s, \overline{U}; \mathcal{M}_{\langle n,n+1\rangle}, s, \overline{V}\right\rangle$ (notice that the info states are the same). The strategy is determined by the following conditions:

If I plays an $\rightarrow$ move and chooses s' in one of the two models, then **II** chooses s' in the other. Notice that this condition ensures that the info states that appear in the current position are the same trough the game;

If I plays a $\forall$ or $\overline{\exists}$ move and chooses an element from the model $\mathcal{M}_{\langle n,n\rangle}$ obtaining the sequence $\overline{U'} = \overline{U}\,\langle a_l, b_l\rangle$, then **II** choose an element from the other model obtaining a sequence $\overline{V'} = \overline{V}\,\langle a'_l, b'_l\rangle$ such that $E_1^{\overline{U}} = E_1^{\overline{V}}$ and $E_2^{\overline{U}} = E_2^{\overline{V}}$. Notice that this can always be achieved as the sequences $\overline{U'}$ and $\overline{V'}$ have length at most n.
The case in which **I** chooses an element from the model $\mathcal{M}_{\langle n,n+1\rangle}$ is analogous.

At the end of the game, the equality $E_i^{\overline{U}} = E_i^{\overline{V}}$ for $\overline{U}$ and $\overline{V}$ the sequences of elements chosen in the two models ensures the winning condition for **II**, as wanted.

□

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Tobias Heindel

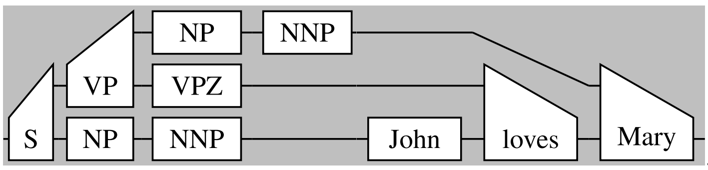
The Chomsky-Schützenberger Theorem with Circuit Diagrams in the Role of Words

Introduction

String diagrams have been gaining popularity over the last decade, especially in cross-disciplinary work on physics, logic, and computation. They arise in semantics of subject and object relative pronouns [10] as the graphical language [11] of compact closed categories. String diagrams are very intuitive, yet have formal semantics and thus bear the potential to convey theoretical subject matter to a wide audience [12].

The present paper considers string diagrams as syntactic entities that play the role of words in formal language theory in the spirit of Lafont's work on Boolean circuits [8]. Alphabets will be generalized to signatures of symbols with non-empty lists of "inputs" and "outputs". The arrows of free PROPS over such signatures can be seen as acyclic layouts of logic gates [11, Theorem 3.12].

We generalize Chomsky grammars in the obvious way to study context-free languages of arrows in free PROPS. Intuitively, context-free languages consist of (compositions of) string diagrams of tree-like shape. As example, consider the string diagram



,

which matches the phrase structure with its sentence, embedded as a branch into a tree. Note that this is not a tree but a non-trivial acyclic graph.

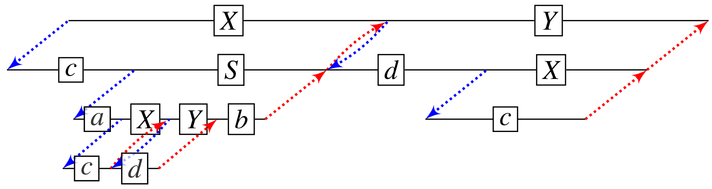
Our main contribution is the Chomsky-Schützenberger theorem for context-free languages of free PROPS, generalizing a classic result of formal language theory. In comparison to similar work on trees [1], the proposed Dyck languages of signatures are naturally seen as languages of matching brackets. The relation to context-free and recognizable graph languages [6] is left for future research.

1 Prelude: `dimension++`

One trait that we can find in formal language theory that extends existing results from words to more general structures is an extra dimension in illustrations of the objects that play the role of words. For example, if we consider the grammar

$$S \to aXYb \qquad\qquad X \to c \mid cS \qquad\qquad Y \to d \mid dX$$

we can use the illustration



in the x-z-plane to represent a parallel derivation: sentential forms are like bead necklaces laid out horizontally and derivation steps are rectangles. We can read off the corresponding leftmost derivation by following the solid lines in direction of the x-axis, switching z-coordinate along dotted arrows. We apply a production whenever we encounter a blue arrow, replacing the variable to the right of the source by the sentential form to the right of its target that forms the opposite side of a rectangle in the x-z-plane (delimited by the next red arrow).

In the present paper, circuit diagrams [5] will play the role of words. To guide the intuition about context-free grammars of circuit diagrams, we think of symbols as (placeholders for) gates that are connected by wires and of productions as implementation of non-terminal gates by more complex circuits, which drive the derivation of circuit layouts of basic gates.

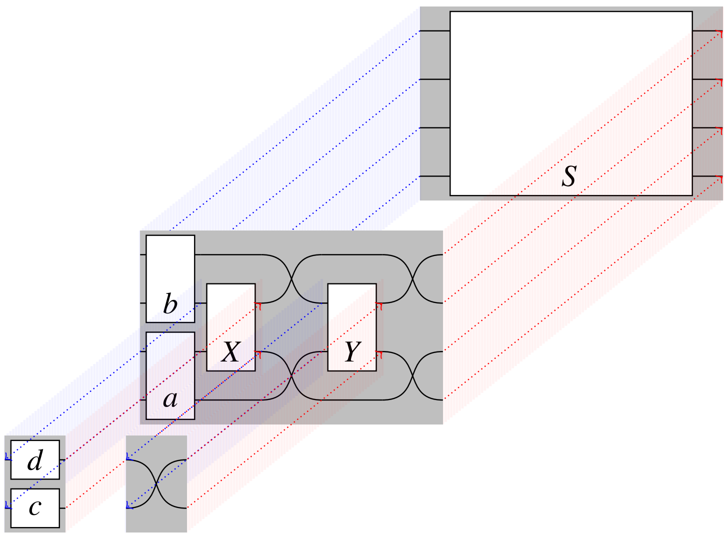


Figure 1: Parallel derivation of a circuit grammar

The extra dimension of circuit diagrams w.r.t. words is evident in Figure 1, which happens to be the illustration of a derivation in a circuit grammar—to be defined, after formalizing sequential and parallel composition of circuit diagrams in symmetric monoidal categories.

2 Preliminaries and notation

We start with notational conventions for symmetric monoidal categories and a definition of signature that induces free product and permutation categories (PROP) [9, 13].

Given a locally small category C and objects $A, B \in C$, the homset of arrows from A to B is denoted by $C(A, B)$. We shall use diagrammatic composition of arrows, denoted by the semicolon, i.e., the composition of arrows $f\colon A \to B$ and $g\colon B \to C$ in C, is denoted $f; g$. The identity on an object $A \in C$ is $\mathrm{id}_A\colon A \to A$. The monoidal product $\otimes$ of any PROP $(C, \otimes, I, \gamma)$ binds stronger than composition, i.e., $f; g \otimes h; k = f; (g \otimes h); k$ whenever the compositions are defined for arrows f, g, h, k in C.

A *signature* is a graph on the natural numbers, given by a triple $\mathbf{\Sigma} = (\Sigma, s, t)$ of a set Σ of *symbols* and two functions $s, t \colon \Sigma \to \mathbb{N}$, mapping symbols to their *arity* and *coarity*, respectively; we call it *alphabet-like* if both, arity and coarity of every symbol are positive. For a symbol $a \in \Sigma$, we write $a \colon m \to n$ if $s(a) = m$ and $t(a) = n$. The coproduct of signatures (Σ_1, s_1, t_1) and (Σ_2, s_2, t_2) is $(\Sigma_1 + \Sigma_2, [s_1, s_2], [t_1, t_2])$, the coproduct in the comma category $\mathbf{Set}/\mathrm{Id}_{\mathbf{Set}} \times \mathrm{Id}_{\mathbf{Set}}$. The *opposite* of a signature $\mathbf{\Sigma} = (\Sigma, s, t)$, denoted $\mathbf{\Sigma}^{\mathrm{op}}$, is (Σ, t, s). A *sub-signature* of $\mathbf{\Sigma}$ is a signature $\mathbf{\Upsilon} = (\Upsilon, i, o)$ such that $\Upsilon \subseteq \Sigma$, $i \subseteq s$, and $t \subseteq o$ (identifying functions with their graphs).

We fix a signature $\mathbf{\Sigma} = (\Sigma, s, t)$ for the remainder of the paper. The free PROP with generators $\mathbf{\Sigma}$ is denoted by $\mathcal{F}\mathbf{\Sigma}$. For any sub-signature $\mathbf{\Upsilon} \subseteq \mathbf{\Sigma}$, we assume $\mathcal{F}\mathbf{\Upsilon}$ to be a symmetric monoidal subcategory of $\mathcal{F}\mathbf{\Sigma}$ in the obvious manner. Finally, arrows of free PROPs are often called *cicuit diagrams*.

3 Grammars in free PROPs

We define the analogue of Chomsky grammars and formal languages in the setting of free PROPs, focussing on the context-free case. A *language* over a signature is a set of arrows in its free PROP. The basic idea of deriving in a context-free grammar consists in replacing a (non-terminal) symbol of the signature by an arrow in the free PROP that matches the arity and co-arity of the symbol and is specified by the grammar. The basic principle at work is rewriting or reduction in context, which is common in formalisms of theoretical computer science, such as the λ-calculus, configuration graphs of automata in (coloured) product categories [4], or graph rewriting [7], especially in relation to symmetric monoidal theories [3].

A *production* in a free PROP over $\mathbf{\Sigma}$ is a set of pairs of $\mathcal{F}\mathbf{\Sigma}$ arrows that share domain and codomain, i.e., a subset

$$R \subseteq \bigcup_{m,n \in \mathbb{N}} \mathcal{F}\mathbf{\Sigma}(m,n) \times \mathcal{F}\mathbf{\Sigma}(m,n).$$

Productions are thought of as directed and their components are called *left* and *right hand side*, respectively. Re-using notation for productions, we have the following examples (cf. Figure 1).

$$\begin{aligned} S &\to a \otimes b ; \mathrm{id}_1 \otimes X \otimes \mathrm{id}_1 ; \gamma \otimes \gamma ; \mathrm{id}_1 \otimes Y \otimes \mathrm{id}_1 ; \gamma \otimes \gamma \\ X &\to c \otimes d \\ Y &\to \gamma \end{aligned}$$

A *rewriting context* for a production (l, r) is a quadruple (f, i, j, g) of arrows f, g in $\mathcal{F}\mathbf{\Sigma}$ and natural numbers i, j such that $f ; \mathrm{id}_i \otimes l \otimes \mathrm{id}_j ; g$ is defined in $\mathcal{F}\mathbf{\Sigma}$. The *derivation relation* of a production (l, r), denoted by $\Rightarrow_{l,r}$, relates two arrows h and k in $\mathcal{F}\mathbf{\Sigma}$ if $h = f ; (\mathrm{id}_i \otimes l \otimes \mathrm{id}_j) ; g$ and $k = f ; (\mathrm{id}_i \otimes r \otimes \mathrm{id}_j) ; g$ hold for some rewriting context (f, i, j, g). A derivation of the above productions is illustrated in Figure 1 (in its parallel form).

The definition of grammar is as expected.

Definition 1 (Circuit grammar)**.** A *circuit grammar* is a quadruple $\mathcal{G} = (\mathbf{\Sigma}, \mathbf{\Upsilon}, P, S)$ where

- $\mathbf{\Sigma} = (\Sigma, s, t)$ and $\mathbf{\Upsilon} = (\Upsilon, \iota, o)$ are signatures of *terminals* and *variables*, respectively, such that $\Upsilon \cap \Sigma = \varnothing$;
- P is a finite set of productions in the free PROP $\mathcal{F}(\mathbf{\Sigma} \cup \mathbf{\Upsilon})$ whose left hand sides do *not* belong to $\mathcal{F}\mathbf{\Sigma}$ where $\mathbf{\Sigma} \cup \mathbf{\Upsilon}$ is the component-wise union $(\Sigma \cup \Upsilon, s \cup \iota, t \cup o)$; and
- $S \in \Upsilon$ is the *start symbol*.

The grammar $\mathcal{G}$ is *context-free* if left hand sides of all productions are variables.

Concerning syntacticity of grammars, it can be shown that each production has a corresponding expression of the following specification.

$$\mathtt{f} ::= u \mid \gamma \mid \mathrm{id}_0 \mid \mathrm{id}_1 \mid (\mathtt{f};\mathtt{f}) \mid (\mathtt{f} \otimes \mathtt{f}) \quad (u \in \Sigma \cup \Upsilon)$$

An arrow f in $\mathcal{F}\boldsymbol{\Sigma}$ is *derivable* in a grammar $(\boldsymbol{\Sigma}, \boldsymbol{\Upsilon}, P, S)$ if

$$S \Rightarrow_{l_1,r_1} f_1 \cdots \Rightarrow_{l_n,r_n} f_n = f$$

holds for some sequence of productions $(l_i, r_i) \in P$ and arrows f_i in $\mathcal{F}(\boldsymbol{\Sigma} \cup \boldsymbol{\Upsilon})$ $(i = 1, \ldots, n)$. The *language* of a grammar $\mathcal{G}$, denoted by $L(\mathcal{G})$, is the set of all $\mathcal{F}\boldsymbol{\Sigma}$-arrows that are derivable. Finally, a *context-free circuit language* is the language of some context-free circuit grammar.

4 The theorem

We shall introduce natural counterparts of Dyck languages and rational sets to generalize the Chomsky-Schützenberger Theorem.

Theorem 1 (Chomsky-Schützenberger). *A language L over an alphabet Σ is context-free if, and only if, there exists an alphabet Ξ, a rational set R over $\Xi + \Xi = \{0, 1\} \times \Xi$, and a homomorphism $h \colon (\Xi + \Xi)^* \to \Sigma^*$ such that $L = h(D_\Xi \cap R)$.*

Concerning the Dyck language, note that it corresponds to the least sub-monoid $\mathcal{D}_\Xi$ of the free monoid over $\Xi + \Xi = \{0, 1\} \times \Xi$ that contains the word $(0, u)w(1, u)$ whenever $u \in \Xi$ is a letter and the word w belongs to $\mathcal{D}_\Xi$. Replacing sub-monoid by monoidal subcategory, the *Dyck category* over $\boldsymbol{\Xi}$ is the least monoidal subcategory $\mathcal{D}_{\boldsymbol{\Xi}}$ of the free PROP $\mathcal{F}(\boldsymbol{\Xi} + \boldsymbol{\Xi}^{\mathrm{op}})$ such that $(0, u); f; (1, u)$ is a $\mathcal{D}_{\boldsymbol{\Xi}}$-arrow whenever $u \colon m \to n$ is a symbol of $\boldsymbol{\Xi}$ and $f \colon n \to n$ is a $\mathcal{D}_{\boldsymbol{\Xi}}$-arrow. The *Dyck language*, denoted by $D_{\boldsymbol{\Xi}}$, is the set of arrows of the Dyck category.

Monoidal rational sets in a free PROP $\mathcal{F}\boldsymbol{\Xi}$ are the elements of the least set $\mathcal{R}$ such that

- $\mathcal{R}$ contains all finite sets of $\mathcal{F}\boldsymbol{\Xi}$-arrows;
- $L; L' \in \mathcal{R}$ whenever $L, L' \in \mathcal{R}$ where
$$L; L' = \left\{ f; g \;\middle|\; \begin{array}{l} f \in L, g \in L', \\ f; g \text{ is defined.} \end{array} \right\};$$
- $L \otimes L' \in \mathcal{R}$ whenenver $L, L' \in \mathcal{R}$ where $L \otimes L' = \{f \otimes g \mid f \in L, g \in L'\}$;
- $L^{*_\otimes} := \bigcup_{k \in \mathbb{N}} L^{k_\otimes} \in \mathcal{R}$ whenever $L \in \mathcal{R}$ where $L^{0_\otimes} = \{\mathrm{id}_0\}$ and $L^{k_\otimes} = L^{(k-1)_\otimes} \otimes L$; and
- $L^{*_;} := \bigcup_{k \in \mathbb{N}} L^{k_;} \in \mathcal{R}$ whenever $L \in \mathcal{R}$ where $L^{0_;} = \{\mathrm{id}_1\}^{*_\otimes}$ and $L^{i_;} = L^{(i-1)_;}; L$.

Note that we also have a monoidal version of the Kleene star, as otherwise one could specify only PROP languages of arrows of bounded path width.

Theorem 2. *A* PROP *language L over an alphabet-like signature $\boldsymbol{\Sigma}$ is context-free if, and only if, there exists a signature $\boldsymbol{\Xi}$, a monoidal rational set R over $\boldsymbol{\Xi} + \boldsymbol{\Xi}^{\mathrm{op}}$, and a functor $\mathcal{H} \colon \mathcal{F}(\boldsymbol{\Xi} + \boldsymbol{\Xi}^{\mathrm{op}}) \to \mathcal{F}\boldsymbol{\Sigma}$ such that $L = \mathcal{H}(D_{\boldsymbol{\Xi}} \cap R)$.*

The proof re-uses the ideas of Ref. [2]. In particular the encoding of derivations in words over an extended alphabet can be adapted to encodings of derivations in circuit grammars—at least if the signature is alphabet-like. It is an open problem whether this restriction can be dropped.

5 Conclusion

Besides the rather natural notion of context-free languages of circuit diagrams, which are considered elsewhere [14], we have introduced Dyck languages in free PROPs that naturally fit the intuition of matching brackets. The main contribution is the Chomsky-Schützenberger theorem for languages of arrows in circuit PROPs for alphabet-like signatures; this generalization hinges on the notion of monoidal rational set, involving a monoidal Kleene star, similar to Ref. [4]. The theorem illustrates that free PROPs are a suitable setting for the development of classic results of language theory.

To the author's knowledge, this is the first time that the Chomsky-Schützenberger theorem has been developed for a non-trivial class of graph-like structures that do not consist of trees [1]. Besides future work on the relation to Courcelle's work [6], the notion of look-ahead in parsing offers itself as a promising research field, as part of a formal language theory for PROPs and PROs à la Chomsky.

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Nino Javashvili

Derivation Models According to Otar Tchiladze Text Corpus

Language constantly develops - changes morphologic structure of a word, separate words or word patterns also change semantically. The most obvious are the transformations of the language units. A language is able to derive new words. Derivation is an important part of lingual knowledge. It implies formation of new lexical units, which are created by adding derivation affixes to the root of a noun.

The paper presents the peculiarities of derivation in the novels of the famous Georgian writer Otar Tchiladze. There are shown some problems of word production, also some ways and forms of using word-formation.

The elements that take part in word formation differ in semantics and activeness. Therefore, it is more convenient to consider not separate derivative elements but the models that include these elements.

There are various models of word formation of natural language. Formation of new words can be regular or irregular, productive or unproductive. The model of word formation differs into productive and unproductive models. The model is productive if new words are produced after it in a language.

Productive word-formation is the topical one today. “Word-formation” has broad meaning in Georgian. It also means derivation that on its turn means creating new words not only by affixation but also by composition. Learning productive means of word-formation helps to develop word-formation process in a language. In the process of composing the rules and means of word-formation differ in activeness.

Derivative models are based on text corpus according to Otar Tchiladze novels. The corpus was created at the department of Language and Speech Systems of Archil Eliashvili Institute of Control Systems of the Georgian Technical University in the frames of the project “The full (morphologic, syntactic, semantic) annotation of the Georgian Language” supported by The Rustaveli Foundation.

A computer database of derivative affixes was created in the frames of the project, where there are all morphemes that are necessary for building up derivative constructions. It is possible to order affixes by some rules and form new words from a large list of the words provided with proper information automatically. This type of base makes it easy to discover the deviations of root or an affix itself that are caused by the phonetic or other language processes. The base is a pilot version and it is going to be filled and improved in the process of working.

The base of affixes is based on the works of Georgian scientists: “The Dictionary of Georgian Morphemes and Modal Elements”, “Georgian Noun Root Dictionary”, the elements of derivation by A. Shanidze that was lately expanded by L. Margvelani.

The base unites morphemes that are used in contemporary Georgian language. For now, there are about 270 morphemes, some of them are Georgian and some from other languages that are established in Georgian. It is quite simple to add new morphemes in the base, which is rather important as vocabulary changes in a language permanently. It is well

It is well known that every writer has his own style of writing. Some language elements of style of one writer may coincide with the style of the other writer. However, the structure and the speech order would be different.

Nowadays spoken language and printed media is full of barbarisms and new terms, as well as, with new composed words. The research showed that Otar Tchiladze mostly uses Georgian affixes for derivation though he has published some of his novels in this century. From all the affixes that have the same meaning, he chooses the Georgian ones rather than the ones from foreign languages that are commonly used in the Georgian language, e.g. prefix anti- occurs only once in corpus; suffix -ing is only used in a word mit'ingi (meeting). He never uses such affixes as super-, ex-, extra-, dis-.

The most productive affixes of foreign origin are -ist and -ism. The first occurs in corpus 36 times and the other one 22 times.

One of the productive Georgian affixes is -ul-, which mainly forms gerund. It occurs 2416 times in corpus. Here it should be mentioned that word-formation with infix or only prefix is quite rare. Exceptions are the words formed with ara- (no) prefix. It occurs in corpus 26 times. Circumfixes are more common in Georgian language. Names of purpose e.g. sa-qur-e (earring), sa-pul-e (purse); former condition na-sopl-ar-i (a place, where once was a village), na-kval-ev-i (a path, where once someone had passed and left footprints); names of profession me-baγ-e (gardener), me-zγva-ur-i (sailor), etc.

The paper presents the full model of word-formation and statistical data according to the corpus of the novels by Otar Tchiladze. There will also be shown analysis of the words that is characteristic to the writer only.

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Tableaux Calculus for Unranked Logics

After long time of stagnation, tableaux-based reasoning methods became popular with the development of Semantic Web. All major Description Logic reasoners (e.g. Racer [6], FaCT++ [14], Pellet [12], etc.) use tableaux as their main reasoning method (see e.g. [7, 1]).

Tableaux calculus [13] is based on the principle of refutation. When a formula is given, it is negated and according to some rules decomposed to subformulas. This decomposition produces a tree of formulas. If every branch of the tree is closed (meaning that the branch contains contradictory formulas), then the given formula is valid. Tableau has advantage over other proof systems in that it can also build a model for satisfiable formula, or find a counter-example for non valid formula.

There are many refinements and modification of the tableaux calculus in the literature (see e.g. [10, 3]). This includes tableaux for intuitionistic, temporal, modal, substructural, nonmonotonic, many-valued logics and the like. To the best of our knowledge, there is no tableaux method for unranked logics defined in the literature and our aim is to provide such.

Unranked terms are first order terms in which function symbols do not have a fixed arity: The same symbol may have a different number of arguments in different places and some variables (aka sequence variables) can be instantiated by finite (possible empty) sequences of unranked terms.

In recent years, usefulness of sequence variables and unranked symbols has been illustrated in practical applications related to XML [9], knowledge representation [5], automated reasoning, rewriting, functional, functional logic, and rule-based programming, Common Logic [2], just to name a few. There are systems for programming with sequence variables. Probably the most prominent one is Mathematica, with a powerful rule-based programming language that uses (essentially first order, equational) unranked matching with sequence variables. Unranked function symbols and sequence variables bring a great deal of expressiveness in this language, permitting writing a short, concise, readable code.

Unification procedure for unranked terms has been given in [8]. The unranked terms $f(\overline{x}, x, \overline{y}, \overline{z})$ and $f(f(\overline{x}), x, a)$, where $\overline{x}, \overline{y}, \overline{z}$ are sequence variables and x is an individual variable unifies in three different ways with the substitutions $\{\overline{x} \mapsto (), x \mapsto f(), \overline{y} \mapsto (), \overline{z} \mapsto (f(), a)\}$, $\{\overline{x} \mapsto (), x \mapsto f(), \overline{y} \mapsto f(), \overline{z} \mapsto a\}$ and $\{\overline{x} \mapsto (), x \mapsto f(), \overline{y} \mapsto (f(), a), \overline{z} \mapsto ()\}$. Note that unranked terms may have infinitely many unifiers. For example, $f(\overline{x}, a)$ and $f(a, \overline{x})$ have the unifiers $\{\overline{x} \mapsto ()\}$, $\{\overline{x} \mapsto a\}$, $\{\overline{x} \mapsto (a, a)\}$, etc. Unranked unification, when one of the terms is ground (term without variables), called matching, is finitary.

In this talk we present an extended traditional tableaux inference system to work with formulas built over unranked terms. Unranked unification is used in tableaux as a mechanism that decides whether a path can be closed. It selects terms for replacement in quantification rules. We show, that the calculus is sound and complete, thus non-terminating in general. As it was mentioned above, unranked unification is not finitary in general, that is another reason of non-termination of the given algorithm. Finally, we illustrate the potential of the extended calculus in Web-related applications. In such applications unification problem is reduced to matching and is thus finitary. More details about this work can be found in [4].

We extend classical first-order tableaux for unranked logic. To reduce number of inference rules in the calculus, we consider formulas only in *negation normal form* (NNF). Classical first-order semantics defined for our language allows us to skolemize and transform formulae to NNF in a standard way.

The unranked tableaux calculus for formulae in negation normal form consists of the following rules

$$\frac{A \wedge B}{\begin{array}{c}A\\B\end{array}}\wedge \qquad \frac{A \vee B}{A \quad\quad B}\vee \qquad \frac{\forall x A}{A[x \mapsto t]}\forall_i \qquad \frac{\forall \overline{x} A}{A[\overline{x} \mapsto \overline{s}]}\forall_s$$

Remark 1. Let us clarify the difference between the rules:

- The $\wedge$-rule chooses to decompose either or both operands in the single path. In contrast, the $\vee$-rule creates alternative paths for each operand.
- In the $\forall_i$ quantification rule the individual variable must be replaced by an individual term, while in case of the $\forall_s$-rule the sequence variable is replaced by an arbitrary sequence of terms, including the empty one.

A path of a tableaux is *closed* if it contains both, formula and its negation (modulo unification); otherwise it is *open*. A tableaux is closed if all of its paths are closed.

The tableaux calculus is sound and complete. Moreover, it is confluent; in other words, backtracking over the rules is not necessary. The only "backtracking" points are substitutions in the $\forall_i$ and $\forall_s$ quantification rules.

We demonstrate basic reasoning capabilities of our calculus using the Clique of Friends example from [11]. This example illustrates some basic reasoning for the Semantic Web. It does not use any particular Semantic Web language itself.

Consider a collection of address books where each address book has an owner and a set of entries, some of which are marked as `friend` to indicate that the person associated with this entry is considered a friend by the owner of the address book.

In the example we consider a collection that contains two address books, the first owned by `Donald Duck` and the second by `Daisy Duck`. Donald's address book has two entries, one for `Scrooge`, the other for `Daisy`, and only `Daisy` is marked as `friend`. Daisy's address book again has two entries, both marked as `friend`. In XML, this collection of address books can be represented in a straightforward manner.

The *clique-of-friends* of Donald is the set of all persons that are either direct friends of Donald (i.e. in the example above only `Daisy`) or friends of friends (i.e. `Gladstone` and `Ratchet`), or friends of friends of friends (none in the example above), and so on. To retrieve these friends, we have to define the relation "being a friend of" and its transitive closure.

We introduce the following abbreviations:

- Fixed arity function symbols: $f_r^o(\texttt{owner}), f_r^n(\texttt{name})$ and $f_r(\texttt{friend})$
- Flexible arity function symbols: $f_u^{ab}(\texttt{address-book})$ and $f_u^e(\texttt{entry})$
- Flexible arity predicate symbols: $p_u^{abs}(\texttt{address-books}), p_u^{fo}(\texttt{friend-of})$ and $p_u^{fof}(\texttt{friend-of-friend})$

Then the above-mentioned XML will be represented as the following fact in knowledge base:

$$\begin{array}{rl}\texttt{XML} \equiv & p_u^{abs}(f_u^{ab}(f_r^o(\texttt{Donald}), f_u^e(f_r^n(\texttt{Daisy}), f_r), f_u^e(f_r^n(\texttt{Scrooge}))),\\ & \quad f_u^{ab}(f_r^o(\texttt{Daisy}), f_u^e(f_r^n(\texttt{Gladstone}), f_r), f_u^e(f_r^n(\texttt{Ratchet}), f_r)))\end{array}$$

We define *friend-of* and *friend-of-friend* relationships in the knowledge base:

$$
\begin{aligned}
p_u^{fo}(\overline{x},\overline{y}) &\equiv p_u^{abs}(\_s, f_u^{ab}(f_r^{o}(\overline{x}), \_s, f_u^{e}(f_r^{n}(\overline{y}), f_r), \_s), \_s),\\
p_u^{fof}(\overline{x},\overline{y}) &\equiv p_u^{fo}(\overline{x},\overline{y}),\\
p_u^{fof}(\overline{x},\overline{y}) &\equiv p_u^{fo}(\overline{x},\overline{z}) \wedge p_u^{fof}(\overline{z},\overline{y}),
\end{aligned}
$$

where $\_s$ is an anonymous sequence variable, that can be instantiated by an arbitrary sequence of terms, including the empty one.

The query to be asked is $KB \rightarrow \exists\overline{x}\ p_u^{fof}(\texttt{Donald},\overline{x})$. Note that p_u^{fof} predicate can be represented in a single formula as $p_u^{fo}(\overline{x},\overline{y}) \vee (\neg p_u^{fo}(\overline{x},\overline{y}) \wedge p_u^{fo}(\overline{x},\overline{z}) \wedge p_u^{fof}(\overline{z},\overline{y}))$. Then the query to refute (after transforming to NNF) will be:

$$
\begin{aligned}
KB\ \ &\wedge\forall\overline{x}\ \neg p_u^{fo}(\texttt{Donald},\overline{x})\\
&\wedge\forall\overline{x}\ (p_u^{fo}(\texttt{Donald},\overline{x}) \vee \neg p_u^{fo}(\texttt{Donald},\overline{y}) \vee \neg p_u^{fof}(\overline{y},\overline{x}))
\end{aligned}
$$

The refutation consists of the following steps:

$$
\begin{array}{c}
\dfrac{KB \wedge \forall\overline{x}\ \neg p_u^{fo}(\texttt{Donald},\overline{x}) \wedge \ldots}{KB}\ \wedge\\
\dfrac{\forall\overline{x}\ \neg p_u^{fo}(\texttt{Donald},\overline{x})}{\neg p_u^{fo}(\texttt{Donald},\overline{x})\sigma}\ \forall_s\\
\times
\end{array}
$$

where $\sigma = \{\overline{x} \mapsto \texttt{Daisy}\}$. If we would like to find all solutions, then we should continue by decomposing the second formula:

$$
\begin{array}{c}
\dfrac{KB \wedge \forall\overline{x}\ \neg p_u^{fo}(\texttt{Donald},\overline{x}) \wedge \ldots}{KB}\ \wedge\\
\forall\overline{x}\ (p_u^{fo}(\texttt{Donald},\overline{x}) \vee \neg p_u^{fo}(\texttt{Donald},\overline{y}) \vee \neg p_u^{fof}(\overline{y},\overline{x}))\\
\vdots\\
\dfrac{(\neg p_u^{fo}(\texttt{Donald},\overline{y}) \vee \neg p_u^{fof}(\overline{y},\overline{x}))\theta}{\neg p_u^{fo}(\texttt{Donald},\overline{y})\theta \qquad \neg p_u^{fof}(\overline{y},\overline{x})\theta}\ \vee\\
\times \qquad\qquad\qquad \times
\end{array}
$$

where θ is either of substitutions: $\{\overline{x} \mapsto \texttt{Gladstone}, \overline{y} \mapsto \texttt{Daisy}\}$ and $\{\overline{x} \mapsto \texttt{Ratchet}, \overline{y} \mapsto \texttt{Daisy}\}$. From these substitutions we can read off the answer to our query:

$$\{\texttt{Daisy}, \texttt{Gladstone}, \texttt{Ratchet}\}$$

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Evgeny Kuznetsov

Properties of Local Homeomorphisms of Stone spaces and Priestley spaces

Introduction. Exponentiability of objects and morphisms is one of the important good properties for a category. The problem of exponentiability has been studied in many contexts since 1940s. The reader interested in the subject is referred to: the articles [4], [1],[7], [8]; and the books [3], [5],[6].

Due to the existence of important dualities between Stone spaces and Boolean algebras, as well as between Priestley spaces and distributive lattices, our aim is to characterize exponentiable objects and exponentiable morphisms in the categories of Stone spaces and Priestley spaces. The presented work is part of the more extensive program, which aims to study local homeomorphisms of logical and partially ordered topological spaces which are important for logic e.g. Stone spaces, Priestley spaces, Spectral spaces, Esakia spaces. This is motivated by the importance of local homeomorhphisms not only in topology, but in algebraic geometry and other areas of mathematics due to their attractive properties.

Given objects X, Y in the small category $\mathbf{C}$ with finite limits, the object Y^X (if it exists in $\mathbf{C}$) is said to be an *exponential* of Y by X, if for any object A in $\mathbf{C}$ there is a natural bijection between the set of all morphisms from $A \times X$ to Y and the set of all morphisms from A to Y^X, i.e. $\mathbf{C}(A \times X, Y) \cong \mathbf{C}(A, Y^X)$. An object X of a category $\mathbf{C}$ is said to be *exponentiable* if the exponent Y^X exists in $\mathbf{C}$ for any object Y. Given object X in $\mathbf{C}$, consider the class $\mathbf{C}/X$ of morphisms $f : Y \to X$ with codomain X in $\mathbf{C}$. Let morphisms between members of the mentioned class be the obvious commutative triangles. It is easy to check that the family together with the defined morphisms between them is a category. It is the case that if $\mathbf{C}$ has all finite limits, then so does $\mathbf{C}/X$. Let us note that the product of two objects of $\mathbf{C}/X$ is a pullback in $\mathbf{C}$ with the obvious projection to X. As in the case of $\mathbf{C}$, given two morphisms $f : Y \to X$ and $g : Z \to X$ the object g^f (if it exists in $\mathbf{C}/X$) is said to be an exponential of g by f, if for any object $h : W \to X$ in $\mathbf{C}/X$ there is a natural bijection between the set of morphisms from $h \times_X f$ to g and the set of morphisms from h to g^f, i.e. $\mathbf{C}/X(h \times_X f, g) \cong \mathbf{C}/X(h, g^f)$. A morphism f of a category $\mathbf{C}$ is said to be *exponentiable morphism* if the exponent g^f exists in $\mathbf{C}/X$ for any morphism g of $\mathbf{C}$.

Note that for categories of sets with additional structure and structure preserving maps, the problem of exponentiability reduces to finding appropriate corresponding structure of the same kind on the set of structure-preserving maps. In the following subsections we state the main result already obtained regarding exponentiable objects and morphisms in the categories of Stone spaces and Priestley spaces. For brevity, the supporting lemmas and propositions are omitted.

We conclude the section with definition of the abovementioned notion of local homeomorphism between topological spaces. A map $f : X \to B$ between topological spaces X and B is said to be a *local homeomorphism* if each point x in X has an open neighborhood which is mapped homeomorphically by f onto an open subset of B. For more information about local homeomorphisms see [3], [6].

Local homeomorphisms as Exponentiable maps of Stone spaces. A compact, Hausdorff, and zero-dimensional topological space is called a *Stone space*. The first category we are interested in is the category of Stone spaces and continuous maps. Let us denote the mentioned category by **Stone**. Our investigation of exponentiability of objects in **Stone** showed that only the finite spaces are exponentiable (unlike the case of the category of all topological spaces where only core-compacts are exponentiable, that is the spaces where every neighborhood U of any point x has a sub-neighborhood V, such that every open cover of U contains a finite subcover of V; such spaces can be infinite [2],[4]). Note that by [1] compact Hausdorff topological space is exoponentiable iff it is finite. Below for Stone spaces and Priestley spaces we have similar results:

Proposition 1. *A Stone space X is exponentiable in* **Stone** *if and only if X is finite.*

After that we are able to prove the full characterization of exponentiable maps of Stone spaces. That is the following result holds:

Proposition 2. *The map $f : X \to B$ between Stone spaces is exponentiable in* **Stone**$/B$ *if and only if f is a local homeomorphism.*

Exponentiability in Priestley spaces. A partially ordered topological space $(X, \leq)$ is called a Priestley space, if X is compact topological space and for any pair $x, y \in X$ with $x \not\leq y$, there exists a clopen up-set U of X such that $x \in U$ and $y \notin U$. It turns out that the topology on a Priestley space is compact Hausdorff and zero-dimensional, i.e. is a Stone topology. The second category we are interested in is the category of Priestley spaces and continuous order-preserving maps. Let us denote this category by **PS** (**P**riestley **S**paces). Investigation of exponentiability of objects in **PS** showed that, similarly to the case of Stone spaces, only finite spaces are exponentiable in **PS**. Hence the following:

Proposition 3. *A Priestley space X is exponentiable in* **PS** *if and only if X is finite.*

Due to this fact, given a Priestley space B we get the following corollary about exponentiability of $\pi_2 : X \times B \to B$ in **PS**$/B$:

Corollary 3.1. *$\pi_2 : X \times B \to B$ is exponentiable in* **PS**$/B$ *if and only if X is finite.*

$$
\begin{tikzcd}
(X \times B)_b \arrow[r] \arrow[d, "b^*(\pi_2)"'] \arrow[dr, phantom, "\lrcorner", very near start] & X \times B \arrow[d, "\pi_2"] \\
1 \arrow[r, "b"'] & B
\end{tikzcd}
\qquad
\begin{tikzcd}
X \times B \arrow[r] \arrow[d, "t_B^*(t_X)"'] \arrow[dr, phantom, "\lrcorner", very near start] & X \arrow[d, "t_X"] \\
B \arrow[r, "t_B"'] & 1
\end{tikzcd}
$$

Moreover, we were able to prove a necessary condition for exponentiability of a map between Priestley spaces. An order preserving map $f : X \to B$ is called an interpolation-lifting map if given $x \leq y$ in X and $f(x) \leq b \leq f(y)$, there exists $x \leq z \leq y$ such that $f(z) = b$.

Proposition 4. *If $f : X \to B$ is exponentiable in* **PS**$/B$ *then f is interpolation-lifting.*

We are still unable to find a necessary and sufficient condition for exponentiability of Priestley maps. Already obtained results draw quite interesting picture of considered categories. Only the smallest part of the considered categories (only finite objects) have such strong property as exponentiability. Further work is in progress, namely we are investigating whether exponentiable morphisms in **PS** are precisely the local homeomorphisms that are also interpolation-lifting maps.

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Stepan Kuznetsov, Valentina Lugovaya and Anastasiia Ryzhova

Recursive enumerability doesn't always give a decidable axiomatization

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It is well-known that if a theory (deductively closed set of formulae) over a well-behaved logic (for example, classical or intuitionistic logic) is recursively enumerable (r.e.), then it has a decidable, and even a primitively recursive axiomatization [2]. This observation, known as *Craig's theorem,* or Craig's trick, is indeed very general. If we denote the deductive closure (set of theorems) for an axiomatization $\mathcal{A}$ by $[\mathcal{A}]$ and let $[\mathcal{A}]$ be recursively enumerated as follows: $\varphi_1, \varphi_2, \varphi_3, \ldots$ ($\varphi_k = f(k)$, where f is a computable function), then the set $\mathcal{A}' = \{\varphi_1, \varphi_2 \wedge \varphi_2, \varphi_3 \wedge \varphi_3 \wedge \varphi_3, \ldots\}$ will be decidable (the decision algorithm, given a formula ψ, starts enumerating $\mathcal{A}'$, compares the elements with ψ, and stops with the answer "no" when the size of the formula exceeds the size of ψ: further formulae will be only bigger), and, on the other hand, $\mathcal{A}'$ serves as an alternative axiomatization for the theory, since $[\mathcal{A}'] = [\mathcal{A}]$.

The only thing we need from the logic for this construction to work is the following property: for any formula ψ there exists, and can be effectively constructed, an equivalent formula ψ' of greater size than ψ. Then we take $\mathcal{A}' = \{\varphi_1, \varphi_2', \varphi_3'', \ldots\}$ as the needed decidable axiomatization: since $'$ increases the size of formula, the n-th formula in this sequence has size at least n; therefore, in our search for a given ψ in $\mathcal{A}'$ we have to check only a finite number of formulae. This works even for substructural systems that don't enjoy $\psi \leftrightarrow \psi \wedge \psi$. For example, once there is an operation $\circ$ that has a unit $\mathbf{1}$, Craig's theorem is valid: $A \leftrightarrow A \circ \mathbf{1} = A'$.

Thus, it looks interesting to find a logic for which Craig's theorem fails. Of course, one could easily construct degenerate examples, like a "logic" without any rules of inference: then $[\mathcal{A}]$ is always $\mathcal{A}$, and if it was r.e., but not decidable, it doesn't have a decidable axiomatization. So we're seeking for an example among interesting, useful logical systems.

And such an example exists—it is the product-free fragment of the *Lambek calculus* [3]. We denote this calculus by $\mathbf{L}$ and present it here as a Gentzen-style sequential calculus; a non-sequential ("Hilbert-style") version also exists [4]. Formulae of $\mathbf{L}$ are built from a set of variables $\mathrm{Var} = \{p_0, p_1, p_2, p_3, \ldots\}$ using two binary connectives, $\backslash$ and $/$. Sequents are expressions of the form $A_1, \ldots, A_n \to B$, where A_i and B are formulae and $n \geq 1$ (empty antecedents are not allowed). The axioms and rules of $\mathbf{L}$ are as follows (here capital Greek letters denote sequences of formulae):

$$\overline{A \to A}$$

$$\frac{A, \Pi \to B}{\Pi \to A \setminus B} \text{ where } \Pi \text{ is non-empty} \qquad \frac{\Pi \to A \quad \Gamma, B, \Delta \to C}{\Gamma, \Pi, A \setminus B, \Delta \to C}$$

$$\frac{\Pi, A \to B}{\Pi \to B \,/\, A} \text{ where } \Pi \text{ is non-empty} \qquad \frac{\Pi \to A \quad \Gamma, B, \Delta \to C}{\Gamma, B \,/\, A, \Pi, \Delta \to C}$$

$$\frac{\Pi \to A \quad \Gamma, A, \Delta \to C}{\Gamma, \Pi, \Delta \to C} \text{ (cut)}$$

Let $\mathcal{A}$ be an arbitrary set of sequents. We say that a sequent $\Pi \to A$ is derivable from $\mathcal{A}$ (denoted by $\mathcal{A} \vdash_{\mathbf{L}} \Pi \to A$), if there exists a derivation tree where inner nodes are applications of rules (including cut: in this setting it is not eliminable), and leafs are instances of axioms or sequents from $\mathcal{A}$. The theory axiomatized by $\mathcal{A}$ (the deductive closure of $\mathcal{A}$) is $[\mathcal{A}] = \{\Pi \to A \mid$

$\mathcal{A} \vdash_{\mathbf{L}} \Pi \to A\}$. Clearly, if $\mathcal{A}$ is r.e., then so is $[\mathcal{A}]$. Finally, $\mathcal{A}_1$ and $\mathcal{A}_2$ are equivalent, $\mathcal{A}_1 \approx \mathcal{A}_2$, if $[\mathcal{A}_1] = [\mathcal{A}_2]$.

Theorem 1. *There exists such a recursively enumerable $\mathcal{A}$ that there is no decidable $\mathcal{A}'$ equivalent to $\mathcal{A}$.*

Let $q = p_0$ and let $\mathcal{E} = \{p_i \to q \mid i \geq 1\}$.

Lemma 1. *If $\mathcal{A} \subsetneq \mathcal{E}$ and $\mathcal{A}' \approx \mathcal{A}$, then $\mathcal{A}' \cap \mathcal{E} = \mathcal{A}$.*

This Lemma immediately yields our goal: if $\mathcal{A}$ is a recursively enumerable undecidable subset of $\mathcal{E}$, it gives undecidability of any $\mathcal{A}'$ equivalent to $\mathcal{A}$.

We prove the Lemma by a semantic argument, via formal language models for $\mathbf{L}$. Let Σ be an alphabet; Σ^+ stands for the set of all non-empty words over Σ. An interpretation w is a function that maps formulae of $\mathbf{L}$ to subsets of Σ^+, defined arbitrarily on variables and propagated as follows:

$$w(A \setminus B) = w(A) \setminus w(B) = \{u \in \Sigma^+ \mid (\forall v \in w(A))\, vu \in w(B)\}$$
$$w(B \,/\, A) = w(B) \,/\, w(A) = \{u \in \Sigma^+ \mid (\forall v \in w(A))\, uv \in w(B)\}$$

A sequent $A_1, \dots, A_n \to B$ is true under interpretation w, if $w(A_1) \cdot \ldots \cdot w(A_n) \subseteq w(B)$, where $M \cdot N = \{uv \mid u \in M, v \in N\}$. The calculus is sound w.r.t. this interpretation: if all formulae of $\mathcal{A}$ are true under w and $\mathcal{A} \vdash_{\mathbf{L}} \Pi \to B$, then $\Pi \to B$ is also true under w. (A weak completeness result, for $\mathcal{A} = \varnothing$, is shown in [1]. Here we need only soundness.)

We consider a countable alphabet, $\Sigma = \{a_1, a_2, \dots\}$.

First, we show that $\mathcal{A} \not\vdash_{\mathbf{L}} p_i \to p_j$ for $i \neq j$, $i, j \geq 1$. Consider an interpretation $w_1(p_i) = \{a_i\}$, $w_1(q) = \Sigma^+$. All sequents from $\mathcal{A}$ are true under w_1, while $p_i \to p_j$ isn't. Therefore, $(p_i \to p_j) \notin \mathcal{A}'$ if $i \neq j$, $i, j \geq 1$.

Second, we show that $\mathcal{A} \not\vdash_{\mathbf{L}} E_1 \setminus E_2 \to p_i$ and $\mathcal{A} \not\vdash_{\mathbf{L}} E_2 \,/\, E_1 \to p_i$ for any $i \geq 0$ and any formulae E_1 and E_2. The counter-interpretation here is as follows: $w_2(p_i) = \{a_i\} \cup \Sigma^{\geq 2}$, $w_2(q) = \{a_j \mid (p_j \to q) \in \mathcal{A}\} \cup \Sigma^{\geq 2}$, where $\Sigma^{\geq 2}$ is the set of all words of length at least 2. All sequents from $\mathcal{A}$ are true under w_2. By induction on A we show that $w_2(A) \supseteq \Sigma^{\geq 2}$ for any formula A. Then, since uv is always in $\Sigma^{\geq 2} \subseteq w_2(E_2)$, we have $w_2(E_1 \setminus E_2) = w_2(E_2 \,/\, E_1) = \Sigma^+$, but $w_2(p_i)$ is not Σ^+ for any i (including 0).

Third, we show that if $\mathcal{A} \vdash_{\mathbf{L}} p_i \to q$, then $(p_i \to q) \in \mathcal{A}$. If not, then interpretation w_2 defined above falsifies $p_i \to q$ keeping all sequents in $\mathcal{A}$ true. This yields $\mathcal{A}' \cap \mathcal{E} \subseteq \mathcal{A}$ (since all sequents in $\mathcal{A}'$ are derivable from $\mathcal{A}$).

Finally, we establish the converse inclusion by contraposition. Let $(p_k \to q) \notin \mathcal{A}'$ and show that $(p_k \to q) \notin \mathcal{A}$. Consider the following interpretation: $w_3(p_i) = \{a_i\} \cup \Sigma^{\geq 2}$, $w_3(q) = \{a_j \mid (p_j \to q) \in \mathcal{A}'\} \cup \Sigma^{\geq 2}$. Evidently, w_3 falsifies $p_k \to q$. It remains to show that all sequents from $\mathcal{A}'$ are true under w_3. There are several possible cases for a sequent from $\mathcal{A}'$.

1. The sequent is of the form $A \to A$ (including $q \to q$ or $p_i \to p_i$). This is an axiom, it is true everywhere.

2. The sequent is of the form $p_i \to q$. Then it is true by definition.

3. The sequent is of the form $p_i \to p_j$, $i \neq j$, $i, j \geq 1$. Then this sequent is not derivable from $\mathcal{A}$ (see above) and therefore cannot belong to $\mathcal{A}'$.

4. The sequent is of the form $E_1 \setminus E_2 \to p_i$ or $E_2 \,/\, E_2 \to p_i$. Again, it couldn't be derivable from $\mathcal{A}$ and couldn't belong to $\mathcal{A}'$.

5. The sequent is of the form $A \to F_1 \setminus F_2$ or $A \to F_2 \,/\, F_1$. As for w_2, for w_3 we have $w_3(F_1 \setminus F_2) = w_3(F_2 \,/\, F_1) = \Sigma^+$. The sequent is true.

Hence, $\mathcal{A}' \nvdash_{\mathbf{L}} p_k \to q$, therefore $(p_k \to q) \notin \mathcal{A}$. This finishes the proof.

Notice that this result is not at all robust: slight modifications of the calculus restore Craig's theorem. First, actually one can increase the size of all formulae, *except* variables, by the following equivalences: $A \,/\, B \leftrightarrow A \,/((A \,/\, B) \setminus A)$ and $B \setminus A \leftrightarrow (A \,/(B \setminus A)) \setminus A$. In our construction, we played on an infinite number of variables, for which such increasing is impossible. Thus, Craig's theorem holds for any fragment of **L** with a finite set of variables. Second, if we allow sequents with empty left-hand sides (and remove non-emptiness restrictions from the rules of **L**), we have $A \leftrightarrow (A \,/\, A) \setminus A$ for any formula A, which also yields Craig's theorem.

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Carol-Rose Little

Cardinal and ordinal meanings of possessed numeral constructions in Ch'ol (Mayan)

This paper investigates a certain underscribed numeral construction in Ch'ol, a Mayan language of Southern Mexico, that can mean either a cardinal (e.g., 'the two') or ordinal (e.g., 'the second') meaning. The numeral constructions, which I call 'possessed numerals', are composed of a numeral and its classifier with a possessive suffix. An example is given in (1).

(1) *I-cha'-k'ejl-el jiñi waj.*
A3-two-CL-RS DET tortilla
(i) 'This tortilla is the second'. or
(ii) 'These tortillas are the two.'

Without context, the numeral underlined could be translated as either a definite cardinal ('the two') or an ordinal ('the second').

I discuss how the possessive morphology contributes to the semantics of these numerals. I propose that the ambiguity in meaning arises from the possessive morphology on the numeral. The possessive morphology tracks whether the numeral picks out a salient set in the discourse and gives back the cardinality of that set or if it picks out an individual/subset from a contextually ordered salient set and gives back the position of an individual from that set.

BACKGROUND. Ch'ol is a head-marking, ergative-absolutive Mayan language of Chiapas Mexico. Absolutive morphemes are glossed with B person markers. Possessive morphemes appear on the head noun and are syncretic with ergative markers, glossed as A person markers. The possessee appears before the possessor, as can be seen in (2).

(2) ***i**-juñ aj-Shenia*
A3-book NC-Shenia
'Shenia's book'

The construction relevant to this paper are possessive constructions with the *-el* relational suffix, which derives a tighter semantic relationship between possessor and possessee, as in the minimal pair in (3).

(3) a. *i-pisil aj-Rosa*
A3-clothes NC-Rosa
'Rosa's clothing/cloth (e.g. her family's laundry, curtains, sheets)
b. *i-pisil-**el** aj-Rosa*
A3-clothes-RS NC-Rosa
'Rosa's clothing/cloth (i.e. that she wears on her body)

When attached to numerals it derives ordinals or definite cardinals as in (4). Note that the preposition *tyi* also optionally appears before as well.

(4) *(tyi) i-cha'-p'ejl-el*
(PREP) A3-two-CL-RS
'the second' or 'the two'

POSSESSED NUMERALS. As in the data in (4), without context the possessed numeral has a cardinal or ordinal meaning. However, with other person, we see that the A prefix changes. With an ordinal meaning, the possessive prefix is third person as in (5a) but first person when modifying first person in (5b).

(5) a. *Tyi* *i-cha'-tyikl-el-oñ.*
PREP A3-two-CL-RS-B1
'I am the second.'

b. *Tyi* *k-cha'-tyikl-el-oñ.*
PREP A1-two-CL-RS-B1
'We are the two.'

Semantically, the cardinal meaning of the possessed numeral is exhaustive. Once the speaker introduces into the context *tyi icha'tyiklel kalobil* 'my two children' in (6a), it is infelicitous to follow it up with 'Actually another one went there too'.

(6) a. *Chumul–Ø-**ob*** *tyi* *i-cha'-tyikl-el* *k-alobil* *tyi* *Estados* *Unidos.*
live-B3-**PL** PREP A3-two-CL-RS POSS.1-child PREP SP:states SP:united
'My two children live in the United States.'

b. # *Che'* *a'* *tsa'* *majl-i-Ø* *yambä* *je'e.*
PART there PRFV go-IT-B3 another.one also
'And another one (of my children) went there, too.'

Thus, the data in (6) provides evidence that the possessed numeral in this context entails a set of *exactly* the number of children I have.

Other the other hand, bare numerals do not have such entailments. The sentence in (7) with the bare numeral *cha'tyikil* can be followed up felicitously with (6b).

(7) *Chumul-Ø-ob* *cha'-tyikil* *k-alobil* *tyi* *Estados* *Unidos.*
live-B3-PL two-CL A1-child PREP SP:states SP:united
'Two of my children live in the United States.'

With a singular predicate as in (8), there are no cardinality entailments of the possessed numeral and (8) can be followed up with (6b).

(8) *Chumul-Ø* *i-cha'-tyikl-el* *k-alobil* *tyi* *Estados Unidos.*
live-B3 A3-two-CL-*el* POSS.1-child PREP USA
My second child lives in the United States.

Descriptively, the relational suffix and third person possessive prefix derive an ordinal numeral interpretation. The addition of a preposition derives definite cardinal interpretations. In the ordinal meaning, the possessive prefix is always third person. For the definite cardinal, the possessive prefix has the same person features as the group it group it describes. So for phrases like 'we are the two' in (5b), the possessive prefix is first person plural. In other words, when the possessive prefix and the entity that the possessed numeral modifies are co-indexed, it results in the cardinal meaning. This provides evidence for third person: when the third person marker is referential with the set of objects that the numeral quantifies, it has a cardinal meaning. When it is not, it has an ordinal meaning. This provides evidence that the third person possessive prefix is referring to something else. A possibility I explore is that is refers to an abstract set to which the ordinal belongs.

PUTTING THE MORPHOLOGY TOGETHER. By thinking about the possessive structures as conveying proper or improper relations, we can think about an analysis for the *-el* morpheme in Ch'ol on whether it conveys a proper or improper relationship between possessor and possessee. A part–whole structure in Ch'ol is given in (9). This is similar to the derived relational meaning of the possessed noun in (3b).

(9) *i-tye'-el* *otyoty*
A3-wood-RS house
'the wood of the house'

Say that *-el* is a transitive morpheme that defines a relation between two arguments. The complement (i.e., *tye'* 'wood') must have a proper subpart relation with the possessor *otyoty* 'house'. Indeed, as this is a part–whole relation the relationship between the wood and the house is one where wood is part of what makes up the house. This would be proper subset relatoin (as there are other things that make up a house, not just wood).

Ordinal numerals are derived from cardinal ones with the same morphology in the part-whole relation in (9). In this possibility the part is the number (three) and the whole is some set. One way to think about three as a part is to think about dividing a set into subsets. The cardinal number 'three' divides a set of entities into a subset and then ascribes a property to last member of that subset. In other words, an ordinal is always a proper part of a set: it picks out a member of a plurality.

Evidence that a proper subset notions may be on the right track to account for ordinals is that 'first' in Ch'ol is suppletive. Adding the possessive morphology on 'one' in Ch'ol does not derive 'first'. Rather the suppletive form *ñaxañ* means first. If the possessed numeral 'one' meant 'first' then there would be counterevidence for concluding that *-el* conveys a proper subset. If an entity is first in a set of one then it is *not* a proper subset of that set. Since the possessed numeral 'one' does not mean 'first', rather a suppletive form is used, this is evidence that a proper subset relation can capture the relationship between ordinal and the set to which it belongs, since if something is second, it will always belong to a plurality (i.e., of at least two members). Indeed, Barbiers (2007) discusses this fact that many languages have suppletive forms for 'first', one reason being that the number 'one' is different from higher numbers as it is the only nonplural one. Thus, so far the possessed numerals with their ordinal meaning seem to be predicted by assuming that *-el* conveys a proper subset relation.

However, the possessed numeral has a definite cardinal meaning. These constructions bring up the possibility of an improper subset meaning for *-el*. In the definite cardinal numeral examples, the possessor clearly references a whole set of individuals. In this case the first person plural inclusive refers to a set of individuals numbering in exactly three. Thus, the numeral is describing the exact number of individuals that the first person plural possessive morphology refers to. Thus, it is not a proper subset relationship in this case. Thus, I propose the relational suffix *-el* defines an improper subset relation between the possessor and possessee. (Indeed, Ionin et al. (2006) define partitive relationships as being improper, partially based on evidence like 'the three of us' in English).

Finally, this work contributes important, new data on how languages without determiners express definiteness or uniqueness. Ch'ol is an NP language as defined by Boskovic (2008). The ordinals do not necessitate a definiteness interpretation, however the cardinal meaning does and is semantically equivalent to English 'the *n*'. However, in Ch'ol both the structure and possessive marking contributes to the definiteness meaning.

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Irina Lobzhanidze

Computational Model of Modern Georgian Language and Searching Patterns for On-line Dictionary of Idioms

Introduction

Any kind of electronic dictionary can be considered as a database; generally, its purpose is to provide adequate explanation or translation of separate words or multi-word expressions (MWE), to store information and to allow user to find appropriate language units. Following Gibbon (2000), there are four major prerequisites to the design of any lexicographic database, i.e. dictionaries:

1. Linguistic specification (of macrostructure and microstructure);
2. Database management system (DBMS) specification;
3. Specification of phases of lexicographic database construction: input, verification and modification;
4. Presentation of and access to lexical information: access, re-formatting, dissemination.

In case of Modern Georgian language, the main problems are associated from one point with linguistic specification, which corresponds to types of lexical information involving linguistic analysis for Modern Georgian Language and from another point – with access to lexical information stored in the database (DB) by end-user. The Modern Georgian language belongs to morphologically rich languages. Descriptions of Georgian morphological structure emphasize large number of inflectional categories; the large number of elements that verb or noun paradigms can contain; the interdependence in the occurrence of various elements and the large number of regular, semi-regular and irregular patterns. It means that the morphologically rich nature of Georgian expresses different levels of information at the word level and affects a compilation of dictionaries, i.e. lexicographic databases for Georgian language. Thus, the main issues, which are worth of mentioning are as follows:

a) representing of verbal forms in dictionary entries caused by the absence of infinitive (verbal noun vs verb in the third person singular);
b) polypersonalism of Georgian verb, which causes inclusion of different verbal patterns in the majority of Georgian printed or electronic dictionaries;
c) searching patterns for verbal forms in electronic and online dictionaries having in mind that it is completely impossible to focus on a lemma for verbal forms and to provide their setting in alphabet order.

Present paper answers the above mentioned issues describing for instance the On-line Dictionary of Idioms prepared under the financial support of the Shota Rustaveli Science Foundation (Projects No Y-04-10, No LE/17/1-30/13) and the morphological analyzer of Modern Georgian Language (Project No AR/320/4-105/11) used for advanced search.

The Dictionary of Idioms is bilingual in Modern Georgian and Modern Greek, includes approximately 12000 entries and reflects the function and meaning of idioms. It combines features of translational and learner's dictionaries. The dictionary is available at http://idioms.iliauni.edu.ge/.

Section 1 Morphological Analyzer of Modern Georgian and Problems of Georgian Lexicography

The Morphological analyzer is developed as bi-directional finite state transducer by means of Xerox Finite State Tools (xfst and lexc), which is sufficient for capturing morphological structure of Modern Georgian Language. The system includes 13 blocks of the existing Part of Speech (PoS) of Modern Georgian language as well as separate blocks for Punctuation and Abbreviations, while the pattern for Verbal Paradigm is subdivided into additional 66 groups as described by Melikishvili (2001) and an additional group for irregular verbs.

The morphotactics of language is encoded in PoS lexicons and alternation rules are encoded in regular expressions.

The morphological transducer developed on the basis of Xerox Finite State Tools (Xfst) has the following structure:

The lexicon data are processed in accordance with the appropriate alternation rules. It allows us to distinguish the appropriate lemma and morphological categories. This resource evaluated against different texts is used for tokenizing, lemmatizing and tagging.

Section 2 Brief Descriptions of Dictionaries

The majority of Georgian electronic dictionaries (monolingual and bilingual, e.g. http://translate.ge/, http://ena.ge/ etc.) share similar lexicographic problems caused by the following:

a) Absence of an infinitive form of the verb, which affects dictionary entries and causes use of different patterns (some dictionaries include entries represented in the form of Verbal nouns so called masdars, e.g. Oniani (1966) etc., others – in the form of Verbs in the third-person singular of the present tense, e.g. Sakhokia (1979) etc., also, there are dictionaries possessing both of the above mentioned forms);

b) Georgian verb template consisting at least of twelve constituents implies existence of preverbs, person and version markers before the root. It makes impossible to find appropriate verb in dictionaries by initial letters of verbal noun or the third-person singular of the present tense i.e. in alphabetic order. And it forces Native and Non-Native speakers of Georgian to acquire grammatical information on Georgian verbal patterns with purpose of searching and translating, which have a negative impact on language acquisition.

Each dictionary stated above selects its own types of access to the data, generally, by special filters, which allow user to look for a word not only in the headword lists, but also in the whole database. This possibility is rather difficult to acquire taking into account that the end-user, generally, has a possibility to find some words without their meaning and cannot use them for his/her purposes.

Section 3 Methods

During the compilation of Morphological Analyzer of Modern Georgian language and the compilation of Online Dictionary of idioms, I have used different kind of approaches:

a) Finite state techniques, especially, xfst and lexc (as described by Beesley, Kartunnen 2003, Koskenniemi 1983 etc.) used for the compilation of the morphological analyzer of Modern Georgian;

b) Approaches of modern corpus based lexicography (as described by Atkins 2008, Sinclair 1996, Ooi 1988 etc.) used for the compilation of the On-line dictionary of idioms by means of TLex system.

Section 4. Findings and Hypothesis

The compilation of any dictionary includes the sequence of stages. In the case of the Online Dictionary of Idioms, we determined the form of the on-line dictionary and the structure of entries, revised the existing units using the concordance from the corpus of Modern Georgian Language[1] and additional one created in TLex system[2], add revised and new entries to TLex system, converted the prepared dictionary to .xml format and launched an on-line version of dictionary. At the same time we had to find solution for the problems described previously. So, special attention was paid to

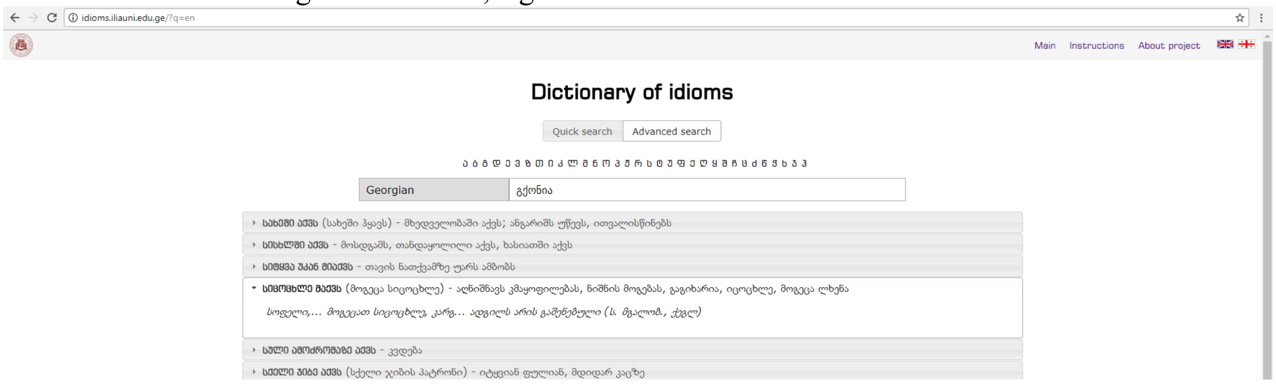
[1] http://corpora.iliauni.edu.ge/

[2] http://tshwanedje.com/

1. The Dictionary entries, which differ from the viewpoint of the elements represented in monolingual and bilingual parts. Most entries include information on lemma sign, derivational variants of use, etymological notes for some entries, definitions, literary citation with indication of literary source;
2. Ordering of entries that is closely connected to the following types of search:
   - Quick Search: Type in keyword or phrase that you are looking for, then press ENTER;
   - Advanced Search: Perform a more extensive search associated with grammatical structure;
   - Alphabetic Search: Browse the dictionary from ა (a) to ჰ (h);
   - Wild Card: * can represents the occurrence of any number of characters

   Such kind of ordering means that if a user wants to find any word or constituent of multiword expression, it is allowed directly from the web.

At the same time the so called Advanced Search is a decision to the issues mentioned above, especially, the absence of infinitive form and the impossibility to find appropriate verbal MWE by initial letters of headwords. This option performs search for any kind of word as it is met in the raw text and gives users possibility to see direct translation of its initial form in our case it is the third person singular for verbs and nominative case singular for nouns, e.g.



Modern Georgian allows forms like: *სიცოცხლე გქონია* 'you are happy', *სიცოცხლე მქონია* 'I am happy' etc. Such kind of forms can be seen in literary sources as well and the user whose knowledge of Modern Georgian is not very high will not be able to find them taking into account that the headwords in the dictionary entries for a verbal form *გქონია* 'you have' are *აქვს* 'has' or *ქონა* 'possession'. The system available online determines the lemma sign for a verb *გქონია:*

გქონია:აქვს+V+Intr+Res1+<DatSubj>+<NomObj>+Subj2Sg+Obj3

And then based on the lemma *აქვს* 'has' provides search in the database and returns all verbal MWE associated with the above-mentioned verb. As a result, the system gives the end-user access to the headword of the appropriate word.

Section 5 Conclusions

The compilation of on-line dictionary is useful for the further development of computational approaches to Georgian language. Taking into account that the compilation of monolingual and bidirectional bilingual dictionary of idioms is over, there is a possibility to represent results of our research and describe the further stages of its development. The dictionary is available at http://idioms.iliauni.edu.ge/.

Keywords: on-line dictionary of idioms, morphological analyzer and generator of Modern Georgian Language, multi-word expressions, verbal patterns

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Sebastian Löbner

Cascades of action levels and the decomposition of action verb meanings

1. Levels of action. The talk is concerned with the mental, or cognitive, ontology of human action as is relevant for the understanding of the meanings of action verbs. It is generally accepted that a particular doing, like Trump's writing his name on a piece of paper, can be seen as constituting acts in an unlimited number of ways. It is, however, controversial in philosophy and semantics whether such a doing should actually be considered one act or as many acts as there are logically different descriptions of it. Castañeda [1] portraits the controversy as one between "unifiers", e.g. Davidson [3] – they take the position that there is just one doing – and "multipliers" like Goldman [5] who claim that what is done at different levels of description is as many different acts; his approach has been characterized as a "fine-grained" view of action. Goldman's is a theory of act tokens. A particular act token, he argues, is in a relationship of "level-generation" to other act tokens by the same agent and at the same time: it "level-generates" acts at higher levels, and it may itself be level-generated by acts at lower levels. As Castañeda [1] points out, a concrete act token can always be considered to generate an infinity of other act tokens. This is due to the fact that higher levels, for example acts that are consequences of the acts at lower levels, may be generated by particular conditions given in the context.

2. Goldman's level-generation, Clark's action ladders. According to Goldman [5], an act (token) is related to other act-tokens by the same agent at the same time in a hierarchical tree structure. There are different types of level-generation, the upward relation ↥ in these trees; Goldman distinguishes simple (x runs 100 m in 9.5 seconds ↥ x breaks the world record over 100 m), conventional (x nods ↥ x agrees), causal (x hits y in the face ↥ x breaks y's nose), and compound level-generation (x jumps, x shoots ↥ x jumpshoots); we omit the controversial type of augmentation generation. The downward relation consists in "x doing A' [higher level] by, or in, doing A". Crucially, all modes of level-generation involve additional necessary conditions. Clark [1] employs a similar, though less differentiated notion of "action-ladder" in his model of social interaction. The actions of an action ladder "begin and end together" (p. 147). Actions at lower levels, cause what is done at higher levels; actions are completed in the ladder bottom-up. Conversely, higher levels are evidence that the lower level actions were performed. Similarly to Goldman, the downward relationship is x does A' by doing A.

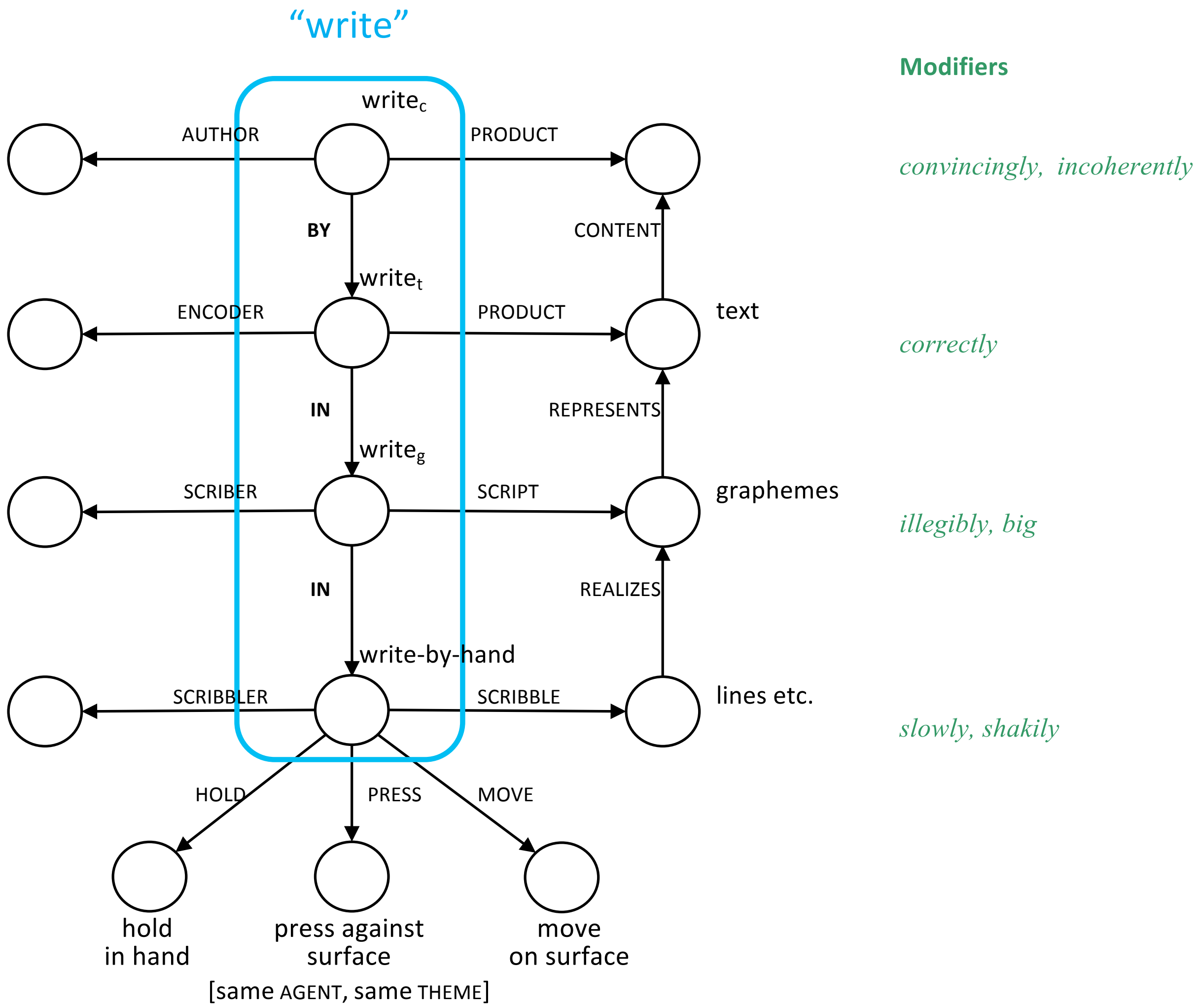
3. From Goldman's theory of act tokens towards a theory of action types. It will be argued that application of Goldman's theory of action to the modeling of the lexical meaning of action verbs allows a better understanding and a more adequate decomposition. At the level of lexical meaning, one is dealing with act[ion] *types* rather than tokens, as the context of action and any particular conditions required for level-generation are not unavailable premises of inference. The condition of context-independence restricts the mechanisms of level-generation to a small number. A further restriction arises from the commitment of frame decomposition to cognitive plausibility.

The arguments for applying Goldmanian level distinctions in the conceptual modeling of verb meanings are these: (1) If the same type of action can systematically be described at multiple levels, independent of context, this needs to be reflected in decomposition as part of cognitive reality. The multiple-level approach is corroborated by the data of verb semantics: (2) The distinction of levels is relevant for modeling the interaction between adverbial modifiers and action verbs – they may apply specifically at different levels. (3) The meanings of action verbs allocate the event denoted at different levels of action, more often then never leaving open what the actual basic doing of the agent is; also, it appears, the level referred to is not necessarily fixed in the lexicon.

The discussion addresses the fact that one and the same action can be conceived of at different levels of what is done – from basic levels of physical doing up to more abstract levels, including social interaction. For example, Eva writes a letter to her friend. She may do this by moving a pen on her favorite stationery, producing written signs that can be read as meaningful text, and thereby communicating with her friend. These are conceived of as different actions; they are carried out with different objectives; the respective agent, although the same person, acts in different roles, and interacts with different material objects and persons in different ways. However, at all these levels of the action, we can describe what Eva does by using the sentence *Eva writes a letter to her friend*. It will be argued that this is not due to an accidental polysemy of the verb *write*, but rather reflects general aspects of the cognitive ontology of action type concepts in the lexicon. Any action type involves a "cascade" of intrinsically related other action types.

4. A frame model of cascades. The essential relational structure of cascades of action can be modeled by frames in the sense of [6], by employing attributes that correspond to the constitutive relations. An act on any non-basic level is necessarily implemented by a lower-level act, the lowest being the physical doing. There is only one such substrate for any act, whence we are entitled to assume an attribute that assigns to a non-basic act the act that level-generates it. There is evidence that it might be indicated to distinguish two such attributes, the IN attribute of IMPLEMENTATION / PHYSICAL BASIS for the cases where x does [higher-level] A' *in* doing A, and a BY attribute for the MEANS of achieving an act A' *by* doing A. These two downward attributes have upward inversions: COUNTS_AS / AMOUNTS_TO / CONSTITUTES for IN, and EFFECT, or CAUSES for BY. These upward and downward attributes apply to conventional and causal level-generation, respectively. "Simple" level-generation (ex. break the world record) is completely contingent on context and will not be found with lexical action-type concepts. Compound level-generation (ex. jumpshoot) can be modeled as part actions constituting the complex whole. The parts correspond to mereological attributes of the next level; notably, mereological attributes are biunique.

5. Application to the semantics of Action verbs: the ‘write’ cascade



The figure displays a cascade involved in one method of writing – writing by hand. The physical level is added for the sake of concreteness. It cannot be considered part of the lexical meaning. Given that the verb can denote writing by hand as well as by a computer or other means, we have to assume that the IN attribute of the mid-level *write*$_g$ is not specified (though present) in the lexical frame. Similarly, the higher levels, though accessible and often made use of, may not be elaborated in the lexical frame. In lexical frames, the generated higher levels are not arbitrary. For ‘write’, the level above the central one will be specified as a level of text production. As all levels displayed here (and maybe more) can just be called “write”, it wouldn’t make sense to distinguish one of the four nodes within the blue box as the predestined referential node of the frame. We assume that a referential node will be chosen in context.

5.2 Cascade levels and modification. As indicated in the figure, adverbial modifiers apply at different levels of the cascade. Some are uniquely related, like *illegibly*, *big*, *shakily*, or *convincingly*. Others can be associated with more than one level: *fast* can relate to the physical level as well as to the content and other levels; *elegantly* may address the grapheme level or the content level.
For adverbials such as *intentionally*, cascade structures matter in other regards: one can do something intentionally at a particular level, but by doing that level-generate an act that is not intended.

5.3 Cascade levels and lexical meaning. A look at the vocabulary of action verbs shows that many verbs specify actions at a level above the concrete physical act. Verbs of motion like *come*, *go*, or *cross* leave open the means and modes of locomotion; speech act verbs do not specify the locutionary level; verbs of causing emotional reactions, like *annoy*, *frighten*, *delight*, *offend* have an unspecified BY argument. Certain verbs of social interaction like *help* leave open by what kind of action the interaction is done; one can do almost anything to help somebody under circumstances (see Engelberg [4] on 'help'). Even verbs of physical action do not fix the bottom level of physical implementation: *eat*, *walk*, *grasp*, *kick*. The concrete BY level, it appears, is generally not addressed by lexicalized meaning. Sæbø [7] discusses two types of answer to *how* questions, like *How did he kill the victim*. One type consists in specifying the method by which the action is implemented; this corresponds to a specification of the IN or BY attribute in the respective cascade.

5.4 Agent roles and products. The levels of action are paralleled in the semantic roles of the acts. At different levels, products of different sorts form the THEME of the action: lines on paper, graphemes, text, content. In a sense similar to the level relations for acts, the products at the different levels build on each other; the lower ones are the substrate of the higher levels. Likewise, the agents of the acts, although usually the same person, act in different roles that build on each other; note that lower level agency can, under circumstances, be delegated to other implementers.

The three threads of cascades can each be considered as building on the same physical lowest stratum, but expand to different levels of description, and into different contexts where they constitute different things in our mental ontology.

6. Philosophical issues. It has been proposed to consider the upward cascade relations as instances of supervenience (e.g. Engelberg [4] for 'help'). The talk will address the relationship between level-generation and supervenience, which will be argued to be a weaker, though concomitant relation.

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Ondrej Majer

Belief Aggregation for Non-standard Reasoners

Aggregated beliefs in Neighborhood Semantics

Our starting point is a non-normal model of (categorical, i.e. non-graded) individual beliefs using neighborhood frames (see e.g. [5], [3]):

Definition 1. A *neighborhood frame* $\mathcal{F}$ is a pair $\langle W, n_i \rangle$, where W is a set of possible worlds and $n_i : W \to \mathcal{P}(\mathcal{P}(W))$ is a neighborhood function, one for i in a given, finite set of agents. A *neighborhood model* is a triple $\langle W, n_i, v \rangle$ such that $\langle W, n_i \rangle$ is a neighborhood frame and $v : At \to \mathcal{P}(W)$ is a valuation function.

As usual, $M, w \models \varphi$ means that φ (a formula of a doxastic logic with a finite set belief operators B_i) is true at the state s in a model M. Truth of atomic formulas is given by the valuation v. Truth of propositional formulas is defined in the standard way as well as the truth conditions for beliefs[1]: $M, w \models B_i\phi$ iff $\|\phi\| \in n_i(w)$, where $\|\phi\| = \{w \in W | \; M, w \models \varphi\}$

In what follows we consider neighborhood frames with the following properties:

(Monotonicity) If $X \in n(w)$ and $X \subseteq Y$ then $Y \in n(w)$.

(Unit) $W \in n(w)$.

(D) If $X \in n(w)$ then for all $W - X \notin n(w)$.

The complete logic of such frames is well-known, all three conditions above can be axiomatized modularly using an axiomatization of classical propositional logic and the following axioms:

E	If $\vdash \phi \leftrightarrow \psi$ then $\vdash B_i\phi \leftrightarrow B_i\psi$	U	If $\vdash \phi$ then $\vdash B_i\phi$
M	$\vdash B_i(\phi \wedge \psi) \to B_i\phi$	D	$\vdash B_i\phi \to \neg B_i\neg\phi$

We want to study aggregated beliefs in that setup, where aggregation is defined in analogy with distributed knowledge in epistemic logic. We say that ϕ is aggregated or distributed beliefs in group G (and write $D_G\varphi$) if it would result from putting together what the members of G believe. For distributed knowledge in normal modal logic the natural way to interpret "putting together" is the intersection of the agent's epistemic accessibility relation. In neighbourhood semantics there are more room to maneuver. Here we look at two variants:

1. Taking the intersection of *pairs of* beliefs held by some (not necessarily different) agents in the group. We denote it $\sqcap_G n_i(w)$ below. Under that reading $D_G\phi$ means that there at least two agents in the group for which ϕ would result from putting together two of their beliefs.

2. Taking the intersection of *all* beliefs of all agents in G. This boils down to close the result of the first variant under intersection. So we denote it $\sqcap^*_G n_i(w)$ below. Here $D_G\phi$ means that ϕ would result from putting together all the beliefs of the group members.

Note that neither of these guarantees consistent aggregation. In case some of the agent's beliefs are mutually inconsistent, then the group will believe in contradiction: $\emptyset$ will be in the aggregated neighbourhood. If the agent's individual beliefs are otherwise closed under logical

[1] There are some alternative interpretations, e.g. evidence based models use a 'monotonic' semantic interpretation in which the neighbourhood requires not the truth-set of the given formula, but only a subset of it, see [2].

consequences (i.e. the neighbourhood are monotonic), then inconsistent aggregation results in explosion. This motivates looking at the result of applying the two operations above in neighbourhoods that are not necessarily upward closed. So taken together we consider four possible scenarios, which all result in different, non-equivalent logics:

	$\sqcap_G n_i(w)$	$\sqcap^*_G n_i(w)$
Monotonic n_i	Variation 1	Variation 2
Arbitrary n_i	Variation 3	Variation 4

In all these variations we use a standard truth condition for aggregated beliefs:

$$M, w \models D_G\phi \text{ iff } \|\phi\| \in n_D(w)$$

The two different understanding of aggregation that we just presented will correspond to two different definitions of n_D. Let A be the set of agents and G one of its non-empty subsets. Let:

$$\sqcap_G n_i(w) = \{X \cap Y : \text{ for } X \in n_i(w), Y \in n_j(w) \text{ for some agents } i, j \in G\}$$

We then define $\sqcap^*_G n_i(w)$ as the smallest set that contains $\sqcap_G n_i(w)$ but is such that if X and Y are in $\sqcap^*_G n_i(w)$ then their intersection is there as well.

Variation 1 This is the version that Eric Pacuit proposes in [5]. The resulting neighbourhood will be monotonic, and contain the unit. It might not, however, satisfy D, nor will it be closed under conjunction. Observe that necessitation (U) for D_G follows from $\wedge'$ and U for the individual beliefs. The usual inclusion axiom that is used in the axiomatization of distributed knowledge for normal beliefs ($B_i\phi \to D_G\phi$) is an instance of $\wedge'$.

Variation 2 This makes the resulting aggregated beliefs a normal modality. Like before, necessitation for D_G follows from necessitation for the individual B_i's. Observe, however, that despite being normal this modality still doesn't validate the D axiom, even if all the individual beliefs do.

Variation 3 We start by considering binary aggregation, i.e. the first variant in the introduction, of individual beliefs that only satisfy the E rule. We are looking at arbitrary neighbourhood functions for the individual beliefs, which might not contain the unit, might not be upward closed or might even be completely empty. Leaving out monotonicity has the direct consequence that D_G will not be closed under logical consequence either. Otherwise the logic of D_G stays the same.

Variation 4 We finally consider the case were we close the aggregated beliefs under conjunctions, but where the individual beliefs are represented by arbitrary neighbourhood. The resulting logic is what one would expect. The loss of monotonicity for the individual beliefs already makes D_G non-normal, but one direction of $\wedge$ remains valid. With this direction in hand we can use the weaker $B_i\phi \to D_{\{i\}}\phi$ in order to capture the relation between individual and aggregated beliefs.

We show that the following axioms for Variation 1 and 3 respectively plus the axioms for belief are together sounds and complete w.r.t. neighbourhood frames with D_G such that $n_D(w)$ is defined as $\sqcap_G n_i(w)$. The same statement holds for Variations 2 and 4 where $n_D(w)$ is defined as $\sqcap^*_G n_i(w)$.

	Variation 1		Variation 2
REG	If $\vdash \phi \to \psi$ then $\vdash D_G\phi \to D_G\psi$	$\wedge$	$\vdash D_G(\phi \wedge \psi) \leftrightarrow D_G\phi \wedge D_G\psi$
$\wedge'$	$\vdash B_i\phi \wedge B_j\psi \to D_{\{i,j\}}(\phi \wedge \psi)$	$Incl'_1$	$\vdash B_i\phi \to D_{\{i\}}\phi$
$Incl$	$\vdash D_G\phi \to D_{G'}\phi$ whenever $G \subseteq G'$	$Incl$	$\vdash D_G\phi \to D_{G'}\phi$ whenever $G \subseteq G'$

	Variation 3		Variation 4
$\wedge'$	$\vdash B_i\phi \wedge B_j\psi \to D_{\{i,j\}}(\phi \wedge \psi)$	$\wedge - I$	$\vdash D_G\phi \wedge D_G\psi \to D_G(\phi \wedge \psi)$
		$Incl'_1$	$\vdash B_i\phi \to D_{\{i\}}\phi$
$Incl$	$\vdash D_G\phi \to D_{G'}\phi$ whenever $G \subseteq G'$	$Incl$	$\vdash D_G\phi \to D_{G'}\phi$ whenever $G \subseteq G'$

Foundation in Non-Standard Probabilities

This part is aimed at looking for a foundation of the different notions of aggregated categorical beliefs above in terms of (non-standard) credence. The connection between partial and full belief is given by the Lockean thesis: a proposition φ is fully believed if the degree of belief in φ is sufficiently high i.e. above a given threshold $r > 1/2$ (see [4] for a recent discussion).

Probability functions are usually defined for a σ-algebra of subsets of a given set Ω. In logical contexts, however, it is often more natural to define probability functions directly for a propositional language. A probability function (for L - the language of classical propositional logic) is a function $p : L \to R$ satisfying the following constraints:

1. Non-negativity. $p(\varphi) \geq 0$ for all $\varphi \in L$.
2. Tautologies. If $\models \varphi$ then $p(\varphi) = 1$.
3. Finite additivity. If $\models \neg\varphi \wedge \psi$, then $p(\varphi \vee \psi) = p(\varphi) + p(\psi)$.

The starting point for individual beliefs is to use a simple form of the Lockean thesis[2]:

$$B\varphi \text{ if and only if } p(\varphi) \geq r > 1/2$$

It is easy to see that the Lockean belief operator B based on classical probabilities satisfies the conditions $\mathsf{E}, \mathsf{M}, \mathsf{D}, \mathsf{U}$ from the previous section. It is not normal however, because $p(\varphi) \geq r$ and $p(\psi) \geq r$ does not imply $p(\varphi \wedge \psi) \geq r$.

Our main motivation for introducing a non-standard framework is to represent agents with possibly inconsistent partial beliefs. Non-standard probabilities (see [6]) are supposed to be a generalization of the classical ones in the same spirit as the Belnap-Dunn four valued logic is a generalization of the classical logic. A formula in Belnap-Dunn logic might be neither true nor false (a truth value gap) or both true and false (a truth value glut). The probabilistic counterpart allows for gaps and gluts having non-zero probabilities.

Definition 2 (Non-standard probabilities)**.** A (non-standard) probability space is a pair $\langle \mathcal{L}, p \rangle$, where $\mathcal{L}$ is the set of formulas of a propositional logic L and p is a (non-standard) probability measure, that is, a function from $\mathcal{L}$ into the real numbers satisfying:

1. for all $\varphi \in \mathcal{L}$, $0 \leq p(\varphi) \leq 1$,
2. for all $\varphi, \psi \in \mathcal{L}$, if $\varphi \vDash_L \psi$ then $p(\varphi) \leq p(\psi)$,
3. for all $\varphi, \psi \in \mathcal{L}$, $p(\varphi \wedge \psi) + p(\varphi \vee \psi) = p(\varphi) + p(\psi)$.

The condition 3. makes the framework non-standard as the probability of a formula and its negation are not complementary in the usual sense, but are connected by a much weaker condition:

[2]This form of Lockean thesis does not allow us to define higher order beliefs, we address this topic in a future research.

$p(\varphi \wedge \neg\varphi) + p(\varphi \vee \neg\varphi) = p(\varphi) + p(\neg\varphi)$. This allows for $p(\varphi \wedge \neg\varphi) > 0$ (positive probability of gluts) and $p(\varphi \vee \neg\varphi) < 1$, i.e. $1 - p(\varphi \vee \neg\varphi) > 0$ (positive probability of gaps).

The axioms above clearly validate M for categorical beliefs. As in the classical case we do not have normality and given the above regarding gluts and gaps, we do lose both U (necessitation), and D (consistency). Lockean beliefs satisfy:

E	If $\vdash \phi \leftrightarrow \psi$ then $\vdash B_i\phi \leftrightarrow B_j\psi$
M	$\vdash B_i(\phi \wedge \psi) \rightarrow B_i\phi$

A standard way of getting a group attitude in this setup is to aggregate the individual probabilities and then apply the Lockean thesis again. We start with a simple linear weighted averaging: assume we have a group G of agents with individual probability functions p_i and equal weights. Then the aggregated probability $p_{G'}$ for a $G' \subseteq G$ is defined as

$$p_{G'}(\varphi) = \sum_{i \in G'} 1/|G'| \cdot p_i(\varphi), \text{ and aggregated belief as } D^{ag}_{G'}\varphi \text{ iff } p_{G'}(\varphi) \geq r$$

It is easy to see that $p_{G'}$ satisfies 1.- 3. from the previous definition. We compare the properties of the group belief operators from the previous section to those of the non-standard Lockean operator. D^{ag}_G satisfies regularity (REG), because if $\vdash \phi \rightarrow \psi$, then $\phi \models \psi$, and according to 2. in the definition of probability $p(\varphi) \leq p(\psi)$ as well as $p_{G'}(\varphi) \leq p_{G'}(\psi)$ for every G'. Distribution over conjunction ($\wedge$) holds for the same reason.

All variations from the previous part preserve an aggregated belief when the set of agents increases ($Incl$). In general this does not hold for $D^{ag}_{G'}$, the reason is that the probabilistic aggregation procedure takes into account not only belief of the agents, but in a sense also their disbelief – the agents added to the original group might push the new group average below the threshold. The weak form of individual belief inclusion ($Incl'_1$) $\vdash B_i\phi \rightarrow D^{ag}_{\{i\}}\phi$ holds, because $p_{\{i\}}(\varphi) = p_i(\varphi)$. The remaining properties ($\wedge'$) and ($\wedge - I$) do not hold in the Lockean setup. The properties of the non-standard Lockean beliefs are summarized in the following table

Lockean belief	
$Incl$	$\vdash D^{ag}_G\phi \rightarrow D^{ag}_{G'}\phi$ whenever $G \subseteq G'$
REG	If $\vdash \phi \rightarrow \psi$ then $\vdash D^{ag}_G\phi \rightarrow D^{ag}_G\psi$
$\wedge - E$	$\vdash D^{ag}_G(\phi \wedge \psi) \rightarrow D^{ag}_G\phi \wedge D^{ag}_G\psi$

Future work

This topic presents an initial stage of a work in progress and there are many ways to proceed. There are more methods of merging of both categorical and partial beliefs to be explored. We will also concentrate on a closer comparison of the categorical and Lockean part and try to establish some correspondence results. Another topic to deal with is the relation of our aggregation procedures for non-standard reasoners to judgment aggregation.

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A Brief History of the Logic of Time

ABSTRACT: Philosophical paradoxes about time, from ancient to contemporary times, have been catalysts for the development of logics of time. Zeno's paradoxes posed questions about the infinite divisibility of time intervals and the coherence of infinitesimals(Salmon, 1980). Aristotle's puzzle in *On Interpretation* about future contingent sentences such as "*tomorrow there will be a sea-battle*" questioned whether such sentences could coherently be ascribed definite truth-values at the present time(Frede, 1970). Diodorus Cronus's "*Master Argument*" proving a principle of plenitude in which all possibilities are realized in time raised questions about the interaction between modality and temporality(White et al., 1984). In modern times the great divide between A-series and B-series theories of time began with McTaggart's(McTaggart, 1908) argument for the "unreality of time". Einstein's(Einstein, 1905) theory of special relativity proving the *relativity of simultaneity* seemed to undermine the intuitive A-series theories of time but the B-series theories of time continued to have their own paradoxes (e.g., the moving present or "*now*"). Gödel's(Gödel, 1946/1995, 1949/1990, 1949/1990, 1952/1990) discovery of values for the equations of Einstein's General Relativity(Einstein, 1915) allowing for time travel seemed further confirmation of McTaggart's view of the *non-objectivity* of time.

Resolving these philosophical paradoxes has led to an evolving series of logics of time(Burgess et al., 1982, 1982b; Burgess, 1984; Kuhn, 1989; Prior, 1957, 1968; van Benthem, 1982, 1984, 2010; Ludlow, 2018). These models involve distinguishing between *time* and *tense*, between the *ordering* and *ontology* of time, and *instants* and *interval* as well as calling attention to a wealth of linguistics distinctions (e.g., *durative/punctual/telic/non-telic/static/dynamic updating* semantics). Reichenbach's(Reichenbach, 1956) incidental, but highly influential, remarks about using three references points to model tense inspired the alternative development of Prior's(Prior, 1968) modal tense logics. These formalizations were precise enough to lead to definability theorem (e.g., Kamp's theorem(Kamp, 1968) that every first-order statement with one free variable is definable on continuous linear order using *since* and *until*) but also to indefinability and incompleteness results. Thomason(Thomason, 1972) proved the incompleteness of tense logic with Löb's Axiom for the past modal operator $\blacksquare$ "*it has always in the past up until now*" $\blacksquare(\blacksquare p) \rightarrow p) \rightarrow \blacksquare p$ and the McKinsey Axiom $\Box\Diamond p \rightarrow \Diamond\Box p$ using the future modal temporal operators, $\Diamond$ "*at least once in the future it will be the case that*" and, $\Box$ "*always in the future from now.*" The progressive tense is indefinable in the standard temporal base language(van Benthem, 2010).

Disciplinary Stages: Logic, Space and Time

	Geometries of Space	Logics of Time and Tense
PARADOXES	Zeno's Paradoxes of space, time and motion [c. 450-430 BC]	Aristotle's Sea Battle, Diodorus Cronos Master Arg. [c. 300 BC]
AXIOMATIZATION OF STANDARD THEORIES	Euclid's Elements [c. 300 BC], Felix Klein's [1872] group-theoretic unification of geometries; Hilbert's axiomatization of geometry [1899]	Aristotle's *Physics*, Reichenbach's model of tenses [1947]. Kamp's definability theorem [1968]. Prior's modal tense logics [1957-1968]
EMERGENCE OF NON-STANDARD THEORIES	Bolyai-Lobachevskian [c. 1830] and Riemannian [1854] non-Euclidean geometries	Einstein's [1905, 1915] relativity of simultaneity, McTaggart [1908] A-series and B-series theories of time
APPLICATIONS OF NON-STANDARD THEORIES	Minkowski's [1915] geometric model of Einstein's Theory of Relativity	Gödel's [1947, 1952] discovery of non-standard topologies and time travel in relativistic universes
PUZZLING OBSERVATIONS & MONSTROSITIES	Peano Curve [1890]. Cantor's Discontinuum [1883]. Weyl-Tile Conundrum [1949]	Tense and Aspect, (Bennett and Partee [1972]), Expletive Negation with *Until, Since, Before*, etc.
EMERGENCE OF NEW PARADIGM	Fractal Geometries. [1970s]	Dynamical Semantics. [1990s]

Careful empirical observations in linguistics have also been the catalysts for showing the inadequacies of the standard models of tense as well as raising a host of linguistic puzzles(Bennett & Partee, 1972). Why is the durative but not the punctual reading of "*until*" consistent with negative polarity (e.g., "*Gödel did not marry until his father died*" is grammatical but "*Gödel married until his father died*" is not)(Mar, Manyakina, & Caffary, 2015). What explains the puzzling synonymy of the so-called "*expletive negation*" constructions such as "*I miss not seeing you*" or the synonymy (in German, but also in Spanish and Hebrew) of such constructions as "*bevor du nicht dein Zimmer aufgeräumt hast, darfst du nicht fernsehen*"?(Cépeda, n.d.) Partee(Partee, 1984) called attention to sentences such as "*Barbara didn't remember that she forgot to turn off the stove until the colloquium started*" which show that some past sentences are more definite than the usual tense logic operators.

Our brief sketch of a history of the logics of time exhibits an evolving dialectic among philosophy, formal logics, and empirical linguistics (Prior, 1957). This pattern of development is shared by the history of other disciplines such as the evolution of the geometries of space (van Benthem, 2010; Mar, 2017)and extensions of classical treatments of the semantic paradoxes into dynamical semantics revealing chaos and fractal images(Mar & Grim, 1991; Mar & St. Denis, 1999; Mar, 2017, 2014, 2006, 2001) .

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Ralf Naumann and Thomas Gamerschlag

Serial verb constructions and covert coordinations in Edo - An analysis in a dynamic frame theory

Data and central issue. Edo is a language spoken in Central Nigeria and belongs to the Kwa-family, in which serial verb constructions (SVCs) are a characteristic part of the grammar. Summarizing the existing consensus, [1] defines an SVC as a monoclausal sequence of verbs which act together as a single predicate, without any overt marker of coordination or subordination and a single value for tense and aspect. For Edo, Aikhenvald's definition must be strengthened in two respects: SVCs have only one subject and at least one internal argument is shared by the verbs. A central difference in Edo is that between a consequential SVC and a covert coordination (CC), [6]. Consider the following two examples taken from [2, p.3].

(1) a. Òzó ghá gbè ẹ̀wé khiẹ́n.
Ozo FUT hit goat sell
'Ozo will kill the goat and sell it.' Consequential SVC (CSVC)
b. Òzó ghá gbè ẹ̀wé khiẹ́n ùhùnmwùn érẹ́n.
Ozo FUT hit goat sell head its
'Ozo will kill the goat and sell its head.' Covert Coordination (CC)

In a CSVC the verbs are either transitive or ditransitive. The subjects and direct objects are always identified with each other, i.e. they are coreferential. By contrast, an indirect object of a ditransitive verb is never identified with any argument of the other verb. In particular, the indirect objects are not identified if both verbs are ditransitive.

(2) Úyi hàé Ìsọ̀kẹ̀n ígHó dó-rhié.
Uyi pay Isoken money steal
'Uyi paid Isoken the money and stole it.' [6, p.137]

Despite the fact that the subjects are always identified, it is not possible to have a subject pronoun before V2 in a CSVC, (3-a). This restriction does not hold for a CC, as shown by (3-b).

(3) a. Òzó$_i$ mú èmá (*Ọ́$_i$) kpẹ̀ẹ́.
Ozo carry drum he beat [6, p.64]
b. Òzó$_i$ gbọ́ọ́ ívìn Ọ́$_i$ bòló ọ́kà.
Ozo plant coconut he peel corn
'Ozo planted coconut and he peeled the corn.' [6, p.65]

If the object arguments in a CC are coreferential, there is a pronoun after the second verb, (4-a). For a CSVC, such a pronoun is not admissible. (4-b) can only be interpreted as a CC, having the same meaning as (4-a).

(4) a. Òzó$_i$ lé ízẹ̀$_j$ Ọ́$_i$ rrí ọ́rẹ̀$_j$.
Ozo cook rice he eat it
'Ozo cooked rice and he ate it.' [6, p.65]
b. *Òzó lé ízẹ̀$_i$ rrí ọ́rẹ̀$_i$.
Ozo cook rice eat it [6, p.65]

Semantically, CSVCs and CCs differ in the following respect. For a CSVC, the action expressed by the first verb is done with the intention to carry out an action expressed by the second verb. For example, (1-a) can only be true if Ozu killed the goat with the intention of selling it afterwards. If he killed the goat by accident or decided to sell it only after the killing, (1-a) is false. No corresponding restriction exists for a CC. For example, (4-a) is true if Ozu cooked the rice and ate it afterwards.

Informal outline of the analysis: basic assumptions. The semantic theory to be presented in this talk is based on the following assumptions: (i) the semantic relation expressed by CSVCs and CCs in Edo is based on coherence relations which, in turn, are defined as complex relations between events; (ii) these relations are defined in terms of typed attribute-value pairs in a frame theory which is used to semantically model proper names, common nouns and verbs. **CSVCs CCs and coherence relations.** The semantic characterization of CSVCs and CCs in the previous section has shown that they differ in the way the events described by the verbs in both constructions are related to each other. These differences are located at at least two different levels: mereological and (constraints on) participants. Underlying the semantic interpretation of a CSVC is a plan (see [3]). The events described by the verbs in this construction are part of an intended plan, given by an event e, which consists of n component events $e_1, \ldots, e_n$ as its constituent (material) parts s.t. $e_i \sqsubseteq e$ and $e_i < e_j$ for $i < j$ and $\bigsqcup e_i = e$. Hence, an event e is related to two are more events which are linearly ordered and each of which is a material part of e. By contrast, for a CC, only two events are related s.t. the first temporarily precedes the second and, therefore, the two events are mereologically disjoint. In addition to these differences at event structure there is a difference w.r.t. how the participants in the events are related to each other. Whereas in the case of a planned event the actor and the theme are always identical, for a CC only the actors are required to be coreferential. This characterization suggests modelling the relation expressed by CSVCs and CCs in terms of *coherence relations* (CR). One way of modelling such relations is in terms of complex relations between events. We use frame theory to implement this idea. In particular, we assume that event frames for verbs in Edo have an attribute COHERENCE RELATION which specifies a possible relation to the next event described in the text (discourse). The values of this attribute are of type **plan** and **list**. Each type has the attribute MEREOLOGICAL RELATION which specifies the mereological relation between the event at the root of the frame and the current topic event. For a CR of type **plan**, the former is a proper material part of the latter whereas in the case of a CR of type **list** the two are disjoint. Constraints on participants are modeled by requiring the corresponding thematic relations to have the same values.

Coherence relations trigger expectations. After processing the first VP in a CSVC or a CC, a comprehender does not yet know what the relation to the next event will be. The event described by this part can either be a part of a larger plan as in (1-a) where the killing is done with the intention of selling the goat, or it simply is the first event in a succession of events with a shared actor, as in (1-b). However, (s)he knows that this relation will be exactly one of these two possibilities. This knowledge is used by her/him to non-deterministically extend the current information state with those two possibilities so that the next part of the construction is interpreted in relation to this event. How can a decision be made between the two possibilities? **Pronouns as indicators of coherence relations.** Recall from the first section that in a CSVC shared arguments are never overtly realized by a pronoun on the n-th verb for $n > 1$. Indirect objects of ditransitive verbs are realized by proper names or common nouns since they can never be coreferential. By contrast, in a CC coreference of the actor role can be marked by a pronoun and coreference of theme roles is marked by a pronoun after the second verb. Hence, a pronoun is an indicator of a list scenario and excludes a plan scenario. This is similar to the way a word in English can be an indicator of a particular coherence relation in English. For example, [5] found that in a context 'Amanda amazed Brittany because ...' with the implicit causality verb 'amaze' the connective 'because' is an indicator of an **explanation** relation since it raises the probability for this coherence relation to (almost) 1. Hence, if after processing say 'Ozu le ize' a pronoun is encountered ('O'), the comprehender knows that the speaker describes a list scenario and not a scenario of type **plan**. If no pronoun is encountered, both types of coherence relations (**plan** as well as **list**) remain options. It is only the region after the second verb which allows for a decision. If a pronoun is encountered, the scenario described must be of type **list**, and of type **plan** otherwise. Thus, one function of a pronoun consists in eliminating possibilities related to coherence relations.

Outline of the formalization. We define a probabilistic dynamic update semantics with frames. **Models.** A probabilistic world model with frames is a tuple $\langle W, D, I, \{f_{d,\sigma,w}\}_{d\in D,\sigma\in\Sigma,w\in W}\rangle$. W is a finite set of possible worlds which is used to represent (epistemic) uncertainty. An example is the uncertainty w.r.t. to the value of the COHERENCE RELATION-attribute in event frames. The domain $D = \{D_\sigma\}_{\sigma\in\Sigma}$ is the union of finite domains D_σ based on a partially ordered sort hierarchy $\langle\Sigma, \sqsubseteq_\Sigma\rangle$ with basic sorts like 'event' (e) or 'individual' (d). D is structured by a (material) part relation $\sqsubseteq$. $F = \{f_{d,\sigma,w}\}_{d\in D,\sigma\in\Sigma,w\in W}$ is the domain of frames. Each frame is of a sort σ and is a (generated) submodel of a possible world w, namely, the information associated with a particular object d in that world which is the root of the frame. F_w is the set of frames in world w and $f_{w,d}$ is the frame in w with root d. A frame is related to a set of relations on $D \times D$. Each relation R corresponds to a finite path (chain of attributes) ≥ 0 starting at the root d. The domain of R is given by the source-sort of the first attribute in the path and the range of R by the target-sort of the last attribute in the path. For path of length 0, one defines the shift: $\lambda Q_\sigma.\lambda x.\lambda y.Q_\sigma(x) \wedge x = y$. Each R must always be satisfied at the root. Hence, a frame f_w with root d corresponds to a complex property $Q_{f_w} = Q_0 \cap Q_1 \cap \ldots \cdot Q_n$ s.t. each Q_i is the domain of a relation R and one has $Q_{f_w}(d)$ is true in w iff $Q_i(d)$ is true in w for each $1 \leq i \leq n$. Using this fact, we define a relation θ s.t. $Q \in \theta(f)(d)$ iff $Q(w)(d)$. $\theta(f)(d) = \sigma$ means that $Q_f(d)$ is of type σ. $\theta(f)(d)\pi$ denotes the set of properties which hold at the end of path π in the frame f with root d.

Information states in a frame theory. An information state s consists of a set of possibilities i. A possibility i consists of a world w, two stacks/lists (following Incremental dynamics, [4]) and two functions γ_1 and γ_2. The stack c_1 assigns values to discourse parameters which are variable. Examples are speech time, speaker, hearer etc. In the present context we are interested in the parameter 'topic event', which is assumed to be located at the 0-th position of c_1. The stack c_2 consists of those objects which are introduced by common nouns, proper names and verbs. o is the root of f. We define two projection functions (see also [3]): p_i which yields the i-th element on a stack counted from the top of the stack, i.e. $p_0(c)$ is the top element of c. The projection function p_σ yields the restriction of the stack to objects of type σ. $p_i(p_\sigma(c))$ is the i-th object of type σ on stack c. The distinction between the topic event, which belongs to c_1, and the current top-most event, which is an element of c_2, is motivated by the following reasons: (i) in case of a plan scenario the topic event is the planned event e which remains constant while its component events are (successively) introduced on the stack c_2; by contrast, in a list scenario the topic event is changed with each new verb because the events are not related at the mereological level; (ii) it is used to implement the mereological relation between two events in a plan and list scenario and (iii) arguments provide information about the top-most event on the c_2 stack, independently of the mereological relation to other events either. The functions γ_1 and γ_2 assign to each element on c$_1$ and c_2 its frame f_w in the world w of the possibility i.

Update operations. We provide simplified versions of the most important update operations The difference between the way common nouns (and proper names) and verbs function is reflected in having two update operations.

Update operations for common nouns and proper names. The update operation for cn's and pn's $s[d]$ is a domain extension operator, similar to $s[x]$ in other update semantics. The difference lies in the fact that each element on the stack is paired with a frame. The definition is $s[d] = \{\langle w', c_1', c_2', \gamma_1', \gamma_2'\rangle \mid \exists n \exists i = \langle w, c_1, c_2, \gamma_1, \gamma_2\rangle \in s \wedge w = w' \wedge \gamma_1' = \gamma_1 \wedge c_1' = c_1 \wedge c_2' = c_2\hat{}d \textit{ for } d \in D_d \wedge \gamma_2'(c_2'[i]) = \gamma_2(c_2[i]) \textit{ for } 0 \leq i \leq n-1 \wedge n = |c_2| \wedge \gamma_2'(c_2'[n]) = f_{w,d}\}$.

Update operation for events in CSVS and CCs. The combination of two verbs or clauses in an SVC or a CC is modeled as an update operation (compare the interpretation of '.' in dynamic semantics as function composition of information states: $\lambda p \lambda q \lambda s \lambda s'.\exists s''.p(s)(s'') \wedge q(s'')(s')$). The update operation is a conditional one: it extends the stack c_2 by an event which is required to satisfy the value of the COHERENCE RELATION-attribute of the (so far) top-most event at the root of its frame and it changes the topic event depending on the type of the CR. Constraints between stack elements are expressed using θ. For example, $\theta(\gamma_2(c_2[n]))(c_2[n]) = \theta(\gamma_2(c_2[n-1]))(c_2[n-1])(\text{COHERENCE RELATION})$ requires the newly introduced event to have the same value as the value of the COHERENCE ATTRIBUTE of the event at the previous position. The condition in (5) requires the relation between the topic event and the top-most event to respect the mereological relation set up by the COHERENCE RELATION-attribute.

(5) $\theta(\gamma_1(c_1[0]))(c_1[0])(\text{MEREOLOGICAL RELATION}) =$
$\theta(\gamma_2(c_2[n]))(c_2[n))(\text{COHERENCE RELATION})(\text{MEREOLOGICAL RELATION})$

The update operation is defined as follows:

(6) $s[e] = \{\langle w', c_1', c_2', \gamma_1', \gamma_2' \rangle \mid \exists n \exists m \exists i = \langle w, c_1, c_2, \gamma_1, \gamma_2 \rangle \in s \wedge w = w' \wedge c_2' = c_2\hat{}e \textit{ for } d \in D_e \wedge \gamma_2'(c_2'[i]) = \gamma_2(c_2[i]) \textit{ for } 0 \leq i \leq n-1 \wedge n = |c_2| \wedge m = |c_1| \wedge \gamma_2'(c_2'[n]) = f_{w,e} \wedge \theta(\gamma_2(c_2[n]))(c_2[n]) = \theta(\gamma_2(c_2[n-1]))(c_2[n-1])(\text{COHERENCE RELATION})$
$\wedge\, \theta(\gamma_1(c_1[0]))(c_1[0])(\text{MEREOLOGICAL RELATION}) =$
$\theta(\gamma_2(c_2[n]))(c_2[n))(\text{COHERENCE RELATION})(\text{MEREOLOGICAL RELATION})$
$\wedge\ \textit{if}\ \theta(\gamma_2(c_2[n]))(c_2[n]) = \textbf{list} \wedge \theta(\gamma_2(c_2[n-1]))(c_2[n-1])(\text{COHERENCE RELATION}) = \textbf{list}\ \textit{then}\ c_1'[0] = c_2'[n] \wedge c_1'([i]) = c_1([i]) \wedge \gamma_1'(c_1'[0]) = \gamma_2'(c_2[n]) \wedge \gamma_1'(c_1'[i]) = \gamma_1(c_1[i])\ for\ i \neq 0\ \textit{else}\ c_1' = c_1 \wedge \gamma_1' = \gamma_1\}$.

Update operation for pronouns. The update operation for pronouns is the sequential composition of three update operations. Similar to DRT, pronouns push a new object on c_2. this is modeled by $s[d]$. Next, it is tested whether the relation between the top-most and the previous event is of type **list**:

(7) $s[CR] = \{\langle w', c_1', c_2', \gamma_1', \gamma_2' \rangle \mid \exists m \exists i = \langle w, c_1, c_2, \gamma_1, \gamma_2 \rangle \in s : w = w' \wedge c_1' = c_1 \wedge m = |c_1| \wedge \gamma_1' = \gamma_1 \wedge c_2' = c_2 \wedge \gamma_2' = \gamma_2 \wedge \theta(\gamma_2(c_2[n-1]))(c_2[n-1]) = \textbf{list} \wedge \theta(\gamma_2(c_2[n-2]))(c_2[n-2])(\text{COHERENCE RELATION}) = \textbf{list}\}$.

In the probabilistic setting of [5] $s[CR]$ implements the raising of $pr(CR)$ to 1 in the following equation:

(8) $pr(\textit{pronoun} = \textit{referent}) = \sum_{CR \in CRs} pr(CR) \cdot pr(\textit{pronoun} = \textit{referent} | CR)$.

This update operation, therefore, has the effect of eliminating one particular type of value from the COHERENCE RELATION attribute. The third update operation uses the fact that a pronoun, being an argument, is always related to a particular thematic relation *TR*. The constraint imposed by $s[TR]$ requires the values of the *TR*-attribute of the current event and the previous event to be the same: $s[Tr] = \{\langle w', c_1', c_2', \gamma_1', \gamma_2' \rangle \mid \exists i = \langle w, c_1, c_2, \gamma_1, \gamma_2 \rangle \in s : w = w' \wedge c_1' = c_1 \wedge c_2' = c_2 \wedge \gamma_1' = \gamma_1 \wedge \gamma_2' = \gamma_2 \wedge \theta(\gamma_2(c_2[n-1]))(c_s[n-1])(\text{TR}) = \theta(\gamma_2(c_2[n-2]))(c_s[n-2])(\text{TR})\}$. This implements the fact that in Edo using a probabilistic setting like that of [5] $pr(\textit{pronoun} = \textit{referent}|\textit{list})$ is 1 for a pronoun in a given argument position.

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Ralf Naumann and Wiebke Petersen

Bridging inferences in a dynamic, probabilistic frame theory

Data and central issue. It is by now a well-known fact that that the semantic processing of an utterance usually involves different sources of information which are used in parallel to arrive at a coherent interpretation of this utterance in the given context. Three principle sources must be distinguished: (i) the (linguistic) meaning of the lexical items; (ii) (non-linguistic) world and situational knowledge and (iii) the prior linguistic context. A prime example of this interplay between different sources of information are bridging inferences. [AL98, 83p.] take bridging to be an inference that two objects or events that are introduced in a text are related in a particular way that isn't explicitly stated, and yet the relation is an essential part of the content of the text in the sense that without this information, the lack of connection between the sentences would make the text incoherent. Examples of bridging inferences are given in (1).

(1) a. Lizzy met a dog yesterday. The dog was very friendly. [AL98, 86p.]
 b. John unpacked the picnic. The beer was warm. [CH77]
 c. A car hit a truck. The windshield shattered. [DK09]
 d. I was at a wedding last week. [Geu11]
  (i) The bride was pregnant.
  (ii) The mock turtle soup was a dream.
 e. I've just arrived. The camel is outside and needs water. [AL98, 86p.]

Common to all bridging inferences is (i) a new discourse referent (dref) is introduced (see [Bur06] for neurophysiological evidence) and (ii) a dependency relation between this dref (corresponding to the bridged DP) and a dref that has already been introduced in the linguistic context (denoting an antecedent object). There are various forms of dependency relations, the most two prominent of which are: (a) identity (1-a) and (b) part-of (e.g. (1-b),(1-c)). Example (1-d), shows that no unique antecedent (car/truck) need be singled out so that one gets an ambiguous bridging inference. Finally, (1-d) and (1-e) show that the dependency can be indirect. E.g., the turtle soup is indirectly related as a part (starter) of the meal which was served at the wedding.

How the various sources of information are related to the factors playing a role in bridging inferences depends on the theory. For example, [AL98] analysis builds on Chierchia's [Chi95] analysis of definite descriptions as anaphoric expressions: 'The N' denotes an N which is related by some dependency relation to an antecedent y. On this account, lexical semantics provides an underspecified relation B which functions as the bridge or dependency relation to an object in the present discourse context. B is computed using world and/or situational knowledge. E.g., [AL98, 89p.] argue that in (1-e) one uses lexical semantics to infer that 'arrive', being a motion verb, defines a mode of transport. Next, one uses world knowledge to infer that camels can be used as such a mode of transport. When taken together, one gets a coherent interpretation of (1-e) because the two sentences are connected by the coherence relation *Result* and the bridging relation $B = \textit{Means-of-transport}$ with $B(x, y), x = \textit{camel}, y = \textit{arrival}$. Hence, one first assumes a form of lexical enrichment, providing B, and then uses both lexical semantic and world knowledge to compute B.

Informal outline of the analysis. We use a dynamic, probabilistic frame theory, which admits of a unified analysis. First, using frames the underspecified relation B is replaced by a (complex) attribute in a frame so that bridging inferences are not restricted to nominal DPs and can involve dependency relations of arbitrary depth. Second, world and pragmatic knowledge, lexical semantic information and discourse information can be combined seamlessly. E.g., semantic information at the lexical level can be combined with other sources of information by extending frames in the lexicon with additional attribute-value pairs representing such non-lexical information. Finally, it becomes possible to apply probabilistic reasoning at the level of frame properties which plays an essential role in determining a unique antecedent object. Frames are typed features structures in the sense that the value of each attribute must be of a particular sort (type). In frame theory the semantic information provided by lexical items like common nouns and verbs is not restricted to sortal information, e.g. it is of sort 'picnic', expressed by 'picnic(x)'. Rather it in addition includes information about attributes and possible values of those attributes in form of sortal restrictions on their target sorts. E.g., for 'picnic', attributes include FOOD and BEVERAGE. Processing a CN or a verb therefore activates a relational structure with sortal (type) restrictions but (mostly) unspecified values. One way of how such values can be specified is by bridged DPs. For example, in (1-b) the beer is a material part of the value of the attribute BEVERAGE of the frame associated with the picnic that was already introduced into the discourse. Hence, one has the path *picnic*? $\bullet$ BEVERAGE $=$ *beer*. In frame theory attributes can be chained so that the dependency relation can be more indirect. E.g., in (1-d) the mock turtle soup is the value of the STARTER-attribute which in turn is an attribute of the value of the MEAL-attribute of the wedding object already introduced into discourse, yielding the following path: *wedding?* $\bullet$ MEAL $\bullet$ STARTER $=$ *soup*. A limiting case is identity where the sort of the bridged DP matches that of the antecedent DP. Hence, in frame theory dependency relations used in bridging inferences can be of arbitrary length starting from 0, identity, to length 1, direct dependency relation, to length > 1, indirect dependency relation. So one gets (*):

(*) Bridging inferences are done at the level of frames. The bridged DP provides a value for some attribute in the frame associated with an object that was already introduced in discourse. Therefore, the sort of the frame associated with the object denoted by the bridged DP must be an admissible sort for an attribute chain of the antecedent object. The relation between this attribute chain and the sort is called the *dependency relation*.

Using (*) only requires lexical information together with access to the information provided by antecedent objects. These sources do in general not yield a unique antecedent object as shown by example (1-c). Let the set of admissible antecedents computed by applying (*) be S. This uncertainty for a comprehender can be (partly) resolved by applying world and situational knowledge involving probabilities. For example, in the case of (1-c) the probability that the windshield of a car shatters in an accident will in general be higher than the corresponding probability for the windshield of a truck. Hence, a decision rule applies according to which one chooses that admissible antecedent, i.e. element from S, for which the probability is highest. Hence, a choice between different accessible antecedent objects can often only be made *after* the bridged DP has been parsed and therefore information which is not directly related to the dependency relation can play a role in singling out a unique antecedent. Furthermore, this example shows that bridging inferences are non-monotonic: bottom-up information encountered after the bridged DP can influence the choice of the antecedent object.

Rough outline of the formalization. We define a probabilistic dynamic update semantics with frames. Therefore, we assume a finite set of possible worlds W and a domain D. D is structured by a (material) part relation $\sqsubseteq$. The extended domain D^* is $D \cup \{\perp\}$ with the additional minimal element $\perp \sqsubseteq d$ for all $d \in D$. The elements in D are assigned sorts of the partially ordered sort hierarchy $\langle \Sigma, \sqsubseteq_\Sigma \rangle$ with basic sorts like 'event' or 'individual'. $F = \{f_{d,\sigma,w}\}_{d \in D, \sigma \in \Sigma, w \in W}$ is the domain of frames. Each frame is of a sort σ and is a (generated) submodel of a possible world w, namely, the information associated with a particular object d in that world which is the root of the frame. F_w is the set of frames in world w. A frame corresponds to a set of relations on $D \times D$. Each relation R corresponds to a finite path (chain of attributes) ≥ 0 starting at the root d. The domain of R is given by the source-sort of the first attribute in the path and the range of R by the target-sort of the last attribute in the path.

The probabilistic component. Following [Gär88], we define two probability measures Pr_w and B, one for the probability of properties and the other for the probability of worlds. In order to define (*) it must be possible to speak for a world w about the probability that among the frames of sort σ those having an attribute (chain) with value of sort σ' is p with $0 \leq p \leq 1$. This is captured by a probability distribution Pr_w. If $S \subseteq D$, then $Pr_w(S)$ represents the probability that an individual belongs to the set S in w. Pr_w reflects the fact that for a comprehender two states of the world can differ in their frequency of a property among the objects. For example, a comprehender may not know the exact probabilities of a windshield shattering in a car accident but he knows that it is between 0.5 and 0.7. This uncertainty is reflected in different elements $w \in W$, representing different possible states of the world for the comprehender, having different proportions of shattered windshields in car accidents. Pr_w is local in the sense that it gives the probability of a property in a particular world w. However, in order to determine a unique antecedent object it is necessary to look at *all* worlds which are epistemically possible for the comprehender and to determine the probability of a property in the whole epistemic state. To this end, we define a probability distribution B over all subsets of W. If V is a subset of W, $B(V)$ is a measure of the probability that the actual world is among those in V. For example, B is used to prefer worlds in which during an accident of a car with a truck the windshield of the former and not the windshield of the latter shattered. Using B and Pr_w, it becomes possible to define the probability that an object in the epistemic state of a comprehender has a certain property w.r.t. to a subset V of W (see [Gär88]):

(2) $Pr_V(Q) = \sum_{w \in W} \frac{Pr_w(Q) \cdot B(\{w\})}{B(V)}$, provided that $B(V) \neq 0$.

Information states in a frame theory. An information state s must model both (i) the local discourse information component as well as (ii) the global world and situational knowledge component. (i) Local discourse level: Information in frames is of two kinds: sortal (non-relational) one and relational information about paths of length ≥ 1. Sortal information by itself does not relate two objects. By contrast information given by paths of length ≥ 1 always relate two or more objects: $R(d_1, d_2)$ where d_2 is the value of an attribute in the frame of d_1. Hence, d_1 and d_2 play different roles. This double perspective is captured by defining the discourse component as a stack c (following Incremental dynamics, [vE01]). Each position on the stack is assigned a pair $\langle d, F_{w,d} \rangle$ of an object $d \in D^*$ and a set of frames $F_{w,d}$, which is the set of frames in w with root d. (ii) The global component is given by a set of worlds W_s and the two probability distributions Pr_w and B. An *information state* s is a triple $\langle \alpha, Pr_w, B \rangle$ s.t. α is a set of possibilities i which are pairs $\langle w, c \rangle$ consisting of a world $w \in W_s$ and a stack c.

Update semantics. We provide simplified update operations for 'a', 'the' and atomic formulas. Update operations directly change the value of the stack component and indirectly the probability component consisting of Pr_w and B. The general idea is to first leave the object component of a stack element relatively unconstrained by imposing only sortal information and to then use the frame component to determine the sort of possible objects for the corresponding stack element.

Update related to domain extension: The update operations for 'a' and 'the' are defined in terms of two atomic update operations $[c^1]$ and $[c^2]$ that extend the domain. They both increment the stack by one element s.t. the new element is $\bot$, i.e. the bottom element of D^*. This captures the intuition that incrementing the stack and getting information about this object are two distinct operations: the first introduces a new topic whereas the second provides factual information about this object. In contrast to other update semantics, 'branching' is introduced at the level of the frame component associated with this object. For $[c^1]$ (corresponding to the indefinite determiner 'a'), this is the set $F_{w,d}$ of all frames in w for the frame component (no factual information is known), thus the new incremented stack is $c\hat{}\langle \bot, F_{w,d} \rangle$. By contrast, in the case of a bridged DP (operation $[c^2]$) the frame component is restricted by the set of frames that have already been introduced in relation to a previous stack element. The new incremented stack is $c\hat{}\langle \bot, F_{w,d,c} \rangle$ where $F_{w,d,c}$ is the set of all frames which are the value of a chain of attributes in a frame $F_{w,d}$. Both update operations do not change the two probability distribution Pr_w and B. This reflects the fact that an update only introduces a new topic and (possibly) imposes a constraint on its frame components. $[c^2]$ can be strengthened by imposing further restrictions on the dependent frame component, e.g. by requiring that the last attribute in R be an instance of the part-of or of the identity relation.

Update related to atomic information. The effect of atomic information is threefold. First, possibilities which do not satisfy this information are eliminated. Hence, atomic information must be satisfied in the world of a possibility. Second, for surviving possibilities the frame components of the arguments are updated. Locally this is again eliminative: only those frames in a component survive which satisfy this information. Finally, the probability distributions are updated.

Singling out a unique antecedent. Update by atomic bottom-up information can yield a unique frame so that a unique antecedent object is determined. However, by itself, the compositional process does in general only determine a set of frames, each of a particular sort which are possible antecedents (the set S from above). In order to determine a unique antecedent the comprehender uses the global probability distribution Pr_V from (2). She calculates the global probability $Pr_{W_s}(Q_\sigma)$ for properties Q_σ s.t. f is of sort σ and an element of $\pi^2(\pi^2(c[i]))$ for $c[i]$ corresponding to the interpretation of a bridged DP and s.t. W_s is the set of worlds underlying her current epistemic state. So world knowledge is used to single out a unique antecedent because this operation is a global one since it depends on the information as a whole and not on a single possibility. In fact, other factors will come into play as well, like accessibility e.g., which have not been considered in this abstract.

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Olga Nevzorova and Vladimir Nevzorov

Ontology-Driven Computational Processing for Unstructured Text

At the present time there are various widely used frameworks for Natural Language Processing (Core NLP Suite, Natural Language Toolkit, Apache OpenNLP, GATE and Apache UIMA, etc.).

Stanford CoreNLP [1] provides a set of natural language analysis tools. It can give the base forms of words, their parts of speech, normalize dates, times, and numeric quantities, mark up the structure of sentences in terms of phrases and word dependencies, indicate which noun phrases refer to the same entities, indicate sentiment, extract particular or open-class relations between entity mentions, etc. Natural Language Toolkit [2] (NLTK) is a leading platform for building Python programs to work with language data. It provides easy-to-use interfaces to over 50 corpora and lexical resources such as WordNet, along with a suite of text processing libraries for classification, tokenization, stemming, tagging, parsing, and semantic reasoning, wrappers for industrial-strength NLP libraries.The Apache OpenNLP [3] library is a machine learning based toolkit for the processing of natural language text. The GATE framework [4] comprises a core library and a set of reusable Language Engineering modules. The framework implements the architecture and provides facilities for processing and visualising resources, including representation, import and export of data.

Unstructured Information Management applications (UIMA) [5] are software systems that analyze large volumes of unstructured information in order to discover knowledge that is relevant to an end user. An example UIMA application might ingest plain text and identify entities, such as persons, places, organizations; or relations, such as works-for or located-at. UIMA enables applications to be decomposed into components. Each component implements interfaces defined by the framework and provides self-describing metadata via XML descriptor files. The framework manages these components and the data flow between them. UIMA additionally provides capabilities to wrap components as network services.

For the Russian language, the general classes of computational linguistic tools have been developed, including those based on semantic technologies. Let us mention some systems. OntosMiner system [9] uses semantic ontologies to analyze natural language text. The outcome is a set of searchable and conceptually structured data, which can be categorized, browsed and visually presented in semantic networks. Tamita parser [10] is the linguistic tool for extracting structured data (facts) from text. The extraction of facts is based on context-free grammars and dictionaries of keywords.

Compreno technology [9] is a universal linguistic platform for applications that solve a variety of applied tasks for NLP. In Compreno project, the ultimate goal is to achieve the syntactic and semantic disambiguation. Semantic and syntactic representations are viewed rather as two facets of the same structure. Another (interrelated) feature of the Compreno parsing technology is that syntactic and semantic disambiguation are processed in parallel from the very start (in contrast to the architecture more usual for the NLP systems — the semantic analysis follows the syntactic one).

However, many of NLP systems are commercial and do not provide a clear enough clarification of the details of the main processes.

This article discusses another framework ("OntoIntegrator" system) developed for ontology-driven computational processing for NLP [6].

An important features of the "OntoIntegrator" system are supporting all the processes necessary to build a solution to the NLP task, including the development of ontologies, linguistic resources and specialized databases. We also developed the original ontology-based method of building solution for NLP tasks. The system is available for various NLP applications for Russian language and can be accessed by contacting developers.

The "OntoIntegrator" system includes the following functional subsystems, such as:

- the "OntoEditor+" subsystem for ontology development;
- the "Text analysis" subsystem;
- the subsystem of external linguistic recourses;
- the ontology subsystem;
- the "Integrator" subsystem.

The "OntoEditor+" subsystem supports the main table functions for development of ontology (addition, modification, deletion, automatic correction; keeping of more than one or compound ontologies, in other words with the general lists of relations, classes, text equivalents and others; an import of the ontologies with the different formats of data; a filtration of ontology; keeping of statistics automatically, searching for chains of relations and others). The functions of the visualization unit support different graphic modes of system, including the graphic mode of the ontology modeling.

The “Text analysis” subsystem contains a base linguistic tools for processing Russian, include tools for tokenization (splitting of text into words), part of speech tagging, grammar parsing (identifying things like noun and verb phrases), word sense disambiguation, named entity recognition, and more.

The subsystem of the external linguistic recourses supports storage of the basic linguistic recourses include the grammatical dictionaries and a set of special linguistic data bases.

The ontology subsystem is used in building solutions to applied NLP problems.

The “Integrator” subsystem implements a building the applied linguistic problem solution using all available system resources. The solution is being built under control of the ontology system that includes domain ontologies, the model ontology and the task ontology.

The ontology system is a connected three-component system. The components of this system are the ontologies mentioned above.

To construct a solution to applied linguistic problems, it is necessary to create a new concept of the task ontology (a task-concept) and to perform the structural decomposition of the new concept into the concepts (logical parts) included in the task ontology. The task ontology contains special classes, objects and object properties (relations) to implement the structural decomposition.

To execute the structural decomposition, a special software module has been developed. Thus, the structural scheme of the solution of the problem is determined. Then the structural elements of solution of the linguistic problem are mapped to the set of the concepts of model ontology. The models are used for computational processing of the task-concept. There are the following classes of computational models such as a model for property assigning, a model for relation defining, and computational processing of basic NLP problems. Many models are open and replenished dynamically. The ontology of models contains classes and instances of objects, and their properties (relations).

For easier interpretation all model-concepts are split to following groups supported by complex visualization mechanisms:

- Basic models providing the minimal functionality of ontology system;
- Syntactical models for extracting syntactical structures from a text (text models);
- Semantic models that create both an adequate interpretation of the decision results for applied task, and defining the connections between structural elements of model-concepts and the sequences of syntactic structures extracted from the input text;
- user-defined models dynamically created.

Thus, a task-concept defines the structural sequence of subtasks implemented at the level of model-concepts. Data for model-concepts are extracted from a input text and are interpreted in structural components (classes, instances, properties) of domain ontology (the third component of the ontological system).

The results obtained are displayed in the view window of the extracted textual models of the processed text and are saved as annotated output text.

At present time we developed a main library of basic and applied linguistic tasks based on ontology-driven technology of "OntoIntegrator" system. The library includes the solutions for such tasks as resolution of various types of polysemy, named entity recognition, annotation of text for special purposes and others. All solutions built are the components that may be integrated into new applications.

The software solutions of our system were used for processing mathematical texts, namely for extracting mathematical terminology from texts, annotating mathematical articles [7], and designing ontology of professional mathematics.

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Sumiyo Nishiguchi

Liberalism and Bouletic/Deontic Modality

1 Decision Maker in Bouletic Modality

The state of *wanting something* reflects personal preference and involves personal decision making. In that sense, wanting act follows the Condition of Liberalism. The condition of Liberalism is that, no matter how other people oppose, personal decisions can be made on certain matters. In actuality, what we want may not come out due to restrictions, but wanting something is a liberal act.

To put things in the possible world semantics, in the best possible worlds for a decision maker, her wants are fulfilled. Her want worlds are the subset of the worlds where her wants are fulfilled. The meaning of the sentence (1a) is expressed as in (1b) which says that, in all the accessible world which accords with Mary's wants at world w_c, she watches a movie.

(1) a. Mary wants to watch a movie.

b. $\forall$w.[BOUL$_m$(w)(w$_c$) $\rightarrow$ watch-a-movie(m,w)]

(m: Mary, w: world, w_c: actual world, BOUL$_x$: bouletic accessibility relation of the individual x)

From the perspective of decision making, the wanter is the only person involved with the wishes. If the speaker *I* is the agent of wanting to watch a movie, the speaker is the single decision maker regarding her preference, as shown in (2). If the first person plural subject *we* wants something unanimously, the group members including the speaker are the decision makers as in (3).

(2) a. I want to watch a movie (Others do not want to).

b. decision maker = {I}

(3) a. We want to watch a movie.

b. decision maker = {I, group member}

Even though others may want something contrary to the wanter, the wanter's desire remains unaffected, as in (4).

(4) a. Dee wants to wear blue even though you want her to wear yellow.

b. decision maker = {Dee}

2 Decision Maker in Deontic Modality

In contrast, the decision maker of deontic modals such as *must, should*, and *ought to* differs from the attitude holder. Traffic laws are imposed on public by the lawmakers: therefore, the decision makers are not drivers but a lawgiver, as shown in (5). If a teacher decides that Mary should submit a homework, the instructor is the decision maker of the deontic modal, in (6). The decision that Mary should study Spanish may be imposed due to the linguistic situation of people in Guatemala in (7).

(5) a. We should follow traffic lights.

b. decision maker = {lawmaker}

(6) a. Mary should submit her homework.

b. decision maker = {instructor}

(7) a. Mary should study Spanish. Otherwise she will not be able to communicate in Guatema

b. decision maker =/= Mary
= people in Guatemala

Thus, we can say that, in use of deontic modals, decision makers are someone else other than the attitude holder or the sentential subject. In case of bouletic modals, decision maker is a wanter.

3 Previous Analyses

Relevantly, van der Auwera and Plungian (1998) classify participant-internal and participant-external modality. According to them, ability modal and necessity modals are participant-internal in that the ability and necessity originates in the participants.

(8) a. Mary can make movies.

b. Mary needs to eat breakfast.

On the other hand, deontic and goal-oriented modality is participant-external. The chairperson and the teleological goal decide the possibility and necessity in (9) respectively.

(9) a. You may be seated.

b. To go to Disney Land, you should take this train.

In addition to their analysis, I would like to add that bouletic modality is participant-internal. In (10), the desire originates in the attitude holder *Mary* and the speaker respectively.

(10) a. Mary wants to play the piano.

b. I want to play the violin.

4 Incorporating Decision Makers

Now that bouletic and deontic modals depend on decision makers, the accessibility relations between possible worlds depend on decision makers. When the group preference is involved as in (12), the group members' social decision is reflected.

(11) a. Mary wants to watch a movie.

b. $\forall$w.[BOUL$_m$(w)(w$_c$) $\rightarrow$ watch-a-movie(m)(w)]

(12) a. We want to watch a movie.

b. $\forall$w.[BOUL$_{s,h}$(w)(w$_c$) $\rightarrow$ watch-a-movie(s,h)(w)]

(s: speaker, h: hearer)

(13) a. Mary should submit homework.

b. $\forall$w.[DEON$_i$(w)(w$_c$) $\rightarrow$ submit-homework(m)(w)]

The deontically and bouletically accessible worlds may differ from each other, so that following example in (14a) is not contradictory.[1]

(14) a. She ought to speak, but I do not want her to.

b. $\forall$w.[DEON$_s$(w)(w$_c$) $\rightarrow$ speak(m)(w)] $\wedge$ $\forall$w.[BOUL$_s$(w)(w$_c$) $\rightarrow$ $\neg$speak(m)(w)]

Such incorporation of modal judges may be reminiscent of Stephenson (2007)'s analysis on epistemic modality.

(15) $[\![must]\!]^{c:w,t,j}$ = [λp$_{<s,<ie,t>>}$.$\forall$w',t',x.Epist$_{w,t,j}$: p(w')(t')(x) = 1]

(Stephenson 2007, 502)

In addition to her analysis, I further claim that bouletic and deontic modals have decision makers. Moreover, the group decision is a social choice (Arrow 1963, Sen 1979, Chevaleyre et al. 2007). The social choice function SCF returns a single choice, which is going to a movie. The decision may not be unanimous but follows Pareto principle, in that when nobody has contrary preference, the mass decision agrees with individual's preferences. Also Independence of Irrelevant Alternatives is adhered because the relative ranking between going to movie and other alternatives only matter to the group decision.

(16) a. decision makers I = {s, h, p}

b. alternatives χ = {go to movie, eat out, relax at home}

c. A profile, a vector of linear orders, or preference R = (R$_s$, R$_h$, R$_p$) $\in$ L(χ)^3

d. Social Choice Function SCW(L(χ)^3) = {go to movie}

Therefore, the group desire is a result of the social choice.

[1] I thank a reviewer for bringing up this example.

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Konstantine Razmadze

Bi-modal Logics of Mappings[1]

Plain maps. Initially, we consider the maps $f : X \to Y$ between two sets. Without loss of generality, suppose that X and Y are disjoint sets. consider a Kripke frame $\mathfrak{F}_f = (W_f, R_f)$, where $W_f = X \sqcup Y$, $R_f = f$, i.e. we say that, pair of points $(x, y) \in W_f \times W_f$ is in the relation R_f, iff $f(x) = y$. The resulting Kripke frames are called Functional Frames. We say that the height of a frame $\mathfrak{F} = (W, R)$ is 2 if there exists $w, u \in W$, such that uRw and for any triple of points $(u, v, w) \in W \times W \times W$ either uRv or vRw fails. We say that a Kripke frame $\mathfrak{F} = (W, R)$ has no branching, if for any triple of points $(u, v, w) \in W \times W \times W$ either uRv or uRw fails. Irreflexive frames of height ≤ 2 are characterized by a formula $\Box\Box\bot$, the no branching property is characterized by a formula $\Diamond p \wedge \Diamond q \to \Diamond(p \wedge q)$. We show that a Kripke frame is a Functional Frame iff it is irreflexsive, non branching frame of height ≤ 2. The mentioned two formulas define the class of Functional Frames. Denote

$$K_f = K + \big(\Box\Box\bot\big) + \big(\Diamond p \wedge \Diamond q \to \Diamond(p \wedge q)\big)$$

Proposition 1. *The modal logic K_f is sound and complete with respect to the class of Functional Frames.*

We show that although the class of Functional Frames is modally definable, the subclasses of injective and surjective functional frames are not. If we extend the modal language by using four temporal operators $\overrightarrow{\Box}$, $\overleftarrow{\Box}$, $\overrightarrow{\Diamond}$and $\overleftarrow{\Diamond}$, then the injective and surjective functional frames become definable. We interpret temporal operators as follows for a Kripke frame $\mathfrak{F} = (W, R)$ and $w \in W$,

1. $w \models \overrightarrow{\Box} p$ iff $\forall u \in W$, wRu implies $u \models p$.
2. $w \models \overleftarrow{\Box} p$ iff $\forall u \in W$, uRw implies $u \models p$.
3. $\overrightarrow{\Diamond} p = \neg\overrightarrow{\Box}\neg p$, $\overleftarrow{\Diamond} p = \neg\overleftarrow{\Box}\neg p$

We show that in the temporal language injective Functional Frames are determined by the formula

$$p \to \overrightarrow{\Box}\overleftarrow{\Box} p,$$

while surjective Function Frames are determined by the formula

$$\overrightarrow{\Diamond}\top \vee \overleftarrow{\Diamond}\top.$$

Order preserving maps. We consider the maps $f : \mathfrak{F}_1 \to \mathfrak{F}_2$ between Kripke frames $\mathfrak{F}_1 = (W_1, R_1)$ and $\mathfrak{F}_2 = (W_2, R_2)$. The *Relational Functional Frame* associated with f is a bi-relational frame $f_R = (W, R, R_f)$, where $W = W_1 \sqcup W_2$, $R = R_1 \sqcup R_2$ and $R_f = f$. We say xRy if either xR_1y or xR_2y.

Note that (W, R_f) is a functional frame. In addition, the Relational Functional Frame f_R possesses the following *coherence property*: for any points $x, y \in W$, if $R_f(x) \neq \emptyset$ and $xRy \vee yRx$, then $R_f(y) \neq \emptyset$.

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[1]We would like to express our appreciation to David Gabelaia (Razmadze Mathematical Institute) for his constant support and encouragement.

Proposition 2. *A bi-relational Kripke frame $\mathfrak{F} = (W, R, R_f)$ is Relational Functional Frame if and only if R_f is irreflexive, its height is less than 3, it is non branching and R_f, R have the coherence property.*

Since we deal with bi-relational frames. We have $\Box$, $\Diamond$ for R_f and $\boxplus$, $\diamondplus$ for R in our bi-modal language. Here $\Box$, $\Diamond$ are interpreted as in Functional Frames and the $\boxplus$, $\diamondplus$ are interpreted as follows

- For any formula φ in a language, $\boxplus\varphi$ is satisfiable in $w \in W$ if φ is satisfiable in all R-successors of w.
- For any formula φ in a language, $\diamondplus\varphi$ is satisfiable in $w \in W$ if there exists an R-successor $u \in W$ of w, such that φ is satisfiable in u.

Coherence property in the Relational Functional Frames corresponds to the following formulas

$$\Diamond\top \to \boxplus\Diamond\top$$

$$\diamondplus\Diamond\top \to \Diamond\top$$

We show that the class of Relational Functional Frames is modally definable in the bi-modal language. Let K_R be defined as follows

$$K_R = K_{\Box\boxplus} + \big(\Box\Box\bot\big) + \big(\Diamond p \wedge \Diamond q \to \Diamond(p \wedge q)\big) + \big(\Diamond\top \to \boxplus\Diamond\top\big) + \big(\diamondplus\Diamond\top \to \Diamond\top\big)$$

Proposition 3. *Bi-modal logic K_R is sound and complete with respect to Relational Functional Frames.*

We show that the class of p-morphic Relational Functional Frames is also modally definable. Let the bi-modal logic of p-morphic Relational Functional Frames be denoted by K_p.

Proposition 4.

$$K_p = K_R + \big(\diamondplus\Diamond p \leftrightarrow \Diamond\diamondplus p\big)$$

We also axiomatize the bi-modal logic of order preserving Relational Functional Frames denoted by K_o. Moreover, we show the following:

Proposition 5. *K_R and K_o have the finite model property.*

Continuous maps. Now instead of relations, we equip the domain and co-domain of a Functional Frame with topological structure. Suppose $f : X_1 \to X_2$ is a map between topological spaces (X_1, τ_1) and (X_2, τ_2). Let us introduce $f_\tau = (X, \tau, R_f)$ topological structure, where $X = X_1 \sqcup X_2$, τ is a topology generated by $\tau_1 \sqcup \tau_2$, and $R_f = f$. A topological structure $f_\tau = (X, \tau, R_f)$ is called a *Topological Functional Frame* if (X, R_f) is a Functional Frame and $R_f^{-1}(X) = \{x \in X \mid \exists y \in X \text{ with } yR_f x\}$ is clopen (coherence property). Again due to existence of two, topological and function structures, we have two kinds of modal operators in our language, $\Box$, $\Diamond$ and $\boxplus$,$\diamondplus$ respectively. The operator $\Diamond$ is interpreted as f^{-1}. The operator $\boxplus$ is interpreted as topological *Interior* operator and the operator $\diamondplus$ - as topological *Closure* operator. We show that Topological Functional Frames are modally definable. The bi-modal logic of Topological Functional Frames is denoted by $S4_R$. We axiomatize this logic as follows:

$$S4_R = K_R + (\boxplus p \to p) + (\boxplus p \to \boxplus\boxplus p).$$

Proposition 6. *Bi-modal logic $S4_R$ is sound and complete with respect to Topological Functional Frames.*

Furthermore, we characterize the subclasses of continuous, open and interior Topological Functional Frames modally and axiomatize the corresponding bi-modal logics.

The following literature was used:[1], [2], [3].

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Kerstin Schwabe

The uniform representation of German embedded polar interrogatives, a typology of their embedding predicates and adaptors

The talk will present a typology of German *ob*-predicates like *argwöhnen* 'suspect' as in (1), that is, of predicates that embed *ob*-clauses, a uniform analysis of *ob*-clauses and quantifiers that adapt *ob*-clauses to different verb classes.

(1) ... die Gesundheitsbehörden müssen stets *argwöhnen*, ob sich eine neue Epidemie anbahnt. ZDB 898: DWDS TS 2003
'The health authorities always have to suspect whether a new epidemic is looming.'

The typology is based on the ZAS Data Base of German clause embedding predicates which contains about 1790 synchronical annotated and exemplified predicates including 666 *ob*-predicates – cf. Stiebels et al. (2017). The typology is more exhaustive than Wunderlich's (1976) and Karttunen's (1977) characterizations since it also contains predicates that embed reports of indirect speech acts – see (5) below. It is more elaborated since its classification is more fine-grained and includes the compositionally derived Logical Form of each construction type. The talk will show that the majority of *ob*-predicates denote eventualities that are located on a 'route' from an individual's α question state QS_α 'α wants [(α knows that σ) $\vee$ (α knows that $\neg\sigma$)]' to her or his answer state AS_α '(α knows that σ) $\vee$ (α knows that $\neg\sigma$)'. There is an interactive and a non-interactive epistemic route as well a deontic route.

Interactive epistemic route: $QS_\alpha > QA_\alpha > AA_\beta > BS_\alpha > AS_\alpha$

The interactive epistemic route includes, in addition to QS_α and AS_α, a question act QA_α, an answer act AA_β of the addressee β and a believe state BS_α of the question state holder α. The answer act is either a proper answer act pAA_β – the addressee of the question act believes to render the true answer – or an improper answer act $ipAA_\beta$ – an act where the addressee reacts, but does not render a believed true answer.

Question states are denoted by predicates like *sich fragen* 'wonder' or *argwöhnen* 'suspect' – see (1). Question acts are related to by predicates like *fragen* 'ask' or *nachhaken* 'ask further questions'. Proper answer acts pAA are denoted by verbs like *ankündigen* 'announce' or *gestehen* 'confess'. Generally, they embed declaratives, but they also occur with *ob*-interrogatives in particular contexts – see (2).

(2) …, dass der französische Verkehrsminister Jean-Claude Gayssot und sein britischer Kollege in Le Bourget ankündigt, ob und wann die Concorde wieder fliegen darf. ZDB 551: DWDS BZ 2001
'… that the French minister of transport ... will announce whether … the Concorde is allowed to fly.'

Improper answer acts ipAA can be related to by predicates like *verheimlichen* 'conceal', *egal sein* 'do no care' or *nicht sicher sein* 'be not sure' – see (3a, b).

(3) a. *Es ist mir egal, ob ich berühmt bin oder nicht*. ZDB 3233: DWDS BZ 2005
'I do not care if I'm famous or not.'

b. Noch ist nicht einmal sicher, ob das von ihm vermittelte Gespräch zwischen dem israelischen Außenminister Peres und Palästinenserpräsident Arafat überhaupt zu Stande kommt und wann oder wo es stattfindet. ZDB 7896: DWDS BZ 2001

Depending on whether the question state holder believes the answer given by the answer act of the addressee and whether the answer is true, the question state holder is in an answer state AS_α. The latter is denoted by predicates like *wissen* 'know'.

As for the non-interactive epistemic route, it contains – instead of QA_α and AA_β – the research act of the question state holder RsA_α and his or her result state ReS_α.

Non-interactive epistemic route: $QS_\alpha > RsA_\alpha > ReS_\alpha > BS_\alpha > AS_\alpha$

Research acts are denoted by predicates like *abwägen* 'ponder' or *ausprobieren* 'test'. Result states are related to by predicates like *herausfinden* 'find out' or *folgern* 'conclude'.

Whereas the truth of σ or ¬σ of the epistemic question-answer routes is decided on with respect to the actual world, the validity of σ or ¬σ of the deontic question-answer route is determined with respect to a deontic world.

Deontic route: $QS_\alpha > QA_\alpha > AA_\beta > AS_\alpha$

Question acts on a deontic route are denoted by predicates like *bitten* 'ask' or *fragen* 'ask'. Proper answer acts are related to by predicates like *bestimmen* 'determine' or *entscheiden* 'decide'. Improper answer acts are related to by predicates like *verantworten müssen* 'have to account for' or *egal sein* 'do not care' – see (4).

(4) Er ***mag*** es selbst ***verantworten***, ob er sich zum Richter über Leben und Tod aufschwingt. ZDB 8574: DWDS Zeit 2005
'He must account for if he rules over life and death.'

Beside *ob*-predicates denoting an eventuality on a question-answer route, there are *ob*-predicates relating to indirect speech acts or beliefs. Their embedded *ob*-clause often contains a modal particle:

(5) Paul Ehrenfest hat … ***vorgeschlagen***, ob man ***nicht*** so etwas wie Teilchen der Strahlung definieren könnte, ... ZDB 11562: DWDS Zeit 2004
'Paul E. has proposed whether it isn't possible to define particles of radiation ...'

Indirect speech acts are related to by predicates like *vorschlagen* 'propose', *verspotten* 'mock', and *bitten* 'ask'. Indirect beliefs can be related to by verbs like *fürchten* 'fear', *eingestehen* 'admit' or *daran denken* 'think of'.

Like Adger & Quer (2001) in their analysis of unselected *if*-clauses (6), the talk represents *ob*-clauses uniquely as questions that correspond to the set of propositions {σ, ¬σ} – cf. (6iv). As far as unselected *ob*-clauses are concerned, which are only licensed in negative contexts, Adger & Quer suggest that they are a complement of a non-overt determiner Δ that applies them to their matrix clause – see (6v-viii).

(6) The bar tender$_j$ does $[_{XP}$ not $[_{VP}$ $[_{\Delta P}$ Δ $[_{CP}$ if the costumer was drunk$]]_i$ $[_{VP}$ t_j admit $t_i]]]$

i.	$[\![V]\!]$	=	$\lambda p\ \lambda x\ [admit\ (p, x)]$
ii.	$[\![VP]\!]$	=	$\lambda r\ [admit\ (r, bar\ tender)]$
iii.	$[\![\textit{if}]\!]$	=	$\lambda p\ \lambda q\ [(q = p) \vee (q = \neg p)]$
iv.	$[\![\textit{if}\text{-CP}]\!]$	=	$\lambda q\ [(q = come\ m) \vee (q = \neg\ come\ m)]$
v.	$[\![\Delta]\!]$	=	$\lambda R\ \lambda P\ \exists q\ [Rq \wedge Pq]$
vi.	$[\![\Delta P]\!]$	=	$\lambda P\ \exists q\ [[(q = come\ m) \vee (q = \neg\ come\ m)] \wedge Pq]$
vii.	$[\![VP']\!]$	=	$\exists q\ [[(q = come\ m) \vee (q = \neg\ come\ m)] \wedge\ [admit\ (q, bar\ tender)]]$
viii.	$[\![XP]\!]$	=	$\neg\exists q\ [[(q = come\ m) \vee (q = \neg\ come\ m)] \wedge\ [admit\ (q, bar\ tender)]]$

Whereas Adger & Quer regard Δ as a polarity sensitive generalized quantifier, the talk extends it to a neutral generalized quantifier Ψ, which can be applied to an *ob*-clause that is embedded by a predicate like *wissen* 'know' or *sicher sein* 'be certain' – cf. (7) and (8).

(7) Frank weiß, ob Maria kommt.
'Frank knows whether Maria will come.'

$[_{CP} \ldots [_{\mathbf{vP}} [_{VP} \text{Frank} [_{V'} \text{r} [_{V} \text{weiß}]]] [_{\Psi P} \Psi [_{CP} \textit{ob} [_{TP} \text{Maria kommt}]]]]$

i.	$[\![\text{V}]\!]$	=	$\lambda p_{p \in \mathcal{F}}\ \lambda x\ \lambda e\ [\text{know}\ (p, x, e)]$
ii.	$[\![\text{VP}]\!]$	=	$\lambda r\ \lambda e\ [\text{know}\ (r, \text{frank}, e)]$
iii.	$[\![\textit{ob}]\!]$	=	$\lambda q_{q \in \mathcal{P}}\ \lambda p_{p \in \mathcal{P}}\ [(q = p) \vee (\neg q = p)]$
iv.	$[\![\textit{ob}\text{-CP}]\!]$	=	$\lambda p_{p \in \mathcal{P}}\ [(p = \text{come maria}) \vee (p = \neg\text{come maria})]$
v.	$[\![\Psi]\!]$	=	$\lambda R_{R \in \mathcal{Q}}\ \lambda P_{P \in pAP}\ \exists q\ \exists e\ [(P\ (p, e)) \wedge (R\ (p))]$
vi.	$[\![\Psi\text{P}]\!]$	=	$\lambda P_{P \in pAP}\ \exists q\ \exists e\ [(P\ (p, e)) \wedge ((p = \text{come maria}) \vee (p = \neg\text{come maria}))]$
vii.	$[\![\text{VP}]\!]$	=	$\exists q\ \exists e\ [(\text{know}\ (p, \text{frank}, e)) \wedge ((p = \text{cm}) \vee (p = \neg\text{cm}))]$
viii.	$[\![\mathbf{VP}]\!]$	=	$\exists q\ \exists e\ [((\text{know}\ (p, f, e)) \wedge (p = \text{cm})) \vee ((\text{know}\ (p, f, e)) \wedge (p = \neg\text{cm}))$

Ψ relates the *ob*-clause to a predicate like *wissen* 'know', which relates to the set of facts ($\mathcal{F}$) – cf. Hintikka (1976) and Groenendijk & Stokhof (1984). These predicates are objectively veridical (OVP) in terms of Giannakidou (2003) or Schwabe & Fittler (2014). Predicates like *sicher sein* 'be certain' are subjectively veridical (SVP) – cf. Öhl (2016) and Giannakidou (2003). The talk suggests that the derivation of the Logical Form of constructions with a subjectively veridical predicate like (8) is similar to the derivation of constructions with an objectively veridical predicate like (7). Since predicates like *sicher sein* 'be certain' are not objectively veridical, an affirmative context would lead to pragmatic inappropriateness. If, however, (8vii) is in the scope of a non-veridical operator, a felicitous representation results – cf. (8viii).

(8) Frank ist nicht sicher, ob Maria kommt.
'Frank is not certain if Maria will come.'

$[_{CP} \ldots [\ \text{nicht} [_{\mathbf{vP}} [_{VP} \text{Frank} [_{V'} \text{r} [_{V} \text{weiß}]]] [_{\Psi P} \Psi [_{CP} \textit{ob} [_{TP} \text{Maria kommt}]]]]$

i.	$[\![\text{V}]\!]$	=	$\lambda p_{p \in \mathcal{P}}\ \lambda x\ \lambda e\ [\text{be certain}\ (p, x, e)]$
ii.	$[\![\text{VP}]\!]$	=	$\lambda r\ \lambda e\ [\text{be certain}\ (r, \text{frank}, e)]$
iii.	$[\![\textit{ob}]\!]$	=	$\lambda q_{q \in \mathcal{P}}\ \lambda p_{p \in \mathcal{P}}\ [(q = p) \vee (\neg q = p)]$
iv.	$[\![\textit{ob}\text{-CP}]\!]$	=	$\lambda p_{p \in \mathcal{P}}\ [(p = \text{come maria}) \vee (p = \neg\text{come maria})]$
v.	$[\![\Psi]\!]$	=	$\lambda R_{R \in \mathcal{Q}}\ \lambda P_{P \in pAAP \cup ipAAP}\ \exists p\ \exists e\ [(P\ (p, e)) \wedge (R\ (p))]$
vi.	$[\![\Psi\text{P}]\!]$	=	$\lambda P_{P \in pAAP \cup ipAAP}\ \exists p\ \exists e\ [(P\ (p, e)) \wedge ((p = \text{cm}) \vee (p = \neg\text{cm}))]$
vii.	$[\![\mathbf{VP}]\!]$	=	$\exists p\ \exists e\ [(\text{be certain}\ (r, \text{frank}, e)) \wedge ((p = \text{cm}) \vee (p = \neg\text{cm}))]$
viii.	$[\![\text{CP}]\!]$	=	$\neg\exists p\ \exists e\ [(\text{be certain}\ (r, \text{frank}, e)) \wedge ((p = \text{cm}) \vee (p = \neg\text{cm}))]$

A predicate like *glauben* 'believe', which is also subjectively veridical, reveals that subjective veridicality is not a sufficient condition for a subjectively veridical predicate to embed an *ob*-clause.

The predicate has additionally to be antonymous, that is, it must be consistent with (9a) as well as with (9b), while (9b) corresponds to (6viii) or (8viii). However, *glauben* 'believe' is complementary if there is any epistemic activitiy involved. That is, it is only consistent with (9a), which, by the way, implies neg-raising.

(9) a. $\exists p\ [(p = \sigma) \wedge (\text{verb}\ \sigma, \alpha)] \vee \exists p\ [(p = \neg\sigma) \wedge (\text{verb}\ \neg\sigma, \alpha)]$

b. $\forall p\ [[(p = \sigma) \Rightarrow (\neg\ \text{verb}\ \sigma, \alpha)] \wedge [(p = \neg\sigma) \Rightarrow (\neg\ \text{verb}\ \neg\sigma, \alpha)]]$

Ob-clauses that are embedded by verbs like *fragen* 'ask' or *bedenken* 'consider' relate to the question itself, that is, they embed question intensions. They are the complement of the quantifier Ω:

(10) Frank fragt, ob Maria kommt.

i. $[\![V]\!] = \lambda qu\ \lambda x\ \lambda e\ \lambda QA_x\ [\text{say}\ (qu, x, e) \wedge (e \in QA_x)]$

ii. $[\![VP]\!] = \lambda r\ \lambda e\ \lambda QA_x\ [\text{say}\ (qu, f, e) \wedge (e \in \lambda QA_x)]$

v. $[\![\Omega]\!] = \lambda R_{R \in \boldsymbol{Q}}\ \lambda P_{P \in QP}\ \exists p\ \exists qu\ \exists e\ \exists QA_x\ [P\ (qu, QA_x, e) \wedge ((Rp)(qu))]$

vi. $[\![\Omega P]\!] = \lambda P_{P \in QP}\ \exists p\ \exists qu\ \exists e\ \exists QA_x\ [P\ (qu, QA_x, e) \wedge ((mc\ (p) \vee \neg mc\ (p))\ qu)]$

vii. $[\![VP]\!] = \exists p\ \exists qu\ \exists e\ \exists QA_x\ [\text{say}\ (qu, f, e) \wedge (e \in \lambda QA_x) \wedge ((mc\ (p) \vee \neg mc\ (p))\ qu)]$

Returning to the question-answer routes, epistemic as well as deontic QS_α-, QA_α- and RsA_α-predicates are elements of the family of question predicates $\boldsymbol{QP}$, the set of sets of predicates directly relating to questions. They agree with ΩP. Proper AA_β-, ReS_α- and AS_α-predicates are elements of OVP, the set of objectively veridical predicates. Improper AA_β-predicates are elements of the intersection of subjectively veridical and antonymous predicates $SVP \cap AP$ $(ASVP)$. Both, OVPs and ASVPs match with ΨP. ASVPs only agree with ΨPs in non-affirmative contexts. Predicates denoting indirect speech acts agree with ΩP.

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Giuli Shabashvili

Syntactic Realization of Present Perfect Forms in Georgian

In regard of conceptualization of reality/objective environment the forms of perfect tense relates to the speaker's space and show events from the speaker's prospective. Moreover using the proper semantic attributes, forms of perfect tense present the evaluation of experience from the speaker's point of view rather than the real state of the subjects. Thus, according to the temporal approach perfect is subjective tense.

Any verbal form is realized in any syntactic environment. The realization essence is defined by the internal semantic structure of the verb. In Georgian Language Perfect forms are distributed mainly within the hypotactic constructions. In most cases present perfect forms are realized in subordinate clauses. According to the data of Old Georgian Language, this amount constitutes 67%. As for the minimal context, these forms can be met in negative or interrogative clauses. In narration present perfect forms (realized by I resultative screeves) lose the meaning of resultativness and obtains the semantics of evidentiality.

The paper aims to describe and analyze the distribution the forms of I resultative forms according to the following data: temporal adverbs, particles and type of sentence.

The research is conducted by the using of corpus research and statistical methods. As an experiment all variations of the single verb is considered. The verb გაკეთება 'gak'et^{h}eba' (to do) is chosen.

According to the data of Georgian National Corpus (www.gnc.gov.ge) active voice forms of the above mentioned verb is fixed in a large extant. III person forms of perfective aspect are dominated on this point: გაუკეთებია 'gauk'et^{h}ebia' (he has done) is realized in 3667 cases. The continuative forms can be found in relatively less examples - უკეთებია 'uk'et^{h}ebia' (he has been doing) only 152 cases. Next frequently used forms are I person perfective forms - გამიკეთებია 'gamik'et^{h}ebia' (I have done) is counted in 751 cases. As for imperfective meaning მიკეთებია 'mik'et^{h}ebia' (I have been doing) was found only in 91 clauses. Based on the same stem the passive voice forms are produced as well. Only the III person perfective forms are possible to fix - გაკეთებულა 'gak'et^{h}ebula' (it has been done) in 1363 cases.

Due to the discussion the realization of above mentioned verbal forms, it was pointed out that these verbs are distributed in negative subordinate clauses in 80% of total data. Adverbs, particles and pronouns used with these verbs in negative context are as follows: not yet, nothing, nothing anymore, never, never more, nothing can, not. As for minimal context, 10% shows interrogative clauses using the following forms: how many, how many times, how many times it should be, ever. The paper discusses widely the case of narration and minimal context. In this regard the following adverbs are fixed: many times, often, before, numerously.

Analyze of the data reveals that mostly the forms of present perfect tense of the verb to do carry the meaning of the present perfect tense and negative context serves as the strong referential aspect in this regard. Interrogative context also keeps the meaning of perfect tense. Due to the dynamic semantic of the particular verb, evidential meaning is revealed only in a relatively few cases.

Andres Soria Ruiz

Comparative Evaluative Judgments

Broadly expressivist proposals about normative language understand value judgments in *binary* terms, that is, in terms of the expression, on the part of the utterer, of a favorable or unfavorable attitude (sometimes called a *PRO*- or *CON*-attitude) towards the object under evaluation. For theories of this sort, when a speaker utters (1) she expresses a favorable attitude towards volunteering for a charity; and when she utters (2), she expresses an unfavorable attitude towards donating money to a charity.

(1) Volunteering for a charity is good.

(2) Donating money to a charity is bad.

This approach faces a fundamental shortcoming when faced with sentences like the following:

(3) Volunteering for a charity is better than donating money.

When a speaker utters (3), she need not endorse and/or reject neither volunteering nor donating. She is merely comparing the *goodness* of the two actions; and her uttering (3) is compatible with adopting almost any combination of positive and negative attitudes towards either of them (with the exception of being in favor of donating money while being against volunteering). This is shown by the fact that (4)-(6) are acceptable, while (7) is not:

(4) Volunteering for a charity is better than donating money, though both are bad.

(5) Volunteering for a charity is better than donating money, though both are good.

(6) Volunteering for a charity is better than donating money; in fact, volunteering for a charity is good whereas donating money is bad.

(7) ?? Volunteering for a charity is better than donating money; in fact, volunteering for a charity is bad whereas donating money is good[1].

[1] That these combinations are coherent suggests that *good* is a relative adjective, in the sense of Kennedy 2007. Other value adjectives show different patterns of inference. In particular, comparisons using negative value adjectives like *bad* or *ugly* invite the inference that the positive form applies to one or both of the *relata*. An anonymous referee points to the case of *beautiful*, where in order to cancel the inference to

How can the expressivist insight about *absolute* judgments of value (i.e. (1) and (2)) be extended to comparative judgments like (3)?

Value adjectives are gradable, so the literature on gradability in semantics should point to a solution. However, value adjectives are different from run-of-the-mill gradable adjectives—adjectives like *tall* or *rich*. In those cases, it is clear what it means for an object a to possess the relevant property to a higher degree than another object b: it is simply to possess more height, or money. But what about evaluative properties? What is for an object to possess an evaluative property, e.g. *goodness*, to a higher degree than another object? This is what we need to spell out. What we propose is to combine insights from the literature on gradability and meta-ethics to arrive at a model that makes the right predictions both for absolute and comparative value judgments.

As basic elements in our semantics we use Gibbard's (1990, 2003) *hyperplans*. Hyperplans were devised by Gibbard as tools for modeling the close connection between normative judgments and action-planning. In his view, to judge that an action is rational is to adopt a plan to perform that action in the appropriate circumstances. A domain H of hyperplans looks very much like the familiar domain W of possible worlds of intensional semantics (i.e. maximally determined states of affairs), and given the usefulness of understanding informational content in terms of set-theoretical operations over W, it is suggestive to understand *normative* content in terms of set-theoretical operations over H (see Field 2009; Yalcin 2017 for suggestions in this direction).

A hyperplan is a maximally decided planning state: a state that tells you what to do in every conceivable situation that you could find yourself in[2]. We can think of a hyperplan as a *total* function from the set of conceivable situations S to the set of possible actions A. The actual plans adopted by agents however, are less than maximally decided: for many situations, they do not tell you what to do. We can thus conceive of a plan as *partial* function from S to A, or alternatively, as a set of hyperplans that agree on what to do in some situations, but not for others. Conversely, an action a can be defined as the set of hyperplan-situation pairs $\langle h, s\rangle$ such that the agent of h performs action a in s.

Plans and situations can be now employed to give truth-conditions for absolute judgments of value. Following the expressivist tradition, we map the adjective *good* (*bad*) at a context of utterance c to a relation of *support* (*rejection*) by the relevant plan P at c.

We start by defining support and rejection (at a situation s) as follows:

P supports a in s iff $\forall h \in P.\langle h, s\rangle \in a$

P rejects a in s iff $\forall h \in P.\langle h, s\rangle \notin a$

ascribing the positive form to either *relatum* some qualifying particle is needed:

(1) Anna is more beautiful than Berta, ?? but neither of them is beautiful.

(2) Anna is more beautiful than Berta, but *in fact* neither of them is beautiful.

[2]Every hyperplan will tell what to do if your car breaks, if it doesn't, if there's a fire, if your neighbors fight, if you were Ceasar right before crossing the Rubicon, etc.

That is, P supports (rejects) a in a situation s just in case every hyperplan in P is such that the agent of h does (does not) a in s. If neither condition holds, then P is *indifferent* with respect to a.

Now as we've seen, plans are defined for more than one situation, so in order to generalize the notion of support/rejection we need to consider a set of situations, with some restrictions. Intuitively, we want to say that a plan supports, say, smoking, just in case most normal situations in which one could smoke are situations in which one actually smokes. Let us stipulate that, for every action a, there exists a set of a-pertinent situations, that we will loosely define as situations where action a could be performed (think of them as situations in which nothing prevents you from performing action a). Now our definitions of support and rejection by a plan can be generalized to pertinent situations as follows:

$$P \text{ supports } a \text{ iff } \forall h \in P \ \& \ \forall s \text{ s.t. } s \text{ is } a\text{-pertinent}, \langle h, s\rangle \in a \ (\textit{ceteris paribus})$$

$$P \text{ rejects } a \text{ iff } \forall h \in P \ \& \ \forall s \text{ s.t. } s \text{ is } a\text{-pertinent}, \langle h, s\rangle \notin a \ (\textit{ceteris paribus})$$

In words: P supports (rejects) a just in case every hyperplan h in P and a-pertinent situation s are such that the agent of h does (does not) a in s, *ceteris paribus*[3].

Now, in order to derive comparisons from this system, we need to restrict the set of situations that we are considering in a different way. In particular, for any two actions a and b, we need to consider only situations that are a- and b-pertinent. Such restriction on our original plan P delivers a set of "subplans" of P, and all we have to do is consider which of a or b is good (or bad) relative to those subplans:

> $a >_P b$ iff $\forall P' \subseteq P$. and $\forall s \in P'$ s.t. s is both a- and b-pertinent, P' supports a and rejects b.

That is, an action a is better than an action b relative to a plan P just in case, given a choice between a and b, we would consistently choose a over b without modifying our plan, that is, without adopting a different set of hyperplans.

Informally, the idea is that a plan may be such that any number of actions is supported and rejected by it in different situations, but in order to make a comparative judgment, it doesn't matter whether the actions are actually supported or rejected. All that matters is that, having to choose, we would choose one over the other. This predicts the admissibility of (4)-(6). Nonetheless, the proposed truth conditions do rule out a situation like (7), where volunteering is better than donating, and yet donating is supported while volunteering is rejected: if we adopt a plan such that every volunteering-pertinent situation is one where we don't volunteer, and every donating-pertinent situation is one where we do donate, then

[3]This *ceteris paribus* clause is meant to help with the following, immediate problem: for many actions a and b, there will be situations that are both a- and b-pertinent, but where both actions cannot be performed. For instance, I may support jogging and smoking, yet reject doing both at the same time. By our naked definition however, if I end up jogging in a smoking-pertinent situation, then smoking comes out "not good" relative to my plan. The *ceteris paribus* clause is meant to read as: assuming that nothing else is supported by this plan.

if there's any situation that is both vounteering- and donating-pertinent, we will always donate rather than volunteer.

Importantly, this strategy preserves the syntactic primacy of the absolute over the comparative form of value adjectives (see Barker 2002; Benthem 1982; Klein 1980). The expressivist idea that absolute value judgments express outright support or rejection of an action is the starting point from which a semantics for the comparative form is derived.

Finally, disagreement over a claim like (3) reveals different ordering preferences between the plans adopted by the disagreeing speakers. But the proposed semantics does not, by itself, provide an account of disagreement. That depends on exactly how we define the content expressed by the claims involved. For example, a subjectivist could adopt our system and say that a claim like (3) and its negation *describes* the plans of their utterers. On this view, the two speakers would be talking past each other. In order to hold on to a semantics where different speakers may adopt different plans and retain a notion of disagreement, we may recur to the expressivist idea that a normative disagreement is a practical disagreement, that is, not a disagreement about what follows from a certain plan, but about what plan to adopt[4].

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Peter Sutton

Vagueness as a product of learning over a noisy channel

1 Introduction:

This paper investigates the hypothesis that vagueness in predicates arises naturally from a combination of information theoretic pressures based on learning and communication. The semantic bottleneck problem (that learners are exposed to insufficient data to fully resolve the interpretations of expressions) introduces a pressure on languages to encode information with fewer, more equivocal expressions. At the same time, transmission of information between speakers is not perfect (information is transmitted over a noisy channel) and so there is uncertainty about what any particular message is communicating.

We propose a model based on the Iterated Learning paradigm [i.a. 1, 5, 6] (which can model the semantic bottleneck problem over generations of agents), a noise source, and a simple information theoretic learning strategy (which encodes a form of constraint over the level of information learnt to be encoded by a particular predicate, given a set of learning data). Given a set of situations to be described and a set of predicates to encode this information, language evolves between generations of agents with the result that it either remains highly unstable (not a phenomenon we observe in natural languages), or becomes stable. If stable, the language can either be uninformative (e.g. where it contains only one predicate to communicate all situations), or informative (with multiple predicates relative to the size of the situation set). A successful balance between the learning and communication pressures results in a stable, informative language. Our findings indicate two results. When the parameters of the model are set in such a way as to produce stable, non-trivial languages: (1) there is a correlation between the 'narrowness' of the learning bottleneck and the number of predicates that languages have; (2) noise in the information channel results in vagueness (i.e. graded boundaries between predicates modelled as the probability of an agent applies a predicate, given a situation to be described).

2 Background:

Since at least the work of Zipf [11, 12], numerous linguistic phenomena have been analysed in terms of the interaction between information theoretic constraints and the idea that language approximates an optimal tool for communication. Recent examples include arguments that: ambiguity is an optimal feature of human language [8]; languages optimize information density [4, 7]; general efficiency principles can explain a wide range of crosslinguistic syntactic patterns [i.a. 3]; conflict between information-theoretic pressures can explain crosslinguistic count/mass variation [10]; and vagueness evolves in language when boundedly rational agents repeatedly engage in cooperative signalling [2]. This work falls broadly within this Zipfian paradigm.

The model presented in this paper is inspired, methodologically, by Iterated Learning Models (ILMs) [1, 5, 6] and how semantic learning connects to vagueness [9]. In ILMs a competent adult agent provides a sample of her vernacular to a learner. The learner becomes a second generation competent agent and provides a sample of her language to a third generation learner etc.. The cycles progress and the impact of certain parameter values within models can be witnessed on the long-term behaviour of the language. In particular, ILMs model the semantic bottleneck in that learners are not necessarily exposed to all of the competent agent's language.

3 Hypotheses:

It was hypothesised, but not further investigated in [9] that vagueness is an effect of the combination of employing a learning strategy to help to overcome the semantic bottleneck whilst learning in conditions of uncertainty. This paper seeks support for this hypothesis by implementing a Iterated Learning Model with a probabilistic learning strategy for inferring applications of predicates based on a nosy information channel.

Hypotheses: (i) For at least some settings of model parameters (e.g. the ratio of the size of the language to the size of the data set presented to the learner), probabilistic learning results in a stable, informative language. (ii) Items in the language will display the hallmarks of vague predicates, i.e., having graded boundaries.

4 Model:

A computational model was written with Matlab. Elements in the model are: a collection of agents $\mathbb{A} = \{A_1, ..., A_n\}$ where, e.g. A_1 is the first generation agent and A_n is the n^{th} generation agent; an ordered set of situations to be described $\mathbb{S} = \langle s_1, ..., s_n\rangle$. Situations are assumed to form a total order with respect to similarity such that s_1 is most similar to s_2 and least similar to s_n. (Likewise s_n is most similar to s_{n-1} and least similar to s_1.) The intuitive idea behind this was to model something like a colour spectrum of shades. A distance function $D : \langle\mathbb{S} \times \mathbb{S}\rangle \rightarrow \mathbb{N}$ such that $D(s_n, s_{n\pm k}) = \delta \times k$ (where δ is a parameter of the model). A set of predicates $\mathbb{M} = \{m_1, ..., m_n\}$. A set of languages $\mathbb{L} = \{L_{A_1}, ..., L_{A_n}\}$ where L_{A_1} is the language of A_1. (All other languages are derived from L_{A_1} via a string of intergeneration learning events in a way to be made clear below.) Languages are characterised as sets of probability distributions $P(m_i|s_j)$ such that for each $s_j \in \mathbb{S}$, $\sum_{m_i\in\mathbb{M}} P(m_i|s_j) = 1$.

A random sample of situations, S, is generated (the sample size is a parameter of the model). A_k then provides A_{k+1} with a set of situation-predicate pairs d_{A_k} (with a predicate for every situation in the randomly generated set). This set of pairs provides the learning data for A_{k+1}. The size of S relative to $\mathbb{S}$ determines the probability that A_{k+1} will not witness every situation and so will not be exposed to the full extent of L_{A_k}. This models the semantic bottleneck. When noise is present in the model, there are two parameters. One parameter, $\mathcal{N} \in [0, 1]$ sets the probability that for a situation-predicate pair $\langle s_i, m_j\rangle \in d_{A_k}$, the learner A_{k+1} also witnesses $\langle s_{i+1}, m_j\rangle$ and $\langle s_{i-1}, m_j\rangle$. For example, when $\mathcal{N} = 0$, there is no noise in the channel and the learner receives three identical sets d_{A_k}. When $\mathcal{N} = 1$, the learner receives d_{A_k}, but also $d_{A_k,\uparrow}$ and $d_{A_k,\downarrow}$ such that:[1]

$$d_{A_k,\uparrow} = \{\langle s_{i+1}, m_j\rangle : \langle s_i, m_j\rangle \in d_{A_k}\} \tag{1}$$
$$d_{A_k,\downarrow} = \{\langle s_{i-1}, m_j\rangle : \langle s_i, m_j\rangle \in d_{A_k}\} \tag{2}$$

This reflects noise in the channel (the addition of information before reception by the learner). A second parameter, $\mathcal{U} \in [0, 1]$, reflects the extent a learner is able to be certain about the 'true' situation being described (the extent that they are able to be sure that s_i is being described by m_j as opposed to s_{i+1} or s_{i-1}). For example, when $\mathcal{U} = 0$, the learner can be certain of the situation being described (there is no noise). When $\mathcal{U} = 1$, there is maximum perplexity with respect to the situation being described.

From the data set, there is then a learning event in which A_{k+1} develops her own language $L_{A_{k+1}}$ based on d_{A_k} (and $d_{A_k,\uparrow}$ and $d_{A_k,\downarrow}$ in the case of noise). This amounts to inferring probability distribu-

[1] For s_1, noise only impacts $d_{A_k,\uparrow}$. For s_{10}, noise only impacts $d_{A_k,\downarrow}$.

tions, $P(m_j|s_i)$, for each s_i that is witnessed in d_{A_k}, $d_{A_k,\uparrow}$, or $d_{A_k,\downarrow}$:

$$P_{A_{k+1}}(m_j|s_i) = \frac{P_{A_k,d_{A_k}}(m_j,s_i) + (\mathcal{U} \times P_{A_k,d_{A_k,\uparrow}}(m_j,s_i)) + (\mathcal{U} \times P_{A_k,d_{A_k,\downarrow}}(m_j,s_i))}{P_{A_k,d_{A_k}}(s_i) + (\mathcal{U} \times P_{A_k,d_{A_k,\uparrow}}(s_i)) + (\mathcal{U} \times P_{A_k,d_{A_k,\downarrow}}(s_i))} \tag{3}$$

If a situation $s_\varnothing$ is not in the second projections of d_{A_k}, $d_{A_k,\uparrow}$, or $d_{A_k,\downarrow}$, then, the learner, A_{k+1}, searches in both 'directions' along the set of situations for the nearest s that is instantiated in the data set. Where $s_\uparrow$ is the nearest instantiated situations in one direction and $s_\downarrow$ is the nearest instantiated situation in another. For each $m_j \in \mathbb{M}$ (where all probabilities in (4) are for A_{k+1}):

$$P(m_j|s_\varnothing) = \tag{4}$$
$$\frac{\log_2^{-1}(\log_2(P(m_j|s_\uparrow) - D(s_\varnothing, s_\uparrow)) + \log_2^{-1}(\log_2(P(m_j|s_\downarrow) - D(s_\varnothing, s_\downarrow))}{\sum_{m_i \in \mathbb{M}} \log_2^{-1}(\log_2(P(m_i|s_\uparrow) - D(s_\varnothing, s_\uparrow)) + \sum_{m_i \in \mathbb{M}} \log_2^{-1}(\log_2(P(m_i|s_\downarrow) - D(s_\varnothing, s_\downarrow))}$$

An example of this is shown in Figure 1. On the left hand side, s_5, s_6 and s_7 are not instantiated in the learners data set, but $P(m_3|s_4) = P(m_8|s_8) = 1$. The right hand side shows the result of the inference (where distance function parameter $\delta = 1$).

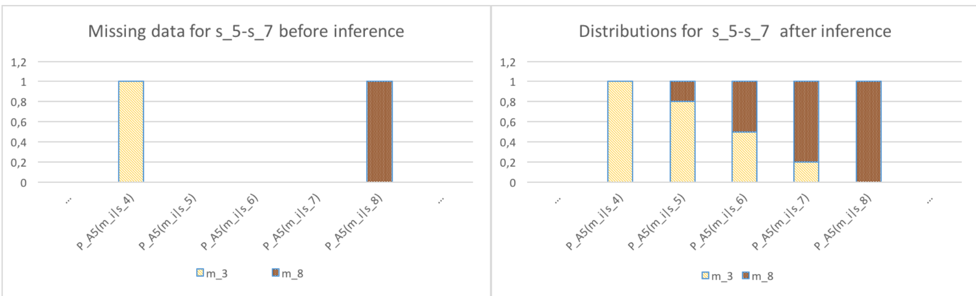

Figure 1: Example of inferring how to apply predicates when situations haven't been witnessed in the data set.

The result of this procedure is that each learner has a probability distribution for applying predicates in each situation. These distributions can then be sampled by a fresh randomly selected set of situations (possibly plus noise) and used as input data for a fresh learner.

By assumption, A_1 has a completely categorical language with not vagueness and a unique predicate for every situation. This is graphically represented in Figure 2. The reason for this assumption is to be sure that any vagueness that emerges is not the result of the input from A_1.

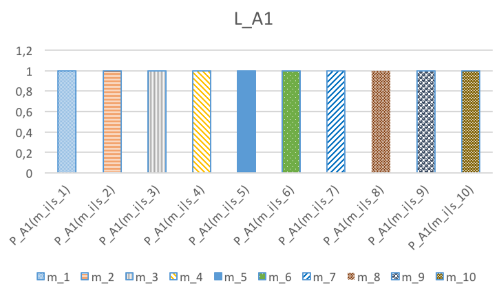

Figure 2: Starting language for A_1

5 Results:

Simulations were run for a space of 10 situations (s_1-s_{10}): (A) without a bottleneck or noise; (B) with a bottleneck, without noise; (C) without a bottleneck, with noise; (D) with a bottleneck, with noise. Results are shown for A_{100}. The distance function parameter was kept at $\delta = 1$. Where relevant, $\mathcal{U}$ was kept at 0.5. Other parameters are described below. In all cases the languages that emerged were relatively stable, where boundary shifts between predicates were small between generations.

(A) Without a bottleneck or noise: The language of A_{100} was exactly as it was for A_1 in Figure 2, namely, with ten categorical predicates. Any bottleneck was in effect removed by setting the sample size to 500 (so that the probability of an agent not witnessing a situation was very low).

(B) With a bottleneck, without noise: The result for A_{100} in Figure 3 was typical of the result for a situation sample size of 30, namely, a reduction to between two and three categorical predicates.

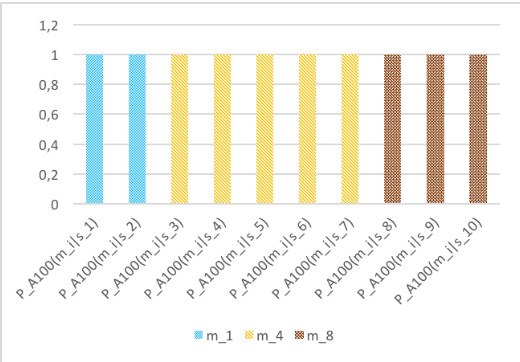

Figure 3: Bottleneck, sample of 30.

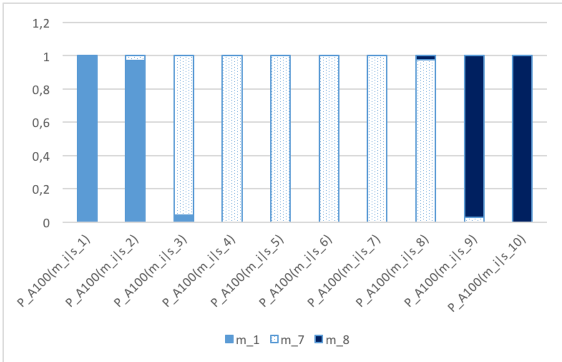

Figure 4: Noise, $\mathcal{N} = 0.1$. No bottleneck.

(C) Without a bottleneck, with noise: In these simulations, as in condition (A), the situation sample size was set to 500 to effectively remove any bottleneck. Figure 4 shows a typical result for A_{100} when $\mathcal{N} = 0.1$ (approximately 10% of signals were noisy), namely a reduction to 2-3 predicates which have graded boundaries, but only marginally so (around 0.98 and 0.02 at the boundary situations). Figure 5 shows a typical result for A_{100} when $\mathcal{N} = 0.3$ (approximately 30% of signals were noisy). On these settings, it was more typical to end up with 2 predicates, and the boundaries between them were more graded (around 0.9 and 0.1 at the boundary situations).

(D) With a bottleneck, with noise: In these simulations, the situation sample size was set to 30, as in condition (B). The noise level was $\mathcal{N} = 0.3$, as in the second example in condition (C). Here, we typically witnessed a reduction to two predicates with graded boundaries (around 0.8-0.85 and 0.2-0.15 at the boundary situations). A typical example outcome for A_{100} is given in Figure 6.

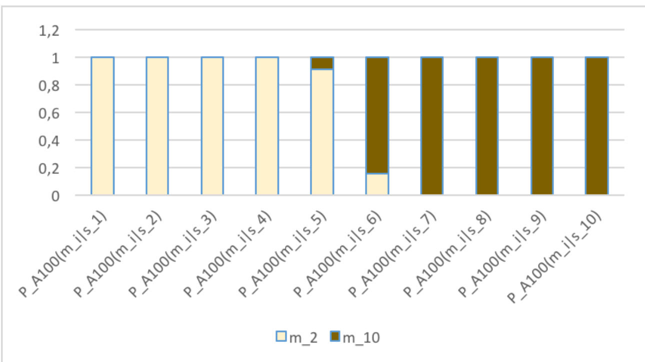

Figure 5: Noise, $\mathcal{N} = 0.3$. No bottleneck.

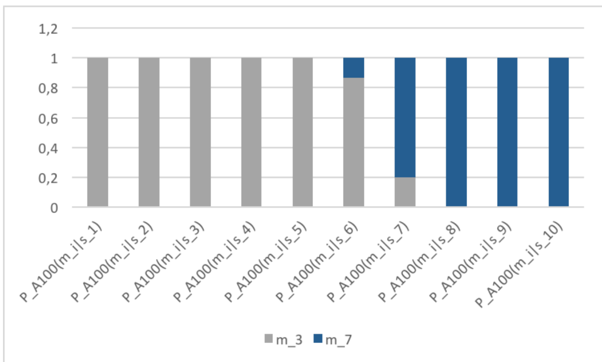

Figure 6: Noise, $\mathcal{N} = 0.3$. Bottleneck, sample of 30.

6 Discussion:

The simulations showed some confirmation for hypothesis (i): for at least some settings of model parameters, probabilistic learning results in a stable, informative language. However, if noise, $\mathcal{N}$ was set too high, or the situation sample size was set too low, the outcome was an uninformative, trivial language (one with only one predicate which applies in every situation). We observed a surprising result regarding hypothesis (ii) (that items in the language will display the hallmarks of vague predicates). The introduction of a bottleneck alone did not result in the common occurrence of vagueness. This was the case despite the fact that agents reasoned probabilistically in cases where they had not witnessed description for a situation in the learning phase. However, when noise was introduced into the model, vague, graded boundaries between predicates invariably emerged. Furthermore, there appears to be a correlation between the noise level $\mathcal{N}$ and the extent to which graded boundaries emerge. A tentative conclusion we might also draw is that when the bottleneck and noise are combined, we see a small increase in the level to which boundaries are graded.

7 Further work and conclusions:

Further work needs to be done to establish with more clarity how the settings of the parameters within the model interact. In particular, we have not yet assessed the impact of changing the value of the distance parameter δ. Some preliminary testing seems to indicate that low δ values tend to make the boundaries of predicates more graded.

Given the simplicity of these simulations, conclusions about natural language are hard to draw. Our results do, however, suggest an enticing possibility. It may well be the case that vagueness in natural language also arises as a byproduct of semantic learning over a noisy communication channel.

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Carla Umbach and Stefan Hinterwimmer

German 'wie'-complements: manners, methods and events in progress

German *wie* ('how') is, first of all, a question word asking for manner or method, as in (1) (for manner/ method readings of English *how* see Saebo 2015). German *wie* occurs in many more positions, most prominently in equative comparison, as in (2). In this paper we focus on interrogative complements headed by *wie*, as in (3), which give rise to three types of readings. First, there is a manner and a method reading, as shown in (3a) and (b). Both manner and method reading allows for clarification questions with *wie*, to be answered by properties of the event or by ways of performing it. There is, however, a third reading which is neither manner nor method and does not allow for *wie* clarification questions. It is close in meaning to bare infinitives and can be paraphrased by a progressive-like form in German. We name it the *eventive reading* of *wie*-complements. With gradable adjectives the difference between the manner (including degree) and the method readings on the one hand and the eventive reading on the other can be seen on the surface, see (4). In the degree reading in (4a) the adverb is fronted together with the question word, whereas in the eventive reading the adverb stays in situ, (4b).

(1) Wie packte Berta ihre Tasche für das Wochenende?
'How did Berta pack her bag for the weekend?'

(2) Anna packte ihre Tasche so wie Berta (ihre Tasche packte).
'Anna packed her bag like Berta did.'

(3) Anna sah, wie Berta ihre Tasche packte.
'Anna saw Berta packing her bag, LIT: how Berta was packing her bag.'

a. <u>Manner reading</u>

Q: … und WIE hat Berta ihre Tasche gepackt?	'How did she do that?'
A: Sehr sorgfältig.	'Very carefully.'

b. <u>Method reading</u>

Q: … und WIE hat Berta ihre Tasche gepackt?	'How did she do that?'
A: Zuerst die Turnschuhe, dann ein Tshirt, dann zwei Romane und oben drauf einen Pullover.	'Running shoes first, then a tshirt, then two novels. And on top a sweater.'

c. <u>Eventive reading</u>

Q: # … und WIE hat Berta ihre Tasche gepackt?	'How did she do that?'
para.: Anna sah, wie Berta ihre Tasche am packen war.	'Anna saw how Berta was packing her ba

(4) a. Anna sah, wie sorgfältig Berta ihre Tasche packte. — Degree reading

b. Anna sah, wie Berta sorgfältig ihre Tasche packte. — Eventive reading
'Anna saw how carefully Berta packed her bag /
how Berta was carefully packing her bag.'

Eventive readings of *wie*-complements occur mostly with perception verbs (*sehen* 'see', *hören* 'hear', …) but also with report verbs (*berichten* 'report', *erzählen* 'tell', …) and cognitive verbs (*erinnern 'remember'*, …). Moreover, eventive readings of complements headed by manner question words are found in many other languages, although with slightly different distribution and usage constraints (e.g. Polish, Russian, French, English, Greek, see Legate 2010). In this talk, we will focus on German.

One of the few references discussing the meaning of eventive readings of *wie*-complements is Falkenberg (1989). He observes that the complement must denote a durative eventuality – states are excluded, cf. (5) – and that they give rise to the imperfective paradox, cf. (6). Combining these data with the progressive-like paraphrase in (3c) there is good evidence that *wie*-complements denote events in progress.

(5) *Anna sah wie Berta müde war.
'Anna saw Berta being tired. / Lit: how Berta was tired.'

(6) Anna sah, wie Berta ihre Tasche packte, aber die halbvolle Tasche dann wieder auspackte.
'Anna saw Berta packing her bag but in the middle of it unpacking the bag again.'

The semantics of *wie*-complements is puzzling for a number of reasons. There is, first of all, the question of how to interpret the eventive reading and, in particular, how to explain that various languages make use of a manner question word in expressing events in progress. Secondly, even if the semantics of interrogative complements is in general well understood, there is no agreement about the denotation of manner question words – are we obliged to add manners to the ontology or is there a more conservative solution? (cmp. the case of *why*, where you would not want to add reasons to the ontology and instead refer to causally related proposition). Thus the semantic analysis has to answer two questions, (i) what is the meaning of *wie* in manner & method readings and, assuming that there is no ambiguity? (ii) what is its role in eventive readings – why use a manner question word to express an event in progress?

Here is our proposal in a nutshell:

A. We start from an interpretation of *wie* as denoting similarity;

B. We assume that in manner & method readings *wie* is base-generated in a low position while in eventive readings it is generated only after the event has been introduced.

C. We interpret manner and method readings as answers to questions involving sets of similar events where features of comparison relate to properties licensed by the event predicate (in the case of manner) and to procedures of realizing an instance of the event predicate (in the case of method).

D. We interpret the eventive reading as a variant of the method reading: While method readings yield sequences of subevents realizing events of a certain type, eventive readings yield events in progress, i.e. initial stages plus possible continuations. Thus while method readings apply at the level of the event type providing different ways of realizing events of this type, eventive readings apply at the level of particular events providing different ways of continuing a given initial stage of an event.

ad (A): We start from the similarity interpretation of *wie* in manner equatives as in (2) (cf. Umbach & Gust 2014). The basic idea is that *wie* creates classes of events. Grossly simplifying technical details (see Gust & Umbach 2015), *wie* denotes a similarity relation between two entities x and y with respect to a set F of features of comparison: $\boldsymbol{\lambda x\ \lambda y.\ \mathrm{sim}(x,y,F)}$. The similarity relation is, again grossly simplifying, implemented such that two items are similar with respect to a given set F of features $f_1 \ldots f_n$ if their values are identical, $\mathbf{sim(x,\ y,\ \{f_1 \ldots f_n\})\ iff\ f_1(x)=f_1(y),\ \ldots\ f_n(x)=f_n(y).}$ Spelt out this way the similarity relation generates sets or classes of items similar to a given item y_0 with respect to a given set of features, $\mathbf{\{x \mid sim(x,y_0,F)\}}$ (note that this notion of similarity is tantamount to indistinguishability with respect to given features F, and is an equivalence relation).

ad (B): We follow standard theories on adverbial positions in German as, e.g. in (7), see Schäfer (2013). We assume that in manner and method readings *wie* is base-generated in the position of verb-related adverbials modifying the event type, e.g. *sorgfältig* in *die Tasche sorgfältig packen* 'pack the bag carefully'. In eventive readings *wie* is generated in the position of event-related adverbials, which is reserved for adverbials characterizing an event only after it has been introduced. Adverbials in this positions are also called 'event-external'.

(7) subject > $\text{adverbial}_{\text{EVENT-RELATED}}$ > direct object > $\text{adverbial}_{\text{VERB-RELATED}}$ > verb

ad (C): On their manner and method readings *wie*-complements are interpreted as answers to a question addressing a manner or method modifier of the event predicate. The modifier is given as a similarity class, and the difference between manner and method is realized via different features of comparison. In the example in (3a), manner features of bag-packing might be SPEED (n minutes) or TIDINESS (low/middle/high) etc., whereas methods of bag-packing are ordered sets of stages: *shoes in, shoes + books in, shoes + books + sweater in, …* (8) shows the interpretation of manner/method readings.

(8) a. There is an event $\boldsymbol{e_0}$ of Anna seeing an eventuality (i.e. event or state) ***e***, where ***e*** is a bag-packing event and is an element of a class of bag-packing events similar with respect to their manner or method, and ***e*** being in this class causes Anna to know an answer to the question of what the manner or method of bag-packing performed by Berta is, that is, Anna's seeing is epistemic, see the discussion in Barwise (1989).

b. $\exists e_0, \exists e.\ \text{see}_e\ (e_0)(\text{Anna})(e)\ \&\ \text{bag-pack}(e)\ \&\ \text{ag}(e, \text{Berta})$

$\&\ \underline{e \in \{e' \mid \text{sim}(e', e, F)\ \&\ \text{bag-pack}(e')\}}$ where F is a set of features or of ordered stages

(Note that even though the similarity conjunct is logically idle it generates a class of items representing manner/method. We are aware of the fact that indexing *see* as being epistemic is not yet satisfactory.)

ad (D): On the eventive reading *wie*-complements denote events in progress. Following Landman (1992) progressive events include an initial stage plus a set of possible (and reasonable) continuations, that is, ways of how the initial stage may develop into a complete event of the respective type. Landman implements continuations in terms of developments in possible worlds. Bonomi (1997) adapts the idea of continuations in an extensional fashion making use of *frames* specifying *natural courses of events*. More specifically, courses of events are partially ordered sets of stages and frames are functions taking a stage (and some contextual facts) and giving a course of events extending the original stage. Now compare methods and courses of events. They are both ordered sets of stages. But there is a crucial difference: While any (reasonable) ordered set of stages may qualify as a method, courses of events interpreting the progressive must include the initial stage of the event up to the time of evaluation. Put it the other way around: methods can be seen as events in progress with a not yet existing initial stage.

(9) a. There is an event $\boldsymbol{e_0}$ of Anna seeing an eventuality ***e***, where ***e*** is the unique stage of bag-packing at evaluation time ***t*** by Berta and is an element of a similarity class of bag-packing-by-Berta events including the interval ***t*** that differ only in their degree of development.

b. $\exists e_0.\ \text{see}\ (e_0)(\text{Anna})(\iota e.\ \text{bag-pack}(e)\ \&\ \text{ag}(e, \text{Berta})\ \&\ t \subset \tau(e)$

$\&\ \underline{e \in \{e' \mid \text{sim}(e', e, F)\ \&\ \text{bag-pack}(e')\ \&\ \text{ag}(e', \text{Berta})\ \&\ \tau(e) \subset \tau(e')\ \}}\ )$,

where F is provided by a Bonomi frame.

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Diego Valota

Computing Spectra of Finite Goedel Algebras through Finite Forests

Given a class of structures $\mathcal{C}$ and a natural number $k \geq 1$, the finite counterparts of the Shelah's classification in classical Model Theory can be given by the following problems ([7]):

- *Spectrum*: counting the number of k-element structures in $\mathcal{C}$;
- *Fine Spectrum*: counting the number of non-isomorphic k-element structures in $\mathcal{C}$;
- *Free Spectrum*: counting the elements of the free k-generated algebra in $\mathcal{C}$ (when $\mathcal{C}$ is variety of algebras).

The variety of Gödel algebras is obtained by adding the prelinearity equation to the class of Heyting algebras. Gödel algebras are the algebraic semantics of Gödel logic, a non-classical logic whose studies date back to Gödel [8] and Dummett [3]. Indeed, Gödel logic can be obtained by adding the prelinearity axiom to Intuitionistic logic. Furthermore, Gödel logic is one of the three major (many-valued) logics in Hajek's framework of Basic Logic, that is the logic of all continuous t-norms and their residua [5].

Given a finite Gödel algebra $\mathbf{A}$, the set of prime filters of $\mathbf{A}$ ordered by reverse inclusion forms a *forest* [1]. Viceversa, given a forest $\mathcal{F}$, the collection of all subforests of $\mathcal{F}$, equipped with properly defined operations, is a finite Gödel algebra. This construction is functorial, meaning that it can be extended to obtain a dual equivalence between the category of finite Gödel algebras and their homomorphisms, and the category of finite forests and open maps [2].

[1] A *forest* is a poset F where the downset of every element is totally ordered. Every downset of F is itself a forest that we call *subforest* of F.

[2] An order-preserving map between forests is *open* (or is a *p-morphism*) when it preserves downsets.

The above duality is an adaptation of the Esakia duality [4] between Heyting algebras and their homomorphisms, and posets and order-preserving open maps, to the case of finite Gödel algebras (see [2, 1] for details and proofs).

In this talk we exploit the category of forests to solve two of the above mentioned Spectra problems when $\mathcal{C}$ is the variety of Gödel algebras $\mathbb{G}$, namely the *Free Spectrum* and the *Fine Spectrum* problem.

Solutions to the *Free Spectrum* problem for $\mathbb{G}$ can be easily found in literature. Indeed, already in 1969 Horn has obtained a recurrence formula to compute the cardinalities of free k-generated Gödel algebras [6]. Another solution to this problem can be achieved by restating the Horn's recurrence in terms of finite forests [2].

Conversely, to the best of our knowledge, the *Fine Spectrum* problem for $\mathbb{G}$ has never been considered before. We introduce an algorithm that given a natural number $k \geq 1$, it generates a set of forests S_k such that for every $\mathcal{F} \in S_k$ the number of subforests of $\mathcal{F}$ is exactly k. That is, given a finite cardinal k we can build the set of finite Gödel algebras with k elements, solving in this way the *Fine Spectrum* problem for $\mathbb{G}$.

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Henk Zeevat

From Experience to Meaning through Semantic Memory

Experience can be seen as the information state built up by updating with all experiences of a subject. Without loss of generalisation, such an information state can be modeled as a collection of frames, sets of points connected by arrows, in which the arrows are labeled by natural attributes and the points by natural classes of objects or of points in some conceptual domain like space, time, color, size, weight, shape, etc.
Assume that part of the experience is verbalised, i.e. that there are subframes in experience which have been evoked by words or complex expressions of some natural language: it is that part of experience that has been acquired by interpreting the utterances of others. This is a partial function f from the set of tokens of subframes of experience to linguistic expressions. It can be turned around to give the set of subframes a word has been known to express: f^{-1} maps words and other expressions to a set of subframe types, with the cardinality of the set of tokens. A lexicon given by experience can then be defined as $l = f^{-1-1}$.
Semantic memory can be thought of as the function that assigns

$$(freq_E(G), freq_E(F))$$

to pairs of subframes F and G, such that F is a subframe of F and G is verbalised (this is necessary if semantic memory is to be semantic). The pair of frequency numbers can be turned into a normal distribution that gives the probability that $p(G|F) = x$ under the assumption that ongoing experience produces fair samples of the natural occurrences of F and G.
$sm(G, F) = h : (0, 1) \rightarrow (0, 1)$ such that $h(x) = p(p(G|F) = x)$
This notion of semantic memory explains whether and to what degree a word activates other words. But more importantly, it helps to build an account of semantic content.
1. Cats and dogs have a very similar structure and for many natural attributes there is a considerable overlap: cat colours can be dog colours, cat shapes can be dog shapes, cat sizes can be dog sizes. This stands in marked contrast to the human, feline and canine competence to tell cats from dogs. And it is clear how people,dogs and cats manage: they take the derived normal distribution of probabilities for the values of a number of attributes for cats and dogs respectively: this gives a reliable discriminator. The normal distributions for the values are determined by semantic memory.
This is a special case: we know cats and dogs are genetically different, making them different biological kinds thus making it the case that cats cannot be partly dogs as well. Our discriminator is a heuristics for a proper distinction in reality. This is typical for all natural kinds, but also for natural phenomena like walking, eating, salty, etc.
Non-natural kinds —with the exception of mathematical notions— are typically vague and subject to cultural variation: blonde, spaghetti bolognese, game, red, tall, hill, etc. Also notions that derive from natual kinds (the border between a dog and what surrounds it, the border between the arm and the hand) are not natural kinds and therefore vague. Semantic

memory provides vague content for all concepts and can help to make natural kind concepts crisp. This is as it should be.
2. The discriminators in (1) are a general possibility for a frame F in the range of semantic memory. Consider F for which attribute α is still undefined. Let $G = F + \alpha = x$ and look at the probability that given $F, F + \alpha = x$ is the case. This gives a probability distribution over values for α which can be notated and defined as follows.
α^F
$\alpha^F = g : X \to (0, 1)$ such that $g(x) = max(sm(F + \alpha = x, F))$
High uncertainty due to lack of experience leads to depressed values for α^F.
Many natural notions that are of this kind. The generic length of men is one.
$length^{man}$
The notion of a possible cause of a type of event is another.
$cause^{event}$
The causes and effects of specific event types are crucial for causal inference.
Affordances can be approached by a natural attribute purpose, the reason why an action is carried by an agent. The affordance of an instrument like cup is a distribution over such purposes. Drinking a hot liquid is the one for which cups were designed, but cups can be used as containers of sugar or of pens.
$purpose^{cup\ as\ instrument\ in\ action}$
$shape^{dog}$
This gives a distribution over the many shapes of dogs. Given the finite number of rather diverse shapes in experience, this example brings in the need of smoothing the distribution by giving large amounts of probability to in-between shapes.
3. Everybody knows that if John is a man he is not 3 meters tall. If conceptual structure is all that is involved, this is not given with the concept man. But, if semantic memory is part of conceptual structure of a concept like man, it is given with the concept and thus should count as analytic. It is dependent on subject's experience of men and such experience can be insufficient. It is crucially independent of knowledge of John, and so is not synthetic.
4. The contention is that adding semantic memory to conceptual structure brings content to the notion of lexical meaning as involved in judgment, even where there are no clear truth-conditional effects, such as in predicates of personal taste or in aesthetic predicates. Like in other cases, such predicates can be analysed by attributes that get their values in experiences. Unlike the cognitive attributes, there is a much weaker prediction about what happens to the values for the same attributes in other people. One can figure out that others are similar or different or one can attempt to educate people in matters of taste, but that gives the limit. In such judgments, one reports ones own experience and since it is predictive of other people's experience it is useful to communicate such experience.
5. Predicates of personal taste are merely one extreme in the possibility of failing intersubjectivity. Cultural factors may make the distinctions between spaghetti bolognese and other kinds of spaghetti quite different and the same holds for distinctions between blonde women and others in say Italy or Sweden. It is only where the distinctions have a strong basis in reality as in the case of natural kinds or a strong conceptual basis as in mathematics that intersubjectivity can be guaranteed (for suitably expert subjects) or that objectivity can arise. But failure of intersubjectivity or objectivity has only a marginal effect on inference and logical consequence (vagueness is a real problem). For determining our actions, we need to know what we want and what can be done. This means we need to decide not just what to do, but what is the case and what is good, overcoming uncertainty.

Dan Zeman

Some Solutions to the Perspectival Plurality Problem for Relativism

In recent literature (Kneer (2015); Kneer, Vicente and Zeman (2017); Zeman (2017)), the less discussed[1] phenomenon of *perspectival plurality* has been shown to pose a serious problem for at least some versions of relativism about predicates of taste (the view that postulates a parameter for perspectives in the circumstances of evaluation with respect to which utterances of sentences are evaluated). Perspectival plurality is the phenomenon whereby sentences containing two or more predicates of taste have to be interpreted by appeal to two or more perspectives. The problem for (certain versions of) relativism stems from this phenomenon getting in tension with a core commitment of such versions, namely that "[i]n a relativist theory, in order to assess a sentence for truth or falsity, one must adopt a stance – that is, truth assessment is always done from *a* particular perspective" (Lasersohn, 2008: 326). In this presentation I want to further the debate by i) showing that the phenomenon applies to a wider range ofexpressions than predicates of taste and ii) investigating and criticizing a number of possible relativist proposals to account for perspectival plurality.

In the works mentioned, perspectival plurality is illustrated with respect to predicates of taste, which is a good place to start. Thus, consider the following scenario: Halloween has just passed, and the neighbors discuss about how their kids spend the holiday. Parents take turns, and when Johnny's father's turn comes, he utters

(1) Johnny played a **silly** prank and had a lot of **tasty** licorice.

In such a context, the most salient interpretation of (1) is that the prank was silly from the father's perspective, while the licorice was tasty from Johnny's perspective. If so, two perspectives are needed for the interpretation of the sentence: predicates of personal taste exhibit perspectival plurality.[2]

Perspectival plurality is present with other perspectival expressions too. Consider (2), which contains aesthetic predicates:

(2) Johnny drew a **nice** portrait of the teacher in the play time and saw an **exquisite** painting in the main exhibition.

In a context in which what is discussed is a school trip to the art museum, the most salient interpretation of (2) as uttered by Johnny's mother (a sophisticated art lover) is that the painting was exquisite from her perspective, while the portrait was nice from Johnny's perspective.

Moral terms follow suit. Consider

(3) Jeremy **ought to** lie, but Immanuel **ought** not **to** lie,

and imagine it uttered by a philosophy student who answers a question in an ethics exam regarding the moral profile of lying in a certain scenario according to various moral views. The most salient interpretation of (3) is that Jeremy ought to lie from a Benthamian perspective, while Immanuel ought not to lie from a Kantian perspective.

[1] Previous engagement with the phenomenon is limited to a few works: Lasersohn (2008), Cappelen and Hawthorne (2009) and Kissine (2012). Their examples, however, are different from those used by Kneer (2015), Kneer, Vicente and Zeman (2017) and Zeman (2017), the latter showing that the phenomenon appears in the absence of expressions that "shift" perspectives (e.g. "for Johnny").

[2] Examples are obviously not limited to conjunctions or other sentences containing logical connectives. "Johnny had a funny-looking, tasty dish" has an interpretation according to which the dish was funny-looking from the speaker's perspective and tasty from Johnny's perspective (or the other way around).

Perspectival plurality also holds for gradable adjectives:

(4) Dumbo is **small**, but Iñaki is **big**.

In a context in which the speaker summarizes a situation in a children book about animals in which Dumbo is pictured as the smallest elephant and Iñaki as the biggest ant, the most salient interpretation of (4) is that Dumbo is small for an elephant, while Iñaki is big for an ant.

Finally, take epistemic modals. Imagine the speaker playing Mastermind with two people simultaneously, and commenting on the epistemic possibilities within the reach of each player. In such a context, the most salient interpretation of

(5) There **might** be a green piece, and there **might** be a red piece too

is that there might be a green piece from the perspective of the first player, while there might be a red piece from the perspective of the second payer. I take these examples to show that predicates of taste, aesthetic adjectives, moral terms, gradable adjectives and epistemic modals, respectively, exhibit perspectival plurality.[3] And given the core commitment of relativism exposed above, this phenomenon is troublesome for relativism about all these expressions.[4]

Several possible solutions can be envisaged on behalf of the relativist. One solution (pursued with a different purpose by MacFarlane (2014) in connection to predicates of taste) is to draw a semantic wedge between *egocentric* (from one's own perspective) and *exocentric* (from somebody else's perspective) uses of the expressions in question and thus provide different semantics accounts for them. Thus, MacFarlane claims that when used exocentrically predicates of taste harbor a variable for perspectives in their logical configuration (as most contextualist views would have it), but when used egocentrically such a variable is missing. This would help with perspectival plurality because sentences like (1) are combinations of egocentric and exocentric uses, and thus the only relativization that is not explicit at the level of logical form is that of egocentric uses, thus canceling the need to appeal to different perspectives when evaluating the respective sentence for truth. Such a distinction can perhaps be generalized to all or most of the expressions involved. However, MacFarlane offers little independent motivation for the distinction, and adopting it would mean that the expressions at stake are ambiguous between one-place predicates (when used egocentrically) and two-place predicates (when used exocentically). A unitary semantic theory is preferable on methodological grounds, while appeal to ambiguity should be made only as a last resort.

[3] Obviously, what a perspective comes down to in each case is different: a standard of taste, an aesthetic standard, a moral standard, a compassion class, a body of knowledge, respectively. From a formal point of view, however, these differences don't matter.

[4] The phenomenon is, in fact, much more pervasive than the examples above show. First, in connection to the very expressions mentioned, perspectival plurality is present in quantified sentences as well as in sentences embedded under attitude verbs. As an example of the former, consider the sentence

Every kid played a silly **prank** and had a lot of **tasty** licorice,

uttered in the context devised for the interpretation of (1) made vivid above, as a means to summarize the parents' discussion. (A structurally similar sentence is discussed in Lasersohn (2008), but his aim is to show that the interpretation highlighted here doesn't exist.) As an example of the latter, consider the following example involving aesthetic predicates from Sæbø (2009, 337):

The mother snipe thinks the **ugliest** baby birds are **beautiful**,

in which the intended interpretation is that the snipe baby birds are beautiful from the mother snipe's perspective but the ugliest from the speaker's perspective. Neither quantified sentences nor embedded ones are tackled here, but a complete account of the expressions in question should obviously take them into consideration.

Second, the range of expressions that exhibit perspectival plurality might be larger than those focused on here. For example, temporal and location expressions, first person pronouns or even common nouns can be said to be perspectival, in a braider sense of the term. I ignore such expressions here, but see Zeman (2017) for discussion.

A second relativist strategy is to paraphrase the problematic examples as logical conjunctions of simpler sentences, with each of them containing one relevant predicate only (this is the approach put forward in Kneer, Vicente and Zeman (2017)). Thus, according to this view, (1) will be paraphrased as

(1') [Johnny played a **silly** prank] & [Johnny had a lot of **tasty** licorice].

This would help with perspectival plurality because each such simple sentence could, in principle, be evaluated with respect to a different perspective: while the first conjunct of (1') will be evaluated with respect to the father's perspective (the speaker), the second conjunct will be evaluated with respect to Johnny's perspective, thus yielding the relevant reading. The view remains relativistic in that a parameter for perspectives is still postulated in the circumstances of evaluation for simple sentences.

This solution holds promise, but it is also problematic in several respects. First, it is not clear that all sentences are paraphrasable as conjunctions of simple sentences: complex sentences involving comparatives, predicates in subject position ("Interesting books are fun"), two predicates in predicative position ("The dog food is astonishingly tasty") etc. might not lend themselves easily to paraphrases, or at least do so while incurring semantic commitments that are not trivial (see though Kneer, Vicente and Zeman (2017) for detailed discussion). Second, the view might not happily align with the predictions of most contemporary syntactic theories: it is doubtful that the syntactic representation of (1) involves breaking it down into two separate sentences connected by "and". As a reply to this latter objection, one could claim that the paraphrasing is done for purposes of truth-evaluation only and thus correspondence with syntax is not a desideratum. Even so, however, this comes close the postulation of an independent level of representation of a sentence (significant only for truth-evaluation); but postulating an additional level of representation is a substantial burden, which should be supported on independent grounds.

A third relativist solution is to postulate not one parameter for perspectives in the circumstances of evaluation, but a *sequence* of them, with each parameter indexed to each occurrence of the relevant expression (this is the idea pursued in Zeman (2017)). As the strategy investigated before, this strategy helps with perspectival plurality because it allows that, in principle, each occurrence of the relevant expression can be evaluated with respect to a different perspective. To illustrate, in this framework the abstract truth-conditions of (1) are given by

(1'') [[Johnny played a silly^1 prank and had a lot of tasty^2 licorice]]$^{c,\ w,\ <p1,\ p2>}$ = 1 iff Johnny played a silly prank in *w*according to the value of p_1 and had a lot of tasty licorice in *w*according to the value of p_2,

wherep_1 and p_2 are the two parameters for perspectives in the sequence introduced, the superscripts on the two predicates of taste represents the order in which they appear and the co-indexing of the parameters with those superscripts signifies that they correspond to the predicates superscripted (p_n corresponds to Φ^n, where Φ is a predicate). Once values are given to p_1 and p_2, we obtain actual readings of (1) – the plural reading made salient in the context of (1) presented above, but also singular readings in which all the relevant predicates are evaluated with respect to *the same perspective* (the difference simply stems from giving different values to the perspectives in the sequence).

While I think this solution is the most promising one, there are challenges to be addressed. First, introducing a sequence of perspectives (an instance of "multiple indexing") is highly unorthodox, so an independent motivation for this departure from orthodoxy has to be given. Second, we need to get clear on how to understand several key notions used in semantics if we postulate sequences of perspectives: what notion of context we end up employing and what does semantic content (the things we assert, believe and report) come down to etc.

From a more general perspective, besides accounting for examples like the ones presented above, working out the details and responding to the particular objections each view faces, the challenge posed by perspectival plurality to semantic theories of the expressions in question has also to do with finding empirically adequate and principled constraints on the interpretations of such sentences, as well as a discussion of the appropriate notion of context that underlies the solutions given. All these are issues to be pursued in future work.

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Yulia Zinova

Explaining meaning: The interplay of syntax, semantics, and pragmatics

1 Introduction

Traditionally, one of the usages of the prefix *po-* (often called *delimitative* or *attenuative*) is associated with some characteristic of an event being lower than the expected value: an event lasting for a short period of time, a small quantity of the theme consumed, etc. According to Filip (2000, pp. 47–48) "[t]he prefix *po-* contributes to the verb the [. . .] meaning of a small quantity or a low degree relative to some expectation value, which is comparable to vague quantifiers like *a little, a few* and vague measure expressions like *a (relatively) small quantity / piece / extent of.*"

(1) Ivan po-guljal po gorodu.
Ivan po-walk.PST.SG.M around town
'Ivan took a (short) walk around the town.'
= example (9c) in Filip 2000

(2) Ivan po-el jablok.
Ivan po-eat.PST.SG.M apple.PL.GEN
'Ivan ate some (not many) apples.'
= example (3) in Kagan 2015 (p. 46)

Although the observations about the low degree on some scale, associated with the discussed usage of the prefix *po-*, are commonly accepted and seem to be well established, examples like (3) do not support it, as there the same verb as in (2) is modified by an adverbial denoting a high degree. If the adverbial is removed, the sentence is neutral with respect tot the quantity of the food eaten.

(3) Kogda do stolicy ostavalos' tridcat' kilometrov, našël stolovuju i očen' plotno
when until capital was left thirty kilometers found canteen and very tight
po-el [. . .]
po-eat.PST.SG.M
'When I was about 30 km away from the capital, I found a canteen and had a very good meal [. . .]'
Anatolij Azol'skij. *Lopušok* (1998)

In addition, there are other usages of the prefix *po-* that are never associated with a 'low degree' component: e.g. a usage that is described by Švedova (1982, p. 365) as 'to complete the action denoted by the derivational base', which is encountered in such verbs as *poblagodarit'* 'to thank'. The distribution of the delimitative and non-delimitative prefix usages over derivational bases and contexts did not receive any explanation so far.

2 Proposal

I propose to use underspecified semantics and probabilistic pragmatic modelling to explain intuitions about the delimitative nature of the prefix *po-* and account for the cases that seem exceptional from the traditional perspective. The general line goes the following way: the prefix *po-* makes the event denoted by the derivational base bounded. The boundaries are imposed by mapping the initial and the final stages of the event to some degrees on the relevant scale, but in case of the prefix *po-* these degrees are not specified by the prefix.

At the same time, most verbs can be prefixed with a range of prefixes and almost all of them are more restrictive with respect to the identification of the initial and final stages of the event than *po-*. I propose to explain the observed inference of 'low intensity' or 'short duration' of the *po*-prefixed verbs by the competition that occurs between various perfective verbs derived from the same base.

3 Pragmatic competition

The following information is contributed by the prefixes with respect to the initial and final stages of the event: while the prefix *po-* remains neutral in this regards (only the presence of the initial and final stages is postulated), the prefix *za-* necessarily identifies the initial stage of the event with the minimum of the scale, the prefix *do-* identifies the final stage of the event to the maximum point on the scale, and the prefix *pere-* (in some of its usages) does both.

Consider the verb *zimovat'* 'to spend winter time'. Four prefixed verb derived from it are commonly used (more can be found in the dictionary, but not in the contemporary texts, as evidenced by the data in Russian National Corpora[1]): (1) *pozimovat'* 'to spend some winter time' describes a finished event of staying in some particular place without imposing further restrictions on the start and the end of the stay; (2) *zazimovat'* 'to stay for the winter' establishes a connection between the start living somewhere and the beginning of the winter; (3) *dozimovat'* 'to spend the rest of the winter' fixes the end point of the stay to be the end of the winter; and (4) *perezimovat'* 'to spend the winter' relates both the start and the end points of the stay to the beginning and the end of the winter, respectively.

A natural assumption with respect to the events of spending winter time is to limit the number of situations a speaker may want to describe to four (Table 1): (1) spending one whole winter (t_1); (2) spending an initial part of the winter (t_2); (3) spending a final part of the winter (t_3); (4) spending some time of the winter without bounding the event duration to the duration of the winter (t_4).

Given the situations specified in Table 1 and the restrictions imposed by particular prefixes, possible interpretations of prefixed verbs are shown on Figure 1: the verb *pozimovat'* 'to spend some winter time' can refer to any of the situations t_1–t_4, the verb *zazimovat'* 'to stay for the winter' can refer to t_1 and t_2, *dozimovat'* 'to spend the rest of the winter' – to t_1 and t_3, and *perezimovat'* 'to spend the winter' – only to t_1. In such a configuration, however, it follows from basic pragmatic and game-theoretic principles (one can use, e.g., Optimality Theory, see Blutner 2000) that the usage of the *za-*, *do-*, and *po*-prefixed verbs would be restricted to the situations t_2, t_3, and t_4, respectively.

[1] Available online at ruscorpora.ru.

	event start = winter start	event end = winter end
t_1	+	+
t_2	+	-
t_3	-	+
t_4	-	-

Table 1: The domain of terminated events related to spending the winter

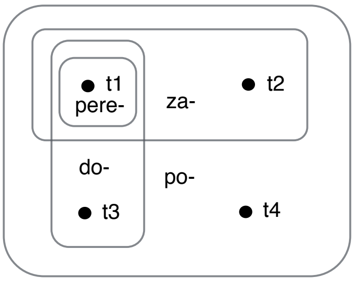

Figure 1: Possible interpretations of the verbs derived from *zimovat'* 'to spend the winter', see also Table 1

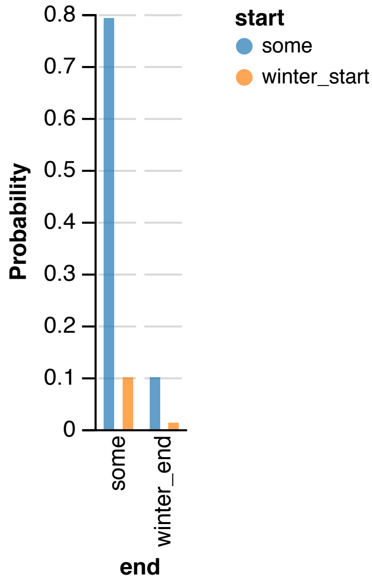

Figure 2: RSA model output

4 Implementation: RSA framework

As a further step, I propose to implement such an approach using the Rational Speech Act model (RSA, Goodman and Frank 2016). For the implementation I have used WebPPL with a basic three-layered RSA model (literal listener, pragmatic speaker, pragmatic listener); a world model with four states shown in Table 1 with a categorical distribution, a flat prior, a meaning function corresponding to the semantics described above, and the optimality parameter alpha 1[2]. Given this model the verb *pozimovat'* is interpreted by a pragmatic listener as 'spend some but not all winter time' with the probability almost 0.8.

5 The influence of syntax

Let us now consider examples (2) and (3). I claim that the difference in the interpretation of the verb *poest'* 'to eat' can be accounted for by using the same pragmatic principles as in the case of the verb *pozimovat'* 'to spend winter time'. The key idea here is that the number of available alternatives depends on the syntactic context: when an object if present, as in (2), the verb *poest'* 'to eat' competes with the verbs *naests'ja* 'to eat until becoming full' and *s"jest'* 'to eat all of smth' and thus acquires the enriched interpretation 'to eat some but not all of smth and not until becoming full'. In an intransitive context, however, there are no alternatives, as both *naests'ja*

[2]This is an arbitrary selected value. By varying this parameter one can model different behaviour: more or less dependent on the rational considerations.

'to eat until becoming full' and *s"jest'* 'to eat all of smth' are obligatory transitive. This results in the observed asymmetry of the interpretations.

6 Results

Underspecified semantics coordinated with pragmatic competition allows to explain the observed inference of 'low intensity' or 'short duration' of the *po*-prefixed verbs by the competition between various perfective verbs derived from the same derivational base: when the semantics of several prefixed verbs overlaps, the usage of the *po*-prefixed verb gets restricted to the 'low degree' situations; when no such competition takes place (e.g. due to the restrictions on the type of the scale), the usage of the *po*-prefixed verb is not constrained further.

In sum, the combination of the underspecified semantics and basic pragmatics allows to deal with phenomena that have not received any explanation so far.

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