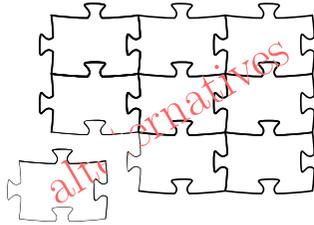# Alternatives in Montague Grammar[*]

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Stanford Construction of Meaning Workshop

February 20, 2015

1 Introduction

- Montague's (1973) type-theoretic framework for natural language semantics forms the cornerstone of all subsequent work in the field.

- Hamblin (1973) proposed a modification/extension of the basic Montagovian framework, in which the semantic value of an expression is not taken to be a single object in its usual domain of interpretation, but rather a **set** of such objects—a set of **alternatives**.

- For instance:
  - The semantic value of a **complete sentence** is a **set of propositions**
  - The semantic value of an expression of **type** e is a **set of individuals**

- The resulting framework is refered to as **alternative semantics**.

- A large number of linguistic phenomena has been analyzed insightfully in this framework:
  - questions (Hamblin, 1973)
  - focus (Rooth, 1985)
  - indeterminate pronouns (Shimoyama, 2001; Kratzer and Shimoyama, 2002)
  - indefinites (Kratzer and Shimoyama, 2002; Menéndez-Benito, 2005; Aloni, 2007)
  - disjunction (Alonso-Ovalle, 2006; Aloni, 2007)

- However, the framework also faces some **fundamental issues**.

- We will be concerned with the following two, in our view the most elementary ones:
  - **The compositionality issue:**
    In alternative semantics, meanings can no longer be composed by means of the standard type-theoretic operations of function application and abstraction.
  - **The entailment issue:**
    In alternative semantics, the type-theoretic notion of entailment no longer works.

- That is, alternative semantics forces us to give up the standard type-theoretic ways of composing and comparing meanings, which lie at the heart of Montague Grammar.[1]

- We will argue that these issues arise from specific assumptions, which can be adapted without affecting the linguistic utility of the framework.

- More specifically, we will show that:
  1. the compositional architecture of the system can be adjusted so that meanings can be composed by means of standard function application and abstraction.
  2. the notion of meaning can be adjusted so that meanings can be compared by means of the standard notion of entailment.
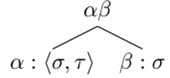
2 Compositionality

- In ordinary semantic theories, the semantic value of an expression α of type τ (notation: $\alpha : \tau$) relative to an assignment g is an object $[\![\alpha]\!]_g \in D_\tau$, where:
  - D_e is a primitive domain of individuals
  - $D_t = \{0, 1\}$ is the set of truth values
  - D_s is a primitive domain of possible worlds
  - $D_{\langle \sigma, \tau \rangle} = \{f \mid f : D_\sigma \to D_\tau\}$

- This allows us to use the standard composition rules provided by type theory, namely:
  - **Function Application:** if $\alpha : \langle \sigma, \tau \rangle$ and $\beta : \sigma$ then $\alpha(\beta) : \tau$
    and $[\![\alpha(\beta)]\!]_g = [\![\alpha]\!]_g([\![\beta]\!]_g) \in D_\tau$
  - **Abstraction:** if $\alpha : \tau$ and $x : \sigma$ then $\lambda x.\alpha : \langle \sigma, \tau \rangle$
    and $[\![\lambda x.\alpha]\!]_g \in D_{\langle \sigma, \tau \rangle}$ is the function mapping $d \in D_\sigma$ to $[\![\alpha]\!]_{g[x/d]} \in D_\tau$

[1]Karttunen's (1977) theory of questions is similar to Hamblin's in that the meaning of a complete question corresponds to a set of propositions. However, when it comes to the meaning of subsentential expressions and the way in which these are composed, the two theories are very different. In particular, Karttunen does not assume that the semantic value of *every* expression is a set of objects in its usual domain of interpretation. For this reason, the compositionality problem that alternative semantics faces does not apply to Karttunen's theory of questions. The entailment problem on the other hand does apply to Karttunen's theory.

- In alternative semantics, the semantic value $[\![\alpha]\!]_g$ of an expression $\alpha : \tau$ is no longer a single object in D_τ, but rather a **set** of such objects: $[\![\alpha]\!]_g \subseteq D_\tau$.[2]

- As a consequence, meanings can no longer be composed by means of the standard type theoretic operations.

- Let us see why, and what could be done about this.

2.1 Function application

- Suppose we want to compute the meaning of an expression $\alpha\beta$, where $\alpha : \langle \sigma, \tau \rangle$ and $\beta : \sigma$.

$$
\begin{array}{c}
\alpha\beta \\
\diagup \quad \diagdown \\
\alpha : \langle \sigma, \tau \rangle \quad \beta : \sigma
\end{array}
$$

- In alternative semantics, we have that $[\![\alpha]\!]_g \subseteq D_{\langle \sigma, \tau \rangle}$ and $[\![\beta]\!]_g \subseteq D_\sigma$.

- Since $[\![\alpha]\!]_g$ is not a function but a set of functions, it cannot simply be applied to $[\![\beta]\!]_g$.

- So standard function application cannot be used to compute the meaning of $\alpha\beta$.

- What we *can* do is apply each element $f \in [\![\alpha]\!]_g$ to each element $d \in [\![\beta]\!]_g$:

  - **Pointwise function application:** if $\alpha : \langle \sigma, \tau \rangle$ and $\beta : \sigma$ then $\alpha\beta : \tau$ and $[\![\alpha\beta]\!]_g = \{ f(d) \mid f \in [\![\alpha]\!]_g \text{ and } d \in [\![\beta]\!]_g \}$

- PFA is taken to be the fundamental composition rule in alternative semantics.

- However, this rule has an important drawback.

- Namely, in computing the meaning of a complex expression $\alpha\beta$ using PFA, the functor α **only has access to each alternative for β in isolation**, never to the whole set.

- Some functors *do* need to access to the whole set at once.

- Take for instance **negation**. The standard treatment of negation in alternative semantics (see, e.g., Kratzer and Shimoyama, 2002) is as follows:

  - $[\![\text{not } \beta]\!]_g = \{ \overline{\bigcup [\![\beta]\!]_g} \}$, where overline denotes set-theoretic complementation.

- To compute $\overline{\bigcup [\![\beta]\!]_g}$ we need access to all the alternatives for β at once.

- This means that negation needs to be treated **syncategorematically**, that is, it cannot be given a meaning of its own.

[2] In some work on alternative semantics, the types that expressions are usually taken to have are systematically adapted: expressions that are usually taken to be of type τ are now rather taken to be of type $\langle \tau, t \rangle$ (see, e.g., Shan, 2004; Novel and Romero, 2010). The usual correspondence between the type of an expression and its semantic value is then preserved. In other work, the usual types are preserved: expressions that are usually taken to be of type τ are still taken to be of type τ (see, e.g., Rooth, 1985; Kratzer and Shimoyama, 2002; Alonso-Ovalle, 2006). In this case, the correspondence between the type of an expression and its semantic value changes: the semantic value of an expression of type τ is no longer a single object in D_τ, but rather a set of such objects. The choice between these two options seems immaterial; for concreteness we assume the second, but our arguments do not hinge on this assumption.

- Other operators that need access to all the alternatives of their argument at once include:

  - conditionals (see, e.g. Alonso-Ovalle, 2006)

  - modals (see, e.g. Aloni, 2007)

  - exclusive strengthening operators (see, e.g. Menéndez-Benito, 2005)

  - existential and universal closure operators (see, e.g. Kratzer and Shimoyama, 2002)

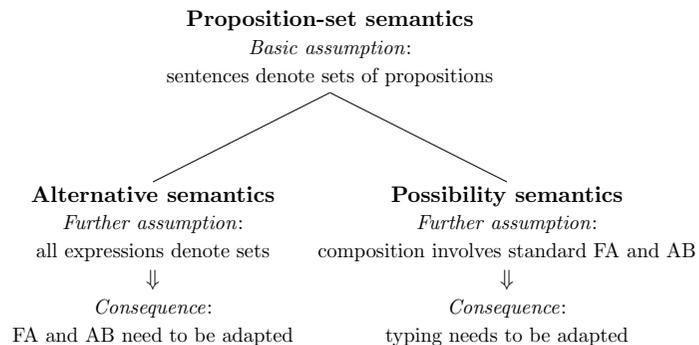 - question-embedding verbs

- These operators cannot be given a meaning of their own, but have to be treated syncategorematically, in terms of a tailor-made composition rule.

- Clearly, it would be preferable if our system contained just a small number of general composition rules.

- The contribution of specific linguistic items should be derived by means of these general rules, given the specific lexical meaning of these items.

2.2 Abstraction

- Now let us consider abstraction.

- If $x : \sigma$ and $\alpha : \tau$, then $\lambda x.\alpha$ should be of type $\langle \sigma, \tau \rangle$.

- We cannot apply the standard abstraction rule, which would identify $[\![\lambda x.\alpha]\!]_g$ with the function mapping every $d \in D_\sigma$ to $[\![\alpha]\!]_{g[x/d]}$.

- This would be a function from D_σ into sets of objects in D_τ.

- What we need for $[\![\lambda x.\alpha]\!]_g$ is a **set of functions** from D_σ to D_τ.

- But **which functions** should be included in this set?

- Kratzer and Shimoyama (2002) propose to include every function $f : D_\sigma \to D_\tau$ such that for any $d \in D_\sigma$, $f(d) \in [\![\alpha]\!]_{g[x/d]}$.

- Shan (2004) points out that this proposal leads to **problematic empirical predictions**, and argues that it is in fact impossible to obtain the right set of functions in a principled way.

- Novel and Romero (2010) propose a way to overcome the problems that Shan pointed out, but this solution requires a number of non-trivial assumptions.

- Moreover, Charlow (2014) points out that Novel and Romero's remedy still fails for cases where the abstraction operator binds into an indefinite in its scope.

- We will not contribute directly to this debate but rather take a more **conservative approach** and investigate whether it is really necessary to depart from the standard composition rules.

3 Composing alternatives using standard composition rules

- In our view, the fundamental feature of alternative semantics to be retained is the assumption that **sentences denote sets of propositions**.

- Not necessarily the stronger assumption that **every expression denotes a set**.

- This stronger assumption forces us to abandon the standard composition operations.

- We will show that the weaker assumption is perfectly compatible with standard function application and abstraction.

- We will lay out a compositional semantic framework based on the following three assumptions:

  1. the semantic value of a complete sentence is a set of propositions;
  2. the semantic value of an expression of type τ is a single object in D_τ;
  3. the fundamental composition rules are standard FA and AB.

- We will refer to the resulting framework as **possibility semantics**.

- In a diagram:

<div align="center">

Proposition-set semantics
Basic assumption:
sentences denote sets of propositions

Alternative semantics　　　　**Possibility semantics**
Further assumption:　　　　　*Further assumption*:
all expressions denote sets　　composition involves standard FA and AB
$\Downarrow$　　　　　　　　　　$\Downarrow$
Consequence:　　　　　　　*Consequence*:
FA and AB need to be adapted　　typing needs to be adapted

</div>

- Given assumptions 1 and 2 above, the **type of complete sentences** in possibility semantics is $\langle\langle s,t\rangle,t\rangle$, which we will abbreviate as T.

- Given that meanings are composed using standard FA and AB (assumption 3) and assuming that proper names are of type e, the following typing naturally suggests itself:

  - John : e
  - walks : $\langle e,T\rangle$
  - likes : $\langle e,\langle e,T\rangle\rangle$
  - not : $\langle T,T\rangle$
  - or : $\langle T,\langle T,T\rangle\rangle$
  - and : $\langle T,\langle T,T\rangle\rangle$
  - everyone : $\langle\langle e,T\rangle,T\rangle$

- Also, assuming as usual that:

  - $[\![\mathsf{John}]\!] = j$
  - $[\![\mathsf{John\ walks}]\!] = \{\{w \mid j \text{ walks in } w\}\}$

  it follows from the assumption that composition is driven by standard FA that:

  - $[\![\mathsf{walks}]\!] = \lambda x.\{\{w \mid x \text{ walks in } w\}\}$

  The meaning of other subsentential expressions similarly follows from our basic assumptions.

- Since FA is not pointwise, there is **no problem with operations that need access to the whole alternative set** of their argument rather than to each alternative individually.

- For instance, **negation** can be treated categorematically as follows:

  - $[\![\mathsf{not}]\!] = \lambda\mathcal{P}_T.\{\overline{\bigcup\mathcal{P}}\}$

- The same holds for other operations that need access to the whole set of alternatives. For instance, the account of may from Aloni (2007) may be formulated as follows, where σ gives the modal base at each world.

  - $\mathsf{may} : \langle T,T\rangle$
  - $[\![\mathsf{may}]\!] = \lambda\mathcal{P}.\{\ \{\ w \mid \sigma(w) \cap p \neq \emptyset \text{ for all } p \in \mathcal{P}\ \}\ \}$

- Sentential **disjunction** can be given the following, alternative-generating treatment, à la Alonso-Ovalle (2006), Simons (2005), Aloni (2007), among others:

  - $[\![\mathsf{or}]\!] = \lambda\mathcal{P}.\lambda\mathcal{Q}.\mathcal{P} \cup \mathcal{Q}$
  - $[\![\mathsf{John\ sings\ or\ Mary\ dances}]\!] = [\![\mathsf{John\ sings}]\!] \cup [\![\mathsf{Mary\ dances}]\!]$
    $= \{\{w \mid j \text{ sings in } w\}\} \cup \{\{w \mid m \text{ dances in } w\}\}$
    $= \{\{w \mid j \text{ sings in } w\}, \{w \mid m \text{ dances in } w\}\}$

- A treatment of **wh-questions** that delivers the same results as Hamblin's original treatment may be implemented straightforwardly as well.

- Though unlike on Hamblin's account, wh-phrases like who should be assigned the same type as generalized quantifiers like $\mathsf{everyone}$ (as in Karttunen, 1977).

  - who : $\langle\langle e,T\rangle,T\rangle$
  - $[\![\mathsf{who}]\!]_g = \lambda P_{\langle e,T\rangle}.\bigcup_{x\in D_e} P(x)$

- For example:

  - $[\![\mathsf{who\ sings}]\!] = \bigcup_{x\in D_e}\{\{w \mid x \text{ sings in } w\}\}$
    $= \{\{w \mid x \text{ sings in } w\} \mid x \in D_e\}$

- Thus, we have seen that:

> Having alternatives in our semantics does not require departing from the standard type-theoretic composition rules of function application and abstraction.

- In particular, in the framework that we sketched:
  - Operations that need access to the whole set of alternatives generated by their argument can be given a categorematic treatment;
  - Abstraction does not require any special assumptions.

4   Entailment

- Type theory does not only come with the operations of **function application** and **abstraction**, which are used to **compose** meanings.

- It also comes with a notion of **entailment**, which is used to **compare** meanings.

- Entailment standardly amounts to **set-theoretic inclusion**.

- It applies **cross categorically**, to expressions of any conjoinable type (ending in t).

- It also gives rise to a principled cross categorical treatment of **conjunction** and **disjunction**.

- Namely, the **conjunction** of two expressions α and β, of any conjoinable type, can be treated as the **meet** of α and β, i.e., their **greatest lower bound** w.r.t. entailment.

- Similarly, the **disjunction** of two expressions α and β, of any conjoinable type, can be treated as the **join** of α and β, i.e., their **least upper bound** w.r.t. entailment.

- Treated as such, conjunction amounts to **intersection** and disjunction to **union**.

- The cross categorical notion of entailment, as well as the cross categorical treatment of conjunction and disjunction as meet and join are **central features** of the standard Montagovian framework, just like the composition rules of function application and abstraction.

- They should not be lost in the transition to alternative/possibility semantics.

4.1   Standard entailment and conjunction fail

- In alternative/possibility semantics, the standard notion of entailment as set inclusion does not give desirable results.

- This was first observed by Groenendijk and Stokhof (1984), who gave it as an argument against Hamblin's and Karttunen's theories of questions.[3]

- However, this problem concerns alternative semantics more generally.

- For instance, we want John walks to entail John moves, but:
  - $\llbracket \mathsf{John\ walks} \rrbracket = \{\{w \,|\, j \text{ walks in } w\}\} \not\subseteq \{\{w \,|\, j \text{ moves in } w\}\} = \llbracket \mathsf{John\ moves} \rrbracket$

- Treating conjunction as intersection also makes wrong predictions.

[3]As mentioned in footnote 1, the compositional architecture of Karttunen's account of questions is very different from Hamblin's, and therefore it does not run into the compositionality issue discussed in Section 2. However, its notion of question meanings is very similar. As a consequence, at the level of complete questions, it shares the issues with entailment and coordination that we are concerned with in the present section.

- For instance, we would expect:
  - $\llbracket \mathsf{John\ sings\ and\ Mary\ dances} \rrbracket = \{\{w \,|\, j \text{ sings in } w \text{ and } m \text{ dances in } w\}\}$

- But what we get is quite different:
  - $\llbracket \mathsf{John\ sings\ and\ Mary\ dances} \rrbracket = \{\{w \,|\, j \text{ sings in } w\}\} \cap \{\{w \,|\, m \text{ dances in } w\}\} = \emptyset$

- In principle there are two ways to react to this problem:
  - Try to adapt the standard type theoretic notions of entailment and conjunction
  - Reconsider the architecture of alternative/possibility semantics

- We will first consider what seems to be the most natural way to adapt the notions of entailment and conjunction, namely to construe them as **pointwise** notions.

- We will find that this is still problematic, which will ultimately lead us to reconsider the architecture of alternative/possibility semantics.

4.2   Pointwise entailment

- We may devise an alternative-friendly notion of entailment as **pointwise inclusion**:
  - $\alpha \models \beta \overset{\text{def}}{\Longleftrightarrow} \forall p \in \llbracket \alpha \rrbracket \, \exists q \in \llbracket \beta \rrbracket \text{ s.t. } p \subseteq q$

- This notion makes reasonable predictions in the most basic cases.

- For instance, since the only alternative for John walks is a subset of the only alternative for John moves, we get that:
  - John walks $\models$ John moves.

- However, there is a fundamental drawback: namely, this notion of entailment **does not amount to a partial order** on the space of meanings.

- In particular, it is **not anti-symmetric**, i.e., two expressions may entail each other without having the same meaning.

- To see this, consider:
  - $\llbracket \mathsf{John\ moves} \rrbracket = \{\{w \,|\, j \text{ moves in } w\}\}$
  - $\llbracket \mathsf{John\ moves\ or\ walks} \rrbracket = \{\{w \,|\, j \text{ moves in } w\}, \{w \,|\, j \text{ walks in } w\}\}$

- Since the proposition that John walks is contained in the proposition that John moves, every alternative for John moves or walks is contained in an alternative for John moves.

- Vice versa, the unique alternative for John moves is clearly contained in one of the alternatives for John moves or walks.

- Thus, the two sentences entail each other, but they have different meanings.

- If the given pointwise notion of entailment is the correct one, then our meanings are too **fine-grained**, i.e., they allow us to make distinctions that are invisible for entailment.

- One specific consequence of this general problem is that conjunction and disjunction can no longer be treated as **meet** and **join** operators w.r.t. entailment.

- Consider conjunction: we want to define $[\![\alpha \text{ and } \beta]\!]$ as the meet of $[\![\alpha]\!]$ and $[\![\beta]\!]$, i.e., as the weakest meaning entailing both $[\![\alpha]\!]$ and $[\![\beta]\!]$.

- However, since entailment is not anti-symmetric, there is not a **unique** weakest meaning entailing both $[\![\alpha]\!]$ and $[\![\beta]\!]$, but rather a whole **cluster** of such meanings.

- The same problem arises for the treatment of disjunction as a join operator.

- Thus, the pointwise notion of entailment considered here is not satisfactory.
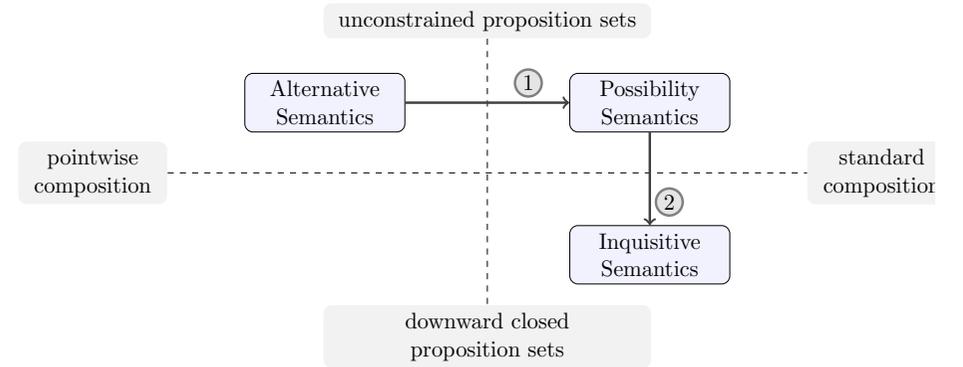
4.3 Pointwise conjunction

- Setting the general problem with entailment aside, we may still try to devise a notion of conjunction that avoids the problematic predictions noted above.

- A natural option would be to construe conjunction as **pointwise intersection**.

  – $[\![\text{and}]\!] = \lambda P.\lambda Q.\{p \cap q \mid p \in P \text{ and } q \in Q\}$

- For the most basic cases, this treatment seems to give the right results:

  – $[\![\text{John sang and Mary danced}]\!]$
    $= \{p \cap q \mid p \in \{\{w \mid j \text{ sand in } w\}\} \text{ and } q \in \{\{w, \mid m \text{ danced in } w\}\}$
    $= \{\{w \mid j \text{ sang in } w\} \cap \{w, \mid m \text{ danced in } w\}\}$
    $= \{\{w \mid j \text{ sang in } w \text{ and } m \text{ danced in } w\}\}$

- However, in some cases, it yields spurious alternatives:

  – $[\![\text{John sang or danced and John sang or danced}]\!]$
    $= \{\{w \mid j \text{ sang in } w\}, \{w \mid j \text{ danced in } w\}, \{w \mid j \text{ sang and danced in } w\}\}$

- In particular, conjunction is **not idempotent**:

  – $[\![\text{John sang or danced and John sang or danced}]\!] \neq [\![\text{John sang or danced}]\!]$

- Thus, even if the general problem concerning entailment and the usual characterization of conjunction as a meet operator is set aside, it is difficult, if not impossible, to devise a satisfactory treatment of conjunction.

5 Recovering standard entailment and conjunction

- We will show that, just like FA and AB, the general type theoretic notions of **entailment and conjunction can be retained** in a framework where sentences denote sets of propositions.

- The key to this result is to construe the meaning of a sentence not as an arbitrary set of propositions, but rather as a set of propositions that is **downward closed**.

- That is, if $p \in [\![\alpha]\!]$ and $q \subseteq p$, $q \in [\![\alpha]\!]$.

- This notion of meaning has been proposed in recent work on **inquisitive semantics** (see, e.g., Ciardelli, Groenendijk, and Roelofsen, 2012, 2013; Roelofsen, 2013a).[4]

- In this framework, the meaning of a question is not taken to lie in its **answerhood conditions**, but rather in its **resolution conditions**.

- The elements of $[\![\alpha]\!]$ are seen as pieces of information that **resolve** the issue that α raises.

- It follows from this that $[\![\alpha]\!]$ should be **downward closed**: if p resolves the issue raised by α, then any stronger proposition $q \subseteq p$ will do so as well.

- Overall, we arrive at the following picture:

- The move from alternative to possibility semantics is driven by the **compositionality** issue.

- The move from possibility to inquisitive semantics is driven by the **entailment** issue.

- The two moves are **independent**, and one could in principle be adopted without the other.

5.1 Inquisitive meanings

- We will refer to downward closed sets of propositions as **inquisitive meanings**.

- We refer to the **maximal elements** of an inquisitive meaning $\mathcal{P}$ as the **alternatives** in $\mathcal{P}$:[5]

  – $\text{ALT}(\mathcal{P}) := \{p \in \mathcal{P} \mid \text{there is no } q \in \mathcal{P} \text{ such that } p \subset q\}$

- The semantic value of a basic sentence will no longer be a singleton set, but rather the downward closure thereof.

---

[4]In initial work on inquisitive semantics, we restricted our attention to a simple, propositional or first-order logical language. One of the contributions of the present paper, as well as Theiler (2014), is to extend the framework to the type-theoretical setting.

[5]Notice that this gives rise to a more constrained notion of alternatives. Namely, since alternatives are defined as maximal elements of a meaning, one alternative can no longer be contained in another. This has consequences, e.g., for the analysis of sentences like Frege lived in Göttingen or in Germany. A comprehensive discussion must be left for another occasion, but Appendix A provides some preliminary remarks, suggesting that the more constrained notion of alternatives is in fact advantageous.

- Example:

  - $\llbracket \text{John walks} \rrbracket = \{\{w \,|\, j \text{ walks in } w\}\}^{\downarrow}$

    where $S^{\downarrow} := \{p \,|\, p \subseteq q \text{ for some } q \in S\}$

- The fragment specified above can easily be adjusted to the present setting.

- For instance, assuming that meanings are composed by means of FA, the entry for walks is:

  - $\llbracket \text{walks} \rrbracket = \lambda x.\{\{w \,|\, x \text{ walks in } w\}\}^{\downarrow}$

- The meaning of other items can be inferred in a similar way.

- A systematic outline of a compositional inquisitive semantics for an elementary fragment of English is provided in Appendix B and in Theiler (2014).

5.2 Entailment, conjunction, and disjunction

- If meanings are downward closed, **entailment** can simply be construed as **inclusion**.

  - $\alpha \models \beta \iff \llbracket \alpha \rrbracket \subseteq \llbracket \beta \rrbracket$

- The problematic predictions that resulted from construing entailment as inclusion in alternative/possibility semantics no longer arise:

  - $\{w \,|\, j \text{ walks in } w\} \subseteq \{w \,|\, j \text{ moves in } w\}$
  - so, every subset of the former proposition is a subset of the latter:
  - $\llbracket \text{John walks} \rrbracket = \{\{w \,|\, j \text{ walks in } w\}\}^{\downarrow} \subseteq \{\{w \,|\, j \text{ moves in } w\}\}^{\downarrow} = \llbracket \text{John moves} \rrbracket$
  - thus, we predict that John walks $\models$ John moves

- Moreover, entailment is now a **partial order** on the space of meanings, as desired:

  - Reflexivity: $\mathcal{P} \subseteq \mathcal{P}$ for every $\mathcal{P}$
  - Transitivity: if $\mathcal{P} \subseteq \mathcal{Q}$ and $\mathcal{Q} \subseteq \mathcal{R}$, then $\mathcal{P} \subseteq \mathcal{R}$
  - Anti-symmetry: if $\mathcal{P} \subseteq \mathcal{Q}$ and $\mathcal{Q} \subseteq \mathcal{P}$, then $\mathcal{P} = \mathcal{Q}$.

- With respect to this ordering, two inquisitive meanings $\mathcal{P}$ and $\mathcal{Q}$ always have:

  - a unique greatest lower bound, or **meet**, given by $\mathcal{P} \cap \mathcal{Q}$
  - a unique least upper bound, or **join**, given by $\mathcal{P} \cup \mathcal{Q}$

- This means that we can recover the standard treatment of **conjunction** and **disjunction**:

  - $\llbracket \text{and} \rrbracket = \lambda \mathcal{P}.\lambda \mathcal{Q}.\mathcal{P} \cap \mathcal{Q}$
  - $\llbracket \text{or} \rrbracket = \lambda \mathcal{P}.\lambda \mathcal{Q}.\mathcal{P} \cup \mathcal{Q}$

- This generalizes to **arbitrary conjoinable types**, yielding a **cross categorical** account of conjunction and disjunction.

- For instance, consider the $\langle e, T \rangle$-type disjunction sing or dance:

  - $\llbracket \text{sing} \rrbracket = \lambda x.\{p \,|\, p \subseteq \{w \,|\, x \text{ sings in } w\}\}$
    $= \{\langle x, p \rangle \,|\, p \subseteq \{w \,|\, x \text{ sings in } w\}\}$
  - $\llbracket \text{dance} \rrbracket = \lambda x.\{p \,|\, p \subseteq \{w \,|\, x \text{ dances in } w\}\}$
    $= \{\langle x, p \rangle \,|\, p \subseteq \{w \,|\, x \text{ dances in } w\}\}$
  - $\llbracket \text{sing or dance} \rrbracket = \llbracket \text{sing} \rrbracket \cup \llbracket \text{dances} \rrbracket =$
    $= \{\langle x, p \rangle \,|\, p \subseteq \{w \,|\, x \text{ sings in } w\} \text{ or } p \subseteq \{w \,|\, x \text{ dances in } w\}\}$
    $= \lambda x.\{p \,|\, p \subseteq \{w \,|\, x \text{ sings in } w\} \text{ or } p \subseteq \{w \,|\, x \text{ dances in } w\}\}$

- Let's take a closer look at the behavior of these operations.

- As in alternative semantics, **disjunction can generate alternatives**:

  - $\llbracket \text{John sings or Mary dances} \rrbracket = \llbracket \text{John sings} \rrbracket \cup \llbracket \text{Mary dances} \rrbracket$
    $= \{\{w \,|\, j \text{ sings in } w\}\}^{\downarrow} \cup \{\{w \,|\, m \text{ dances in } w\}\}^{\downarrow}$
    $= \{\{w \,|\, j \text{ sings in } w\}, \{w \,|\, m \text{ dances in } w\}\}^{\downarrow}$

- This meaning contains **two alternatives**: the proposition that John sings and the proposition that Mary dances.

- However, this behavior is **not stipulated** now; instead, it follows from interpreting disjunction as a join operation in the given semantic framework (cf., Roelofsen, 2013b).

- We could also treat **indefinites** and **wh-phrases** as join operators, which would give them the potential to generate alternatives as well.

  - $\text{someone}, \text{who} : \langle \langle e, T \rangle, T \rangle$
  - $\llbracket \text{someone} \rrbracket = \llbracket \text{who} \rrbracket = \lambda P. \bigcup_{x \in D_e} P(x)$
  - $\llbracket \text{someone walks} \rrbracket = \llbracket \text{who walks} \rrbracket = \{\{w \,|\, x \text{ walks in } w\} \,|\, x \in D_e\}^{\downarrow}$

- Notice that someone and who are taken here to generate exactly the same set of alternatives, as in, e.g., Kratzer and Shimoyama (2002).

- The difference between the two could be captured by means of contraints on what happens with these alternatives in the further derivation. For instance, we could require that:

  - Alternatives generated by who must be evaluated by a question operator
  - Alternatives generated by someone must be evaluated by an existential closure operator

  (See Kratzer and Shimoyama, 2002, and Appendix B for further discussion)

- Finally, consider **conjunction**.

- We restored the standard treatment of conjunction as a meet operator.

- This does not only re-establish the link between entailment and conjunction, but also resolves the **empirical** problems pointed out above.

- Performing intersection now yields the right results:

  - $\llbracket \text{John sings and Mary dances} \rrbracket =$
    $\{\{w \,|\, j \text{ sings in } w\}\}^{\downarrow} \cap \{\{w \,|\, m \text{ dances in } w\}\}^{\downarrow} =$
    $\{p \,|\, p \subseteq \{w \,|\, j \text{ sings in } w\} \text{ and } p \subseteq \{w \,|\, m \text{ dances in } w\}\} =$
    $\{\{w \,|\, j \text{ sings in } w \text{ and } m \text{ dances in } w\}\}^{\downarrow}$

- Moreover, unlike pointwise intersection, plain intersection is **idempotent**. (more generally, it retains fully standard logical features.)

  – $[\![\alpha \text{ and } \alpha]\!] = [\![\alpha]\!] \cap [\![\alpha]\!] = [\![\alpha]\!]$

- This means that we **no** longer get **spurious alternatives**:

  – ALT$[\![$John sang or danced and John sang or danced$]\!] = \{\{w \mid j \text{ sang in } w\}, \{w \mid j \text{ danced in }$

- Thus, we have seen that:

> Having alternatives in our semantics does not require departing from the standard type-theoretic notions of entailment, conjunction and disjunction.

6 Conclusion

- The specific setup of alternative semantics forces us to give up:

  – The standard type theoretic operations to compose meanings, FA and AB.
  – The standard cross categorical notions of entailment, conjunction, and disjunction.

- This leads to a range of problems, both empirical and theoretical.

- We have tried to identify precisely which features of alternative semantics are responsible for these two issues, and what can be done to overcome them.

- The compositionality issue can be resolved by **dropping the assumption that all expressions denote sets**.

- Dropping this assumption does not seem to undermine the general spirit of the framework, nor the empirical coverage of the theories that are formulated within it.

- This adjustment led us to the framework of **possibility semantics**.

- However, this framework still faces the entailment issue.

- This issue can be resolved by reconsidering our basic notion of meaning.

- Namely, rather than assuming that sentences denote arbitrary sets of propositions, we assume that they denote sets of propositions that are **downward closed**, as in inquisitive semantics.

- If sentential meanings are assumed to be downward closed, the entailment issue evaporates:

  – the standard type-theoretic notion of entailment as inclusion can be retained;
  – conjunction and disjunction can be treated as meet and join operations;
  – these operations amount to the general type-theoretic ones.

- These adjustments of alternative semantics lead to a semantic framework that fully preserves the solid type-theoretic foundations of Montague Grammar.

- At the same time, the essential features of alternative semantics are retained as well.

- As a result, linguistic analyses that have been formulated in alternative semantics can generally be imported quite straightforwardly into the adapted framework.

- Thus, the empirical coverage of these analyses is preserved while at the same time the problematic predictions related to the observed framework issues are avoided.

A Nested alternatives and Hurford disjunctions

- In inquisitive semantics, sentential meanings are downward closed sets of propositions.

- The **alternatives** in an inquisitive meaning $\mathcal{P}$ are the **maximal elements** of $\mathcal{P}$.

- This means that one alternative can never be contained in another.

- Thus, the notion of alternatives is more constrained in inquisitive semantics than in alternative semantics.

- Is this a vice or a virtue?

- A comprehensive discussion of this issue is beyond the scope of this paper, but we will offer some preliminary observations, which suggest that the more restricted notion of alternatives in fact has advantages.

A.1 Hurford's constraint

- One empirical domain that is likely to shed light on the issue at hand is that of **disjunctions where one disjunct entails another**.

- Hurford (1974) pointed out that such disjunctions are generally infelicitous, based on examples like the following:

  (1)   a. #John's wife is Asian or Chinese.
         b. #Mary has a dog or a pet.
         c. #Mary was wearing a scarf or a red scarf.
         d. #The director is a bachelor or a man.

- This phenomenon has been discussed quite extensively in subsequent work under the heading of **Hurford's constraint** (e.g. Gazdar, 1979; Simons, 2001; Singh, 2008; Chierchia *et al.*, 2009; Katzir and Singh, 2013).

- Hurford's constraint can be explained in terms of **redundancy** (Simons, 2001; Katzir and Singh, 2013)

- Under the **classical treatment of disjunction**, this explanation goes as follows:

  – if α is entailed by β, then α or β is **equivalent** to just α;
  – hence, if a speaker utters α or β, she could have expressed the same meaning in a simpler way — the second disjunct is redundant;
  – if there is no good reason for this redundancy, the utterance is infelicitous.

- In **inquisitive semantics**, this explanation can still be given, because if α is entailed by β, then α or β is **still equivalent** to just α.

- However, in **alternative semantics** the explanation is no longer available, because if α is properly entailed by β, then α or β is **not equivalent** to just α.

- Thus the more constrained notion of alternatives seems advantageous here.

A.2 Exhaustive strengthening to avoid redundancy

- There are cases where Hurford's constraint seems to be **violated** (see, e.g. Gazdar, 1979; Singh, 2008; Chierchia *et al.*, 2009).

- For instance, the following disjunctive sentences are felicitous, even though in each case one disjunct entails the other:

  (2) a. Mary is having dinner with John or with John and Bill.
       b. Mary read most or all of the books on this shelf.
       c. John and Mary have three or four kids.

- Chierchia *et al.* (2009) argue that the crucial difference between these cases and the previous ones is that the weak disjuncts in (2) can receive an exhaustive interpretation.

  (3) a. Mary is having dinner with John. ⤳ not with Bill, Sue, . . .
       b. Mary read most of the books. ⤳ not all
       c. John and Mary have three kids. ⤳ not four

- Under this exhaustive interpretation, they are no longer entailed by the other disjunct.

- So, after all, Hurford's constraint is satisfied in these cases as well.

- The weak disjuncts in (1) cannot be given an exhaustive interpretation that breaks the entailment between the two disjuncts:

  (4) a. John's wife is Asian. ⤳̸ not Chinese
       b. Mary has an pet. ⤳̸ not a dog
       c. Mary was wearing a scarf. ⤳̸ not a red scarf
       d. The director is a man. ⤳̸ not a bachelor

- Under this explanation of the contrast between (1) and (2), the weak disjuncts in (2) are strengthened so that they are no longer entailed by the other disjunct, and Hurford's constraint is satisfied.

- Recall, however, that Hurford's constraint is just a generalization.

- As shown above, the generalization can be explained in terms of **redundancy**.

- Now, for basic cases like (1) this explanation can assume either a classical or an inquisitive treatment of disjunction.

- However, for some of the cases involving exhaustive strengthening, the explanation crucially requires an **inquisitive** treatment of disjunction.

- Under a classical treatment of disjunction it would no longer go through.

- To see this, consider the following case:

  (5) Mary read most or all of the books on this shelf.

- Prima facie, the disjunction most or all violates Hurford's constraint, because all entails most.

- The reason why this would be bad is that if α is entailed by β, then α or β is **equivalent** to just α, which means that the second disjunct β is redundant.

- It does not matter at this point whether we assume a classical or an inquisitive treatment of disjunction.

- In both cases, most or all is equivalent to most.

- But now exhaustive strengthening of the first disjunct enters the stage.

- The exhaustive interpretation of most is 'most but not all'.

- This exhaustive interpretation **breaks the entailment** between the two disjuncts.

- However, under the classical treatment of disjunction, it **does not break the equivalence** between the disjunction as a whole and the plain non-exhaustive interpretation of the first disjunct.

- That is, classically we have that:

  ((most but not all) or all) $\equiv$ most

- Thus, given the classical explanation of Hurford's constraint, exhaustive strengthening of the first disjunct is not really expected to make the disjunction felicitous.

- On the other hand, under the inquisitive explanation of Hurford's constraint, this *is* expected.

- This is because inquisitively we have that:

  ((most but not all) or all) $\not\equiv$ most

- The expression on the left generates two alternatives, while the one on the right generates just one alternative.

- So, the explanation of Hurford's constraint in terms of redundancy only extends from the basic cases to the full range of cases involving exhaustive strengthening if we assume inquisitive meanings, and in particular an inquisitive treatment of disjunction.

B Sketch of a basic fragment

The purpose of the present appendix is to illustrate the proposal made in the talk by specifying a small compositional fragment of English which implements both the notion of sentence meaning and the compositional architecture that we argued for. We have no pretense to provide here a fully satisfactory analysis of the relevant items and constructions.

B.1 The fragment

- Individual-denoting expressions:

  - $[\![\mathsf{John}]\!]_g = j$
  - $[\![t_i]\!]_g = g(i)$

- Verbs:

  - $[\![\mathsf{sing}]\!] = \lambda x_e.\{\{w \mid x \text{ sings in } w\}\}^{\downarrow}$
  - $[\![\mathsf{dance}]\!] = \lambda x_e.\{\{w \mid x \text{ dances in } w\}\}^{\downarrow}$
  - $[\![\mathsf{like}]\!] = \lambda x_e \lambda y_e.\{\{w \mid x \text{ likes } y \text{ in } w\}\}^{\downarrow}$

- Connectives:

  - $[\![\mathsf{or}_\tau]\!] = \lambda X_\tau \lambda Y_\tau.X \cup Y$
  - $[\![\mathsf{and}_\tau]\!] = \lambda X_\tau \lambda Y_\tau.X \cap Y$
  - $[\![\mathsf{not}_T]\!] = \lambda \mathcal{P}_T.\{\overline{\bigcup \mathcal{P}}\}^{\downarrow}$
  - $[\![\mathsf{not}_{\langle e,T \rangle}]\!] = \lambda P_{\langle e,T \rangle}.\lambda x_e.[\![\mathsf{not}_T]\!](P(x))$

- Quantifiers and wh-phrases:

  - $[\![\mathsf{everyone}]\!] = \lambda P_{\langle e,T \rangle}.\bigcap_{x \in D_e} P(x)$
  - $[\![\mathsf{someone}]\!] = \lambda P_{\langle e,T \rangle}.\bigcup_{x \in D_e} P(x)$
  - $[\![\mathsf{who}]\!] = \lambda P_{\langle e,T \rangle}.\bigcup_{x \in D_e} P(x)$

- Aloni's may:

  - $[\![\mathsf{may}]\!] = \lambda \mathcal{P}_T.\{\{w \mid \sigma(w) \cap p \neq \emptyset \text{ for all } p \in \text{ALT}(\mathcal{P})\}\}^{\downarrow}$

- Structural items:

  - $[\![!]\!] = \lambda \mathcal{P}.\{\bigcup \mathcal{P}\}^{\downarrow}$
  - $[\![?]\!] = \lambda \mathcal{P}_T.\mathcal{P} \cup \{\overline{\bigcup P}\}^{\downarrow}$
  - $[\![\mathsf{exh}]\!] = \lambda \mathcal{P}_T.\{p \mid \forall q \in \text{ALT}(\mathcal{P}), \text{ either } p \subseteq q \text{ or } p \subseteq \overline{q}\}$

Definitions

- **Alt-generating operators:** or, someone, who.

- **Alt-evaluating operators:** !, ?, may.

Rules

- The default mode of composition is **function application**.

- Items of type $\langle e,T \rangle$ may be subjected to standard **quantifier raising**.

- ? is the **interrogative complementizer**.
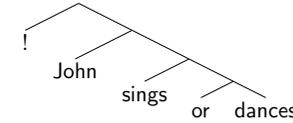
- ! is the **declarative complementizer**.

- In addition, ! can apply freely at type T nodes.

- exh can be optionally inserted after ?.

- Every alt-generating operator comes with specific **constraints** on the alt-evaluating operators in the immediate scope of which it can occur.

- These constraints are spelled out in the following table:

Alt-generating operator	associates with
or	!, ?, may
someone	!
who	?

B.2 Basic declaratives

Basic declaratives receive a meaning which is isomorphic to their standard one: they have one alternative, namely, the set of all worlds where they are true.

(6)   John sings or dances.

- $[\![\mathsf{sings\ or\ dances}]\!] = \lambda x.\{\{w \mid x \text{ sings in } w\}, \{w \mid x \text{ dances in } w\}\}^{\downarrow}$

- $[\![\mathsf{John\ sings\ or\ dances}]\!] = \{\{w \mid j \text{ sings in } w\}, \{w \mid j \text{ dances in } w\}\}^{\downarrow}$

- $[\![!(\mathsf{John\ sings\ or\ dances})]\!] = \{\{w \mid j \text{ sings or dances in } w\}\}^{\downarrow}$

(7)   Someone sings.

- $[\![\mathsf{someone\ sings}]\!] = \bigcup_{x \in D_e}.[\![\mathsf{sing}]\!](x)$
  $= \{\{w \mid x \text{ sings in } w\} \mid x \in D_e\}^{\downarrow}$

- $[\![!(\mathsf{someone\ sings})]\!] = \{\bigcup [\![\mathsf{someone\ sings}]\!]\}^{\downarrow}$
  $= \{\{w \mid \text{someone sings in } w\}\}^{\downarrow}$
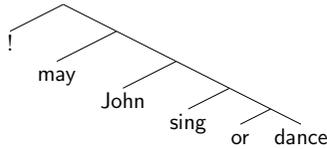
B.3 Exploiting alternatives: may

Operators like may (conditionals, imperatives, question-embedding verbs, ...) can **make use of the alternatives** generated by their argument in a non-trivial way.

(8)   John may sing

- $\llbracket$John sings$\rrbracket = \{\{w \mid j \text{ sings in } w\}\}^{\downarrow}$

- $\text{ALT}\llbracket$John sings$\rrbracket = \{\{w \mid j \text{ sings in } w\}\}$

- $\llbracket$may(John sing)$\rrbracket = \{\{w \mid \sigma(w) \cap p \neq \emptyset \text{ for all } p \in \text{ALT}\llbracket\text{John sings}\rrbracket\}\}^{\downarrow}$
  $= \{\{w \mid \sigma(w) \cap \{w \mid j \text{ sings in } w\} \neq \emptyset\}\}^{\downarrow}$

- $\llbracket!(\text{may(John sing)})\rrbracket = \llbracket\text{may(John sing)}\rrbracket$

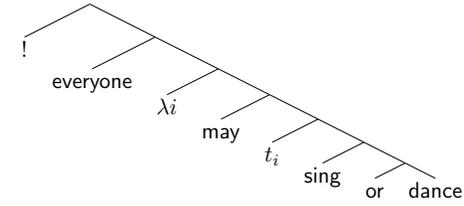(9)   John may sing or dance

- $\llbracket$John sings or dances$\rrbracket = \{\{w \mid j \text{ sings in } w\}, \{w \mid j \text{ dances in } w\}\}^{\downarrow}$

- $\text{ALT}\llbracket$John sings or dances$\rrbracket = \{\{w \mid j \text{ sings in } w\}, \{w \mid j \text{ dances in } w\}\}$

- $\llbracket$may(John sing or dance)$\rrbracket =$
  $\{\{w \mid \sigma(w) \cap p \neq \emptyset \text{ for all } p \in \text{ALT}\llbracket\text{John sings or dances}\rrbracket\}\}^{\downarrow} =$
  $\{\{w \mid \sigma(w) \cap \{w \mid j \text{ sings in } w\} \neq \emptyset \text{ and } \sigma(w) \cap \{w \mid j \text{ dances in } w\} \neq \emptyset\}\}^{\downarrow}$

- $\llbracket!(\text{may(John sing or dance)})\rrbracket = \llbracket\text{may(John sing or dance)}\rrbracket$
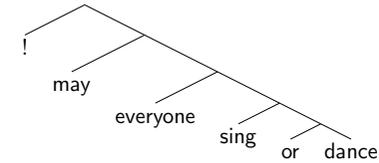
(10)   Everyone may sing or dance

Reading 1.

- $\llbracket(5)\rrbracket = \{w \mid \text{for every } x \in D_e,\ \sigma(w) \cap \{w \mid x \text{ sings in } w\} \neq \emptyset$
  $\text{and } \sigma(w) \cap \{w \mid x \text{ dances in } w\} \neq \emptyset\}$

- If John and Mary are the relevant individuals, this reading requires the modal base to be consistent with the propositions corresponding to:

  - John sings
  - John dances
  - Mary sings
  - Mary dances

- Notice that this reading would be true in a situation in which the only options are that both sing, or that both dance.
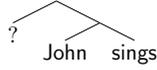
Reading 2.

- $\llbracket(5)\rrbracket = \{w \mid \sigma(w) \cap (\bigcap_{x \in D_e} p_x) \neq \emptyset$
  $\text{where } p_x \in \{\{w \mid x \text{ sings in } w\}, \{w \mid x \text{ dances in } w\}\}$

- Under this, more demanding reading, (10) is only true if every particular combination of people singing and dancing is allowed.

- If John and Mary are the relevant individuals, this reading requires the modal base to be consistent with the propositions corresponding to:

  - Both John and Mary sing
  - John sings and Mary dances
  - John dances and Mary sings
  - Both John and Mary dance

- In particular, this reading would be false in a scenario where the only two options are for both to sing or for both to dance.
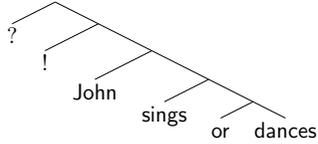
B.4 Interrogatives

(11)   Does John sing?

- $[\![\text{John sings}]\!] = \{\{w \mid j \text{ sings in } w\}\}^{\downarrow}$

- $[\![?(\text{John sings})]\!] = \{\{w \mid j \text{ sings in } w\}\}^{\downarrow} \cup \{\overline{\{w \mid j \text{ sings in } w\}}\}^{\downarrow}$
  $= \{\{w \mid j \text{ sings in } w\}\}^{\downarrow} \cup \{\{w \mid j \text{ does not sing in } w\}\}^{\downarrow}$
  $= \{\{w \mid j \text{ sings in } w\}, \{w \mid j \text{ does not sing in } w\}\}^{\downarrow}$

- This gives us the **standard account of polar questions**.

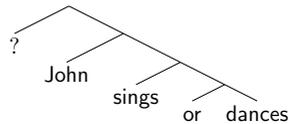- Adding the optional exh would not make any difference in this case.

(12)   Does John sing or dance?

Reading 1.

- $[\![!(\text{John sings or dances})]\!] = \{\{w \mid j \text{ sings or dances in } w\}\}^{\downarrow}$

- $[\![?(!(\text{John sings or dances}))]\!] = \{\{w \mid j \text{ sings or dances in } w\},$
  $\{w \mid j \text{ does not sing or dance in } w\}\}^{\downarrow}$
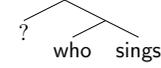
- This gives us the **polar reading** of (12).

Reading 2.

- $[\![\text{John sings or dances}]\!] = \{\{w \mid j \text{ sings in } w\}, \{w \mid j \text{ dances in } w\}\}^{\downarrow}$

- $[\![?(\text{John sings or dances})]\!] = \{\{w \mid j \text{ sings in } w\}, \{w \mid j \text{ dances in } w\},$
  $\{w \mid j \text{ doesn't sing or dance in } w\}\}^{\downarrow}$

- This corresponds to the reading that (12) has when pronounced with **rising intonation** on **both disjuncts** (see e.g., Roelofsen and van Gool, 2010; Pruitt and Roelofsen, 2011).

- To capture the **alternative question reading** of (12), presuppositions need to be added to the framework (see, e.g., Biezma and Rawlins, 2012; Ciardelli *et al.*, 2012; Roelofsen, 2013b).
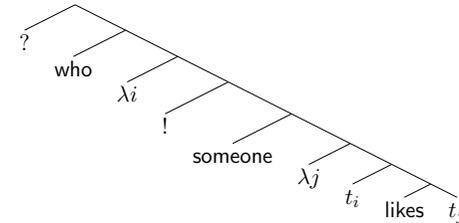
(13)   Who sings?

Reading 1.

- $[\![\text{who sings}]\!] = \bigcup_{x \in D_e} \{\{w \mid x \text{ sings in } w\}\}^{\downarrow}$
  $= \{\{w \mid x \text{ sings in } w\} \mid x \in D_e\}^{\downarrow}$

- $[\![?(\text{who sings})]\!] = \{\{w \mid x \text{ sings in } w\} \mid x \in D_e\}^{\downarrow} \cup \{\{w \mid \text{no one sings in } w\}\}^{\downarrow}$

- The alternatives we get for this meaning are the **Hamblin/Karttunen alternatives**.
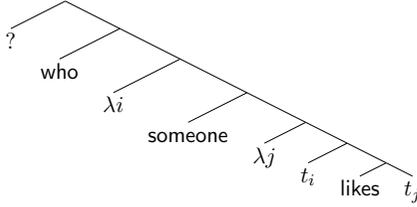
Reading 2.

- $[\![\text{exh}(?(\text{who sings}))]\!] = \{p \mid \forall x \in D_e, \ p \subseteq \{w \mid x \text{ sings in } w\} \text{ or }$
  $p \subseteq \{w \mid x \text{ doesn't sing in } w\}\}$

- The alternatives here are the **Groenendijk & Stokhof alternatives**.

(14)   Who likes someone?

- $[\![t_i \text{ likes } t_j]\!] = \{\{w \mid g(i) \text{ likes } g(j) \text{ in } w\}\}^{\downarrow}$

- $[\![\text{someone}(\lambda j(t_i \text{ likes } t_j))]\!] = \{\{w \mid g(i) \text{ likes } y \text{ in } w\} \mid y \in D_e\}^{\downarrow}$

- $[\![!(\text{someone}(\lambda j(t_i \text{ likes } t_j)))]\!] = \{\{w \mid g(i) \text{ likes someone in } w\}\}^{\downarrow}$

- $\llbracket$who$(\lambda i(!(\text{someone}(\lambda j(t_i \text{ likes } t_j)))))\rrbracket = \{\{w \mid x \text{ likes someone in } w\} \mid x \in D_e\}^{\downarrow}$

- $\llbracket ?(\text{who}(\lambda i(!(\text{someone}(\lambda j(t_i \text{ likes } t_j))))))\rrbracket =$
  $\{\{w \mid x \text{ likes someone in } w\} \mid x \in D_e\}^{\downarrow} \cup \{\{w \mid \text{no one likes anyone in } w\}\}^{\downarrow}$

- Again, this corresponds to the standard Karttunen meaning.

- By adding exh we would obtain the corresponding G&S meaning.

- Notice that our constraints **rule out the following analysis**, preventing (14) from having a reading in common with the question who likes whom?

C Some remarks on generalized quantifiers

Generalized quantifiers in the standard extensional setting.

- In an extensional setup, generalized quantifiers are sets of properties, where properties are sets of individuals.

- So generalized quantifiers are of type $\langle\langle e, t\rangle, t\rangle$.

- A GQ like every man combines with a verb phrase like sings; the latter is of type $\langle e, t\rangle$ and the end result is of type t, reflecting the fact that the extension of a sentence in a world is a truth value.

- One attractive feature of this setup is that GQs can be compounded by means of Boolean operations. For instance, the GQ expressed by every man and some children can be obtained by taking the intersection (meet) of the GQ expressed by every man and that expressed by some children.

Generalized quantifiers in inquisitive semantics.

- In the current framework, GQs are of type $\langle\langle e, T\rangle, T\rangle$.

- A GQ like every man still combines with a verb phrase like sings; the latter, however, is now of type $\langle e, T\rangle$ and the end result is of type T; this reflects the fact that the meaning of the sentence is a set of propositions.

- Notice that in this setting the type of a sentence does not indicate what kind of semantic object it *denotes* in a world, i.e., what its extension is, but rather what kind of semantic object is assigned to it as its *meaning*, which is world-independent.

- Given the meaning of a sentence in this framework, i.e., a set of propositions, we could think of the extension of the sentence in a particular world as the set of those propositions in its meaning that are true in the given world. This notion, however, does not seem to play much of a role. What really matters are meanings. Various kinds of world-dependent semantic objects could be derived from them if needed.

- Some examples:
  - $\llbracket$every man$\rrbracket = \lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\forall x.\forall q \subseteq p(\llbracket$man$\rrbracket(x)(q) \rightarrow P(x)(q))$
  - $\llbracket$some child$\rrbracket = \lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists x.(\llbracket$child$\rrbracket(x)(p) \wedge P(x)(p))$

- If we let $\mathbb{W}$, $\exists\!\!\!\exists$, $\twoheadrightarrow$, $\mathbb{A}$, $\mathbb{W}$, and $\neg\!\!\!\neg$ denote the operations that are assigned to the logical constants in InqB, rather than the ones that are assigned to them in type theory, then we can in fact simply write:
  - $\llbracket$every man$\rrbracket = \lambda P_{\langle e,T\rangle}.\mathbb{W}x.(\llbracket$man$\rrbracket(x) \twoheadrightarrow P(x))$
  - $\llbracket$some child$\rrbracket = \lambda P_{\langle e,T\rangle}.\exists\!\!\!\exists x.(\llbracket$child$\rrbracket(x) \mathbb{A} P(x))$

- How to think about these intuitively?
  - every man takes a property P and delivers a set of propositions/information states p which contain enough information to establish that every man has the property P. That is, for every individual x it should be the case that every substate of p which establishes that x is a man should also establish that x has the property P. Intuitively speaking, the information state p should be such that, whenever we learn of some individual that it is a man, we also know that this individual has the property P.
  - Similarly, some child takes a property P and delivers a set of propositions/information states p which contain enough information to establish of some individaul x that it is a man and has the property P.

Algebraic operations on generalized quantifiers.

- Even though the semantic objects assigned to GQs, which we will also just call GQs, are now more complex, they still have many of the attractive properties of classical GQs.

- For instance, like before, GQs can be compounded by means of Boolean operations.

- Example: the GQ expressed by every man and some children can be obtained by taking the intersection (meet) of the GQ expressed by every man and that expressed by some child.

  The lambda terms given above for these GQs denote functions that map property-proposition pairs to truth values. Such functions can be respresented as sets of property-proposition pairs:
  - $\llbracket$every man$\rrbracket = \{\langle P_{\langle e,T\rangle}, p_{\langle s,t\rangle}\rangle \mid \forall x.\forall q \subseteq p(\llbracket$man$\rrbracket(x)(q) \rightarrow P(x)(q))\}$
  - $\llbracket$some child$\rrbracket = \{\langle P_{\langle e,T\rangle}, p_{\langle s,t\rangle}\rangle \mid \exists x.(\llbracket$child$\rrbracket(x)(p) \wedge P(x)(p))\}$

  Now that we have represented the quantifiers as sets, it is clear that we can take, just like in the classical setting, their intersection and union. For instance:
  - $\llbracket$every man and some child$\rrbracket$
    $= \{\langle P_{\langle e,T\rangle}, p_{\langle s,t\rangle}\rangle \mid \forall x.\forall q \subseteq p(\llbracket$man$\rrbracket(x)(q) \rightarrow P(x)(q)) \wedge \exists x.(\llbracket$child$\rrbracket(x)(p) \wedge P(x)(p))\}$

- If we want we could turn this back into a function:

  - $\llbracket\text{every man and some child}\rrbracket =$

    $\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.(\forall x.\forall q \subseteq p(\llbracket\text{man}\rrbracket(x)(q) \rightarrow P(x)(q)) \wedge \exists x.(\llbracket\text{child}\rrbracket(x)(p) \wedge P(x)(p)))$

- Using the inquisitive logical constants, we can simply write:

  - $\llbracket\text{every man and some child}\rrbracket = \lambda P_{\langle e,T\rangle}.(\forall\!\!\!\forall x.(\llbracket\text{man}\rrbracket(x) \twoheadrightarrow P(x)) \wedge\!\!\!\wedge \exists\!\!\!\exists x.(\llbracket\text{child}\rrbracket(x) \wedge\!\!\!\wedge P(x)))$

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