GENERALIZED LOGICS AND INNER MODELS FOR SET THEORY ABSTRACT

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The constructible universe L is built by a series of stages where each successor stage is the set of (first order) definable subsets of the previous stage.The problem with L is that it misses many canonical objects like $0^{\#}$. One possible attempt to define a rich class of inner models is by imitating the construction of L but using definablility by stronger logic. A classical theorem of Myhill and Scott claims that if we use second order logic we get HOD -The class of sets hereditarily ordinal definable.HOD is not very canonical ,it depends very much on the universe of Set Theory from which we start.

It turns out that using a logic which is strictly between first order and second order yields inner models which are much more canonical and contains many interesting definable objects. We believe that the study of these inner models shades light both on definability in Set Theory and on the relationship between different generalized logics.

In this talk we shall present on-going work, which is joint work with J.Kennedy and J. Vaananen, on the inner models we can get by using logics which are between first order logic and second order logic.

We shall concentrate on two interesting cases. The first is the logic of the quantifier $Q^{\omega}_{\mathrm{cf}}x, y\Phi(x, y)$ which means "The formula $\Phi(xy)$ defines a linear order which has cofinality ω. The model we get is rather canonical (in the presence of large cardinals) and contains many canonically definable objects.

The second is stationary logic (The "aa" logic) which is first order logic with the quantifier $Q_{aa}P\Phi(P)$ which means that there is a closed unbounded set of countable subsets of the universe,P, such that $\Phi(P)$ holds.

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