# Towards a logic of information exchange*
## An inquisitive witness semantics

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## 1 Introduction

Traditionally, the meaning of a sentence is identified with its *truth conditions*. This approach is driven by the age-old attention that philosophy has devoted to the study of argumentation. In terms of truth conditions one defines *entailment*, the crucial notion that rules the soundness of an argument: a sentence $\varphi$ is said to entail another sentence $\psi$ in case the truth conditions for $\varphi$ are at least as stringent as the truth conditions for $\psi$.

But argumentation is neither the sole, nor the primary function of language. One task that language more widely and ordinarily fulfills, among others, is to enable the *exchange of information* between individuals. If we embark on the enterprise of studying this particular use of language, the objects of our study are no longer arguments, or proofs, but rather conversations, or dialogues. Just like in logic we traditionally pursue a formal characterization of well-formed proofs, we would then like to obtain a characterization of well-formed dialogues. Thus, as advocated in [17,18], instead of the notion of entailment, which judges whether a sentence can be inferred from a given set of premises, we need to consider another logical notion, which judges whether a sentence forms a pertinent response to the foregoing discourse. This notion, which we will call *compliance*, may be regarded as the crucial logical relation in the study of information exchange. The aim of this paper is to contribute to a better formal characterization of compliance.

In order to pursue our goal, the first thing we need is a shift in perspective on meaning. For, the static notion of meaning as truth conditions is not particularly suited to understand the dynamics of information exchange, or at the very least, not in its usual form. Such a shift in perspective has been initiated by Stalnaker [33], who gave the notion of meaning a dynamic and conversational twist. He proposed to take the meaning of a sentence to consist in its potential to bring about an information change, enhancing the so-called *common ground* of a conversation, and with it the information states of the conversational participants.

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However, if we take seriously the goal to study information exchange through conversation, this change of perspective is not sufficient. For, Stalnaker's proposal is limited to *assertions*, whose meaning can be identified with their informative potential. In conversation, information exchange takes place through a complex interplay of requests and provisions of information, in a process in which issues are raised, addressed, and sometimes resolved. It is the requests for information that drive the development of the conversation, setting the momentary goal of the exchange and thus eliciting certain reactions from other participants. If we really want to understand how information exchange works, our notion of meaning should thus encompass *inquisitive potential* (the potential to request information) alongside informative potential (the potential to provide information). This simple observation forms the cornerstone of *inquisitive semantics* [6,11,20,32]. This framework is intended to provide new logical foundations for the analysis of discourse, especially the type of discourse that is aimed at the exchange of information. For instance, Farkas and Roelofsen [15] show that inquisitive semantics makes it possible to both simpify and enrich the discourse theory of Farkas and Bruce [14], which in turn builds on much previous work on discourse [23,33,4,12,16,31,22,1,3]. A comparison of inquisitive semantics with classical theories of questions [24,28,21], treatments of questions in dynamic semantics [27,26,17], and an earlier version of inquisitive semantics [18,30], shown to be defective in [6,11], is provided in [9,19].

The most basic implementation of inquisitive semantics, the system $\mathsf{Inq}_B$, will be summarized in section 2.1 below. Its notion of meaning encompasses both informative and inquisitive potential. The question, then, is whether this framework allows us to construe a suitable formal notion of compliance. If we limit our attention to a propositional language, the answer seems to be positive: in section 2.2, we present a natural candidate, which was first defined and discussed in [20] and [8]. Unfortunately, however, examples from [6] and [7] show that the same strategy does not yield satisfactory results in the case of a first-order language. These examples are discussed in section 2.3. We will argue that, in fact, no fully satisfactory notion of compliance can be given based on the notion of meaning used in $\mathsf{Inq}_B$, since sentences with intuitively distinct compliant responses are assigned the same semantic value. These considerations will lead us to devise a more fine-grained semantics, described in section 3, that we will call *inquisitive witness semantics*, $\mathsf{Inq}_W$. While this enriched system coincides with $\mathsf{Inq}_B$ on the treatment of informative and inquisitive content, it allows for the formulation of a notion of compliance that does justice to the formerly problematic cases, assigning them the intended set of compliant responses. However, in the conclusion we will look at an example that shows that our solution is not yet quite general, and needs to be further refined.

## 2 Background

We start with a brief recapitulation of $\mathsf{Inq}_B$. We will first consider the language of propositional logic, and then move on to the first-order setting. More elaborate expositions of $\mathsf{Inq}_B$ can be found in [6,11,20,32].

### 2.1 Propositional InqB

In this section we consider a language $\mathcal{L}_\mathcal{P}$, whose formulas are built up from $\bot$ and a set $\mathcal{P}$ of proposition letters, by means of the binary connectives $\wedge, \vee$ and $\rightarrow$. We use $\neg\varphi$ as an abbreviation of $\varphi \rightarrow \bot$, $!\varphi$ as an abbreviation of $\neg\neg\varphi$, and $?\varphi$ as an abbreviation of $\varphi \vee \neg\varphi$. We refer to $!\varphi$ and $?\varphi$ as the non-inquisitive and the non-informative projection of $\varphi$, respectively.

The basic ingredients for the semantics are *worlds* and *states*.

**Definition 1 (Worlds).**
*A world is a function from $\mathcal{P}$ to $\{0,1\}$. We denote by $W$ the set of all worlds.*

**Definition 2 (States).**
*A state is a set of worlds. We denote by $\mathcal{S}$ the set of all states.*

The meaning of a sentence will be defined in terms of the notion of *support* (just as, in a classical setting, the meaning of a sentence is usually defined in terms of truth). Support is a relation between states and formulas. We write $s \models \varphi$ for '$s$ supports $\varphi$'. Intuitively, the support relation captures the conditions under which a formula $\varphi$ is *redundant* in a state $s$, meaning that $\varphi$ does not provide any information that is not already available in $s$ and does not raise any issues that are not already resolved in $s$. This intuition will be made more precise momentarily, when formal notions of informativeness and inquisitiveness have been introduced, see in particular fact 6 below.[1]

**Definition 3 (Support).**

| | | |
|---|---|---|
| $s \models p$ | *iff* | $\forall w \in s : w(p) = 1$ |
| $s \models \bot$ | *iff* | $s = \emptyset$ |
| $s \models \varphi \wedge \psi$ | *iff* | $s \models \varphi$ *and* $s \models \psi$ |
| $s \models \varphi \vee \psi$ | *iff* | $s \models \varphi$ *or* $s \models \psi$ |
| $s \models \varphi \rightarrow \psi$ | *iff* | $\forall t \subseteq s :$ *if* $t \models \varphi$ *then* $t \models \psi$ |

It follows from the above definition that the empty state supports any sentence $\varphi$. Thus, we may think of $\emptyset$ as the *absurd* state. The following two facts bring out two basic properties of the support relation.

[1] For further discussion of the notion of support we refer to [11,19]. $\mathsf{Inq}_B$ can also be presented in such a way that support is not the basic notion [10,32]. Rather, this alternative mode of presentation starts with a direct recursive definition of the propositions expressed by the formulas in the language. The proposition expressed by a formula $\varphi$ then determines in which states $\varphi$ is informative and/or inquisitive, and in which states $\varphi$ is supported.

**Fact 1 (Persistence)** *If $s \models \varphi$ then for every $t \subseteq s$: $t \models \varphi$*

**Fact 2 (Singleton states behave classically)** *For any $w$ and $\varphi$:*

$$\{w\} \models \varphi \iff w \models \varphi \text{ in classical propositional logic}$$

It can be derived from definition 3 that the support-conditions for $\neg\varphi$, $!\varphi$, and $?\varphi$ are as follows.

**Fact 3 (Support for negation and the projection operators)**

1. $s \models \neg\varphi$ *iff* $\forall w \in s : w \not\models \varphi$
2. $s \models !\varphi$ *iff* $\forall w \in s : w \models \varphi$
3. $s \models ?\varphi$ *iff* $s \models \varphi$ *or* $s \models \neg\varphi$

In terms of support, we define the *proposition* expressed by a sentence.

**Definition 4 (Propositions, entailment, and equivalence).**

- $[\varphi] \quad := \quad \{s \in \mathcal{S} \mid s \models \varphi\}$
- $\varphi \models \psi$ *iff for all $s$: if $s \models \varphi$, then $s \models \psi$*
- $\varphi \equiv \psi$ *iff* $\varphi \models \psi$ *and* $\psi \models \varphi$

We will refer to the maximal elements of $[\varphi]$ as the *alternatives* for $\varphi$.

**Definition 5 (Alternatives).** *Let $\varphi$ be a sentence.*

1. *Every maximal element of $[\varphi]$ is called an* alternative *for $\varphi$.*
2. *The* alternative set *of $\varphi$, $[\![\varphi]\!]$, is the set of alternatives for $\varphi$.*

The following result guarantees that the alternative set of a sentence completely determines the proposition that the sentence expresses, and vice versa.

**Fact 4 (Propositions and alternatives)** *For any state $s$ and sentence $\varphi$:*

$$s \in [\varphi] \iff s \text{ is contained in some } \alpha \in [\![\varphi]\!]$$

*Example 1 (Disjunction).* Inquisitive semantics differs from classical semantics in its treatment of disjunction. To see this, consider figures 1(a) and 1(b). In these figures, it is assumed that $\mathcal{P} = \{p, q\}$; world 11 makes both $p$ and $q$ true, world 10 makes $p$ true and $q$ false, etcetera. Figure 1(a) depicts the classical meaning of $p \vee q$: the set of all worlds that make at least one of $p$ and $q$ true. Figure 1(b) depicts the alternative set of $p \vee q$ in $\mathsf{Inq}_B$. It consists of two alternatives. One alternative is made up of all worlds that make $p$ true, and the other of all worlds that make $q$ true.

We think of a sentence $\varphi$ as expressing a proposal to update the common ground of a conversation—formally conceived of as a set of possible worlds—in such a way that the new common ground supports $\varphi$. In other words, given fact 4, a sentence proposes to update the common ground in such a way that the resulting common ground is contained in one of the alternatives for $\varphi$.

Worlds that are not contained in any state supporting $\varphi$ will not survive any of the updates proposed by $\varphi$. In other words, if any of the updates proposed by $\varphi$ is executed, all worlds that are not contained in $\bigcup[\varphi]$ will be eliminated. Therefore, we refer to $\bigcup[\varphi]$ as the *informative content* of $\varphi$.

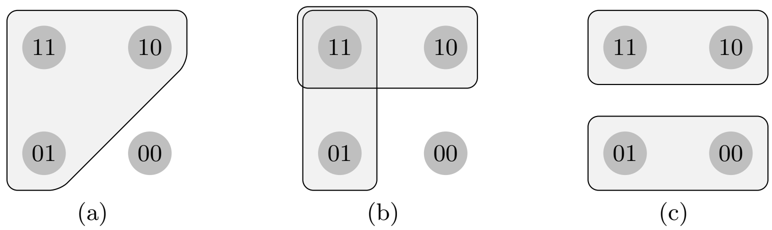


**Fig. 1.** (a) classical picture of $p \vee q$, (b) inquisitive picture of $p \vee q$, and (c) polar question $?p$.

**Definition 6 (Informative content).** $\mathit{info}(\varphi) := \bigcup[\varphi]$

Classically, the informative content of $\varphi$ is captured by the set of all worlds in which $\varphi$ is classically true. We refer to this set of worlds as the *truth set* of $\varphi$.

**Definition 7 (Truth sets).**
*The truth set of $\varphi$, $|\varphi|$, is the set of all worlds where $\varphi$ is classically true.*

The following result says that, as far as informative content goes, $\mathsf{Inq}_B$ does not diverge from classical propositional logic. In this sense, $\mathsf{Inq}_B$ is a conservative extension of classical propositional logic.

**Fact 5 (Informative content is classical)** *For any $\varphi$: $\mathit{info}(\varphi) = |\varphi|$*

A sentence $\varphi$ is informative in a state $s$ iff it proposes to eliminate at least one world in $s$, i.e., iff $s \cap \mathsf{info}(\varphi) \neq s$. On the other hand, $\varphi$ is inquisitive in $s$ iff in order to reach a state $s' \subseteq s$ that supports $\varphi$ it is not enough to incorporate the informative content of $\varphi$ itself into $s$, i.e., $s \cap \mathsf{info}(\varphi) \not\models \varphi$, which means that $\varphi$ requests a response from other participants that provides additional information.

**Definition 8 (Inquisitiveness and informativeness in a state).**

- $\varphi$ *is* informative *in $s$ iff* $s \cap \mathit{info}(\varphi) \neq s$
- $\varphi$ *is* inquisitive *in $s$ iff* $s \cap \mathit{info}(\varphi) \not\models \varphi$

As mentioned above, the support relation intuitively captures when a formula $\varphi$ is redundant in a state $s$. The following fact establishes that this intuition is reflected by the system in a very precise way: $\varphi$ is supported by $s$ just in case it is neither informative nor inquisitive in $s$.

**Fact 6 (Support as redundancy)**

- *$s \models \varphi$ iff $\varphi$ is neither informative nor inquisitive in $s$.*

Besides notions of informativeness and inquisitiveness *relative to a state* we may also define absolute notions of informativeness and inquisitiveness.

**Definition 9 (Absolute inquisitiveness and informativeness).**

- $\varphi$ *is* informative *iff it is informative in at least one state.*
- $\varphi$ *is* inquisitive *iff it is inquisitive in at least one state.*

By persistence (fact 1), an alternative characterization of informativeness and inquisitiveness can be given in terms of informative content, as follows.

**Fact 7 (Inquisitiveness, informativeness, and informative content)**

- $\varphi$ *is* informative *iff* $\mathsf{info}(\varphi) \neq W$
- $\varphi$ *is* inquisitive *iff* $\mathsf{info}(\varphi) \not\models \varphi$

Finally, by fact 4, inquisitiveness can also be characterized in terms of the alternative set for a formula.

**Fact 8 (Inquisitiveness and alternatives)**

- $\varphi$ *is* inquisitive *iff* $[\![\varphi]\!]$ *contains at least two alternatives.*

*Example 2 (Disjunction continued).* As in the classical setting, $p \vee q$ is *informative*, in that it proposes to eliminate worlds where both $p$ and $q$ are false. But it is also *inquisitive*, in that it proposes to move to a state that supports $p$ or to a state that supports $q$, while merely eliminating worlds where both $p$ and $q$ are false is not sufficient to reach such a state. Thus, $p \vee q$ requests a response that provides additional information. This inquisitive aspect of meaning is not captured in the classical setting.

The fact that disjunction is hybrid in the formal system, does not as such embody an *empirical* claim that indicative disjunctions in natural language are both informative and inquisitive. As is shown in [19], the formal system is equally applicable in case it can be argued that, e.g., in a language like English, the semantic properties of informativeness and inquisitiveness are strictly divided over the two syntactic sentential categories of indicative and interrogative sentences, and that hybrid sentences do not exist. Inquisitive semantics offers a general logical framework that can be used to formulate and compare different linguistic theories that are concerned with informative and inquisitive aspects of meaning, but it is not a linguistic theory in itself.

**Definition 10 (Questions, assertions, and hybrids).**

- $\varphi$ *is a* question *iff it is not informative;*
- $\varphi$ *is an* assertion *iff it is not inquisitive;*
- $\varphi$ *is a* hybrid *iff it is both informative and inquisitive.*

*Example 3 (Questions, assertions, and hybrids).* We saw above that $p \vee q$ is both informative and inquisitive, i.e., hybrid. Figure 1(a) depicts the alternative set of $!(p \vee q)$, which consists of exactly one alternative. So $!(p \vee q)$ is an assertion. Figure 1(c) depicts the alternative set of $?p$. Together the alternatives for $?p$ cover the entire logical space, so $?p$ does not propose to eliminate any world. That is, $?p$ is a question.

The framework of propositional basic inquisitive semantics makes it possible to express a wide range of different types of questions. Next to simple polar questions like $?p$, it can also deal with conditional questions like $p \to ?q$, alternative questions like $?(p \vee q)$, and choice questions like $?p \vee ?q$.

### 2.2 Compliance

We now move on to consider a particular notion of compliant responses. In order to motivate this notion, consider the question in (1) and the responses in (1a-d).

(1) Is Mary going to the party?
 a. Yes, she is going.
 b. John is going.
 c. Yes, she is going, and John is going with her.
 d. Yes, she is going; are you going as well?

We would like to have a notion of compliance under which (1a) is a basic compliant response to (1), but (1b-d) are not. (1b) should not count as a basic compliant response because it does not resolve the issue raised by (1), (1c) should not count as a basic compliant response because it provides more information than is necessary to resolve the issue raised by (1), and (1d) should not count as a basic compliant response because, besides providing exactly enough information to resolve the issue raised by (1), it also raises a new issue. Thus, basic compliant responses are those responses that provide exactly enough information to resolve the given issue and do not raise any new issues.

**Definition 11 (Basic compliant responses).**
*$\psi$ is a basic compliant response to $\varphi$ just in case:*

1. *$\psi$ is an assertion*
2. *$\psi \models \varphi$*
3. *There is no assertion $\xi$ such that $\psi \models \xi$, $\psi \not\equiv \xi$, and $\xi \models \varphi$*

Equivalently, a basic compliant response to a formula $\varphi$ may be characterized as an assertion whose informative content coincides with one of the alternatives for $\varphi$. For, a response to $\varphi$ is issue-resolving just in case it provides enough information to establish a state that supports $\varphi$. A basic compliant response is defined as a *minimally informative* issue-resolving assertion, that is, one that establishes a *minimally informed* state that supports $\varphi$. By definition, these states are precisely the alternatives for $\varphi$.

**Fact 9 (Basic compliant responses)**
*$\psi$ is a basic compliant response to $\varphi$ iff $[\![\psi]\!] = \{\alpha\}$ for some $\alpha \in [\![\varphi]\!]$.*

If $\varphi$ is a question, then the basic compliant responses may be viewed as the basic answers to the question, corresponding to those answers that are usually called *direct* or *principal* answers in various erotetic frameworks [2,29,25,34]. Thus, a theory of compliance is also automatically a theory of answerhood for questions.

It should be emphasized that basic compliant responses are not supposed to be the *only* responses to a given sentence that are predicted to be compliant. In terms of basic compliant responses, a more general notion of compliant responses can be defined (see [20]). In the case of questions, among the non-basic compliant responses we find partial answers, as well as sub-questions. For our present purposes, however, considering the general notion of compliance is of little interest. For, the problem we will focus on concerns essentially the determination of the set of *basic* compliant responses to a sentence.

As long as we restrict ourselves to the language of propositional logic, the notion of basic compliant responses, and the more general notion of compliance that it gives rise to, seem to give satisfactory results (again, see[20]). However, we will see right below that this is no longer generally the case if we move to the first-order setting.

### 2.3 First-order InqB

Let $\mathcal{L}$ be a first-order language. The worlds that make up a *state* will now be first-order models for $\mathcal{L}$. We will assume that all worlds in a state share the same domain and the same interpretation of individual constants and function symbols. This assumption is enacted using the notion of a discourse model.

**Definition 12 (Discourse models).**
*A discourse model* $\mathbb{D}$ *for* $\mathcal{L}$ *is a pair* $\langle D, I\rangle$*, where* $D$ *is a domain and* $I$ *an interpretation of all individual constants and function symbols in* $\mathcal{L}$*.*

**Definition 13 ($\mathbb{D}$-worlds and $\mathbb{D}$-states).**
*Let* $\mathbb{D} = \langle D, I\rangle$ *be a discourse model for* $\mathcal{L}$*. Then:*

– *A* $\mathbb{D}$*-world* $w$ *is a model* $\langle D_w, I_w\rangle$ *such that* $D_w = D$ *and* $I_w$ *coincides with* $I$ *as far as individual constants and function symbols are concerned. The set of all* $\mathbb{D}$*-worlds is denoted by* $W_{\mathbb{D}}$*.*
– *A* $\mathbb{D}$*-state is a set of* $\mathbb{D}$*-worlds. The set of all* $\mathbb{D}$*-states is denoted by* $\mathcal{S}_{\mathbb{D}}$*.*

Thus, a $\mathbb{D}$-state $s$ is a set of first-order models for $\mathcal{L}$ that are all based on the same discourse model $\mathbb{D}$. This means that all the models in $s$ share the same domain, and assign the same interpretation to individual constants and function symbols. The interpretation of *predicate symbols* is not fixed by $\mathbb{D}$, and may therefore differ from model to model in $s$. This amounts to the assumption that the domain of discourse and the interpretation of individual constants and function symbols are common knowledge among the participants, and that the exchange of information only concerns the denotation of the predicate symbols.[2]

The definitions below assume a fixed discourse-model $\mathbb{D} = \langle D, I\rangle$ for $\mathcal{L}$. Moreover, for any assignment $g$, we denote by $|\varphi|_g$ the *truth set* of $\varphi$ relative to $g$, i.e., the set of worlds $w$ such that $w \models_g \varphi$ in classical first-order logic.

[2] This simplifying assumption is made here for ease of exposition. A similar semantics without this assumption is also conceivable, but the extra complexity involved would not be relevant to the issue we shall be concerned with.

**Definition 14 (Support in first-order $\mathsf{Inq}_B$).**
*Let $s$ be a $\mathbb{D}$-state, $g$ an assignment, and $\varphi$ a formula in $\mathcal{L}$.*

| | | |
|---|---|---|
| $s \models_g \varphi$ | *iff* | $s \subseteq \vert\varphi\vert_g$ *for atomic* $\varphi$ |
| $s \models_g \bot$ | *iff* | $s = \emptyset$ |
| $s \models_g \varphi \wedge \psi$ | *iff* | $s \models_g \varphi$ *and* $s \models_g \psi$ |
| $s \models_g \varphi \vee \psi$ | *iff* | $s \models_g \varphi$ *or* $s \models_g \psi$ |
| $s \models_g \varphi \to \psi$ | *iff* | $\forall t \subseteq s:$ *if* $t \models_g \varphi$ *then* $t \models_g \psi$ |
| $s \models_g \forall x.\varphi$ | *iff* | $s \models_{g[x/d]} \varphi$ *for all* $d \in D$ |
| $s \models_g \exists x.\varphi$ | *iff* | $s \models_{g[x/d]} \varphi$ *for some* $d \in D$ |

**Definition 15 (Propositions, entailment, and equivalence).**

- $[\varphi]_g \quad := \quad \{s \in \mathcal{S}_{\mathbb{D}} \mid s \models_g \varphi\}$
- $\varphi \models \psi$ *iff for all* $\mathbb{D}$, $s$ *and* $g$*: if* $s \models_g \varphi$*, then* $s \models_g \psi$
- $\varphi \equiv \psi$ *iff* $\varphi \models \psi$ *and* $\psi \models \varphi$

All the basic logical notions defined in the propositional setting, like informativeness, inquisitiveness, questions, assertions, and hybrids, carry over immediately to the first order setting. As is to be expected, given the inquisitive nature of disjunction, the existential quantifier is inquisitive as well. A state $s$ may well embody the information that at least one object in the domain has the property $P$, without supporting the existential $\exists x.Px$. For, the former merely requires that in every world $w \in s$ there be some object $d \in D$ such that $d \in I_w(P)$. In order to support $\exists x.Px$, on the other hand, there must be some object $d \in D$ such that in every $w \in s$: $d \in I_w(P)$. In other words, what is required to support the existential $\exists x.Px$ is that there be a specific object which is known in $s$ to have the property $P$.

The inquisitive nature of existential quantification makes it possible to express mention-some questions in the logical language. But, witnessing the status of inquisitive semantics as a general logical framework, it is equally possible to express mention-all questions: $\forall x.?Px$ is only supported in a state $s$ if the full denotation of the predicate $P$ is known, i.e., if all worlds in $s$ agree on the denotation of $P$.

In effect, this means that two of the main rival theories of questions in natural language semantics, the Hamblin analysis [24] and the partition analysis [21], may both be formulated and compared within the logical framework of inquisitive semantics. At the same time, the framework provides the means to express certain types of questions, such as conditional questions, that are not, or at least not obviously, within the reach of either of these two theories.

### 2.4 The boundedness problem

All the basic properties of the propositional system having to do with informative and inquisitive content still hold in the first order setting. For instance, the classical treatment of informative content is still preserved (fact 2).

However, one feature of the system is not preserved: the proposition expressed by a sentence is no longer fully determined by the alternative set of that sentence (fact 4). In other words, it is no longer the case that every state supporting $\varphi$ is contained in a maximal state supporting $\varphi$. In fact, as shown by Ciardelli [6,7], there are first-order formulas that do not have any maximal supporting states.

*Example 4 (The boundedness formula).* Consider a language which has a unary predicate symbol $P$, a binary function symbol $+$, and the set $\mathbb{N}$ of natural numbers as its individual constants. Consider the discourse-model $\mathbb{D} = \langle D, I\rangle$, where $D = \mathbb{N}$, $I$ maps every $n \in \mathbb{N}$ to itself, and $+$ is interpreted as addition. Let $x \leq y$ abbreviate $\exists z(x + z = y)$, let $B(x)$ abbreviate $\forall y(P(y) \to y \leq x)$, and for every $n \in \mathbb{N}$, let $B(n)$ abbreviate $\forall y(P(y) \to y \leq n)$. Intuitively, $B(n)$ says that $n$ is greater than or equal to any number in $P$. In other words, $B(n)$ says that $n$ is an *upper bound* for $P$.

A $\mathbb{D}$-state $s$ supports a formula $B(n)$, for some $n \in \mathbb{N}$, if and only if $B(n)$ is true in every world in $s$, that is, if and only if $n$ is an upper bound for $P$ in every $w$ in $s$. Now consider the formula $\exists x.B(x)$, which intuitively says that there is an upper bound for $P$. This formula, which Ciardelli refers to as the *boundedness formula*, does not have a maximal supporting state. To see this, let $s$ be an arbitrary state supporting $\exists x.B(x)$. Then there must be a number $n \in \mathbb{N}$ such that $s$ supports $B(n)$, i.e., $B(n)$ must be true in all worlds in $s$. Now let $w^*$ be the $\mathbb{D}$-world in which $P$ denotes the singleton set $\{n + 1\}$. Then $w^*$ cannot be in $s$, because it does not make $B(n)$ true. Thus, the state $s^*$ which is obtained from $s$ by adding $w^*$ to it is a proper superset of $s$ itself. However, $s^*$ clearly supports $B(n+1)$, and therefore also still supports $\exists x.B(x)$. This shows that any state supporting $\exists x.B(x)$ can be extended to a larger state which still supports $\exists x.B(x)$, and therefore no state supporting $\exists x.B(x)$ can be maximal.

This example shows that our notion of basic compliant responses, which makes crucial reference to maximal supporting states, does not always yield satisfactory results in the first-order setting. At first sight, it is tempting to conclude from this observation that there must be something wrong with the given notion of basic compliant responses. However, the problem is deeper than that. Namely, the following example, again from [6,7], shows that the very notion of meaning assumed in $\mathsf{Inq}_B$ is not fine-grained enough to serve as a basis for a suitable notion of compliance in the first-order setting.

*Example 5 (The positive boundedness formula).* Consider the following variant of the boundedness formula: $\exists x(x \neq 0 \wedge B(x))$. This formula says that there is a *positive* upper bound for $P$. Intuitively, it differs from the ordinary boundedness formula in that it does not license $B(0)$ as a compliant response. However, in terms of support, $\exists x(x \neq 0 \wedge B(x))$ and $\exists x.B(x)$ are equivalent. Thus, support is not fine-grained enough to capture the fact that these formulas intuitively do not have the same range of compliant responses.

## 3 An inquisitive witness semantics

In this section we will develop a first-order inquisitive witness semantics, $\mathsf{Inq}_W$, which explicitly reflects the idea that an existentially quantified sentence like $\exists x.Px$ is supported in a state if and only if there is a specific witness in that state which is known to have the property $P$.

This idea is not entirely new. For instance, when informally describing the clause for existential quantification in $\mathsf{Inq}_B$, Ciardelli [7] writes that "an existential will only be supported in those states where a specific witness for the existential is known." However, in $\mathsf{Inq}_B$, states merely encode a certain body of information. To know a witness for a certain property is simply to know that the property holds of a specific individual. To say that a sentence *introduces a witness*, then, is just to say that the sentence provides the information that a certain individual has a certain property. But notice that, on this notion of witnesses, (i) a sentence may introduce infinitely many witnesses and (ii) these witnesses need not even be mentioned explicitly by the sentence. To deal with compliance, we need a stricter notion of witnesses: only individuals that are explicitly mentioned in the conversation should count as such. So, we need to devise a system which keeps track of the mentioned individuals, alongside the information that has been provided about them.[3]

### 3.1 Witnesses, states, and support

In developing such a system, the first question to ask is what our formal notion of witnesses should be. The simplest answer would be that witnesses are objects in the domain $D$. This is indeed sufficient for the simplest cases of existential quantification. For instance, it would be reasonable to think of a state $s$ as supporting a sentence $\exists x.Px$ just in case there is a specific object $d \in D$ which is known in $s$ to have the property $P$. However, this notion of witnesses as objects in $D$ is not general enough. In particular, it becomes problematic when we consider formulas where an existential quantifier is embedded under a universal quantifier. For instance, it would not be appropriate to think of a state $s$ as supporting a sentence $\forall x.\exists y.Rxy$ just in case there is a specific object $d \in D$ which is known in $s$ to stand in the relation $R$ with all other objects in $D$. Intuitively, this is not what $\forall x.\exists y.Rxy$ requires.

To avoid problems of this sort, we will take witnesses to be *functions* from $D^n$ to $D$, where $n \geq 0$. Notice that some of these functions are 0-place functions into $D$, which can simply be identified with objects in $D$. So witnesses *can* still be objects in $D$. But they can be other things as well.

In the definitions below, we will assume a fixed first-order language $\mathcal{L}$ and a fixed discourse-model $\mathbb{D} = \langle D, I \rangle$ for $\mathcal{L}$.

[3] In the concluding section of the paper, we will see that to completely solve the problem under investigation, it is not sufficient to keep track of the mentioned individuals and the information provided about them, but that we should also keep track of which part of the information provided is about which individual. The system to be introduced below achieves the former, but not the latter.

**Definition 16 (Witnesses).**

- *For any $n \in \mathbb{N}$, let $D^\star_n$ be the set of functions $\delta\colon D^n \to D$.*
- *Then $D^\star = \bigcup_{n \geq 0} D^\star_n$ is the set of all witnesses based on $D$.*

The next step is to reconsider our notion of a state. Before, states were sets of worlds, reflecting a certain body of information. Now states will not only reflect a certain body of information, but also contain a set of witnesses. We will assume that the set of witnesses available in a state always includes the identity function. The rationale behind this assumption will become clear in a moment, when we define how witnesses are put to use in the semantics.

**Definition 17 (States with witnesses).**

- *A $\mathbb{D}$-state is a pair $\langle V, \Delta \rangle$, where $V$ is a set of $\mathbb{D}$-worlds and $\Delta$ is a finite set of witnesses based on $D$, which contains the identity function $id : D \to D$.*
- *The set of all $\mathbb{D}$-states is denoted by $\mathcal{S}_\mathbb{D}$.*
- *If $s = \langle V, \Delta \rangle$ is a $\mathbb{D}$-state, then $\mathsf{worlds}(s) := V$ and $\mathsf{witn}(s) := \Delta$.*

We will often drop reference to $\mathbb{D}$, and simply refer to $\mathbb{D}$-states as states. The set of all states is partially ordered by the following *extension* relation.

**Definition 18 (Extension).** *Let $s$ and $t$ be two states. Then we say that $s$ is an extension of $t$, $s \geq t$, iff $\mathsf{worlds}(s) \subseteq \mathsf{worlds}(t)$ and $\mathsf{witn}(s) \supseteq \mathsf{witn}(t)$.*

Notice that there is a minimal state, namely $\mathsf{top} := (W, \{id\})$, of which any other state is an extension. The extension relation will be used in the support definition, in particular in the clause for implication: a state $s$ supports an implication iff every extension of $s$ that supports the antecedent, supports the consequent as well.

Before turning to the definition of support, however, we introduce two more auxiliary notions. The first is the notion of a *witness feed*. The role of these witness feeds will be similar to that of assignments: they will be used to store certain information in evaluating whether or not a certain formula is supported by a certain state. In particular, they play a role in evaluating existentially quantified formulas in the scope of one or more universal quantifiers. This will be further explained once we have specified the support relation.

**Definition 19 (Witness feeds).** *A witness feed $\varepsilon$ is a finite subset of $D$.*

Finally, we assume that the interpretation $I$ of individual constants and function symbols in our discourse model $\mathbb{D}$ is extended in the following natural way to an interpretation of all terms $t \in \mathcal{L}$: if the free variables occurring in $t$ are, orderly, $x_1, \ldots, x_n$, then $I(t)$ is the function $D^n \to D$ which maps a tuple $(d_1, \ldots, d_n) \in D^n$ to the element $d \in D$ denoted by the term $t$ in $\mathbb{D}$ when $x_i$ is interpreted as $d_i$ for all $i = 1, \ldots, n$.

We now have all the necessary ingredients to state the support relation.

**Definition 20 (Support in $\mathsf{Inq}_W$).**
*Let $s$ be a $\mathbb{D}$-state, $g$ an assignment, $\varepsilon$ a witness feed, and $\varphi$ a formula in $\mathcal{L}$.*

$$
\begin{array}{lll}
s \models_{g,\varepsilon} R(t_1,\ldots,t_n) & \textit{iff} & \textit{(i) } \mathsf{worlds}(s) \subseteq |R(t_1,\ldots,t_n)|_g \\
 & & \textit{(ii) } I(t_i) \in \mathsf{witn}(s) \textit{ for } i = 1,\ldots,n \\
s \models_{g,\varepsilon} \bot & \textit{iff} & \mathsf{worlds}(s) = \emptyset \\
s \models_{g,\varepsilon} \varphi \wedge \psi & \textit{iff} & s \models_{g,\varepsilon} \varphi \textit{ and } s \models_{g,\varepsilon} \psi \\
s \models_{g,\varepsilon} \varphi \vee \psi & \textit{iff} & s \models_{g,\varepsilon} \varphi \textit{ or } s \models_{g,\varepsilon} \psi \\
s \models_{g,\varepsilon} \varphi \to \psi & \textit{iff} & \forall t \geq s : \textit{if } t \models_{g,\varepsilon} \varphi \textit{ then } t \models_{g,\varepsilon} \psi \\
s \models_{g,\varepsilon} \forall x.\varphi & \textit{iff} & s \models_{g[x/d],\varepsilon\cup\{d\}} \varphi \textit{ for all } d \in D \\
s \models_{g,\varepsilon} \exists x.\varphi & \textit{iff} & s \models_{g[x/\delta(e_1,\ldots,e_n)],\varepsilon} \varphi \textit{ for some } \delta \in \mathsf{witn}(s) \textit{ and } e_1,\ldots,e_n \in \varepsilon
\end{array}
$$

We will use $s \models_g \varphi$ as an abbreviation of $s \models_{g,\emptyset} \varphi$. The clauses that have changed w.r.t. $\mathsf{Inq}_B$ are those for atomic formulas, implication, universal quantification, and existential quantification. Let us look at these four clauses in some detail.

*Atoms.* For a state $s$ to support an atomic sentence $R(t_1,\ldots,t_n)$, the sentence has to be true in all worlds in $\mathsf{worlds}(s)$, as before, but moreover, for every term $t_i$, the function $I(t_i)$ that it denotes must be available as a witness in $\mathsf{witn}(s)$. To illustrate this, consider the formula $R(a, f(b))$ where $a$ and $b$ are individual constants and $f$ is a unary function symbol. Suppose $I(a) = d_1$ and $I(f(b)) = d_2$: then a state $s$ supports the sentence $R(a, f(b))$ if and only if (i) for every $w \in \mathsf{worlds}(s)$ we have that $\langle d_1, d_2\rangle \in I_w(R)$, and (ii) $d_1$ and $d_2$ are available as witnesses in $\mathsf{witn}(s)$.

Recall that in uttering a sentence, a speaker proposes to update the common ground of the conversation in such a way that it comes to support the sentence. Thus, in particular, in uttering $R(a, f(b))$, a speaker proposes to add $d_1$ and $d_2$ to the witness set of the common ground. In this sense, we can think of atomic sentences like $R(a, f(b))$ as introducing new witnesses. We will see that other sentences, in particular existentials, may request a response that introduces new witnesses.

*Implication.* In order to determine whether a state $s$ supports an implication $\varphi \to \psi$ we have to consider all extensions $t$ of $s$ that support $\varphi$. An extension $t$ of $s$ is a state such that $\mathsf{worlds}(t) \subseteq \mathsf{worlds}(s)$ and $\mathsf{witn}(t) \supseteq \mathsf{witn}(s)$. Thus, it may be that all the extensions of $s$ that support $\varphi$ contain certain witnesses that are not contained in $s$ itself. This means that if $\psi$ requires certain witnesses, as long as we need to introduce them to support $\varphi$, it is not necessary for $s$ as such to already contain them for the implication to be supported in $s$.

To illustrate this, let us show that $\mathsf{top} \models_{g,\varepsilon} Pa \to \exists x.Px$. Given the atomic clause, every $t \geq \mathsf{top}$ that supports $Pa$ must be such that $I(a) \in \mathsf{witn}(t)$. In other words, every $t \geq \mathsf{top}$ that supports $Pa$ contains a witness, namely $I(a)$, which is known to have the property $P$. It follows that $t \models_{g,\varepsilon} \exists x.P(x)$, which in turn means that $\mathsf{top} \models_{g,\varepsilon} Pa \to \exists x.Px$, even though $\mathsf{top}$ itself does not contain any witnesses besides the identity function.

*Universal quantification.* The clause for universal quantification is very much like the clause we had in $\mathsf{Inq}_B$. Only now the witness feed plays a role as well. In determining whether a state $s$ supports a formula $\forall x.\varphi$ we do not only set the current assignment $g$ to $g[x/d]$, but we simultaneously augment the current witness feed $\varepsilon$ with the same object $d$. Then we check whether $\varphi$ is supported by $s$ relative to the adapted assignment and the augmented witness feed. As we will see below, the augmented witness feed is put to use when $\varphi$ contains an existential quantifier.

*Existential quantification.* In checking whether $s \models_{g,\varepsilon} \exists x.\varphi$ holds, we have to check whether $s \models_{g[x/d],\varepsilon} \varphi$ holds for some object $d \in D$ which is obtained by applying some witness $\delta \in \mathsf{witn}(s)$ to objects $e_1, \ldots, e_n$ in the witness feed. We call this element $d$ a witness *for* the existential. This means that in uttering $\exists x.Px$, a speaker requests a response that introduces a suitable witness and establishes of this witness that it has the property $P$. The fact that the set of witnesses always contains the identity function *id* ensures that any element $e$ of the witness feed can always be used *itself* as a witness for an existential, since $e$ can be obtained by applying *id* to $e$. The invariable presence of the identity function in the witness set, required by our definition of states, is designed precisely to make the elements of the current witness feed directly available as witnesses for an existential.

*Example 6 (Interaction between existentials and universals).* Consider the sentence $\forall x.\exists y.Rxy$. In order to determine whether $s \models_g \forall x.\exists y.Rxy$, we have to check whether $s \models_{g[x/d],\{d\}} \exists y.Rxy$ for all $d \in D$. And this means that we have to verify whether for every $d \in D$, there is a witness $f \in \mathsf{witn}(s)$ such that $s \models_{g[x/d][y/f(d,\ldots,d)],\{d\}} Rxy$. This witness $f$ may be an element of the domain, a unary function, or a function of higher arity. It may also be the identity function, which, as we saw, means that the element $d$ itself may serve as a witness for the existential. This, then, is how universal and existential quantifiers interact: universal quantifiers add objects to the current witness feed, and these objects then may serve as input for functional witnesses that may be needed for existentials in the scope of a universal. In this way, the witness that is required for the embedded existential in $\forall x.\exists y.Rxy$ may functionally depend on the value of $x$ under the current assignment. We will return to this example in section 3.5, where we illustrate the relevance of $\mathsf{Inq}_W$ for natural language semantics.

As in $\mathsf{Inq}_B$, support is *persistent*. That is, if a state $s$ supports a formula $\varphi$ relative to a certain assignment $g$ and a certain witness feed $\varepsilon$, then any extension of $s$ also supports $\varphi$ relative to $g$ and $\varepsilon$.

**Fact 10 (Persistence)** *If* $s \models_{g,\varepsilon} \varphi$ *and* $t \geq s$, *then* $t \models_{g,\varepsilon} \varphi$

Also as in $\mathsf{Inq}_B$, we take $\neg\varphi$ to be an abbreviation of $\varphi \to \bot$, and $!\varphi$ an abbreviation of $\neg\neg\varphi$. The derived clauses for $\neg\varphi$ and $!\varphi$ read as follows.

**Fact 11 (Support for negation)**

– $s \models_{g,\varepsilon} \neg\varphi$ *iff for all* $w \in \textsf{worlds}(s)$*:* $w \not\models_g \varphi$ *classically*
– $s \models_{g,\varepsilon} !\varphi$ *iff for all* $w \in \textsf{worlds}(s)$*:* $w \models_g \varphi$ *classically*

### 3.2 Propositions, entailment, and equivalence

Based on the notion of support, we define the proposition expressed by a formula, and the notions of entailment and equivalence, just as in $\textsf{Inq}_B$. Recall that our definitions assume a fixed first-order language $\mathcal{L}$ and a fixed discourse-model $\mathbb{D} = \langle D, I\rangle$ for $\mathcal{L}$.

**Definition 21 (Propositions, entailment, and equivalence).**

1. $[\varphi]_g := \{s \in \mathcal{S}_{\mathbb{D}} \mid s \models_g \varphi\}$
2. $\varphi \models \psi$ *iff for all* $\mathbb{D}$*, s and g: if* $s \models_g \varphi$*, then* $s \models_g \psi$
3. $\varphi \equiv \psi$ *iff* $\varphi \models \psi$ *and* $\psi \models \varphi$

In $\textsf{Inq}_B$, states were sets of possible worlds, ordered by inclusion, and we referred to *maximal* states supporting $\varphi$ as *alternatives* for $\varphi$, where maximality was determined by the inclusion-order. Thus, alternatives for $\varphi$ in $\textsf{Inq}_B$ were *minimally informed* states supporting $\varphi$. In $\textsf{Inq}_W$, states are ordered by the extension relation, $\geq$, and alternatives for $\varphi$ will be defined as $\geq$-minimal states supporting $\varphi$. Thus, in $\textsf{Inq}_W$ alternatives for $\varphi$ are states that support $\varphi$ with a minimum amount of information and a minimal set of witnesses.[4]

**Definition 22 (Alternatives).** *Let $\varphi$ be a formula and g an assignment.*

1. *Every $\geq$-minimal element of $[\varphi]_g$ is called an* alternative *for $\varphi$ relative to g.*
2. *The* alternative set *of $\varphi$ relative to g, $[\![\varphi]\!]_g$, is the set of alternatives for $\varphi$ relative to g.*

We also introduce notions of *factive* support, entailment, and equivalence, which ignore witness issues.

**Definition 23 (Factive support, entailment, and equivalence).**

1. $V \models^\star_g \varphi$ *iff there is a state s with* $\textsf{worlds}(s) = V$ *such that* $s \models_g \varphi$
2. $\varphi \models^\star \psi$ *iff for all* $V, g$*: if* $V \models^\star_g \varphi$*, then* $V \models^\star_g \psi$
3. $\varphi \equiv^\star \psi$ *iff* $\varphi \models^\star \psi$ *and* $\psi \models^\star \varphi$

Witness sensitivity was introduced in order to be able to discriminate sentences that differ in the responses they license. Factive notions disable this sensitivity, thus taking into account only inquisitive and informative content of sentences. Not surprisingly, then, the system that we obtain by disregarding witness issues in $\textsf{Inq}_W$ is precisely our good old $\textsf{Inq}_B$.

[4] We do not know at this point whether the equivalent of fact 4 holds for $\textsf{Inq}_W$, that is, whether any state that supports a formula $\varphi$ relative to an assignment $g$ must be an extension of some alternative for $\varphi$ relative to $g$.

**Fact 12 (Factive support and support in $\mathsf{Inq}_B$)**

$$V \models^{\star}_g \varphi \text{ in } \mathsf{Inq}_W \quad \Longleftrightarrow \quad V \models_g \varphi \text{ in } \mathsf{Inq}_B$$

Clearly, this also means that factive entailment and equivalence in $\mathsf{Inq}_W$ amount to entailment and equivalence in $\mathsf{Inq}_B$. We say that a formula is witness-insensitive in case it is supported by a state as soon as it is factively supported by the information available in that state.

**Definition 24 (Witness insensitivity).**
*$\varphi$ is witness insensitive iff for all $s, g$: if $\mathit{worlds}(s) \models^{\star}_g \varphi$, then $s \models_g \varphi$*

**Fact 13 (Partial characterization of witness insensitivity)**

1. *An atomic formula is witness insensitive iff it does not contain any individual constant or a function symbol;*
2. *$\bot$ is witness insensitive;*
3. *If $\varphi$ and $\psi$ are witness insensitive, so are $\varphi \vee \psi$ and $\varphi \wedge \psi$;*
4. *If $\psi$ is witness insensitive, so is $\varphi \to \psi$;*
5. *$\exists x.\varphi$ is not witness insensitive for any $\varphi$;*
6. *$\forall x.\varphi$ is witness insensitive iff $\varphi$ is witness insensitive.*

Given that negation $\neg\varphi$ is defined as $\varphi \to \bot$, and non-inquisitive projection $!\varphi$ as $\neg\neg\varphi$, item 2 and 4 above guarantee that negation and non-inquisitive projection block witness sensitivity of their complement.

### 3.3 Informativeness and inquisitiveness

As before, we define the informative content of a sentence $\varphi$ relative to an assignment $g$ as the set of worlds that are contained in at least one state that supports $\varphi$ relative to $g$.

**Definition 25 (Informative content).** $\mathit{info}_g(\varphi) \coloneqq \bigcup\{\mathit{worlds}(s) \mid s \in [\varphi]_g\}$.

Also as before, the informative content of a sentence $\varphi$ relative to an assignment $g$ always coincides with the *truth set* of $\varphi$ relative to $g$, $|\varphi|_g$, i.e., the set of worlds that satisfy $\varphi$ in classical first-order logic relative to $g$. So as far as informative content is concerned, $\mathsf{Inq}_W$ does not diverge from classical first-order logic.

**Fact 14 (Informative content is classical)** *For any $\varphi, g$: $\mathit{info}_g(\varphi) = |\varphi|_g$*

In terms of the informative content of a formula, we define whether it is informative and/or inquisitive.

**Definition 26 (Inquisitiveness and informativeness in a state).**

- $\varphi$ *is* informative in $s$ w.r.t. $g$ *iff* $\mathit{worlds}(s) \cap \mathit{info}_g(\varphi) \neq \mathit{worlds}(s)$
- $\varphi$ *is* inquisitive in $s$ w.r.t. $g$ *iff* $\mathit{worlds}(s) \cap \mathit{info}_g(\varphi) \not\models^{\star}_g \varphi$

As before, we also define absolute notions of informativeness and inquisitiveness.

**Definition 27 (Absolute inquisitiveness and informativeness).**

- $\varphi$ *is* informative *iff for some* $g$: $\mathit{info}_g(\varphi) \neq W$
- $\varphi$ *is* inquisitive *iff for some* $g$: $\mathit{info}_g(\varphi) \not\models^{\star}_g \varphi$

The following fact reports that the notions of informativeness and inquisitiveness in $\mathsf{Inq}_W$ correspond exactly with those in $\mathsf{Inq}_B$.

**Fact 15 (Informativeness and inquisitiveneness in $\mathsf{Inq}_W$ and $\mathsf{Inq}_B$)**

- *$\varphi$ is informative in $\mathsf{Inq}_W$ iff $\varphi$ is informative in $\mathsf{Inq}_B$*
- *$\varphi$ is inquisitive in $\mathsf{Inq}_W$ iff $\varphi$ is inquisitive in $\mathsf{Inq}_B$*

All notions in $\mathsf{Inq}_B$ that are defined in terms of informativeness and inquisitiveness, such as the notions of assertions, questions, and hybrids, remain precisely the same in intension and extension. Only, now within each class there is a further distinction between witness sensitive and witness insensitive formulas.

*Compliance.* The notion of basic compliant responses that we had in $\mathsf{Inq}_B$ carries over straightforwardly to $\mathsf{Inq}_W$. Recall that in $\mathsf{Inq}_B$, the basic compliant responses to a sentence $\varphi$ were characterized as those responses that provide precisely enough information to establish a state that supports $\varphi$, without raising any new issues. In $\mathsf{Inq}_W$, states do not only contain information but also witnesses, and support sometimes requires the presence of such witnesses. Thus, in $\mathsf{Inq}_W$ the basic compliant responses to $\varphi$ are naturally characterized as those responses that provide precisely enough information and precisely enough witnesses to establish a state that supports $\varphi$, without raising any new issues. This is captured by our earlier definition of basic compliant responses, definition 11, provided that the notion of entailment from $\mathsf{Inq}_B$ is replaced by the notion of entailment from $\mathsf{Inq}_W$.

### 3.4 The boundedness problem resolved

Now that we have discussed some of the basic logical properties of $\mathsf{Inq}_W$, let us return to the problem that we set out to resolve. The boundedness formula was problematic for $\mathsf{Inq}_B$ in two crucial ways: first, it provided an example of a formula to which we could associate no basic compliant responses; second, it was semantically equivalent to its positive variant which, intuitively, licenses a different range of compliant responses. Let us start by observing that the latter problem no longer arises in $\mathsf{Inq}_W$: thanks to the witness machinery, $\mathsf{Inq}_W$ is fine-grained enough to detect the differences between the two boundedness formulas.

**Fact 16 (The boundedness formulas)**
*The boundedness formula and the positive boundedness formula are not equivalent in $\mathsf{Inq}_W$.*

*Proof.* Consider a state $s$ such that:

- $\mathsf{worlds}(s) = \{w\}$, where $I_w(P) = \{0\}$
- $\mathsf{witn}(s) = \{id, 0\}$

This state factively supports both $\exists x.B(x)$ and $\exists x.(x > 0 \wedge B(x))$. However, while the boundedness formula is supported in $s$ *tout court*, $s \models \exists x.B(x)$, the positive boundedness formula is not, $s \not\models \exists x.(x > 0 \wedge B(x))$. So, the boundedness formula and the positive boundedness formula are not equivalent in $\mathsf{Inq}_W$ (although they are *factively* equivalent). □

Now that we ensured that the two boundedness formulas can be distinguished, we would like our semantics to predict for each of them its expected range of basic compliant responses. The following fact shows that $\mathsf{Inq}_W$ achieves this as well.

**Fact 17 (Basic compliant responses to the boundedness formulas)**

- *For any $n \geq 0$, $B(n)$ is a basic compliant response to $\exists x.Bx$*
- *For any $n > 0$, $B(n)$ is a basic compliant response to $\exists x.(x \neq 0 \wedge Bx)$, but $B(0)$ is not a basic compliant response to $\exists x.(x \neq 0 \wedge Bx)$.*

How can $\mathsf{Inq}_W$ get this prediction right? Let us examine this more closely. In $\mathsf{Inq}_B$, an assertion entails another simply in case the informative content of the former entails the informative content of the latter. Now, for any number $n$, the informative content of $B(n)$ entails the informative content of $B(n+1)$. Therefore, $B(n) \models B(n+1)$ in $\mathsf{Inq}_B$. Thus, we have an infinite chain $B(0), B(1), B(2), \ldots$ of weaker and weaker responses, all of which resolve the issue raised by $\exists x B(x)$. None of these responses is minimally informative, and so, none is predicted to be a basic compliant response.

In $\mathsf{Inq}_W$, on the other hand, entailment is more demanding. For an assertion $\psi$ to entail another assertion $\chi$, entailment of informative content is no longer sufficient: it should also be the case that $\psi$ introduces all the witness that $\chi$ introduces. This prevents $B(n)$ from entailing $B(n+1)$, since, unlike the latter, the former does not introduce the witness $n+1$. Thus, from the point of view of $\mathsf{Inq}_W$, every assertion $B(n)$ constitutes a *minimal* way to resolve the issue raised by $\exists x.Bx$, where the minimality concerns now not only the information provided, but also the witnesses introduced.

Notice that, by means of factive entailment, $\mathsf{Inq}_W$ is still capable of accounting for the fact that the informative content of one compliant response may entail the informative content of another. If this happens, the stronger response will be quantitatively preferred pragmatically over the weaker one. For instance, $B(1)$ and $B(135)$ are both basic compliant responses to $\exists x.Bx$. However, $B(1)$ factively entails $B(135)$. If the information state of the responder supports $B(1)$, then it would be misleading for her to actually choose $B(135)$ as a response. In general, if $\psi$ and $\chi$ are two basic compliant responses to $\varphi$, and $\psi$ factively entails $\chi$, then $\psi$ is preferred over $\chi$ as a response to $\varphi$.

**Definition 28 (Comparing basic compliant responses).**
*Let $\varphi$ be an inquisitive initiative, let $\psi$ and $\chi$ be two basic compliant responses to $\varphi$, and let $\sigma$ be an information state, i.e., a set of worlds. Then:*

1. *$\psi$ is* preferred *over $\chi$ as a response to $\varphi$ iff $\psi \models^\star \chi$ and $\chi \not\models^\star \psi$.*
2. *$\psi$ is an* optimal response *to $\varphi$ in $\sigma$ iff*
   - *$\sigma \subseteq \mathsf{info}(\psi)$, and*
   - *for every basic compliant response $\xi$ to $\varphi$ that is preferred over $\psi$, $\sigma \not\subseteq \mathsf{info}(\xi)$.*

To illustrate the notion of an optimal response, consider an information state consisting of three worlds, one where the highest element of $P$ is 5, one where it is 14, and one where it is 3. The optimal response to $\exists x.Bx$ in this information state is $B(14)$. This accounts for the intuition that, on the one hand, any response $B(n)$ with $n < 14$, even though compliant, would be *qualitatively* inappropriate, while any response $B(n)$ with $n > 14$ would be *quantitively* dispreferred. The only optimal response in this scenario is $B(14)$.

### 3.5 An example from natural language

In this section we will briefly illustrate the potential of the proposed notion of compliance by means of an example from natural language. Consider a quantified question like *Who does every man like?*. As has been discussed widely in the literature (e.g., [5,13,21]), such questions allow for different types of responses, e.g., *Mary*, *himself*, or *his mother*. If this question is formally represented as $\forall x.\exists y.Rxy$ these different types of responses are accounted for in a uniform way. We illustrate this below for $\mathsf{Inq}_W$. However, since boundedness issues do not play a role in this particular example, the same results are also obtained in $\mathsf{Inq}_B$.

Consider what is needed for a state $s$ to support $\forall x.\exists y.Rxy$. If we assume that $\mathsf{witn}(s)$ does not contain any witnesses, apart from the identity function, which is always an element of $\mathsf{witn}(s)$, then we must have that $\langle d, d\rangle \in I_w(R)$ for every $d \in D$ and every $w \in \mathsf{worlds}(s)$. The $\geq$-minimal state that satisfies this condition is one of the alternatives for $\forall x.\exists y.Rxy$. It is also the unique alternative for the response *himself* ($\forall x.Rxx$). Thus, this is a basic compliant response.

Now consider a state $s$ such that $\mathsf{witn}(s)$ contains an object $m$, and such that $\langle d, m\rangle \in I_w(R)$ for every $d \in D$ and every $w \in \mathsf{worlds}(s)$. The $\geq$-minimal state that satisfies these conditions is another alternative for $\forall x.\exists y.Rxy$. It is also the unique alternative for the response *Mary* ($\forall x.Rxm$). Thus, this is another basic compliant response.

Finally, consider a state $s$ such that $\mathsf{witn}(s)$ contains a 1-place function $f$ which maps every individual in $D$ to his mother, and such that $\langle d, f(d)\rangle \in I_w(R)$ for every $d \in D$ and every $w \in \mathsf{worlds}(s)$. The $\geq$-minimal state that satisfies these conditions is again one of the alternatives for $\forall x.\exists y.Rxy$. It is also the unique alternative for the response *his mother* ($\forall x.R(x, f(x))$). Thus, this is yet another basic compliant response.

## 4 Conclusions

In this paper, we addressed a problem that arises when the notion of compliance introduced in [20] is extended from a propositional setting to a first-order setting. This notion of compliance is formulated within the basic inquisitive semantic system, $\mathsf{Inq}_B$. We argued that the problem does not lie in the particular way the notion of compliance is defined, but rather in the fact that the system $\mathsf{Inq}_B$ itself is not fine-grained enough to capture compliance conditions, since sentences with intuitively different sets of compliant responses are assigned the same semantic value. As a consequence, no satisfactory notion of compliance can be defined based on $\mathsf{Inq}_B$.

To fix this problem we developed a more sophisticated semantics, $\mathsf{Inq}_W$, in which the conversational context is taken to include not only a certain body of information, but also a certain set of witnesses. Along with information, sentences may then request or provide certain witnesses. This semantic refinement allows us to assign a different range of compliant responses to sentences having the same informative and inquisitive content, but requesting different sorts of witnesses. This extra semantic sensitivity allowed us to correctly characterize the set of basic compliant responses for those examples that were problematic for $\mathsf{Inq}_B$.

But how general is our solution? Is the system $\mathsf{Inq}_W$ rich enough to account for *all* semantic distinctions that are relevant to determine compliant responses? Unfortunately, not quite. In fact, just before submitting this paper, we became aware of a problematic case. Consider the setting of the earlier boundedness formulas, but now suppose that our language has two unary predicates, $P$ and $Q$. Let $B_P(x) \coloneqq \forall y(P(y) \to y \leq x)$ and let $B_Q(x) \coloneqq \forall y(Q(y) \to y \leq x)$. Consider the following two sentences:

1. $\exists x B_P(x) \wedge \exists x B_Q(x)$
2. $\exists x(B_P(x) \wedge B_Q(x))$

Intuitively, these two sentences have different basic compliant responses. Any sentence of the form $B_P(n) \wedge B_Q(m)$, with $n$ and $m$ two natural numbers, should count as a basic compliant response to the former, while only sentences of the form $B_P(n) \wedge B_Q(n)$ should count as basic compliant responses to the latter. However, the two come out exactly equivalent, not only in $\mathsf{Inq}_B$, but also in $\mathsf{Inq}_W$. It can be shown that the only basic compliant responses predicted by our semantics are those of the form $B_P(n) \wedge B_Q(n)$. What is going on here is that the witnesses for one formula interfere with the witnesses for another in a way they are not meant to. $\mathsf{Inq}_W$ is still too coarse, in that it pools all witnesses together in one witness set. What we need, it seems, is a system that treats witnesses in a more refined way, keeping track of the properties or relations that they are witnesses for. Evidently, the development of such a further refinement is a task for future work.

## References


1. Asher, N., Lascarides, A.: Logics of Conversation. Cambridge University Press (2003)
2. Belnap, N.: Questions, their presuppositions, and how they can arise. In: Lambert, K. (ed.) The logical way of doing things. Yale University Press (1969)
3. Büring, D.: On d-trees, beans, and accents. Linguistics and Philosophy 26, 511–545 (2003)
4. Carlson, L.: Dialogue Games. Reidel, Dordrecht (1983)
5. Chierchia, G.: Questions with quantifiers. Natural Language Semantics 1(2), 181–234 (1993)
6. Ciardelli, I.: Inquisitive semantics and intermediate logics (2009), Master Thesis, University of Amsterdam
7. Ciardelli, I.: A first-order inquisitive semantics. In: Aloni, M., Bastiaanse, H., de Jager, T., Schulz, K. (eds.) Logic, Language, and Meaning: Selected Papers from the Seventeenth Amsterdam Colloquium. Springer (2010)
8. Ciardelli, I., Cornelisse, I., Groenendijk, J., Roelofsen, F.: Computing compliance. In: He, X., Horty, J., Pacuit, E. (eds.) Logic, Rationality, and Interaction, pp. 55–65. Springer (2009)
9. Ciardelli, I., Groenendijk, J., Roelofsen, F.: Inquisitive semantics: a new notion of meaning (2012), forthcoming in *Language and Linguistics Compass*.
10. Ciardelli, I., Groenendijk, J., Roelofsen, F.: Inquisitive semantics, NASSLLI lecture notes (2012), available via www.illc.uva.nl/inquisitive-semantics
11. Ciardelli, I., Roelofsen, F.: Inquisitive logic. Journal of Philosophical Logic 40(1), 55–94 (2011)
12. Clark, H.: Arenas of Language Use. University of Chicago Press, Chicago (1992)
13. Engdahl, E.: Constituent questions: The syntax and semantics of questions with special reference to Swedish. Reidel, Dordrecht (1986)
14. Farkas, D., Bruce, K.: On reacting to assertions and polar questions. Journal of semantics 27, 81–118 (2010)
15. Farkas, D., Roelofsen, F.: Polar initiatives and polarity particle responses in an inquisitive discourse model (2012), presented at the Amsterdam Colloquium, December 2011, and at the Newcastle Workshop on the Syntax of Answers to Polar Questions, June 2012, available via www.illc.uva.nl/inquisitive-semantics
16. Ginzburg, J.: Dynamics and the semantics of dialogue. In: Seligman, J. (ed.) Language, Logic, and Computation. vol. 1 (1996)
17. Groenendijk, J.: The logic of interrogation. In: Matthews, T., Strolovitch, D. (eds.) Semantics and Linguistic Theory. pp. 109–126. Cornell University Press (1999)
18. Groenendijk, J.: Inquisitive semantics: Two possibilities for disjunction. In: Bosch, P., Gabelaia, D., Lang, J. (eds.) Seventh International Tbilisi Symposium on Language, Logic, and Computation. Springer-Verlag (2009)
19. Groenendijk, J.: Erotetic languages and the inquisitive hierarchy (2011), in a Festschrift for Martin Stokhof, available via www.vddoes.net/Martin/articles/Jeroen.pdf
20. Groenendijk, J., Roelofsen, F.: Inquisitive semantics and pragmatics (2009), Presented at the Workshop on Language, Communication, and Rational Agency at Stanford, May 2009, available via www.illc.uva.nl/inquisitive-semantics
21. Groenendijk, J., Stokhof, M.: Studies on the Semantics of Questions and the Pragmatics of Answers. Ph.D. thesis, University of Amsterdam (1984)

22. Gunlogson, C.: True to Form: Rising and Falling Declaratives as Questions in English. Ph.D. thesis, University of California at Santa Cruz (2001)
23. Hamblin, C.I.: Mathematical models of dialogue. Theoria 37, 130–155 (1971)
24. Hamblin, C.L.: Questions in Montague English. Foundations of Language 10, 41–53 (1973)
25. Harrah, D.: The logic of questions. In: Gabbay, D., Guenthner, F. (eds.) Handbook of Philosohical Logic, Second Edition, vol. 8, pp. 1–60. Kluwer (2002)
26. Hulstijn, J.: Structured information states: raising and resolving issues. In: Benz, A., Jäger, G. (eds.) Proceedings of MunDial (1997)
27. Jäger, G.: Only updates: on the dynamics of the focus particle *only*. In: Dekker, P., Stokhof, M. (eds.) Proceedings of the 10th Amsterdam Colloquium (1996)
28. Karttunen, L.: Syntax and semantics of questions. Linguistics and Philosophy 1, 3–44 (1977)
29. Kubiński, T.: An outline of the logical theory of questions. Akademie Verlag (1980)
30. Mascarenhas, S.: Inquisitive semantics and logic (2009), Master Thesis, University of Amsterdam
31. Roberts, C.: Information structure in discourse. In: Yoon, J., Kathol, A. (eds.) OSU Working Papers in Linguistics, vol. 49, pp. 91–136. Ohio State University (1996)
32. Roelofsen, F.: Algebraic foundations for inquisitive semantics. In: van Ditmarsch, H., Lang, J., Shier, J. (eds.) Proceedings of the Third International Conference on Logic, Rationality, and Interaction. pp. 233–243. Springer-Verlag (2011)
33. Stalnaker, R.: Assertion. Syntax and Semantics 9, 315–332 (1978)
34. Wiśniewski, A.: The logic of questions as a theory of erotetic arguments. Synthese 109(1), 1–25 (1996)