Gradability in MTT-Semantics

Stergios Chatzikyriakidis
Department of Philosophy, Linguistics and Theory of Science
stergios.chatzikyriakidis@gu.se

Zhaohui Luo
Royal Holloway, University of London
zhaohui.luo@hotmail.co.uk

Introduction. Formal semantics in Modern Type Theories (MTT-semantics) has been proposed as an alternative to Montague Semantics, and various semantic accounts have been given within this paradigm for a wide range of linguistic phenomena [7, 8, 1, 10, 2]. In this paper, we provide an additional test case for MTT-semantics, i.e., gradable adjectives. We claim that the rich typing structures of MTTs provide us with a more natural formalization of various issues related to gradable adjectives, such as the representation of a polymorphic standard of comparison function extending over the Degree universe for positive gradable adjectives, and the use of indexed types to represent common nouns that are indexed with degree parameters to naturally capture the class dependency of the gradable property. We further implement our account in the proof assistant Coq, checking its formal correctness and inferential properties.

Gradable Adjectives. We start by proposing that the arguments of gradable adjectives are not simple types, but rather types indexed by degree parameters (dependent types). In MTT-semantics, the universe CN of common nouns is refined into sub-universes of CNs each of which is indexed by a degree. For example, the collection represented by the common noun *human* may be refined into the family of types indexed by heights: $HHuman : Height \to Type$ and $HHuman(n)$ is the type of humans of height n.[1] We can then define a function *height* that returns the value of the height-index of a human; i.e., $height(i, h)$ is the height of human h:

(1) $height : \Pi i : Height.\ HHuman(i) \to Height$

(2) $height(i, h) = i.$

With these in line, we may consider the semantic interpretation of *tall* as follows, to mean that the height of the human concerned is bigger than some given standard n:

(3) $tall : \Pi i : Height.\ HHuman(i) \to Prop$

(4) $tall(i, h) = height(i, h) \geq n$

The above definition for *tall* specifies that for any i of type $Height$, *tall* takes a human argument indexed with i and returns the proposition saying that i, the height of the human, is bigger

[1] Informally, this family of types of humans are refined from the type $Human$ of all humans. Formally, we'd have $HHuman(i) \leq Human$.

than or equal to a natural number n, which stands for the contextually restricted parameter – humans taller than n are regarded as tall. In a similar fashion, we can define the comparatives, where the RHS of (6) is the same as $i > j$:

(5) $taller\_than : \Pi i, j : Height.\ HHuman(i) \to HHuman(j) \to Prop$

(6) $taller\_than(i, j, h_1, h_2) = height(i, h_1) > height(j, h_2).$

From this definition, we can easily prove that, for example, if $height(i, h_1) \geq height(j, h_2)$ and $tall(j, h_2)$, then $tall(i, h_1)$.

But where do we get the contextual parameter n in (4)? In what we have provided so far, it is just a number, which does not depend on anything. A more proper way is to posit that the value is dependent on the noun, the adjective, and sometimes even some other contextual information, which in MTT-semantics are represented as a type, a predicate and a context (in type theory), respectively. We use polymorphism and type dependency in MTTs to realise this. First, we introduce the universe of (totally ordered) degree types, $Degree$. As examples of degrees, one would find in $Degree$ the degree types such as $Height$, $Weight$ and $Width$, among many others. Then, for $D : Degree$, we introduce the indexed universe $\text{CN}_\text{G}(D)$, which is a subuniverse of CN, consisting of the CNs with the indexed degree. For instance, we'd have $Human : \text{CN}_\text{G}(Height)$. Besides its introduction rules like $Human : \text{CN}_\text{G}(Height)$, the general rule of CN_G are given below, the second of which says that $\text{CN}_\text{G}(D)$ is a subtype of CN.

$$\frac{D : Degree}{\text{CN}_\text{G}(D) : Type} \qquad \frac{D : Degree \quad A : \text{CN}_\text{G}(D)}{A : \text{CN}}$$

We can now introduce the polymorphic standard, STND. First, for any common noun A, let $ADJ(A)$ be the type of syntactic forms of adjectives whose semantic domain is A. For instance, $TALL : ADJ(Human)$, where $TALL$ strands for the syntax of *tall*. Then, STND takes a degree D, a D-indexed common noun A and (the syntax of) an adjective whose domain is A, and returns the relevant standard for the adjective:

(7) $\text{STND} : \Pi D : Degree\ \Pi A : \text{CN}_G(D).\ ADJ(A) \to D$

We can now give a revised definition for an adjective like *tall* whose type is (3):

(8) $tall(i, h) = height(i, h) \geq \text{STND}(Height, Human, TALL).$

Note that indexing on the noun by means of a degree gives one for free the fact that we are not talking about tallness in general but tallness with respect to the relevant class (represented by the type $Human$ in the above example).[2] Furthermore, the polymorphic STND function is a more straightforward interpretation of Kennedy's context sensitive function from measure functions (adjectives basically) to degrees [4]. One may consider standards that are dependent on contextual information: for example, whether it is regarded as an expensive car might depend on where the expensive car in question is considered. In that case, the $STND$ function may take an additional parameter of locations that would take this into account.

[2]This does the work that is achieved by using the . combinator [5] to compose comparison classes with adjectives in the work of [4]. To give an example, one needs to compose the comparison class, say basketball player, $BB : e \to t$ and $tall : e \to d$ to $BB(tall) : e \to d$. Normal functional application will not work here, so the . combinator is used to remedy this. This additional, and not well-motivated, extra machinery is not needed here.

Gradable Nouns. In the context of gradability, another issue we want to discuss is that of gradable nouns, i.e. gradability cases where the relevant gradable element is not an adjective, but rather a noun, as (9) illustrates.

(9) John is an enormous idiot. / He is a big stamp collector.

Can we extend the usage of indexed types to abstract nouns like *idiot*? The account we propose here is one where the distinction between nouns and adjectives are clear, with adjectives being predicates and nouns being types, and at the same time, we assume that abstract nouns like *idiot* in line with gradable adjectives involve a degree parameter, albeit an abstract one. A natural way to capture this duality is to use Σ-types and assume that the first projection is actually the abstract parameter. We consider the type family $IHuman : Idiocy \to Type$ indexed by idiocy degrees of type $Idiocy : Degree$ and, then, the CN *idiot* can be represented by means of (10): an idiot is a triple (i, h, p) where h is a human whose idiocy degree i is bigger than or equal to the standard of being an idiot.

(10) $Idiot = \Sigma i : Idiocy.\ IHuman(i) \times (i \geq \mathrm{STND}(Idiocy, Human, IDIOTIC))$

Note that this account has not only similarities with the ideas proposed in [3] but also brings out a connection with gradable adjectives in the sense that they both involve a degree parameter. However, we note that these two constructions are clearly different in terms of their formal status and, in particular, in MTT-semantics, CNs are types rather than predicates.

Let us now consider *enormous idiot*. What we want to get in this case is someone who is an idiot to a high degree. This means that this degree must be (much) higher than the degree of idiocy needed for someone to be considered an idiot (the standard $\mathrm{STND}(Idiocy, Human, IDIOTIC)$ in (10)). Exceeding $\mathrm{STND}(Idiocy, Human, IDIOTIC)$ is an idiot and, to be an enormous idiot, one needs to exceed a higher degree of idiocy. In general, *enormous* can be interpreted as having the following type, where $PHY_D : \mathrm{CN_G}(D)$ is the type of physical objects indexed by D:

$$\begin{aligned} enormous \ : \ & \Pi D : Degree \\ & \Pi A : D \to \mathrm{CN_G}(D) \\ & \Pi p : (\Sigma d{:}D.\ d \geq \mathrm{STND}(D, PHY_D, ENORMOUS)). \\ & A(\pi_1(p)) \to Prop \end{aligned}$$

Enormous idiot is then the following Σ-type (we suppress the D and A arguments as implicit), where $q : \pi_1(h) \geq \mathrm{STND}(Idiocy, Human, IDIOTIC)$:

(11) $EnormousIdiot = \Sigma h : Idiot.\ enormous((\pi_1(h), q), \pi_1(\pi_2(\pi_1(h))))$

In the longer version of the paper, we plan to discuss other issues with respect to gradability, such as the relative vs absolute distinction and degree adverbs. We will also look at the predictions this general treatment of gradability gives with respect to adjectival vagueness [9, 6] from the perspective of MTT-semantics.

References

[1] Stergios Chatzikyriakidis and Zhaohui Luo. An account of natural language coordination in type theory with coercive subtyping. *Constraint Solving and Language Processing 2012, LNCS 8114*, 2013.

[2] Stergios Chatzikyriakidis and Zhaohui Luo. Adjectival and adverbial modification: The view from modern type theories. *Journal of Logic, Language and Information*, 26(1):45–88, 2017.

[3] Camelia Constantinescu. Big eaters and real idiots: evidence for adnominal degree modification. In *Proceedings of Sinn und Bedeutung 17*, pages 183–200.

[4] Christopher Kennedy. Vagueness and grammar: The semantics of relative and absolute gradable adjectives. *Linguistics and philosophy*, 30(1):1–45, 2007.

[5] Angelika Kratzer and Irene Heim. *Semantics in generative grammar*, volume 1185. Blackwell Oxford, 1998.

[6] Daniel Lassiter and Noah D Goodman. Adjectival vagueness in a bayesian model of interpretation. *Synthese*, 194(10):3801–3836, 2017.

[7] Zhaohui Luo. Type-theoretical semantics with coercive subtyping. In *Semantics and linguistic theory*, volume 20, pages 38–56, 2010.

[8] Zhaohui Luo. Formal semantics in modern type theories with coercive subtyping. *Linguistics and Philosophy*, 35(6):491–513, 2012.

[9] Peter R Sutton. Towards a probabilistic semantics for vague adjectives. In *Bayesian Natural Language Semantics and Pragmatics*, pages 221–246. Springer, 2015.

[10] Tao Xue and Zhaohui Luo. Dot-types and their implementation. In *International Conference on Logical Aspects of Computational Linguistics*, pages 234–249. Springer, 2012.

A Coq Code

```
(* Degree is type of names of degrees -- a Tarski universe *)
(* Here is an example with three degrees. *)
Require Import Omega.

Inductive Degree : Set := HEIGHT | AGE | IDIOCY.
Definition D (d : Degree) := nat. (* example for simplicity *)
Definition Height := D(HEIGHT).
Definition Age := D(AGE).
Definition Idiocy := D(IDIOCY).
(* Universe CN_G of indexed CNs *)
Definition CN_G (_:Degree) := Set.
Parameter Human : CN_G(HEIGHT).
Parameter height : Human->Height.
(* Type of physical objects indexed with a degree *)
Parameter PHY : forall d: Degree, CN_G(d).

(* ADJ(D,A) of syntax of adjectives whose domain is A : CN_G(d) *)
Parameter ADJ : forall d:Degree, CN_G(d)->Set.
```

```
Parameter TALL SHORT : ADJ HEIGHT Human.
Parameter IDIOTIC : ADJ IDIOCY Human.
Parameter ENORMOUS : forall d: Degree,  ADJ d  (PHY(d)).
(* STND *)
Parameter STND : forall d:Degree, forall A:CN_G(d), ADJ d A -> D(d).

(* semantics of tall, taller_than *)
Definition tall (h:Human) := ge (height h) (STND HEIGHT Human TALL).
Definition taller_than (h1:Human) (h2:Human) := gt (height h2) (height h1).

(* Some simple theorems *)
Parameter John Mary Kim : Human.
Theorem TALLER:
  taller_than Mary John /\ height Mary = 170 -> gt (height John) 170.
  cbv. intro. omega. Qed.

Theorem trans:
  taller_than Mary John /\ taller_than Kim Mary -> taller_than Kim John.
  cbv. intro. omega. Qed.

(* Definition for Idiot *)
Parameter IHuman : Idiocy -> Type.
Definition Idiot:=
  sigT (fun x: Idiocy=> prod (IHuman x) (ge x (STND IDIOCY Human IDIOTIC))).

Parameter enormous :
  forall d: Degree,
  forall A : D(d) -> CN_G(d),
  forall p : (sig (fun d1: D d  => d1 > (STND  d (PHY(d)) (ENORMOUS d)))),
  A(proj1_sig p) -> Prop.
```