Deontic and Epistemic Modals in Suppositional [Inquisitive] Semantics[*]

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Abstract

In Groenendijk and Roelofsen (2015) a suppositional semantics for implication is proposed within the general framework of inquisitive semantics. Our aim is to extend this semantic approach to epistemic and deontic modals, but, for the purposes of this short paper, we bracketed off inquisitive aspects of meaning. To illustrate the semantics we discuss a semantic solution to a Jackson inspired puzzle which involves the interaction of implication and both types of modals.

1 Introduction

Our starting point is the suppositional treatment of implication in Groenendijk and Roelofsen (2015), within the general framework of inquisitive semantics. Our overall aim is to extend the coverage of the semantics to a propositional language with epistemic and

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deontic modals. To distinguish between the modals, epistemic possibility and necessity are designated standardly with $\Diamond\varphi$ and $\Box\varphi$. Deontic permission and obligation are designated by $\Diamond\!\!\!v\,\varphi$ and $\boxed{v}\,\varphi$, marked with a v, which signifies that deontic rules can be *violated*.

Deontic modals have been studied within the general framework of inquisitive semantics (see Aher (2011, 2013)), and the same holds for epistemic modals (see Ciardelli et al. (2009, 2014); Roelofsen (2013)), but these analyses were carried out in two quite different extensions of the basic inquisitive system (see, e.g., Ciardelli et al. (2012, 2013)). Our aim is to bring implication, epistemic and deontic modals together within a single semantic system, and to study their interaction. To serve conciseness of this paper, we focus on suppositional and informative aspects of the meaning of these operators and bracket off inquisitive features, which explains the title of the paper.

As for epistemic modals, we follow the simple treatment of epistemic possibility in update semantics proposed in Veltman (1996), see also Groenendijk et al. (1996), and turn it into a suppositional approach with minimal changes, ignoring dynamic aspects. This means that we do not follow the alternative attentive treatment of epistemic possibility proposed in Ciardelli et al. (2014), which is arguably also simple, but adds attentive content to informative and inquisitive content as an additional aspect of meaning.

Ciardelli et al. argue specifically in relation to Veltman's analysis, that what is obtained there as *semantic* content of $\Diamond\varphi$, can be obtained from its purely attentive meaning and *pragmatic* principles. This point could apply for the analysis we will propose here. Also, more generally, it has been argued that the suppositional phenomena that we target can be accounted for pragmatically. To the extent that this is the case, we aim to flesh out an alternative semantic approach for comparison.

As for deontic modals, we follow Anderson (1967), which is particularly suitable in the context of this paper, since in such an approach deontic statements are looked upon as a kind of implication.[1] In doing so we are also in the footsteps of the analysis of deontic modals in Aher (2013) in radical inquisitive semantics, the predecessor of this system.

Epistemic and deontic modals, alongside implication, have been extensively studied in philosophy, logic and linguistics. There are many aspects of their meaning that our simple analysis will not touch upon, or that it may even be at odds with. We believe that the suppositional aspects of meaning that we study here for the basic case are orthogonal to other aspects of meaning that one may care to take into account, and that, in principle, the way we propose to deal with suppositional aspects in the basic case can be carried over to other more sophisticated non-suppositional analyses.

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[1]Recently, there has also been renewed interest in Andersonian deontic modals, see for example Barker (2010).

We will first discuss the notion of meaning that we use, and how we bracket off inquisitiveness. Next we define the notion of supposability we need for the interpretation of the operators $\rightarrow, \Diamond$ and $\diamondsuit$. Then we present and discuss the clauses for implication and epistemic possibility. In order to deal with deontic modals, we extend information states with 'deontic information'. Finally, we discuss examples involving the interplay of epistemic and deontic modals.

2 Background

Inquisitive semantics is information-based. Sentences are evaluated relative to information states, where information states are sets of possible worlds. For a standard propositional language, a possible world can be identified with a binary valuation for all atomic sentences in the language. We denote the set of all possible worlds by ω, and refer to the empty set as *the absurd state.*

In basic inquisitive semantics the interpretation for the language is given by defining recursively when a state σ *supports* a sentence φ, denoted as $\sigma \models \varphi$. The proposition expressed by φ consists of the set of all states that support it: $[\varphi] := \{\sigma \mid \sigma \models \varphi\}$. The union of all states that support φ is called the *informative content* of φ: $info(\varphi) := \bigcup[\varphi]$.

Crucially, due to the interpretation of disjunction in inquisitive semantics as: $\sigma \models \varphi \vee \psi$ iff $\sigma \models \varphi$ or $\sigma \models \psi$, it need not be the case that the state that equals $info(\varphi)$ is itself a state that supports φ. By definition, φ is *inquisitive* iff $info(\varphi) \notin [\varphi]$. Most typically, the classical tautology $p \vee \neg p$, where $info(p \vee \neg p) = \omega$, is inquisitive. Any state that supports $p \vee \neg p$ either consists only of worlds where p is true, or only of worlds where p is false. Hence, $info(p \vee \neg p)$, the union of all such states, is not itself a state that supports $p \vee \neg p$. For this reason, in inquisitive semantics, $p \vee \neg p$, abbreviated as $?p$, is not a tautological statement, but a non-tautological inquisitive *question.* However, in the disjunction-free fragment of a propositional language, no sentence φ is inquisitive, for any sentence in this fragment it holds that $info(\varphi) \in [\varphi]$, i.e., that no disjunction-free sentence is inquisitive.

In Groenendijk and Roelofsen (2015), the interpretation for the language is given by defining by simultaneous recursion the three semantic relations of (i) when a state *supports* a sentence, $\sigma \models^{+} \varphi$; (ii) when it *rejects* it, $\sigma \models^{-} \varphi$; and (iii) when it *dismisses a supposition* of it, $\sigma \models^{\circ} \varphi$. The *proposition* expressed by a sentence, is then determined by the triple: $[\varphi] := \langle[\varphi]^{+}, [\varphi]^{-}, [\varphi]^{\circ}\rangle$, where $[\varphi]^{+} := \{\sigma \mid \sigma \models^{+} \varphi\}$, and likewise for the other two components. Just focusing on $[\varphi]^{+}$, the (support) *informative content* of φ is defined as $info(\varphi) := \bigcup[\varphi]^{+}$. And, as in the basic case, a sentence is (support) inquisitive in case $info(\varphi) \notin [\varphi]^{+}$, but now with the proviso $info(\varphi) \neq \emptyset$. This still holds for $p \vee \neg p$, but unlike

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in the basic case, it now also holds for the sentence $\neg(p \wedge \neg p)$ that is classically equivalent with it. The equivalence does not hold in basic inquisitive semantics, where no sentence $\neg\varphi$ is inquisitive. In suppositional inquisitive semantics no sentence is inquisitive in the fragment of a propositional language that is both disjunction-free and conjunction-free. For any sentence φ in this fragment, it holds that if $\mathit{info}(\varphi) \neq \emptyset$, then $\mathit{info}(\varphi) \in [\varphi]^+$, i.e., $\mathit{info}(\varphi) \models^+ \varphi$.

The strategy in this paper is to focus on the informative and suppositional aspects of the meanings assigned to conditional statements, to statements expressing epistemic possibility or necessity, and to statements expressing deontic possibility or necessity, while completely ignoring inquisitive aspects of meaning, Technically, the way to do this is to restrict ourselves to a language that is disjunction-free and conjunction-free. So the only operators we are considering are $\{\neg, \rightarrow, \Diamond, \diamondsuit\}$, where $\square$ and $\boxed{v}$ are defined as the dual of $\Diamond$ and $\diamondsuit$ respectively.

The original motivation behind the suppositional enrichment of the basic system was to give a *semantic* account of certain intuitions concerning implication. We will show below how the semantics accounts for the most relevant intuitions that (1a) and (1b) *contradict* each other, and that (1c) *suppositionally dismisses* both (1a) and (1b).

(1) a. If Abe goes to the party, Bea will go. $\hspace{4cm} p \rightarrow q$
     b. If Abe goes to the party, Bea will not go. $\hspace{3cm} p \rightarrow \neg q$
     c. Abe won't go. $\hspace{7.5cm} \neg p$

2.1   Postulating properties of propositions

It is a foundational features of basic inquisitive semantics that *support is persistent*: if $\sigma \models \varphi$, then for all $\tau \subseteq \sigma \colon \tau \models \varphi$, and hence the absurd state supports any sentence: $\emptyset \models \varphi$, for all φ. In the suppositional system we **postulate** that *the absurd state dismisses every sentence*, and that *dismissal is fully persistent*. I.e., if a state dismisses a supposition of a sentence, then so does any more informed state. The intuitive motivation for these postulates is that in the semantics, suppositional dismissal of a sentence in a state is always caused by the impossibility to coherently make a certain *supposition* in that state, and as the information available in a state grows, it may become impossible to coherently make that supposition, and once that is the case, it remains the case by further growth of information, and it is impossible to coherently make any supposition in the absurd state.

It may typically play a role, also for support and rejection of a sentence φ in a state σ, that certain suppositions can be coherently made in σ. And $\varphi \rightarrow \psi$, $\Diamond\varphi$, and $\diamondsuit\varphi$ share this semantic feature. In case of $\varphi \rightarrow \psi$ and $\diamondsuit\varphi$, it will be a prerequisite for both support and rejection in σ that φ is supposable in σ, if not, suppositional dismissal

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results. As for $\Diamond\varphi$, it can be looked upon as just a check of the supposability that φ. Rejection of $\Diamond\varphi$ then requires non-supposability of φ.

Given the non-persistent feature of supposability, it may then typically hold that whereas φ is supposable in σ, this no longer holds in some more informed state $\tau \subset \sigma$. This implies that support and rejection are not fully persistent. A state σ can support or reject $\varphi \to \psi$, $\Diamond\varphi$, or $\lozenge\varphi$, whereas some stronger state $\tau \subset \sigma$ no longer does. However, we **postulate** that *support and rejection are persistent modulo suppositional dismissal*: for any φ, if $\sigma \models^+ \varphi$ and $\tau \subseteq \sigma$, then $\tau \models^+ \varphi$ or $\tau \models^\circ \varphi$, and similarly for rejection.

It may even hold for certain sentences, $\Diamond\varphi$ is a case in point, that whereas it is supported by a state σ, it is rejected by some more informed state $\tau \subset \sigma$, but then it cannot fail to be the case that τ simultaneously suppositionally dismisses the sentence. The same can happen in the opposite direction, here $\Box\varphi$ is a case in point, some state σ can reject a sentence, where some $\tau \subset \sigma$ supports it, but then τ also suppositionally dismisses it.

So, support and dismissal need not exclude each other, and the same holds for rejection and dismissal. However, as is to be expected, we **postulate** that *support and rejection are mutually exclusive*. Finally, we have seen that we take the absurd state to dismiss every sentence, but we ban it from support and rejection: we **postulate** that *the absurd state neither supports nor rejects any sentence*.

We have postulated that the set of states that dismiss a supposition of φ is never empty, it will always be the case that $\emptyset \in [\varphi]^\circ$. To distinguish *suppositional* sentences, we define:

Definition 1. [Suppositionality] φ is *suppositional* iff for some non-absurd state σ: $\sigma \models^\circ \varphi$

Below, and in the full inquisitive system, only sentences with $\to$, or $\Diamond$, or $\lozenge$ can be suppositional.

3   Implication in suppositional [inquisitive] semantics

Before we turn to the treatment of implication for the non-inquisitive fragment of a propositional language, we first state the semantic clauses for atomic sentences and negation, where this restriction of the language plays no role.

3.1   Atomic sentences and negation

For support/rejection of an atomic sentence p in a state σ, we require, as is to be expected, that p is true/false in every world $w \in \sigma$. Only, in view of the general constraint on

propositions that the absurd state does not support or reject any sentence, we add the condition that σ is not the absurd state. And we declare that σ suppositionally dismisses p iff it is absurd. So, under the definition of suppositionality, atomic sentences are not suppositional.

The clauses for negation are straightforward: support/rejection of $\neg\varphi$ switches to rejection/support of φ, and suppositional dismissal of φ is preserved by $\neg\varphi$, which means that $\neg\varphi$ is suppositional if and only if φ is. Clearly double negation holds: φ is equivalent with $\neg\neg\varphi$.[2]

Definition 2 (Atomic sentences).

$\sigma \models^+ p$ iff $\sigma \neq \emptyset$ and $\forall w \in \sigma: w(p) = 1$

$\sigma \models^- p$ iff $\sigma \neq \emptyset$ and $\forall w \in \sigma: w(p) = 0$

$\sigma \models^\circ p$ iff $\sigma = \emptyset$

Definition 3 (Negation).

$\sigma \models^+ \neg\varphi$ iff $\sigma \models^- \varphi$

$\sigma \models^- \neg\varphi$ iff $\sigma \models^+ \varphi$

$\sigma \models^\circ \neg\varphi$ iff $\sigma \models^\circ \varphi$

Just for simplicity, if the language contains only a single atomic sentence p, a state can include at most two different worlds, depicted in an obvious way in the figures below, which indicate what the maximal states are that support, reject, and suppositionally dismiss p.

Fig. 1: Supporting p Fig. 2: Rejecting p Fig. 3: Dismissing p

3.2 Supposability

As we said above, under our suppositional analysis, in evaluating support, rejection, and suppositional dismissal in a state σ of $\varphi \to \psi$, $\Diamond\varphi$, and $\diamondsuit\!\!\!\!\diamond\,\varphi$, it plays a role whether φ is *supposable* in σ. In our non-inquisitive setting, this central notion can be defined as follows.

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[2]Negation behaves very differently here from how it does in basic inquisitive semantics, where the maximal supporting states for φ and $\neg\varphi$ cannot overlap. Here, the maximal states that support and reject a suppositional sentence φ may overlap, but states in the overlap cannot fail to dismiss a supposition of φ. Also, in basic inquisitive semantics, negation blocks inquisitiveness, hence φ is not equivalent with $\neg\neg\varphi$ in case φ is inquisitive.

Definition 4 (Supposability for non-inquisitive φ).
φ is *supposable* in σ iff $\sigma \cap info(\varphi) \models^+ \varphi$.

Recall that since φ is not inquisitive, $info(\varphi) = \emptyset$ or $info(\varphi)$ is bound to support φ. And since support is persistent modulo suppositional dismissal, it can then only be the case that $\sigma \cap info(\varphi)$ does not support φ, because it dismisses a supposition of φ, i.e., $\sigma \cap info(\varphi) \models^{\circ} \varphi$. This will invariably be the case when $\sigma \cap info(\varphi) = \emptyset$, since the absurd state suppositionally dismisses every sentence. So, supposability of φ in σ implies that φ is consistent with σ. And for sentences φ which are not suppositional, i.e., which are only dismissed by the absurd state, supposability of φ in σ boils down to consistency with σ. This holds for example for an atomic sentence p and its negation.

However, when φ is itself suppositional, it can also hold that $\sigma \cap info(\varphi)$ does not support φ and $\sigma \cap info(\varphi) \models^{\circ} \varphi$, while $\sigma \cap info(\varphi) \neq \emptyset$, because a prerequisite for support of φ is the supposability of some sentence χ and χ is not supposable in $\sigma \cap info(\varphi)$. As we will see in more detail below, a case in point is $\Diamond(p \to q)$. Taking $\varphi = p \to q$, and $\chi = p$, we have an example of a sentence φ which requires the supposability of χ. So, according to the definition, for $\varphi = p \to q$ to be supposable in a state σ it should also hold that p is supposable in σ. So, this is then also a prerequisite for support of $\Diamond(p \to q)$.

3.3 Implication

Apart from inquisitive features which we leave out of the picture here, there are two crucial features in the semantic treatment of implication in suppositional inquisitive semantics. The first is that $\varphi \to \psi$ is deemed neither supported nor rejected, but dismissed in a state σ, when the antecedent φ is not supposable in σ, according to the definition given above.

The second feature lies in the non-classical way in which support and rejection in a state σ are treated. In both cases we consider the state $\tau = \sigma \cap info(\varphi)$ that results from *supposing* the antecedent φ in σ. Then if the supposed state τ supports the consequent ψ, the implication $\varphi \to \psi$ as a whole is taken to be supported by σ as such, and if τ rejects the consequent ψ, the implication $\varphi \to \psi$ as a whole is taken to be rejected by σ as such.

Finally, $\varphi \to \psi$ is not only dismissed in σ, when φ is not supposable in σ, but also when if we coherently can and do suppose the antecedent φ in σ, we reach a state where the consequent ψ is dismissed. This additional element in the dismissal of $\varphi \to \psi$ is only relevant when the consequent ψ is itself suppositional.

This leads to the following clauses for implication in suppositional inquisitive semantics, when restricted to the non-inquisitive fragment of the full propositional language.

Definition 5 (Implication in suppositional [inquisitive] semantics)**.**

$\sigma \models^+ \varphi \to \psi$ iff $\sigma \cap \mathit{info}(\varphi) \models^+ \varphi$ and $\sigma \cap \mathit{info}(\varphi) \models^+ \psi$

$\sigma \models^- \varphi \to \psi$ iff $\sigma \cap \mathit{info}(\varphi) \models^+ \varphi$ and $\sigma \cap \mathit{info}(\varphi) \models^- \psi$

$\sigma \models^\circ \varphi \to \psi$ iff $\sigma \cap \mathit{info}(\varphi) \not\models^+ \varphi$ or $\sigma \cap \mathit{info}(\varphi) \models^\circ \psi$

The three figures below jointly depict the meaning of the simplest example of an implication $p \to q$. The light-gray area in Fig. 4 corresponds to the maximal state that supports $p \to q$. Any substate of it will still support it, except when it is completely included in the darker area, in which case we end up in a state that suppositionally dismisses it. The maximal dismising state is shown in Fig. 6. All substates of it also dismiss $p \to q$, including the absurd state. Fig. 5 is read similarly to Fig. 4.
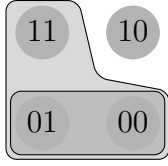

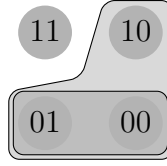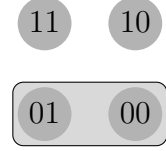Fig. 4: Supporting $p \to q$ Fig. 5: Rejecting $p \to q$ Fig. 6: Dismissing $p \to q$

It is not difficult to see that the states that reject $p \to q$ are the same as the states that support $p \to \neg q$, and the states that reject $p \to \neg q$ support $p \to q$. So, this accounts for the intuition that (1a) and (1b) contradict each other. Also the states that dismiss $p \to q$ are the same as those that dismiss $p \to \neg q$. The maximal state that does so is also the maximal state that supports $\neg p$. We thereby account for the intuition that (1c) suppositionally dismisses both (1a) and (1b).

4   Epistemic possibility as a supposability check

In Veltman (1996) a semantics for an epistemic possibility operator $\Diamond$, pronounced as *might*, is presented in the framework of update semantics. The update of an information state σ with a sentence $\Diamond \varphi$ either succeeds or fails. It only succeeds when φ is consistent with the information in σ, otherwise it fails. In the first case the update of σ with $\Diamond \varphi$ has no effect, it leaves the state as it is. In the second case the update of σ with $\Diamond \varphi$ would lead to the absurd state.

We say *would* lead to the absurd state, because the addressee of an utterance of $\Diamond \varphi$ whose information state is inconsistent with φ will of course not actually perform the update. What she will rather do is signal in response to the speaker that, apparently

unlike his state, her state is inconsistent with φ. It is for this reason that $\Diamond\varphi$ is referred to by Veltman as a 'consistency test'. The primary conversational function of an utterance of $\Diamond\varphi$ is for a speaker to check with the hearer(s) whether they agree that φ is consistent with their current information states.

There could be other conversational functions of $\Diamond\varphi$, such as drawing attention to a certain possibility or possibilities, but these are not directly modeled in Veltman's semantics, and the same holds for the suppositional semantics proposed here. In Ciardelli et al. (2009, 2014) it is precisely this attentive effect of $\Diamond\varphi$ that is taken as the hallmark of its semantics, and its conversational function as a consistency check is explained pragmatically. The attentive analysis gives rise to a semantic explanation for free choice effects for disjunctive sentences which involve epistemic possibility. That is also within the power of our suppositional semantics, for both epistemic and deontic possibility, but this can only be shown in the full inquisitive version of the semantics.

In the spirit of Veltman's proposal, we treat $\Diamond\varphi$ as a *supposability check* relative to information states.[3] We let a state σ support $\Diamond\varphi$ if φ is supposable in σ, according to the definition of supposability given above. In case φ is not supposable in σ, we deem $\Diamond\varphi$ to be dismissed in σ. If, unlike φ, $\neg\varphi$ *is* supposable in σ, then σ rejects $\Diamond\varphi$. When neither φ nor $\neg\varphi$ is supposable, we let σ dismiss $\Diamond\varphi$. In this case, dismissal occurs either because σ is absurd or because a supposition of φ fails in σ. When φ is not suppositional, checking the supposability of φ in σ boils down to checking consistency of φ with σ.

Definition 6 (Epistemic possibility in suppositional [inquisitive] semantics)**.**

$\sigma \models^+ \Diamond\varphi$ iff $\sigma \cap \mathit{info}(\varphi) \models^+ \varphi$

$\sigma \models^- \Diamond\varphi$ iff $\sigma \cap \mathit{info}(\varphi) \not\models^+ \varphi$ and $\sigma \cap \mathit{info}(\neg\varphi) \models^+ \neg\varphi$

$\sigma \models^\circ \Diamond\varphi$ iff $\sigma \cap \mathit{info}(\varphi) \not\models^+ \varphi$ or $\sigma \cap \mathit{info}(\varphi) \models^\circ \varphi$

The suppositional nature of the semantics proposed for $\Diamond\varphi$ manifests itself in three ways. First, because it constitutes a supposability check. Secondly, for any informative sentence φ, it holds that $\Diamond\varphi$ is suppositional, so there is a non-absurd state that dismisses it. Thirdly, rejection of $\Diamond\varphi$ implies dismissal, in line with the postulate that rejection is persistent modulo suppositional dismissal.

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[3]The semantics as such is neutral as to who beholds the information state relative to which we evaluate a sentence. There is a lively debate going on concerning so-called *contextualist* analyses of epistemic possibility. Such analyses share the idea that the state relative to which $\Diamond\varphi$ is evaluated need not be the state of the speaker/hearer, but that, as (Yanovich, 2013, p. 29) puts it, it can be "some body of knowledge determined by the evaluation world and the context of utterance". Our semantics as such does not model this, but there is also nothing that excludes extending it in a way that it does. And when this is done, a contextualist analysis may profit from the fact that it is then not consistency, but supposability relative to such a contextual body of information that the semantics will cover.

Below we depict the meaning of the simple example $\Diamond p$, only reckoning with a single atomic sentence in the language. It can be compared with the pictures above of the meaning of p as such. Note that $\Diamond p$ is supported in the ignorant state that consists of all worlds. What is not completely clear from the picture as such is that not all states that dismiss $\Diamond p$ also reject it, the absurd state is the exception. For this example, and the one given next, rejection of $\Diamond \varphi$ and φ is fully the same. That is not always so in the full language, but what does hold generally is that $info(\neg \Diamond \varphi) = info(\neg \varphi)$.

Fig. 7: Supporting $\Diamond p$ Fig. 8: Rejecting $\Diamond p$ Fig. 9: Dismissing $\Diamond p$

We have seen in the discussion of $\Diamond \varphi$, that except when φ is suppositional, it boils down to a consistency check, as in Veltman's update semantics. Since $p \to q$ is suppositional, we can observe the additional suppositional features of $\Diamond \varphi$ by inspecting the meaning of $\Diamond(p \to q)$, depicted below. A state σ supports $\Diamond(p \to q)$ when $p \to q$ is supposable in σ. This is so as long as σ contains a world w: $w(p) = w(q) = 1$. As soon as that is not the case, $\Diamond(p \to q)$ is dismissed in σ. Furthermore $\Diamond(p \to q)$ is not only dismissed but also rejected in σ when $\neg(p \to q)$, which is equivalent with $p \to \neg q$, is supposable in σ. This is so as long as σ contains a world w: $w(p) = 1$ and $w(q) = 0$. In particular when σ only contains worlds w: $w(p) = 0$, i.e., when σ supports $\neg p$, σ neither supports, nor rejects $\Diamond(p \to q)$, but only dismisses it.
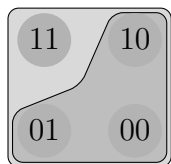
Fig. 10: Supporting $\Diamond(p \to q)$ Fig. 11: Rejecting $\Diamond(p \to q)$ Fig. 12: Dismissing $\Diamond(p \to q)$
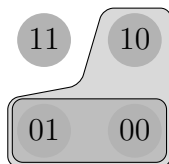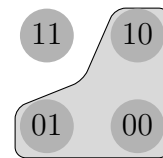
4.1   Epistemic necessity as a non-supposability check

Unfortunately, we do not have sufficient space here to include a proper discussion of the semantics that results for $\Box \varphi$ when defined as the dual of $\Diamond \varphi$. In a nutshell, parallel to the conversational function of $\Diamond \varphi$ as a supposability check for φ, the semantics of $\Box \varphi$ turns it into *non-supposability check for $\neg \varphi$*. What the semantics suggests is that the

primary conversational function of an utterance of $\Box\varphi$ is for a speaker to check with the hearer(s) whether they agree that $\neg\varphi$ *is not supposable*. The less information a hearer has, the easier it becomes to reject $\Box\varphi$, like it is the case for support of $\Diamond\varphi$.

This has also interesting effects when we consider the interplay of epistemic necessity and implication, as in the simple case of $p \to \Box q$, which comes out equivalent with $\Box(p \to q)$. This modal implication is much easier to reject than $p \to q$, having no information concerning p and q already suffices in case of $p \to \Box q$, because this is enough to support $p \to \Diamond\neg q$, but that is not enough to support $p \to \neg q$, which is needed to reject $p \to q$.[4]

5 Deontic suppositional [inquisitive] semantics

To be able to state an information-based semantics for deontic modalities, we enrich our information states in a way that they can also embody *deontic information*.

5.1 Worlds with a deontic dimension

We will define worlds (and information states) in such a way that they have both an *ontic dimension* and a *deontic dimension*. The ontic dimension of a world is given by an *ontic possibility*: a possible assignment of truth values to the atomic sentences. So, what used to be called the set of possible worlds ω is now called the set of ontic possibilities. The deontic dimension of a world is given by a *deontic possibility*: a function d which determines for every possible world v whether a rule is violated in v.[5] Δ denotes the set of all deontic possibilities.

For the purposes of this paper, we make two assumptions concerning possible worlds. The first is the innocent assumption that two worlds cannot *only* differ in their ontic dimension. If this could be the case, then the assignment of a truth value to an atomic sentence in a world could depend on its deontic dimension, whence an atomic sentence could carry deontic information.

The second assumption is that whether a world v is characterized by a deontic possibility d as being a world where some deontic rule is violated is *independent of the deontic dimension of v*.[6] This means that, technically, the set of deontic possibilities Δ can be

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[4]See von Fintel and Gillies (2010) for an overview of recent literature on epistemic *must*.

[5]What these rules are and how we learn them is left implicit. We only model the results of having learned them.

[6]This assumption is not innocent. We disregard the realistic possibility that a world v is a violation because the actual deontic rules that hold in v violate some more general deontic rule, set by a higher authority. In not reckoning with this possibility we typically restrict ourselves to the most basic situation

taken to be the set of functions d which specify for each *ontic possibility* $o \in \omega$ whether it violates a deontic rule. Indirectly, the deontic dimension d of a world w then also determines which *worlds v* are such that a deontic rule is violated in v, viz., all worlds v that have an ontic possibility o as their ontic dimension such that o is a violation according to the deontic dimension d of w.

Under these two assumptions concerning the nature of worlds, we can identify the set of possible worlds with the Cartesian product of the set of ontic possibilities ω and the set of deontic possibilities Δ.[7]

Definition 7 (Possible worlds as pairs of an ontic and a deontic possibility)**.**

- The set of *ontic possibilities* ω is the set of all valuations for the atomic sentences.

- The set of *deontic possibilities* Δ is the set of all functions d that specify for each ontic possibility $o \in \omega$ whether o is a violation of some deontic rule or not.

- The set of *possible worlds* $\Omega = \omega \times \Delta$.

The two assumptions we made concerning worlds justify the following two notation conventions.

Definition 8 (Two notation conventions)**.**

- Let w be a world with ontic possibility o, and p an atomic sentence, then by $w(p)$ we mean the truth value assigned by o to p.

- Let w be a world with deontic possibility d, then $bad(w)$ denotes the set of all worlds $v \in \Omega$ with ontic possibility o such that o is a violation according to d.

By the first convention, the formulation of the clauses for atomic sentences (and the other clauses) can remain as it is.[8]

The second convention introduces the auxiliary notion of a *deontic predicate bad*, which facilitates the statement of the semantic clauses for deontic modalities. Jointly these two conventions guarantee that in the semantic clauses for deontic modalities we do not have to explicitly refer to one of the two components of which our worlds consist.[9]

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where only a single authority is in play.

[7]The contents of ω and Δ are fully determined by the set of atomic sentences in the language.

[8]This implies that no sentence in the language without a deontic modality can provide deontic information.

[9]In principle, this makes the formulation of the semantics neutral with respect to the (non-innocent) assumption that gives rise to constructing worlds as pairs consisting of an ontic and a deontic possibility.

5.2 States with a deontic dimension

As usual, we take states to be sets of worlds, i.e., $\sigma \subseteq \Omega = \omega \times \Delta$, where the ontic dimension of a state σ is determined by the ontic dimension of the worlds in σ, i.e., the ontic dimension of a state is characterized by a subset of the set of ontic possibilities ω. Likewise, the deontic dimension of a state σ is determined by the deontic dimension of the worlds in σ, i.e., the state's deontic dimension is characterized by a subset of the set of deontic possibilities Δ.

We also make an assumption concerning information states, viz., that the ontic and deontic dimension of an information state σ are *epistemically independent*. This is enforced by requiring that for every ontic possibility o that occurs in some world $w \in \sigma$, and for every deontic possibility d that occurs in some world $v \in \sigma$, there is a world $u \in \sigma$ such that $u = \langle o, d \rangle$. The conceptual motivation for the epistemic independence of ontic and deontic information is that by obtaining information about the actual state of affairs, we do not obtain information about which possible states of affairs are violations, and vice versa.

Definition 9 (Information states with an ontic and deontic dimension)**.**
An *information state* σ is a subset of the set of worlds $\Omega = \omega \times \Delta$ such that for any two worlds $w, v \in \sigma$ where $w = \langle o, d \rangle$ and $v = \langle o', d' \rangle$, there exists some world $u \in \sigma$ such that $u = \langle o, d' \rangle$.

We can view a state as a pair of a set of ontic possibilities and a set of deontic possibilities.

Fact 1 (Two views of states)**.**
For any information state σ there is some $\alpha \subseteq \omega$ and some $\delta \subseteq \Delta$ such that $\sigma = \alpha \times \delta$.

We use this fact in depicting a state σ as a two-dimensional matrix where the rows are determined by the ontic possibilities in σ, and the columns by its deontic possibilities. A cell in the matrix determined by a row headed with an ontic possibility o and a column headed by a deontic possibility d then reflect the value of $d(o)$, violation/not, which we indicate by coloring the cell red/green.

Alternatively, we could consider worlds to be primitive entities in our model; let the model contain a valuation function for the atoms relative to worlds that takes care of their ontic dimension; and let the deontic predicate *bad*, which clearly has the flavour of an accessibility relation, be the component in the model that takes care of the deontic dimension of worlds by specifying for each world w which worlds v are such that a deontic rule that holds in w is violated in v. So, our models then become more like models in standard deontic logic. In principle this may lead to an enrichment of the semantics presented here, but it would also rob us of the simple pictures of states that we can use here to illustrate the semantics.

5.3 Semantics for permission

A central notion in the semantics, which makes use of the deontic predicate *bad*, characterizes when a *world* w in a state σ is such that according to the deontic information σ embodies, w is a world where some deontic rule is violated.

Definition 10 (Violation worlds in a state)**.** Let σ be a deontic information state, $w \in \sigma$.

w is a *violation world* in σ iff $\forall v \in \sigma : w \in bad(v)$.

Relative to the matrices we will use to depict states, a world $w \in \sigma$ is a violation world in σ if the whole row in the matrix that is headed by the ontic possibility o in w is colored red (and if the whole row for o is colored green, we may call w a non-violation world).

Our suppositional semantics for the deontic modality $\diamondsuit \varphi$ can be stated informally as:

- Our information supports $\diamondsuit \varphi$ if, when supposing that φ, no world is a violation world.

- Our information rejects $\diamondsuit \varphi$ if, when supposing that φ, every world is a violation world.

- Our information neither supports nor rejects but dismisses $\diamondsuit \varphi$ if φ is not supposable.

The Andersonian nature of the semantics can be observed from the fact that the support, rejection, and dismissal conditions specified informally above for $\diamondsuit \varphi$, are the same as the corresponding conditions for the implication $\varphi \to safe$, where *safe* is a special sentence that is supported in a state σ if no world in σ is a violation world in σ, rejected if every world in σ is a violation world in σ, and neither supported nor rejected, but dismissed, when σ is absurd. Of course, we take it that implication is interpreted here in the suppositional way defined above. The modality $\diamondsuit \varphi$ will thus inherit the suppositional features of implication. Likewise it will inherit the non-standard features that support and rejection of implication have in our semantics.

Without introducing such a special sentence *safe*, the semantics for $\diamondsuit \varphi$ can be directly stated in a way that immediately reflects the informal characterization given above.

Definition 11 (Permission in suppositional [inquisitive] semantics)**.**

$\sigma \models^+ \diamondsuit \varphi$ iff $\sigma \cap info(\varphi) \models^+ \varphi$ and $\forall w \in \sigma \cap info(\varphi) : \forall v \in \sigma : w \notin bad(v)$

$\sigma \models^- \diamondsuit \varphi$ iff $\sigma \cap info(\varphi) \models^+ \varphi$ and $\forall w \in \sigma \cap info(\varphi) : \forall v \in \sigma : w \in bad(v)$

$\sigma \models^\circ \diamondsuit \varphi$ iff $\sigma \cap info(\varphi) \not\models^+ \varphi$

The meaning of $\diamondsuit p$ is depicted by Figs. 13-15. For brevity we assume that p is the only atomic sentence in the language. Then there are only two ontic possibilities which determine the two rows in the matrices, and there are four deontic possibilities which

determine the four columns and color the ontic possibilities in the rows either red (violation) or green (no violation). So, when you ignore the demarcated areas in the matrices, what is depicted is the deontically ignorant state.

σ_1	d_1	d_2	d_3	d_4
o_1	1	1	1	1
o_2	0	0	0	0

Fig. 13: Supporting $\diamondsuit p$

σ_2	d_1	d_2	d_3	d_4
o_1	1	1	1	1
o_2	0	0	0	0

Fig. 14: Rejecting $\diamondsuit p$

σ_3	d_1	d_2	d_3	d_4
o_1	1	1	1	1
o_2	0	0	0	0

Fig. 15: Dismissing $\diamondsuit p$

The worlds as such are not depicted in the matrices. There are eight of them corresponding to all pairs $\langle o_i, d_j \rangle$ that can be reconstructed from the picture. Note that by the definition of a state, a subset of those eight pairs only counts as a state when in its picture either a whole row or a whole column from the picture of the ignorant state is missing. All demarcated areas in the three matrices satisfy this condition.

If we consider $info(p)$, then we obtain the four worlds that remain when we ignore the second row. Now first consider when p is not supposable in σ. This is only the case if none of those four worlds in $info(p)$ is in σ, i.e., when only the second row remains. This is depicted in Fig. 15, where the demarcated area in the matrix corresponds to the maximal state that suppositionally dismisses $\diamondsuit p$.

Next consider support of $\diamondsuit p$. The maximal substate of the ignorant state that supports $\diamondsuit p$ corresponds to the largest demarcated area in Fig. 13.[10] I.e., only the four worlds remain with deontic possibility d_1 or d_2, which have in common that they color the ontic possibility o_1 where p is true green, as opposed to d_3 and d_4 which color it red. If in the state with these four worlds with deontic possibility d_1 or d_2 we suppose that p, this leaves us only with the two worlds $\langle o_1, d_1 \rangle$ and $\langle o_1, d_2 \rangle$. In the state consisting of those two worlds no world is a violation world. This means that by the joint force of the support clause for $\diamondsuit p$ and the definition of the deontic predicate *bad*, the state that corresponds to the largest demarcated area in Fig. 13 supports $\diamondsuit p$, and it is the maximal state that does. Any substate of it (delete a column or a row within the demarcated square) still supports $\diamondsuit p$, except when we delete the first row, which would leave us with the demarcated sub-area, where we end up in a state that suppositionally dismisses $\diamondsuit p$.

The rejection of $\diamondsuit p$, i.e., the support of $\neg \diamondsuit p$, shown by Fig. 14, is explained analogously. Note that in the maximal states that support $\diamondsuit p$ and reject $\diamondsuit p$, that correspond

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[10]For reasons made explicit in von Wright (1968), we don't want a deontically ignorant state to support $\diamondsuit p$, and thus define permission as is standard in the Andersonian tradition, i.e., as strong permission.

to $info(\diamondsuit p)$ and $info(\neg\diamondsuit p)$, not all deontic possibilities occur, but all ontic possibilities do occur. This means that $\diamondsuit p$ and $\neg\diamondsuit p$ only provide deontic information, and do not provide ontic information.

5.4   Semantics for obligation

When we define $\boxed{v}\varphi$ as the dual of $\diamondsuit\varphi$, we obtain the following fact.

Fact 2 (Obligation in suppositional [inquisitive] semantics)**.**

$\sigma \models^{+} \boxed{v}\varphi$ iff $\sigma \cap info(\neg\varphi) \models^{+} \neg\varphi$ and $\forall w \in \sigma \cap info(\neg\varphi)\colon \forall v \in \sigma\colon w \in bad(v)$

$\sigma \models^{-} \boxed{v}\varphi$ iff $\sigma \cap info(\neg\varphi) \models^{+} \neg\varphi$ and $\forall w \in \sigma \cap info(\neg\varphi)\colon \forall v \in \sigma\colon w \notin bad(v)$

$\sigma \models^{\circ} \boxed{v}\varphi$ iff $\sigma \cap info(\neg\varphi) \not\models^{+} \neg\varphi$

Informally, this amounts to the following characterization of obligation:

- Our information supports $\boxed{v}\varphi$ if, when supposing that $\neg\varphi$, every world is a violation world.

- Our information rejects $\boxed{v}\varphi$ if, when supposing that $\neg\varphi$, no world is a violation world.

- Our information neither supports nor rejects, but dismisses $\boxed{v}\varphi$ if $\neg\varphi$ is not supposable.

The picture of meaning of $\boxed{v}p$ is given by Figs. 16-18. Note that in order to obtain nice pictures we changed the order of the deontic possibilities in the columns as compared to Figs. 13-15. This means that, unlike what is suggested at a first glance, support of $\diamondsuit p$ and $\boxed{v}p$ do not exclude each other: Fig. 13 and Fig. 16 share the column headed by d_2.

σ_4	d_1	d_3	d_2	d_4
o_1	1	1	1	1
o_2	0	0	0	0

σ_5	d_1	d_3	d_2	d_4
o_1	1	1	1	1
o_2	0	0	0	0

σ_6	d_1	d_3	d_2	d_4
o_1	1	1	1	1
o_2	0	0	0	0

Fig. 16: Supporting $\boxed{v}p$ Fig. 17: Rejecting $\boxed{v}p$ Fig. 18: Dismissing $\boxed{v}p$

Before we turn to the illustrations below, consider a simple case of epistemic and deontic modal interaction called *Kant's law* in McNamara (2014). He argues that $\boxed{v}p$ ought not to imply $\Diamond p$. We obtain this straightforwardly. Recall that for a state to support $\Diamond p$, p

must be supposable. But $\boxed{v}p$ requires that $\neg p$ is supposable. The disjoint supposability conditions allow us to construct a counter-example to Kant's law: consider a state σ where $\neg p$ holds and every world in σ is a violation world, this state supports $\boxed{v}p$ but not $\Diamond p$ (nor $\overline{\Diamond} p$, covering the deontic version as well).

6   Illustrations

We illustrate the semantics by considering a situation inspired by a puzzle for deontic modals in Jackson (1985). Imagine that you receive a request to write a review and so face the issue whether you ought to accept the request to write a review or not, i.e., whether $\boxed{v}p$ holds. We entertain the intuition that there is a strong tendency to answer affirmatively, unless there is a good reason against it, such as when you know already that you will not write the review. We can thus motivate the deontic rule in (2a) that specifically holds in academia.[11] We take it that the situation is also governed by the instantiation in (2b) of the general rule that accepting to do something obligates you to do it. The relevant issue then becomes whether (2c) or (2d) holds. Note that (2a) is particularly suitable for studying the interaction between implication and both types of modals.

(2)   a.   If it is possible that you write the review, you ought to accept the request.
            $\Diamond q \rightarrow \boxed{v}p$
      b.   If you accept the request to write a review, you ought to write it.
            $p \rightarrow \boxed{v}q$
      c.   It is possible that you write the review.
            $\Diamond q$
      d.   It is *not* possible that you write the review.
            $\neg \Diamond q$

The case is semantically unproblematic when (2c) holds where, intuitively, you *ought to accept the request and write the review*. The more puzzling case, at least for certain analyses of implication and deontic modals, arises when (2d) holds, which also means that $\neg q$ holds according to your information. A case in point is Jackson's Dr. Procrastinate who never finishes assignments. In her case it intuitively holds that she *ought not to*

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[11]Ciardelli et al. (2014) argue that in English *might*-sentences in the antecedent of an implication are difficult to interpret, and a paraphrase of (2a) with *might* can indeed sound odd. But an analogous sentence like: "If Maria might apply for the job, I ought not to do so," seems perfectly fine. The issue is also dicussed in Yanovich (2013), and he provides an example with the same flavour: "If Bill might be in Boston, we should send a team there."

accept.

This becomes particularly transparent when we observe that, intuitively, rule (2b), where p makes q obligatory, implies that if you do not bring about q, then you ought not to bring about p either, as that would necessarily lead to a violation of the obligation.[12] In other words, $p \to \boxed{v} q$ intuitively implies that $\neg q \to \boxed{v} \neg p$. By similar reasoning, the converse holds also, suggesting that $p \to \boxed{v} q$ and $\neg q \to \boxed{v} \neg p$ are equivalent. We will refer to entailments of this form as instances of *modal contraposition*. When a semantics gives rise to modal contraposition for (2a), then if (2d) holds, it follows that, as desired, $\boxed{v} \neg p$ holds.

Contraposition of (2a) is not a common feature of standard deontic modal logics. For example, it does not hold for the standard deontic logics in von Wright (1951). However, it does hold for certain plausible deontic modal logics which are in this tradition.[13]

In the widely accepted Kratzer semantics for modals and implication,[14] there is a puzzle because the antecedent of an implication is taken to restrict the modal base (a set of worlds) for the overt modal in its consequent.[15] Then, when $\neg \Diamond q$ holds, restricting to the antecedent $\Diamond q$ of (2a) results in the modal base for the consequent $\boxed{v} p$ being the empty set. But then, from (2d) and (2a), it follows that $\boxed{v} p$ vacuously holds. So, due to the nature of the semantics, when $\Diamond q \to \boxed{v} p$ holds, it has the same effect as $\boxed{v} p$ itself. This counter-intuitively predicts that regardless of whether $\Diamond q$ or $\neg \Diamond q$ holds, $\boxed{v} p$ holds and, thus, $\boxed{v} \neg p$ cannot. To escape from this semantic predicament one can appeal to pragmatic reasoning regarding vacuous truth, but we aim to explore a *semantic solution*.

6.1 The deontic situations according to our system

With respect to the two atomic sentences p and q, there are four ontic possibilities we have to reckon with, viz., 11, 10, 01 and 00. To reduce the number of deontic possibilities in consideration from 16 to 8, we ignore the intuitively strange ontic possibility 01, where

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[12]Although it fits the concrete example we are discussing, it is not essential that p and q are situations you can 'bring about'. From: "If your mother is seriously ill, you ought not to go on a holiday", it also follows that: "If you go on a holiday, it ought not to be the case that your mother is seriously ill". Regarding this example, the only thing you can 'bring about' is collecting more information on the state of health of your mother.

[13]More precisely, as Frank Veltman has pointed out to us, modal contraposition of (2b) holds if implication is strict and the accessibility relation is transitive and weakly symmetric, i.e., if xRy and yRz, then zRy. Weak symmetry corresponds to the intuitive idea that all 'ideal worlds' are equally ideal.

[14]See Kratzer (1977, 1979, 2012) among others.

[15]There's an alternative approach put forward in Frank and Kamp (1997) and Kaufmann and Schwager (2011) which always assumes a covert necessity operator over the consequent. See Kratzer (2012) for arguments why such a solution is problematic for similar examples.

you do not accept the request, but write a review anyway. This restriction plays no other role in the story than brevity.

Concerning ontic information, there are two distinct relevant cases of information states σ to be distinguished: the case where the ontic possibilities in σ are $\{11, 10, 00\}$ and the case where the ontic possibilities in σ are $\{10, 00\}$. In both of these two cases one can choose to neither accept nor write. The first can be considered the standard case, as one can accept the request and write, but also accept the request and neglect to write. The second case applies to people like Dr. Procrastinate who never finish writing tasks, so when she accepts, she will not write. The deontically ignorant states that pertain to the two relevant situations are depicted by the matrices for σ_7 and σ_8.

σ_7	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
o_1	11	11	11	11	11	11	11	11
o_2	10	10	10	10	10	10	10	10
o_3	00	00	00	00	00	00	00	00

σ_7: Ignorant state for $11, 10, 00$

σ_8	$d_{1,3}$	$d_{2,4}$	$d_{5,7}$	$d_{6,8}$
o_2	10	10	10	10
o_3	00	00	00	00

σ_8: Ignorant state for $10, 00$

We will show that in our semantics, if we assume the two deontic rules (2a) and (2b) to apply, we get a straightforward semantic account for our intuitions concerning the two ontic situations we distinguished above. We will see that in our suppositional semantics, the two rules (2a) and (2b) suffice to account for the intuition that when the ontic possibilities in a state σ are $\{11, 10, 00\}$, then under the rules it holds that $\boxed{v}\, p$ and $\boxed{v}\, q$, whereas when the ontic possibilities in σ are $\{10, 00\}$, then according to the rules, it holds that $\boxed{v}\, \neg p$. In order to check this, we first need to see which rules govern the situation, i.e., whether either of the rules is dismissed.

6.2 The standard case

First, consider the situation where the ontic possibilities in σ are $\{11, 10, 00\}$. In this case, neither of the two deontic rules (2a) and (2b) is dismissed. To start with the latter, according to the dismissal clause for implication, $p \to \boxed{v}\, q$ is not dismissed in σ iff (i) p is supposable in σ and (ii) when we suppose p in σ, then $\boxed{v}\, q$ is not dismissed. According to the dismissal clause for deontic necessity, the latter is the case iff $\neg q$ is supposable. So, $p \to \boxed{v}\, q$ is not dismissed in σ iff we can suppose that both p and $\neg q$ in σ. This holds in

the standard case, due to the presence of the ontic possibility 10, and still holds in the Dr. Procrastinate case with ontic possibilities $\{10, 00\}$.

For σ to support $p \to \boxed{v} q$, it should hold that if we do suppose that both p and $\neg q$, which leaves us only with the single ontic possibility 10 in σ, this ontic possibility should be colored red by all deontic possibilities in σ. So, in order to arrive at the maximal state with the ontic possibilities $\{11, 10, 00\}$ where $p \to \boxed{v} q$ is supported, we have to eliminate the four deontic possibilities such that they do not color 10 red from the picture of the deontically ignorant state σ_7, and then we arrive at the picture of the deontic state σ_9. Note that σ_9 neither supports $\boxed{v} p$ nor $\boxed{v} q$.

σ_9	d_1	d_2	d_3	d_4
o_1	11	11	11	11
o_2	10	10	10	10
o_3	00	00	00	00

$$\sigma_9 \models^+ p \to \boxed{v} q$$

σ_{10}	d_2	d_4	d_6	d_8
o_1	11	11	11	11
o_2	10	10	10	10
o_3	00	00	00	00

$$\sigma_{10} \models^+ \Diamond q \to \boxed{v} p$$

σ_{11}	d_2	d_4
o_1	11	11
o_2	10	10
o_3	00	00

$$\sigma_{11} \models^+ p \to \boxed{v} q, \Diamond q \to \boxed{v} p$$

It follows from similar considerations that $\neg q \to \boxed{v} \neg p$ is supported in σ iff both $\neg q$ and $\neg\neg p$, i.e., p, are supposable in σ and that if we do suppose both $\neg q$ and p in σ, leaving us with only the single ontic possibility 10 in σ, it should be colored red in all deontic possibilities in σ. In other words, $p \to \boxed{v} q$ and $\neg q \to \boxed{v} \neg p$ have the same dismissal and support conditions, and in fact the same holds for their rejection conditions. So, we obtain the following fact.

Fact 3 (Modal contraposition). $p \to \boxed{v} q$ and $\neg q \to \boxed{v} \neg p$ are equivalent.

This fact becomes particularly enlightening when we consider the Dr. Procrastinate case where the ontic possibilities are $\{10, 00\}$, whence the antecedent of $\neg q \to \boxed{v} \neg p$ is supported. But for now we continue with the standard case where the ontic possibilities are such that this is not the case.

Next we consider the academia-specific rule (2a). According to the dismissal clause for implication, $\Diamond q \to \boxed{v} p$ is not dismissed in σ when it allows for the ontic possibilities $\{11, 10, 00\}$: $\Diamond q$ is supposable, since q is, and supposing $\Diamond q$ in σ, which leaves us in σ as such, does not dismiss $\boxed{v} p$, since $\neg p$ is supposable in σ. Then, for σ to support $\Diamond q \to \boxed{v} p$ it has to hold that σ as such supports $\boxed{v} p$. I.e., when we suppose that $\neg p$ in σ, which leaves us only with the ontic possibility 00 in σ, it should be colored red by

every deontic possibility in σ. So, in order to arrive at the maximal state with the ontic possibilities $\{11, 10, 00\}$ where $\Diamond q \to \boxed{v} p$ is supported, we have to eliminate the four deontic possibilities that do not color 00 red from the picture of the deontically ignorant state σ_7, and we then arrive at the picture of the deontic state σ_{10}.This state supports $\boxed{v} p$, but not $\boxed{v} q$.

However, we do arrive at the desired result that σ supports both $\boxed{v} p$, and $\boxed{v} q$, if we do not consider the two rules separately, but jointly. If we do so, we arrive at the picture of the state σ_{11}, where only two deontic possibilities remain, which is the maximal state that supports both deontic rules (2a) and (2b) at the same time. If we now inspect the picture of state σ_{11}, it can easily be seen that not only $\boxed{v} p$, but also $\boxed{v} q$ is supported in it. If we suppose that $\neg p$, which leaves us only with the ontic possibility 00 in σ, we see that it is colored red by both deontic possibilities in σ; and if we suppose that $\neg q$, which leaves us only with the ontic possibility 10, we see that it is also colored red by both deontic possibilities in σ. As desired, if we assume both deontic rules (2a) and (2b), and the ontic possibilities in state σ are $\{11, 10, 00\}$, then you ought to accept the request and write.

The state σ_{11} has the interesting feature that it still reckons with the deontic possibility that accepting and writing is a violation, next to the possibility that it isn't. This is as it should be. It reflects that there could be additional deontically relevant features in the situation that we have not reckoned with that would also turn worlds w such that $w(p) = w(q) = 1$ into violation worlds, leading to a situation of *deontic conflict*. E.g., writing the review may interfere with other academic duties.

6.3   The Dr. Procrastinate case

Consider the case where the ontic possibilities in σ are $\{10, 00\}$. Here it is the case that the deontic rule in (2a), $\Diamond q \to \boxed{v} p$, is suppositionally dismissed, because the antecedent is not supposable in σ: $info(\Diamond q) = \omega$, whence $\sigma \cap info(\Diamond q) = \sigma$, and σ does not support $\Diamond q$, but rejects and suppositionally dismisses it. But, as was already observed above, the deontic rule (2b) is not suppositionally dismissed in σ when the ontic possibilities in σ are $\{10, 00\}$, since it contains the ontic possibility 10. Then, as we saw above, for σ to support $p \to \boxed{v} q$ it has to hold that all deontic possibilities in σ color the ontic possibility 10 red. So, to arrive at the maximal state with ontic possibilities $\{10, 00\}$ that supports $p \to \boxed{v} q$, we have to eliminate the deontic possibilities where 10 is not red from the ignorant state σ_8. Thus arriving at the picture of state σ_{12}.

Since, as we also saw above, $p \to \boxed{v} q$ and $\neg q \to \boxed{v} \neg p$ are equivalent, σ_{12} is also the maximal state with ontic possibilities $\{10, 00\}$ that supports $\neg q \to \boxed{v} \neg p$. As is then to be expected, since σ supports $\neg q$, the state depicted here supports the desideratum $\boxed{v} \neg p$.

$$\begin{array}{c|cc} \sigma_{12} & d_{1,3} & d_{2,4} \\ \hline o_2 & \color{red}{10} & \color{red}{10} \\ o_3 & \color{green}{00} & \color{red}{00} \end{array}$$

$$\sigma_{12} \models^+ p \to \boxed{v}q \text{ and } \sigma_{12} \models^\circ \Diamond q \to \boxed{v}p$$

When in this state it is supposed that p, leaving us only with the ontic possibility 10, then this ontic possibility is colored red by every deontic possibility in the state. Thus, you ought not to accept the request if your information excludes the ontic possibility 11, where you also write if you accept. What plays a decisive role here, is that in the Dr. Procrastinate case only the general deontic rule (2b) plays a role, because the specific rule (2a) is suppositionally dismissed, and modal contraposition holds.

This may seem a happy end, but the problem with Dr. Procrastinate is, of course, that she doesn't realize herself that of the two cases discussed above, the Dr. Procrastinate case applies to her.

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