Radical inquisitive semantics[*]

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1 Inquisitive semantics: propositions as proposals

Traditionally, the meaning of a sentence is identified with its *informative* content. In much recent work, this notion is given a dynamic twist, and the meaning of a sentence is taken to be its potential to change the 'common ground' of a conversation. The most basic way to formalize this idea is to think of the common ground as a set of possible worlds, and of a sentence as providing information by eliminating some of these possible worlds.

Of course, this picture is limited in several ways. First, when exchanging information sentences are not only used to provide information, but also—crucially—to *raise issues*, that is, to indicate which kind of information is desired. Second, the given picture does not take into account that updating the common ground is a *cooperative* process. One conversational participant cannot simply change the common ground all by herself. All she can do is *propose* a certain change. Other participants may react to such a proposal in several ways. In a cooperative conversation, changes of the common ground come about by mutual agreement.

In order to overcome these limitations, *inquisitive semantics* starts with a different picture. It views propositions as proposals to update the common ground. Crucially, these proposals do not always specify just one way of updating the common ground. They may suggest alternative ways of doing so, among which

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the addressee is then invited to choose. Formally, a proposition consists of one or more *possibilities*. Each possibility is a set of possible worlds and embodies a possible way to update the common ground. If a proposition consists of two or more possibilities, it is *inquisitive*: it invites other participants to provide information in such a way that one or more of the proposed updates may be established. Inquisitive propositions raise an issue. They indicate which kind of information is desired. In this way, inquisitive semantics directly reflects the idea that information exchange consists in a cooperative dynamic process of raising and resolving issues.

Our starting point here is the implementation of inquisitive semantics that is provided in (Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2009).[1] We will argue that this system, to which we will refer as *conservative* inquisitive semantics, only partially captures the central underlying conception of sentences as expressing proposals to update the common ground. Subsequently, an enriched implementation will be presented and illustrated in some detail.

2 Positive and negative responses

Conceiving of a proposition as a set of possibilities makes it possible to characterize, at least to some extent, what the positive responses to a given sentence are. For instance, (1) expresses a proposition consisting of two possibilities, corresponding with the two positive responses in (2):

(1) Pete will play the piano, and Sue will sing or Mary will dance.

(2) a. Yes, Pete will play the piano and Sue will sing.
    b. Yes, Pete will play the piano and Mary will dance.

However, if sentences are taken to express proposals, then preferably our semantics should not only allow us to characterize positive responses, which accept the proposal in question, but also *negative* responses, which *reject* the proposal. For instance, besides characterizing the responses in (2) as positive responses to (1), we would also like to characterize the responses in (3) as negative responses to (1).

(3) a. No, Pete will not play the piano.
    b. No, Sue will not sing and Mary will not dance.

[1] The historical context of the paper will be discussed in more detail in a separate section.

Conservative inquisitive semantics does not provide the means to establish such a characterization. It formally represents a proposal as a set of possibilities, and these possibilities correspond to positive responses. Negative responses are not always just negations of positive responses. For instance, the negative responses in (3) are not simply obtained by negating the positive responses in (2). Nor do they correspond in any other systematic way with the possibilities for (1).

Thus, in order to characterize both positive and negative responses, the semantics really has to be enriched. One way to do this is to respresent a proposal not just as a set of possibilities, but rather as a set of possibilities plus a set of *counter-possibilities*, where possibilities correspond to positive responses and counter-possibilities to negative responses. This approach will be explored below. We will see that it deals with the above examples in a straightforward way, and that it has some interesting further consequences, especially for the interpretation of conditional sentences.

3 Radical inquisitive semantics

We consider a propositional language.

Definition 1 (Language)**.** We consider a language whose formulas are built up from a finite set of proposition letters $\mathcal{P}$, using the standard operators $\neg, \wedge, \vee$ and $\rightarrow$, and an additional operator $\div$. We will refer to $\div$ as *inversion*, and for any formula φ, we will refer to $\div\varphi$ as the *inverse* of φ. Finally, we will use $?\varphi$ as an abbreviation of $\varphi \vee \div\varphi$.

The basic ingredients of the semantics are possible worlds and possibilities.

Definition 2 (Possible worlds and possibilities)**.** A *possible world* is a function from $\mathcal{P}$ to $\{0, 1\}$. A *possibility* is a set of possible worlds.

For any possibility α, $\overline{\alpha}$ will denote the *complement* of α, i.e., the set of all worlds not in α. For any formula φ, $|\varphi|$ will denote the possibility consisting of all worlds that make φ true in a classical setting. We will refer to $|\varphi|$ as the *truth-set* of φ.

Definition 3 below recursively defines, for every sentence φ in our language, the *proposition* $\lceil\varphi\rceil$ expressed by φ, and the *counter-proposition* $\lfloor\varphi\rfloor$ for φ. Both $\lceil\varphi\rceil$ and $\lfloor\varphi\rfloor$ will be sets of possibilities. We will refer to the elements of $\lceil\varphi\rceil$ as the *possibilities for* φ, and to the elements of $\lfloor\varphi\rfloor$ as the *counter-possibilities for* φ. The clauses of the definition will be illustrated right below.

Definition 3 (Radical inquisitive semantics)**.**

1. $\lceil p \rceil \;\coloneqq\; \{\, |p| \,\}$

   $\lfloor p \rfloor \;\coloneqq\; \{\, \overline{|p|} \,\}$

2. $\lceil \neg\varphi \rceil \;\coloneqq\; \{\, \bigcap_{\alpha \in \lceil \varphi \rceil} \overline{\alpha} \,\}$

   $\lfloor \neg\varphi \rfloor \;\coloneqq\; \lceil \varphi \rceil$

3. $\lceil \varphi \vee \psi \rceil \;\coloneqq\; \lceil \varphi \rceil \cup \lceil \psi \rceil$

   $\lfloor \varphi \vee \psi \rfloor \;\coloneqq\; \{\alpha \cap \beta \mid \alpha \in \lfloor \varphi \rfloor \text{ and } \beta \in \lfloor \psi \rfloor\}$

4. $\lceil \varphi \wedge \psi \rceil \;\coloneqq\; \{\alpha \cap \beta \mid \alpha \in \lceil \varphi \rceil \text{ and } \beta \in \lceil \psi \rceil\}$

   $\lfloor \varphi \wedge \psi \rfloor \;\coloneqq\; \lfloor \varphi \rfloor \cup \lfloor \psi \rfloor$

5. $\lceil \varphi \rightarrow \psi \rceil \;\coloneqq\; \{\gamma_f \mid f \in \lceil \psi \rceil^{\lceil \varphi \rceil}\,\}$ where $\gamma_f \;\coloneqq\; \bigcap_{\alpha \in \lceil \varphi \rceil} (\alpha \Rightarrow f(\alpha))$

   $\lfloor \varphi \rightarrow \psi \rfloor \;\coloneqq\; \{\alpha \Rightarrow \beta \mid \alpha \in \lceil \varphi \rceil \text{ and } \beta \in \lfloor \psi \rfloor\}$

6. $\lceil \div\varphi \rceil \;\coloneqq\; \lfloor \varphi \rfloor$

   $\lfloor \div\varphi \rfloor \;\coloneqq\; \lceil \varphi \rceil$

The clause for implication is defined in terms of a two-place operator $\Rightarrow$, which remains to be specified. Notice that $\Rightarrow$ takes two possibilities as its input and yields a third possibility as its output. For concreteness and simplicity, we will define $\Rightarrow$ as material implication here. But in principle, any more sophisticated existing analysis of non-inquisitive conditionals could be ‘plugged in’ here. We will return to this point in section 4.1.

Definition 4 ($\Rightarrow$)**.** $\alpha \Rightarrow \beta \;\coloneqq\; \overline{\alpha} \cup \beta.$

The remainder of this section is entirely devoted to explaining and illustrating the semantics specified in definition 3. We start with the clauses for atomic sentences, disjunction and conjunction. These clauses will immediately give us a handle on our initial example, (1), which was used to motivate our move to a more radical inquisitive semantics.

3.1 Atoms, disjunction, and conjunction

A natural translation of example (1) into our formal language is:

(4) $\quad p \wedge (q \vee r)$

According to the atomic clause of definition 3, the proposition expressed by the first conjunct p is $\{\, |p| \,\}$, and similarly for the atomic sentences q and r. The clause for disjunction tells us that:

(5) $\quad \lceil q \vee r \rceil = \lceil q \rceil \cup \lceil r \rceil = \{\, |q|, |r| \,\}$

And the clause for conjunction yields:

(6) $\quad \lceil p \wedge (q \vee r) \rceil = \{\, |p \wedge q|, |p \wedge r| \,\}$

So there are two possibilities for $p \wedge (q \vee r)$. The sentence is inquisitive, and its inquisitiveness can be resolved by providing the information that p and q are the case or the information that p and r are the case. This explains the fact that (2a) and (2b) are positive responses to (1).

Now let us turn to the counter-proposition for (4). First, the atomic clause says that the counter-proposition for p is $\{\, \overline{|p|} \,\}$, which is the same as $\{\, |\neg p| \,\}$, and similarly for q and r. The clause for disjunction tells us that:

(7) $\quad \lfloor q \vee r \rfloor = \{\, |\neg q| \cap |\neg r| \,\} = \{\, |\neg q \wedge \neg r| \,\}$

And the clause for conjunction yields:

(8) $\quad \lfloor p \wedge (q \vee r) \rfloor = \lfloor p \rfloor \cup \lfloor q \vee r \rfloor = \{\, |\neg p|\,,\ |\neg q \wedge \neg r| \,\}$

Thus, there are two counter-possibilities for (4), corresponding exactly to the two negative responses in (3a) and (3b).

3.2 Negation and inversion

Consider a simple sentence involving negation:

(9) Sue will not sing or dance.

a. *Primary positive response:*
Right, she will not sing or dance.

b. *Primary negative responses:*
No, she wíll sing.
No, she wíll dance.

Suppose that we translate (9) into our logical language as $\neg(q \vee r)$. The proposition expressed by a negated sentence $\neg\varphi$ always consists of a single possibility, which is the intersection of the complements of all the possibilities for φ (or, equivalently, the complement of the union of all the possibilities for φ). Thus, for the particular case of $\neg(q \vee r)$, we get:

(10) $\lceil \neg(q \vee r) \rceil = \{\, |\neg q| \cap |\neg r| \,\} = \{\, |\neg q \wedge \neg r| \,\}$

That is, the proposition expressed by $\neg(q \vee r)$ consists of a single possibility, which corresponds with the primary positive response in (9a).

Now let us turn to the counter-proposition for $\neg(q \vee r)$. The counter-clause for negation tells us that, in general, the counter-proposition for $\neg\varphi$ is the proposition expressed by φ. Thus for the case of $\neg(q \vee r)$ we get:

(11) $\lfloor \neg(q \vee r) \rfloor = \lceil q \vee r \rceil = \{\, |q|, |r| \,\}$

This means that there are two counter-possibilities for $\neg(q \vee r)$, corresponding to the two primary negative responses in (9b).

The final clause of definition 3 says that the proposition expressed by the inverse $\div\varphi$ of a sentence φ is the counter-proposition for φ itself. And vice versa, the counter-proposition for the inverse of φ is the proposition expressed by φ itself.

Notice that negation and inversion are closely related. First, for any sentence φ, the counter-proposition for $\div\varphi$ is exactly the same as the counter-proposition for $\neg\varphi$, namely the proposition expressed by φ itself:

Fact 5 (Counter-possibilities, negation, inversion)**.** For any φ: $\lfloor \neg\varphi \rfloor = \lfloor \div\varphi \rfloor = \lceil \varphi \rceil$

The proposition expressed by $\div\varphi$ is not generally the same as the one expressed by $\neg\varphi$. This is easy to see: the proposition expressed by $\neg\varphi$ always consists of a single possibility, while the proposition expressed by $\div\varphi$ may in principle contain any number of possibilities. However, as long as φ does not involve implication, there is a straightforward connection between $\lceil \neg\varphi \rceil$ and $\lceil \div\varphi \rceil$:

Fact 6 (Possibilities for negation and inversion)**.**
For any φ that does not involve implication:

$\lceil \neg\varphi \rceil = \{\, \bigcup \lceil \div\varphi \rceil \,\} = \{\, \bigcup \lfloor \varphi \rfloor \,\}$

That is, whenever φ does not contain implication, the single possibility for $\neg\varphi$ is the union of all the possibilities for $\div\varphi$, which is the same as the union of all the counter-possibilities for φ itself.

3.3 Atomic, conjunctive, and disjunctive questions

Recall that $?\varphi$ is defined as an abbreviation of $\varphi \vee \div\varphi$. To illustrate what the consequences are of this definition, let us first consider the interpretation of an atomic polar question $?q$. By definition, $?q$ abbreviates $q \vee \div q$. So $\lceil ?q \rceil = \lceil q \vee \div q \rceil$. The clause for disjunction tells us that $\lceil q \vee \div q \rceil = \lceil q \rceil \cup \lceil \div q \rceil$. By the clause for inversion, we have that $\lceil q \rceil \cup \lceil \div q \rceil = \lceil q \rceil \cup \lfloor q \rfloor$, which, by the atomic clause, amounts to $\{\, |q| \,\} \cup \{\, \overline{|q|} \,\}$, and that is the same as $\{\, |q|,\ |\neg q| \,\}$. So:

(12) $\lceil ?q \rceil = \{\, |q|, |\neg q| \,\}$

This means that there are two positive responses to the atomic question $?q$, corresponding to *yes* and *no*.

As for the counter-proposition expressed by $?q$, we have, by definition of $?q$ as an abbreviation of $q \vee \div q$, that $\lfloor ?q \rfloor = \lfloor q \vee \div q \rfloor$. By the counter-clause for disjunction, we have that $\lfloor q \vee \div q \rfloor = \{\, \alpha \cap \beta \mid \alpha \in \lfloor q \rfloor \text{ and } \beta \in \lfloor \div q \rfloor \,\}$, which reduces to $\{\, \alpha \cap \beta \mid \alpha \in \{\, |\neg q| \,\} \text{ and } \beta \in \{\, |q| \,\} \,\}$. So, what we end up with is:

(13) $\lfloor ?q \rfloor = \{\, \emptyset \,\}$

This means that there are no non-contradictory negative responses to $?q$.

Thus, the analysis of atomic polar questions does not yield any surprises. However, as questions are defined in terms of inversion here, and inversion is a novel concept, our general treatment of questions is bound to diverge from previous analyses. In conservative inquisitive semantics, for instance, questions are defined in terms of negation rather than inversion. That is, $?\varphi$ is defined as an abbreviation of $\varphi \vee \neg\varphi$ rather than $\varphi \vee \div\varphi$. This happens to give exactly the same results for *atomic* polar questions. But the two theories start to make different predictions as soon as we go beyond the atomic case. Take, for instance, a simple conjunctive polar question:

(14) Will both Sue and Mary sing?

We translate (14) into our logical language as $?(q \wedge r)$. In radical inquisitive semantics, there are not just two, but three possibilities for this sentence, $|q \wedge r|$, $|\neg q|$, and $|\neg r|$, which correspond to the following primary positive responses:

(15) *Primary positive responses in radical inquisitive semantics:*

a. Yes, both Sue and Mary will sing. $q \wedge r$

b. No, Sue won't sing. $\neg q$

c. No, Mary won't sing. $\neg r$

In conservative inquisitive semantics, the proposition expressed by $?(q \wedge r)$ consists of just two possibilities, $|q \wedge r|$ and $|\neg(q \wedge r)|$, corresponding to the following primary positive responses:

(16) *Primary positive responses in conservative inquisitive semantics:*

a. Yes, both Sue and Mary will sing. $q \wedge r$

b. No, either Sue or Mary won't sing. $\neg q \vee \neg r$

So, both systems generate the same *yes*-answer, but radical inquisitive semantics generates more specific primary *no*-answers than conservative inquisitive semantics does. In conservative inquisitive semantics (15b) and (15c) cannot be classified as positive responses in any straightforward way, because the proposition assigned to $?(p \wedge q)$ is too coarse-grained. This is clearly a shortcoming, which, for all we know, is shared by any other previous account of questions.

3.4 Conditionals with disjunctive consequents

The first example that we will use to illustrate the clause for implication is the conditional in (17). Notice that the consequent of this conditional is disjunctive (and therefore inquisitive). We want to derive that the positive and negative responses to (17) are the ones specified in (17a) and (17b), respectively.

(17) If Pete plays the piano, then Sue will sing or Mary will dance.

a. *Positive responses:*
Yes, if Pete plays the piano, Sue will sing.
Yes, if Pete plays the piano, Mary will dance.

b. *Negative response:*
No, if Pete plays the piano, Sue won't sing and Mary won't dance.

We translate (17) into our formal language as (18), and we will show that the proposition expressed by (18) and the counter-proposition for (18) are the ones in (18a) and (18b), respectively, which correspond exactly with the positive and negative responses specified in (17a) and (17b).

(18) $p \to (q \vee r)$

a. $\lceil p \to (q \vee r) \rceil = \{ |p \to q|,\ |p \to r| \}$
b. $\lfloor p \to (q \vee r) \rfloor = \{ |p \to (\neg q \wedge \neg r)| \}$

First consider the proposition $\lceil p \to (q \vee r) \rceil$. For convenience, let us repeat the clause for implication:

(19) $\lceil \varphi \to \psi \rceil \ := \ \{ \gamma_f \mid f \in \lceil \psi \rceil^{\lceil \varphi \rceil} \}$ where $\gamma_f \ := \ \bigcap_{\alpha \in \lceil \varphi \rceil} (\alpha \Rightarrow f(\alpha))$

The idea behind this clause is the following. The proposal expressed by a sentence can in general be *realized* in one or more ways. That is, if a proposal consists of just one possibility, then it proposes just one update, and it can be realized in exactly one way, namely by establishing that update. If a proposal consist of several possibilities, it proposes several possible updates, and this means that it can be realized in several ways, namely by establishing either one (or more) of the proposed updates. What we take to be the 'positive responses' to a given proposal are sentences that do exactly this: they realize one of the proposed updates.

Now, under this perspective, a conditional sentence $\varphi \to \psi$ can be thought of as expressing a proposal to establish a certain *implicational dependency* between the ways in which φ may be realized and the ways in which ψ may be realized, or, in more neutral terms, between the possibilities for φ and the possibilities for ψ. Such a dependency links every possibility $\alpha \in \lceil \varphi \rceil$ to some possibility $f(\alpha) \in \lceil \psi \rceil$, in such a way that for all $\alpha \in \lceil \varphi \rceil$, $\alpha \Rightarrow f(\alpha)$ holds.

How many potential implicational dependencies there are depends on the number of possibilities for φ and ψ. If there are m possibilities for φ and n possibilities for ψ then there are n^m functions from $\lceil \varphi \rceil$ to $\lceil \psi \rceil$. Each of these functions f links every possibility $\alpha \in \lceil \varphi \rceil$ to some possibility $f(\alpha) \in \lceil \psi \rceil$. Thus, each of these functions corresponds with a potential implicational dependency between the possibilities for φ and the possibilities for ψ.

In order to establish the implicational dependency corresponding to some function f from $\lceil \varphi \rceil$ to $\lceil \psi \rceil$, we have to establish that $\alpha \Rightarrow f(\alpha)$ holds for all $\alpha \in \lceil \varphi \rceil$. This means that we have to establish $\bigcap_{\alpha \in \lceil \varphi \rceil} (\alpha \Rightarrow f(\alpha))$.Notice that this intersection is a possibility, which is called γ_f in the clause for implication. For each function $f\colon \lceil \varphi \rceil \to \lceil \psi \rceil$, then, there is a corresponding possibility γ_f, and together, these possibilities make up the proposition expressed by $\varphi \to \psi$.

Now let us return to our example, $p \to (q \vee r)$. We have already seen that $\lceil p \rceil = \{ |p| \}$ and $\lceil q \vee r \rceil = \{ |q|, |r| \}$. Thus, there are two functions from $\lceil p \rceil$ to $\lceil q \vee r \rceil$, one that maps $|p|$ to $|q|$, and another one that maps $|p|$ to $|r|$. Call the first one f_q

and the second one f_r. Then, the clause for implication tells us that $\lceil p \to (q \vee r)\rceil$ consists of two possibilities, γ_{f_q} and γ_{f_r}, where $\gamma_{f_q} = |p| \Rightarrow |q| = |p \to q|$ and $\gamma_{f_r} = |p| \Rightarrow |r| = |p \to r|$. Thus, we obtain the desired result:

(20) $\quad \lceil p \to (q \vee r)\rceil = \{\, |p \to q|,\ |p \to r| \,\}$

Next we turn to the counter-proposition for $p \to (q \vee r)$. Again, let us pause one moment to repeat the counter-clause for implication, and briefly explain the intuition behind it.

(21) $\quad \lfloor \varphi \to \psi \rfloor \ := \ \{\alpha \Rightarrow \beta \mid \alpha \in \lceil \varphi \rceil \text{ and } \beta \in \lfloor \psi \rfloor\}$

As specified above, we think of $\varphi \to \psi$ as expressing a proposal to establish a certain implicational dependency between the possibilities for φ and the possibilities for ψ. Rejecting such a proposal, then, amounts to saying that none of the potential dependencies could possibly be established. This means that there must be some way of realizing φ that leads to the *rejection* of ψ. Thus, to reject $\varphi \to \psi$, we must point out that the realization of some possibility α for φ implies the realization of some counter-possibility β for ψ. This is why for every $\alpha \in \lceil \varphi \rceil$ and every $\beta \in \lfloor \psi \rfloor$, $\alpha \Rightarrow \beta$ is a counter-possibility for $\varphi \to \psi$, corresponding to a negative response. So if there are m possibilities for φ and n counter-possibilities for ψ then there are (at most) $m \times n$ counter-possibilities for $\varphi \to \psi$.

Returning to our concrete example, the counter-possibilities for $p \to (q \vee r)$ are possibilities of the form $\alpha \Rightarrow \beta$, where $\alpha \in \lceil p \rceil$ and $\beta \in \lfloor q \vee r \rfloor$. Recall that $\lfloor q \vee r \rfloor = \{\, |\neg q \wedge \neg r| \,\}$. So, since there is only one possibility for the antecedent p and only one counter-possibility for the consequent $q \vee r$, there is also only one counter-possibility for the implication as a whole, which indeed corresponds with the negative response in (17b):

(22) $\quad \lfloor p \to (q \vee r)\rfloor = \{\, |p \to (\neg q \wedge \neg r)| \,\}$

3.5 Conditionals with disjunctive antecedents

If we reverse the antecedent and the consequent of example (17) we arrive at (23). Notice that the number of the positive and negative responses is also reversed.

(23) If Sue sings or Mary dances, then Pete will play the piano.

a. *Positive response:*
Yes, if Sue sings, Pete will play, and if Mary dances, he'll play too.

b. *Negative responses:*
No, if Sue sings Pete will not play.
No, if Mary dances Pete will not play.

We translate (23) as (24). The proposition expressed by (24) is (24a) and the counter-proposition for (24) is (24b), which correspond exactly with the positive and negative responses specified in (23a) and (23b).

(24) $(q \vee r) \rightarrow p$

a. $\lceil (q \vee r) \rightarrow p \rceil = \{\, |q \rightarrow p| \cap |r \rightarrow p| \,\}$
b. $\lfloor (q \vee r) \rightarrow p \rfloor = \{\, |q \rightarrow \neg p|, |r \rightarrow \neg p| \,\}$

Since there is only a single possibility $|p|$ for the consequent of (24), there is only one function f that maps both possibilities $|q|$ and $|r|$ for the inquisitive antecedent of (24) to $|p|$. Thus, the single possibility γ_f for (24) is $(|q| \Rightarrow |p|) \cap (|r| \Rightarrow |p|)$, which, as (24a) reports, is the same as $|q \rightarrow p| \cap |r \rightarrow p|$.

The counter-possibilities for (24) are of the form $\alpha \Rightarrow \beta$, where α is a possibility for the antecedent, $q \vee r$, and β is a counter-possibility for the consequent, p. The possibilities for $q \vee r$ are $|q|$ and $|r|$, and the only counter-possibility for p is $|\neg p|$. So there are two counter-possibilities for (24): $|q| \Rightarrow |\neg p|$ and $|r| \Rightarrow |\neg p|$, which, as (24b) reports, can also be written as $|q \rightarrow \neg p|$ and $|r \rightarrow \neg p|$.

3.6 Conditional questions

Next consider the conditional question in (25). The positive responses that we would like to derive are listed in (25a), and the negative response, which is a denial of the antecedent of the conditional question, is given in (25b).[2]

(25) If Pete plays the piano, will Sue sing?

a. *Positive responses:*
Yes, if Pete plays the piano, then Sue will sing.
No, if Pete plays the piano, then Sue will not sing.

b. *Negative response:*
Well, Pete will not play the piano.

[2] There is an ongoing controversy in the literature about the exact status of responses that deny the antecedent of a conditional question, such as (25b) (see, for instance, Isaacs and Rawlins, 2008).

We translate (25) into our logical language as $p \to ?q$, which expresses the proposition specified in (26a) and has the counter-proposition in (26b):[3]

(26) $p \to ?q \;\equiv\; p \to (q \vee \div q) \;\equiv\; p \to (q \vee \neg q)$

a. $\lceil p \to ?q \rceil = \{\, |p| \Rightarrow |q|,\ |p| \Rightarrow |\neg q| \,\} = \{\, |p \to q|,\ |p \to \neg q| \,\}$

b. $\lfloor p \to ?q \rfloor = \{\, |p| \Rightarrow (|q| \cap |\neg q|) \,\} = \{\, |p| \Rightarrow \emptyset \,\} = \{\, |\neg p| \,\}$

As is to be expected, $p \to ?q$ is inquisitive: $\lceil p \to ?q \rceil$ consists of two possibilities, which correspond to the two positive responses in (25a).

Perhaps more surprisingly, whereas we saw earlier that atomic questions do not license any sensible negative response, we now see that conditional questions do. In particular, the counter-proposition for $p \to ?q$ contains exactly one possibility, which corresponds with the negative response in (25b).

3.7 Denying the antecedent of a conditional assertion

The present framework also offers a new perspective on responses that deny the antecedent of a conditional assertion, like our earlier example (17). Such responses do not count as negative responses to the conditional itself, but they do count as negative responses to the 'question behind' the conditional. If we take the question behind any sentence φ to be $?\varphi$, then the question behind (17) is (27), which is translated into our logical language as (28). The first two positive responses to (27) are also positive responses to (17), the third positive response to (27) is a negative response to (17), and the negative response to (27) denies the antecedent of (17).

(27) Will Sue sing or Mary dance, if Pete plays the piano?

a. *Positive responses:*
Yes, if Pete plays the piano, Sue will sing.
Yes, if Pete plays the piano, Mary will dance.
No, if Pete plays the piano, Sue won't sing and Mary won't dance.

b. *Negative response:*
Well, Pete will not play the piano.

(28) $?(p \to (q \vee r))$

[3] Incidentally, we could just as well have translated (25) as $?(p \to q)$, which is equivalent with $p \to ?q$ in the present system (in the sense that it expresses exactly the same proposition, and has exactly the same counter-proposition).

a. $\lceil ?(p \to (q \vee r)) \rceil = \{\, |p \to q|,\ |p \to r|,\ |p \to (\neg q \wedge \neg r)| \,\}$

b. $\lfloor ?(p \to (q \vee r)) \rfloor = \{\, |\neg p| \,\}$

In general we may distinguish three types of responses to a sentence φ. First, there are the positive and negative responses specified directly by the proposition expressed by φ and the counter-proposition for φ. Together these responses also form the positive responses to the question $?\varphi$ behind φ. The third class of responses to φ are the negative responses to $?\varphi$, which can be looked upon as responses that refuse the question behind φ.

3.8 Refusals and supposition failure

The following definition characterizes responses to a sentence φ that refuse the question behind φ, which we generally take to be $?\varphi$.

Definition 7 (Refusal)**.** ψ refuses the question behind φ iff $\bigcup \lceil \psi \rceil \subseteq \bigcup \lfloor ?\varphi \rfloor$.

In many cases $\lfloor ?\varphi \rfloor = \{\emptyset\}$, and there is no sensible way to refuse the question behind φ. But as we have seen, for conditional questions, for example, there may be non-absurd counter-possibilities, and hence the question behind a conditional sentence can be sensibly refused. E.g., according to the definition of refusal, $\neg p$ is a refusal of the question behind $p \to q$.

Another way to put this is that $\neg p$ goes against the possibility to suppose the antecedent of $p \to q$, that $\neg p$ reports *supposition failure* of $p \to q$. We explicitly define the notion of the supposition of a sentence φ as the complement of the union of the counter-possibilities for φ. At the same time we introduce a notation for the informative content of φ, given by $\bigcup \lceil \varphi \rceil$.

Definition 8 (Information and supposition)**.**

1. The *supposition* of φ is $\mathsf{sup}(\varphi) = \overline{\bigcup \lfloor ?\varphi \rfloor}$.

2. The *informative content* of φ is $\mathsf{info}(\varphi) = \bigcup \lceil \varphi \rceil$.

Note that in case there are no (non-absurd) counter-possibilities for $?\varphi$, the supposition of φ corresponds to ω, the set of all worlds, the supposition of φ is trivial. Having the notion of the supposition of a sentence at hand gives us another way of saying that ψ refuses the question behind φ.

Proposition 9 (Refusal is inconsistence with supposition)**.**
ψ refuses the question behind φ iff $\mathsf{info}(\psi) \cap \mathsf{sup}(\varphi) = \emptyset$.

In case φ is itself a question $?\chi$, the question behind it is $??\chi$. In general the two are not fully equivalent. For example, in case $\lfloor ?\chi \rfloor = \{\emptyset\}$, we have that $\lceil ??\chi \rceil = \lceil ?\chi \rceil \cup \{\emptyset\}$, but nothing changes with respect to the counter-possibilities: $\lfloor ??\chi \rfloor = \emptyset$. As long as there is a single counter-possibility α for $?\varphi$, whether α is absurd or not, the situation will be like this. The only thing that changes with respect to $??\varphi$ is that now α is added to the possibilities for $??\varphi$, α remains the only counterpossibility for $??\varphi$.

However, that there is no difference between the counter-possibilities for $??\varphi$ and for $?\varphi$, no longer needs to be the case if there is more than one counter-possibility for $?\varphi$. But all counter-possibilities for $?\varphi$ remain to be counter-possiblities for $??\varphi$, and any newly added counter-possibility for $??\varphi$ is included in one of the counter-possibilities for $?\varphi$. This means that the following holds.

Lemma 10 (?-Iteration)**.** For every sentence $\mathsf{sup}(?\varphi) = \mathsf{sup}(??\varphi)$

Proof. The counter-propositions for $?\varphi$ and $??\varphi$ are as in (a) and (b), respectively:

(a) $\lfloor ?\varphi \rfloor = \{\alpha \cap \beta \mid \alpha \in \lceil \varphi \rceil,\ \beta \in \lfloor \varphi \rfloor\}$

(b) $\lfloor ??\varphi \rfloor = \{\alpha \cap \beta \cap \gamma \mid \alpha \in \lceil \varphi \rceil,\ \beta \in \lfloor \varphi \rfloor,\ \gamma \in \lceil \varphi \rceil \cup \lfloor \varphi \rfloor\}$

For any counter-possibility $\alpha \cap \beta \in \lfloor ?\varphi \rfloor$ where $\alpha \in \lceil \varphi \rceil$ and β in $\lfloor \varphi \rfloor$, one of the choices for γ in forming a counter-possibility $\alpha \cap \beta \cap \gamma \in \lfloor ??\varphi \rfloor$ is that $\gamma = \alpha$ or $\gamma = \beta$, in which case $\alpha \cap \beta \cap \gamma = \alpha \cap \beta$. This means that we always have that $\lfloor ?\varphi \rfloor \subseteq \lfloor ??\varphi \rfloor$.

Furthermore, for all other choices of $\gamma \in \lceil \varphi \rceil \cup \lfloor \varphi \rfloor$, where $\gamma \neq \alpha$ and $\gamma \neq \beta$, it will be the case that $\alpha \cap \beta \cap \gamma \ \subset\ \alpha \cap \beta$. Hence, it will always hold for any $\alpha \in \lfloor ?\varphi \rfloor$: $\alpha \notin \lfloor ?\varphi \rfloor$, that there is some $\beta \in \lfloor ??\varphi \rfloor$: $\alpha \subseteq \beta$. Since we have also seen that $\lfloor ?\varphi \rfloor \subseteq \lfloor ??\varphi \rfloor$, we can conclude that φ: $\bigcup \lfloor ?\varphi \rfloor = \bigcup \lfloor ??\varphi \rfloor$. □

Concerning questions behind questions this results in the fact that:

Proposition 11 (Questions behind questions)**.**
ψ refuses the question behind φ iff ψ refuses the question behind $?\varphi$

We can also put this result as follows:

Theorem 12 (Supposition and Question)**.** φ and $?\varphi$ have the same supposition.

Putting any number of question marks in front of a sentence makes no change in supposition. On the basis of this fact, we can also show that no matter how many times we take the inversion of a sentence, the supposition is preserved.

Theorem 13 (Supposition and inversion)**.** φ and $\div\varphi$ have the same supposition.

This clearly shows the presuppositional nature of suppositions. Being preserved under negation, and questions, is taken to be the hallmark of presuppositions. For our notion of the supposition of a sentence, and the standard notion of negation it does not hold that $\neg\varphi$ and φ always have the same supposition. As we have just shown, it does hold for inversion. What is interesting, we think, is that this rests upon the more basic fact that φ and $?\varphi$ have the same supposition, which in turn is triggered by the notion of the refusal of a sentence as relating to the question behind a sentence.

3.9 Ramsey on conditionals

In the present framework, the conditional assertions in (29) and (30) contradict each other in a sense: (30) is a negative response to (29), i.e., it rejects the proposal expressed by (29). And vice versa, (29) rejects the proposal expressed by (30).

(29) If Pete plays the piano, then Sue will sing.

(30) If Pete plays the piano, then Sue will not sing.

This corresponds exactly to what Frank Ramsey wrote in his famous footnote in 1929, which is generally referred to as the *Ramsey test* for conditionals:

> If two people are arguing "If *p* will *q*?" and are both in doubt as to *p*, they are adding *p* hypothetically to their stock of knowledge and arguing on that basis about *q*; so that in a sense "If *p*, *q*" and "If *p*, $\neg q$" are contradictories.

In fact, in line with Ramsey's footnote, radical inquisitive semantics takes (29) and (30) to be the two opposing answers to the conditional question in (31) (which is the 'question behind' both (29) and (30)):

(31) If Pete plays the piano, will Sue sing?

Moreover, the framework sheds new light on Ramsey's 'precondition', which says that the people who are arguing "If *p* will *q*?" are both in doubt as to *p*. In particular, it makes predictions about certain situations in which this precondition is *not* met, namely those situations in which one of the participants is in a position to *deny p*. We have seen that this corresponds to being able to report supposition failure. In that case, (31) will be contested immediately with the negative response in (32), rejecting the conditional question and preventing other participants from hypothetically updating their 'stock of knowledge' with p.

(32) Pete will not play the piano.

So our semantics predicts that if two people are to argue, or more cooperatively, are to investigate "If p will q?", then neither of them should be in a position to deny p. Only if the supposition that p does not fail for either of them, it makes sense for both of them to hypothetically update their stock of knowledge with p, and investigate on that basis whether q.

4 Minimal change semantics for conditionals

4.1 Unconditionals

Consider (33), in a sense the reverse of the conditional question (25):[4]

(33) Whether Pete plays the piano or not, Sue will sing.

Sentences of this kind are referred to as *concessive conditionals*, or *unconditionals*. Rawlins (2008) argues that they are conditional sentences whose antecedent is a question. This suggests translating (33) as $?p \to q$, which abbreviates $(p \vee \div p) \to q$. In our system, this is equivalent with $(p \vee \neg p) \to q$ and with $(p \to q) \wedge (\neg p \to q)$. From these equivalences, it should immediately be clear that we predict (33) to license the following positive and negative responses:

(34) a. *Positive response:*
Yes, if Pete plays the piano Sue will sing,
and if he doesn't play, she will sing too.

b. *Negative responses:*
No, if Pete plays the piano, Sue won't sing.
No, if Pete doesn't play the piano, Sue won't sing.

In principle, these predictions are correct. However, there is a subtlety to note here. We defined $\Rightarrow$ as material implication, and as a result of this the proposition expressed by $(p \to q) \wedge (\neg p \to q)$ is actually the same as the one expressed by q.

[4]The benefit of using inquisitive semantics to analyze this type of sentences was pointed out to us by Stefan Kaufmann, and the particular analysis to be presented below follows, in essence, his insight. See Kaufmann (2009) for a slightly different account, largely in the same spirit.

In a classical setting, and in conservative inquisitive semantics, these two formulas are in fact completely equivalent. That is not the case here, because the two formulas are assigned different counter-propositions. But we do predict that (33) expresses exactly the same proposition as (35), and this is clearly a problematic prediction.

(35) Sue will sing.

However, identifying the source of the problem, and fixing it in the obvious way, leads to a promising analysis. The source of the problem is that $\Rightarrow$ is defined as material implication. And the obvious fix is to redefine it along the lines of a more sophisticated existing analysis of conditionals. Suppose for instance, that we make the standard assumption, originating in the work of Stalnaker (1968) and Lewis (1973), that $\Rightarrow$ is sensitive to a *similarity order* between worlds:

(36) $\alpha \Rightarrow \beta \;:=\; \{w \mid \textsc{min}_w(\alpha) \subseteq \beta\}$

where $\textsc{min}_w(\alpha)$ is the set of worlds that belong to α and do not differ more from w than any other world in α.

Under this assumption, $\lceil q \rceil$ and $\lceil ?p \rightarrow q \rceil$ differ in exactly the right way. The former still consists of a single possibility containing all worlds where q holds. $\lceil ?p \rightarrow q \rceil$, however, becomes stronger:

(37) $\lceil ?p \rightarrow q \rceil = \{\, \gamma \,\}$

where $\gamma = \{w \mid \textsc{min}_w|p| \subseteq |q| \text{ and } \textsc{min}_w|\neg p| \subseteq |q|\,\}$

To see whether w belongs to γ we should not just check whether q holds at w, but rather we should look at all p-worlds that minimally differ from w and all $\neg p$-worlds that minimally differ from w, and check whether q holds in all those worlds. In the terms of our original natural language example, we should not just check whether Sue sings at w, but we should look at all worlds minimally different from w where Pete plays the piano, and at all worlds minimally different from w where Pete doesn't play the piano, and check whether Sue sings in all those worlds. This indeed appears to be the correct analysis of (33).

4.2 Dependency statements and questions

Unconditionals can function as negative responses to 'dependency statements'. Consider the following example:

(38) Whether Sue will sing depends on whether Pete will play the piano.

a. *Positive responses:*
Yes, Sue will sing if and only if Pete will play the piano.
Yes, Sue will sing if and only if Pete will not play the piano.

b. *Negative responses:*
No, whether Pete will play the piano or not, Sue will sing.
No, whether Pete will play the piano or not, Sue will not sing.

'Whether Pete plays the piano' and 'whether Sue sings' are translated into our logical language as $?p$ and $?q$, respectively. To say that $?q$ depends on $?p$ is to say that p implies q and $\neg p$ implies $\neg q$, or vice versa, that p implies $\neg q$ and $\neg p$ implies q. Thus, (38) as a whole is translated as a disjunction $\delta := \delta_1 \vee \delta_2$, where:

(39)
$$\delta_1 := (p \to q) \wedge (\neg p \to \neg q)$$
$$\delta_2 := (p \to \neg q) \wedge (\neg p \to q)$$

There is a unique possibility for δ_1 and a unique possibility for δ_2, and together these two possibilities constitute the proposition expressed by δ:

(40)
$$\lceil \delta \rceil = \left\{ \begin{array}{c} |p| \Rightarrow |q| \ \cap \ |\neg p| \Rightarrow |\neg q| \\ |p| \Rightarrow |\neg q| \ \cap \ |\neg p| \Rightarrow |q| \end{array} \right\}$$

The elements of $\lceil \delta \rceil$ correspond exactly with the positive responses in (38a). To compute the counter-possibilities for δ we first have to compute the counter-possibilities for δ_1 and for δ_2, and then take pairwise intersections. This gives us:

(41)
$$\lfloor \delta \rfloor = \left\{ \begin{array}{c} |p| \Rightarrow |q| \ \cap \ |\neg p| \Rightarrow |q| \\ |p| \Rightarrow |\neg q| \ \cap \ |\neg p| \Rightarrow |\neg q| \\ \emptyset \end{array} \right\}$$

The first two counter-possibilities correspond to the two unconditional negative responses in (38b); the third counter-possibility, $\emptyset$, does not correspond to any sensical response and can therefore be ignored.

Now consider the dependency question behind (38):

(42) Does whether Sue will sing depend on whether Pete will play the piano?

There are four positive responses to (42): the two that count as positive responses to (38) and the two that count as negative responses to (38). There are no sensical negative responses to (42). This is exactly what our semantics predicts:

(43) a. $$\lceil ?\delta \rceil = \left\{ \begin{array}{c} |p| \Rightarrow |q| \;\cap\; |\neg p| \Rightarrow |q| \\ |p| \Rightarrow |\neg q| \;\cap\; |\neg p| \Rightarrow |\neg q| \\ |p| \Rightarrow |q| \;\cap\; |\neg p| \Rightarrow |\neg q| \\ |p| \Rightarrow |\neg q| \;\cap\; |\neg p| \Rightarrow |q| \end{array} \right\}$$

b. $\lfloor ?\delta \rfloor = \{ \emptyset \}$

4.3 Strengthening the antecedent: a tension resolved

In the literature on conditionals there has always been a tension between two particular rules of inference, *strengthening the antecedent* (STA) and *simplification of disjunctive antecedents* (SODA).STA says that strengthening the antecedent of a conditional is truth-preserving. For instance, it allows us to infer from $q \rightarrow r$ that $(p \wedge q) \rightarrow r$, for any p. It is widely agreed, based on examples like the following, that this should *not* be a valid inference rule.

(44) If Mary has an essay to write, she will study late in the library.
$\not\rightsquigarrow$ If the library is closed and Mary has an essay to write, she will study late in the library.

Indeed, the fact that material and strict implication validate STA is often presented as a decisive shortcoming of these analysis, and as a basic argument in favor of an order-sensitive semantics, along the lines of Stalnaker (1968) and Lewis (1973), which does not validate STA.

SODA is a weaker inference rule than STA: it does not make a claim about strengthening the antecedent in general, but is concerned with one particular way of strengthening the antecedent: replacing a disjunctive antecedent with one of the individual disjuncts. For instance, it allows us to infer from $(p \vee q) \rightarrow r$ that $p \rightarrow r$ and $q \rightarrow r$. And this *is* widely agreed to be a valid inference pattern.

A standard order-sensitive semantics, however, does not validate SODA. Thus, there is a dilemma: one important feature of order-sensitive semantics is that it does not validate STA, but at the same time one of its weaknesses is that it does not validate SODA either. As we have seen, this dilemma is resolved if an order-sensitive semantics of conditionals is incorporated into the general framework proposed here, which does not equate semantic meaning with informative content, but takes inquisitive content into account as well. The inquisitive treatment of disjunction and implication validates SODA, while the stronger STA remains invalid.

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