Implicatures of modified numerals: quality or quantity?

Liz Coppock Ivano Ciardelli Floris Roelofsen

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1 Introduction

1.1 Implicatures of modified numerals: the basic empirical picture

- We will be concerned with three types of modified numerals:
  - **at least** n
  - **more than** n
  - n **or more**
- Many authors have observed that these contrast with each other, as well as with bare numerals, both in the **quantity implicatures** and the **ignorance implicatures** that they give rise to:

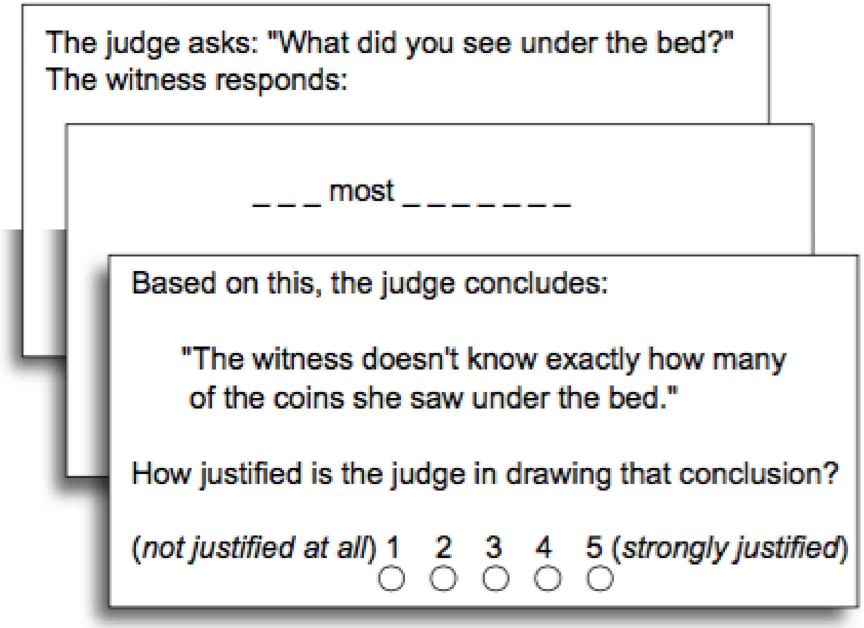
	quantity implicatures	ignorance implicatures
n	yes	no
more than n	no	no
at least n	no	yes
n or more	no	yes

- For instance, using an example from Nouwen (2010):

(1) a. A hexagon has six sides. $\rightsquigarrow$ exactly six $\not\rightsquigarrow$ ignorance
  b. A hexagon has more than five sides. $\not\rightsquigarrow$ exactly six $\not\rightsquigarrow$ ignorance
  c. A hexagon has at least six sides. $\not\rightsquigarrow$ exactly six $\rightsquigarrow$ ignorance
  d. A hexagon has six or more sides. $\not\rightsquigarrow$ exactly six $\rightsquigarrow$ ignorance

- Note that the ignorance implicature of *at least six* and *six or more* is not just that the speaker does not know exactly how many sides a hexagon has, but also that she considers it **possible** that it has **precisely six** sides.
- Westera & Brasoveanu (2014) argue based on experimental data that this basic empirical picture, which is assumed in most work on the topic, is actually a bit **too simplistic**. See Figure (1).
- Their data shows that some explicit QUDs, namely yes/no questions, can eliminate the ignorance implicature for superlative modifiers.
- However, the difference remains in place in some contexts, including 'how many' questions, although comparative modifiers also signal ignorance implicatures in such contexts to a lesser degree.
- It seems that the ignorance inferences triggered by *more than* are of a somewhat different, less obligatory nature, given this and in view of examples like:

(2) a. I grew up with more than two parents.
  b. ??I grew up with at least two parents.



POLAR Did you find {at most / less than} ten of the diamonds under the bed?
WHAT What did you find under the bed?
HOWMANY How many of the diamonds did you find under the bed?

APPROX Approximately how many of the diamonds did you find under the bed?
EXACT Exactly how many of the diamonds did you find under the bed?
DISJUNCT Did you find eight, nine, ten, or eleven of the diamonds under the bed?

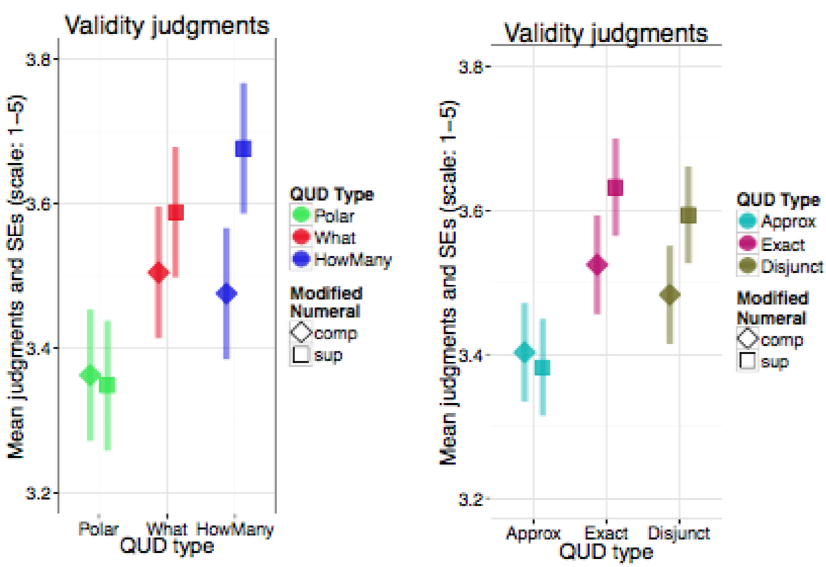


Figure 1: Westera & Brasoveanu's (2014) design and results

1.2 Quality or quantity?

- Two approaches have been explored in the literature to explain the observed empirical contrasts.
  - One approach (e.g., Mayr, 2013b; Schwarz, to appeara) tries to derive all the data from a particular way of computing **quantity** implicatures. Differences between the various kinds of bare/modified numerals are accounted for on this approach by assuming that they activate different pragmatic alternatives.
  - Another approach (Coppock & Brochhagen, 2013) is to derive the ignorance implicatures of *at least n* and *n or more* as **quality** implicatures. The standard Gricean quality maxim, however, does not suffice for this purpose. Rather, Coppock & Brochhagen (2013) invoke a quality maxim that is not only concerned with the *informative* content of the uttered sentence, but also with its *inquisitive* content, i.e., the semantic alternatives that it introduces. Differences between the various kinds of bare/modified numerals are accounted for on this approach by assuming that they introduce different semantic alternatives.
- Note that in other empirical domains (e.g., free choice effects of disjunction under modals or in the antecedent of a conditional), these two approaches have also both been pursued.
- We will suggest that, in the domain of modified numerals, a **combination** of the two approaches is in fact needed.
- We will develop such a combined account, and show that it improves on earlier proposals which placed the entire explanatory burden either on quantity or on quality.

1.3 Structure of the paper

- Previous approaches
  - Quantity-based (Schwarz, including challenges)
  - Quality-based (C&B, including challenges)
- Proposal: a combined approach
- Conclusion

2 Previous approaches

2.1 Quantity-based

- We focus on the proposal of Schwarz (to appeara), but see Mayr (2013a) and Kennedy (2015) for closely related proposals.
- Summary:
  - Horn scale: $\langle$ *at least*, *only* $\rangle$
  - Horn scale: $\langle$ 1, 2, 3, ... $\rangle$
  - Alternatives for *Al hired at least two cooks*:

$$\begin{array}{cccccc}
 & & & \ldots & \ldots & \\
 & & [3] & [4 & \ldots &) \\
 & [2] & [3 & 4 & \ldots &) \\
[1] & [2 & 3 & 4 & \ldots &) \\
[1 & 2 & 3 & 4 & \ldots &)
\end{array}$$

  - Innocent Exclusion Based Recipe for deriving scalar/ignorance implicatures:

* Start with the assumption that the speaker believes p:

$$0_p = \{\Box p\}$$

* Now derive primary quantity implicatures: The speaker does not have sufficent evidence for any stronger alternative in A:

$$1_{p,A} = 0_p \cup \{\neg\Box q : q \in A \,\&\, q \subset p\}$$

* Secondary implicatures are then computed for all alternatives that are innocently excludable.

$$2_{p,A} = 1_{p,A} \cup \{\Box\neg q : \neg\Box q \in 1_{p,A} \,\&\, q \text{ is innocently excludable relative to } 1_{p,A}\}$$

  where p is *innocently excludable* relative to S iff $\Box\neg p$ is an element of every maximal subset of $\{\Box\neg q : \neg\Box q \in S\}$ consistent with S.
* Idea: Look at an element, look at stronger things, try to negate as many as possible, remaining consistent with original elements. There may be various maximal sets that you can get while remaining consistent. Look at their common core. All the things that they have in common are innocently excludable.
* With symmetric alternatives, no alternative is innocently excludable. Hence ignorance.

Challenges

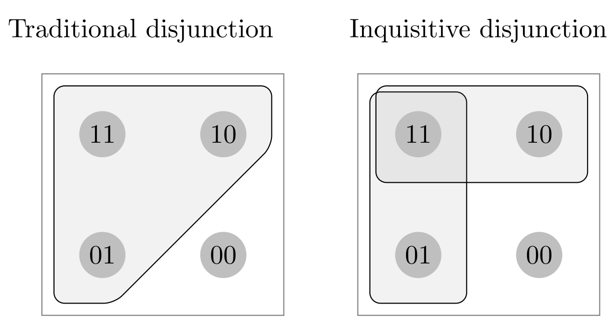
- Unclear how to distinguish *more than* from *at least*. If numerals form a Horn scale, then something akin to what is done for *at least* needs to be done to block scalar implicatures here as well.
- In certain configurations, *at least* cannot be replaced by its presumed pragmatic alternative *only*.

  (3) He gave three people a raise, {at least/*only}.

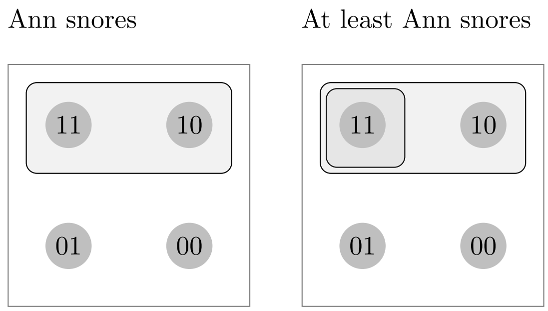
  The ignorance implicature still arises in these configurations.
- *Only* presupposes *at least*, but doesn't entail it. So the ordinary meaning of *only* (*not more than*) is not actually stronger than the ordinary meaning of *at least*. How do we compute strength of alternatives?

2.2 Quality-based

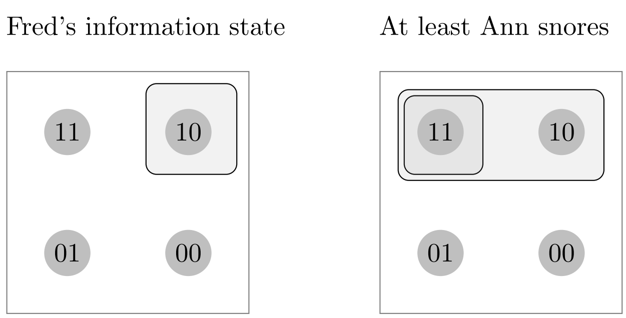
- Traditional vs. inquisitive disjunction



- C&B's analysis: An *at least* clause denotes the set of possibilities that are as strong or stronger than the prejacent according to the pragmatically-determined strength ranking.



- The analysis of modified numerals depends on how you analyze numerals.
  - Two-sided analysis of numerals:
    - *At least two apples fell*: {**[2,...)**, **[3,...)**, **[4,...)**,...}
    - *At most two apples fell*: {**[0-2]**, **[1-2]**, **[2]**}
  - One-sided analysis of numerals:
    - *At least two apples fell*: {**[2]**, **[3]**, **[4]**, ...}
    - *At most two apples fell*: {**[0]**, **[1]**, **[2]**}
- Sincerity Maxim: Don't bring up an issue that you already know how to resolve. (More technically: If a speaker expresses a proposition with multiple alternatives, then the speaker's information state, once restricted to the proposition expressed, should still contain multiple alternatives.)



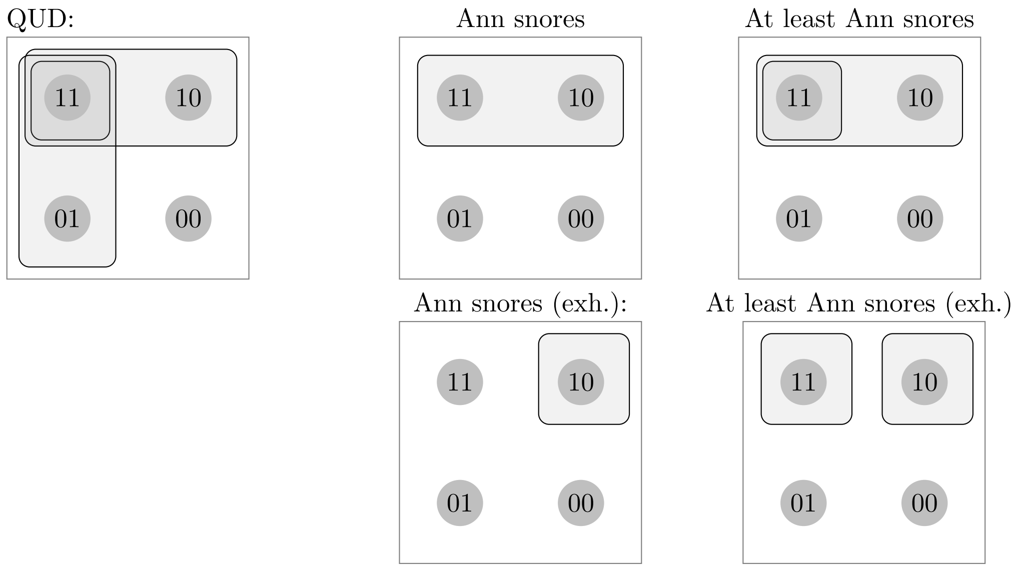
Fred should not assert *At least Ann snores* in this case. Fred should be ignorant with respect to the issue he raises.

- The effect of the Maxim of Quantity can be computed by "exhaustifying" the proposition; for the inquisitive setting we used Balogh's (2009) recipe:

  To exhaustify a proposition P with respect to a question Q:
  For each possibility p in P:

  - For each world w in p:
    If w is an element of an answer to Q that is not entailed by p then take w out of p.

  Examples:



Challenges

- Coppock & Brochhagen (2013) capture the fact that *at least* generates ignorance implicatures but no quantity implicatures, and the fact that *bare numerals* exhibit exactly the opposite pattern. They also predict the lack of ignorance implicatures for *more than*. However, they **do not predict the lack of quantity implicatures for *more than***.

- The effects of the QUD documented by Westera & Brasoveanu (2014) are not accounted for.

- As pointed out by Schwarz (to appearb), the **ignorance implicature** that Coppock & Brochhagen (2013) predict **for *at least* n is too weak**. In particular, it does not imply that the speaker should consider n itself a viable option.

  - Although Coppock & Brochhagen (2013) take inspiration from Buring (2008), who at an informal level assimilates *at least* n with n *or more*, in the formal account of Coppock & Brochhagen these two expressions actually differ in inquisitive content. Following Schwarz's (to appearb) suggestion, we will restore the equivalence, i.e., assign the same content to *at least* n and n *or more*, which actually differs from the content assigned to either of these expressions on Coppock & Brochhagen's original account.

- Framework issue:

  - Coppock & Brochhagen formulate their account in 'unrestricted' inquisitive semantics, Inq_U, an extension of the basic inquisitive semantics framework, Inq_B. Technically, the difference between the two is that in Inq_B propositions are *downward closed*, while in Inq_U they can be arbitrary sets of states (hence the name 'unrestricted').
  - While Inq_U is richer in expressive power than Inq_B, it is less well-behaved / well-understood from a logical point of view. In particular, it does not come with a suitable notion of entailment. As a consequence, it does not come with the usual algebraic operations on meanings, like *meet* and *join*, either.
  - One question, then, is whether an account of scalar modifiers along the lines of Coppock & Brochhagen (2013) really needs the full expressive power of Inq_U, or whether the theory could also be formulated in Inq_B.

3 Proposal: a combined approach

3.1 Bare numerals

- There are two possible accounts for bare numerals, based on a one-sided or two-sided semantics, respectively. For our purposes it is not necessary to choose between these two options.
- Under a two-sided semantics for bare numerals, it follows directly that n is interpreted as *exactly n*.
- Under a one-sided semantics for bare numerals, this is derived as an implicature.

3.2 More than

- For *more than* we propose an account that is very close to Schwarz's proposal for *at least*.
- The only difference is that we assume that the pragmatic alternatives that are taken into account when computing quantity implicatures are partly determined by the question that is being addressed.
- We consider two types of questions that may be addressed:
  - How many people did John invite?
  - Did John invite more than five people?
- In the context of a *how many* question, the pragmatic alternatives that are taken into account are the ones obtained by replacing (i) the numeral n with some other numeral and/or (ii) *more than* with *exactly*/*only*.
- In the context of a polar question, these pragmatic alternatives are not activated/deemed relevant.
- Semantically, *more than n* is interpreted as [n+1,...) and *exactly n* is of course interpreted as [n].
- Using the standard innocent exclusion mechanism, then, we derive:
  - In the context of a *how many* question: ignorance implicatures, and lack of quantity implicatures (because of symmetry of pragmatic alternatives).
  - In the context of a *polar* question: lack of ignorance implicatures, and lack of quantity implicatures (since there are no relevant pragmatic alternatives).

3.3 At least

- We assume that all possibilities are downward-closed; no nested possibilities.
- We adopt the proposal from Schwarz's critique of C&B that *at least n* be analyzed more along the lines Büring suggested, with the same meaning as *n or more*. Example:
  - *At least two apples fell*: {**[2]**, **[3,...)** }
- Horn alternatives for *at least n*: $\{$*at least* $m \mid m \in \mathbb{N}\}$
- Further assumption: The QUD constrains what Horn-alternatives are 'active'.

3.4 Pragmatic assumptions

- Quality:[1]

[1] These maxims are only assumed to be in force in specific types of conversation. What we primarily have in mind here is a conversation in which the participants exchange information in a fully cooperative way.

1. **Informative sincerity** (Gricean Quality)

   If a speaker utters a sentence φ, her information state s should be contained in the informative content of φ:[2]

   $$\Box_s \varphi$$

2. **Inquisitive sincerity** (adapted from Groenendijk & Roelofsen, 2009)[3,4]

   If a speaker utters a sentence φ that is inquisitive, then her information state should not already resolve it:

   $$\text{if } \varphi \text{ is inquisitive then } s \notin [\varphi]$$

- Example illustrating inquisitive sincerity:
  - Suppose A says:

    (4) Is it raining?

  - Since this sentence is inquisitive, A's information should not already resolve it.
  - This means that A should consider both rain and non-rain worlds possible.
  - In other words, A should be **ignorant** as to whether it rains or not.
- Notation: we write $\mathsf{sincere}(\varphi, s)$ if φ can be sincerely uttered given the information available in s.
- Adhering to Gricean intuitions, we assume that Quantity is about alternative *expressions* that the speaker could have used (as opposed to alternative meanings that the speaker could have expressed, as under Balogh's treatment). But only expressions that are relevant to the QUD are considered.
- We adopt an Innocent Exclusion based recipe for deriving implicatures, but now:
  - The Gricean Quality requirement, $\Box_s \varphi$, is replaced by $\mathsf{sincere}(\varphi, s)$, which also encompasses inquisitive sincerity;
  - We do not let $1_{\varphi,A}$ include 0_φ, which restricts the range of Horn alternatives that are considered for innocent exclusion in the final step.
- So the recipe runs as follows:
  - The first step, as before, is to compute the quality implicature:

    $0_\varphi = \{\mathsf{sincere}(\varphi, s)\}$
  - The second step, also as before, is to compute primary quantity implicatures, based on the assumption that any pragmatic alternative for φ that would have been more informative was apparently not sincerely utterable:

    $1_{\varphi,A} = \{\neg\mathsf{sincere}(\psi, s) : \psi \in A \;\&\; \mathsf{info}(\psi) \subseteq \mathsf{info}(\varphi)\}$
  - Finally, again as before, we compute secondary quantity implicatures, based on the assumption that whenever the primary quantity implicatures entail that $\neg\Box_s \psi$ for some Horn alternative ψ, and moreover ψ is 'innocently excludable' given $0_\varphi \cup 1_{\varphi,A}$, then we can conclude that $\Box_s \neg\psi$.

    $2_{\varphi,A} = \{\Box_s \neg\psi : \psi \in A \;\&\; 1_{\varphi,A} \models \neg\Box_s \psi \;\&\; \psi \text{ is innocently excludable given } 0_\varphi \cup 1_{\varphi,A}\}$

    where ψ is innocently excludable given S iff $\Box_s \neg\psi$ is an element of every maximal subset of $\{\Box_s \neg\psi : S \models \neg\Box_s \psi\}$ that is consistent with S.

[2] In our setting, $\Box_s \varphi$ means that $s \subseteq \mathsf{info}(\varphi)$.

[3] The original formulation of the inquisitive sincerity maxim makes reference to the common ground: "If a speaker utters a sentence φ that is inquisitive w.r.t. the common ground, then φ should be inquisitive w.r.t. the speaker's information state as well." For our current purposes this is not necessary. Thus, for presentational purposes we have simplified the formulation somewhat.

[4] Coppock & Brochhagen proposed a stronger sincerity maxim, which they call the maxim of *interactive sincerity*. On their account this is needed because the predictions that inquisitive sincerity delivers are too weak. On our present account, inquisitive sincerity delivers the right predictions, and interactive sincerity would do so as well.

4 Examples

(5) Q: How many apples did John eat?
A: John ate at least three apples.

- Semantics: {[3], [4,...)}
- Horn-alternatives:
  - *John ate least four apples* – {[4], [5,...)}
  - *John ate least five apples* – {[5], [6,...)}
  - etc.
- $0_\varphi = \{\mathsf{sincere}(\varphi, s)\} = \{s \subseteq [3, ...) \textit{ and } s \not\subseteq [3] \textit{ and } s \not\subseteq [4, ...)\}$
- $1_{\varphi,A} = \{\neg\mathsf{sincere}(\psi, s) : \psi \in A \textit{ and } \mathsf{info}(\psi) \subseteq \mathsf{info}(\varphi)\}$

$$= \left\{ \begin{array}{l} \neg\mathsf{sincere}(\textit{John ate at least four apples}, s), \\ \neg\mathsf{sincere}(\textit{John ate at least five apples}, s), \\ \neg\mathsf{sincere}(\textit{John ate at least six apples}, s), \\ \dots \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \neg(s \subseteq [4, ...) \textit{ and } s \not\subseteq [4] \textit{ and } s \not\subseteq [5, ...)), \\ \neg(s \subseteq [5, ...) \textit{ and } s \not\subseteq [5] \textit{ and } s \not\subseteq [6, ...)), \\ \neg(s \subseteq [6, ...) \textit{ and } s \not\subseteq [6] \textit{ and } s \not\subseteq [7, ...)), \\ \dots \end{array} \right\}$$

$$= \left\{ \begin{array}{l} s \not\subseteq [4, ...) \textit{ or } s \subseteq [4] \textit{ or } s \subseteq [5, ...), \\ s \not\subseteq [5, ...) \textit{ or } s \subseteq [5] \textit{ or } s \subseteq [6, ...), \\ s \not\subseteq [6, ...) \textit{ or } s \subseteq [6] \textit{ or } s \subseteq [7, ...), \\ \dots \end{array} \right\}$$

Note that there is no $\psi \in A$ such that $1_{\varphi,A} \models \neg\Box_s\psi$.

- So: $2_{\varphi,A} = \emptyset$
- $\Rightarrow$ ignorance implicature, and no scalar implicature
- Note: ignorance is already implied at the level of quality implicatures, 0_φ, and then reinforced at the quantity level, $1_{\varphi,A}$.
- We will see below that in the case of comparative modifiers (*more than*) ignorance only arises, if at all, at the quantity level.
- This could explain Westera and Brasoveanu's finding that superlative modifiers have stronger ignorance implicatures than comparative modifiers in contexts asking for an exact number, and also that the ignorance implicatures of superlative modifiers seem to be of a more obligatory nature, as witnessed by cases like (6):

  (6) a. I grew up with more than two parents.
  b. ??I grew up with at least two parents.

(7) Q: How many apples did John eat?
A: John ate three apples.

- Assume a one-sided reading.
- Semantics: {[3,...)}

- Stronger Horn-alternatives:
  John ate four apples – [4,...)
  John ate five apples – [5,...)
  etc.

- $0_\varphi = \{\mathsf{sincere}(\varphi, s)\} = \{s \subseteq [3, ...)\}$

- $1_\varphi = \{\neg\mathsf{sincere}(\psi, s) : \psi \in A \textit{ and } \mathsf{info}(\psi) \subseteq \mathsf{info}(\varphi)\}$
  $$= \left\{ \begin{array}{l} \neg(s \subseteq [4, ...)), \\ \neg(s \subseteq [5, ...)), \\ ... \end{array} \right\}$$

- Note that all stronger Horn alternatives are ψ's in A such that $1_\varphi \models \neg\Box_s\psi$.

- Recall: $2_{\varphi,A} = \{\Box_s\neg\psi : \psi \in A \;\&\; 1_{\varphi,A} \models \neg\Box_s\psi \;\&\; \psi$ is innocently excludable given $0_\varphi \cup 1_{\varphi,A}\}$

- So $2_{\varphi,A} = \{\Box_s\neg\textit{John ate four apples}, \Box_s\neg\textit{John ate five apples}, ...\}$

- $\Rightarrow$ scalar implicature, and no ignorance implicature

(8) Q: How many apples did John eat?
A: John ate more than two apples.

- Semantics: {[3,...)}

- Stronger Horn-alternatives:
  John ate more than three apples – [4,...)
  John ate more than four apples – [5,...)
  etc.
  John ate exactly three apples – [3]
  John ate exactly four apples – [4]
  etc.

- $0_\varphi = \{\mathsf{sincere}(\varphi, s)\} = \{s \subseteq [3, ...)\}$

- $1_\varphi = \{\neg\mathsf{sincere}(\psi, s) : \psi \in A \textit{ and } \mathsf{info}(\psi) \subseteq \mathsf{info}(\varphi)\}$
  $$= \left\{ \begin{array}{l} \neg(s \subseteq [4, ...)), \\ \neg(s \subseteq [5, ...)), \\ ... \\ \neg(s \subseteq [3]), \\ \neg(s \subseteq [4]), \\ ... \end{array} \right\}$$

- Again all stronger Horn alternatives are elements ψ of A such that $1_\varphi \models \neg\Box_s\psi$.

- But in this case, none of them are innocently excludable.

- So $2_{\varphi,A} = \emptyset$.

- $\Rightarrow$ ignorance implicature, and no scalar implicature

- Note, as anticipated above, that the ignorance implicature only arises at the quantity level, unlike in the case of *at least*.

(9) Q: Did John eat at least three apples?
A: Yes, he ate at least three apples.

- Cf. *Did John eat an apple or a pear?* – the alternatives generated by disjunction are flattened before the question operator applies, resulting in a polar question with two basic answers: 'yes' (he ate an apple or a pear) and 'no' (he didn't eat either) (cf., Roelofsen & Farkas, 2015).

- No Horn-alternatives are relevant, so no scalar implicature can arise.

(10) Q: Did John eat more than three apples?
A: Yes, he ate more than three apples.

- Again, no Horn-alternatives are relevant, so no scalar implicature can arise.
- Nor do we derive ignorance.

5 Conclusion

This proposal allows us to:

- predict ignorance with respect to the prejacent of *at least* (cf. Schwarz's critique of C&B)
- get a three-way contrast between superlative modifiers, comparative modifiers, and numerals without appeal to a two-sided analysis (in contrast to Schwarz's proposal)
- avoid the prediction that *at least* should produce quantity implicatures when *only* is not a grammatical alternative (in contrast to Schwarz's proposal)

With it, we have:

- reconciled Westera & Brasoveanu's (2014) findings with the achievements of the C&B account
- brought that work in line with recent theorizing on inquisitive semantics using downward-closed possibilities
- shown that inquisitive sincerity can interact with Horn-based quantity in a non-trivial way, something that may be fruitful to consider in other empirical domains as well.

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