On the semantics and logic of declaratives and interrogatives

Ivano Ciardelli · Jeroen Groenendijk · Floris Roelofsen

Abstract In many natural languages, there are clear syntactic and/or intonational differences between *declarative* sentences, which are primarily used to provide information, and *interrogative* sentences, which are primarily used to request information. Most logical frameworks restrict their attention to the former. Those that are concerned with both usually assume a logical language that makes a clear syntactic distinction between declaratives and interrogatives, and usually assign different types of semantic values to these two types of sentences.

A different approach has been taken in recent work on *inquisitive semantics*. This approach does not take the basic syntactic distinction between declaratives and interrogatives as its starting point, but rather a new notion of meaning that captures both informative and inquisitive content in an integrated way. The standard way to treat the logical connectives in this approach is to associate them with the basic algebraic operations on these new types of meanings. For instance, conjunction and disjunction are treated as *meet* and *join* operators, just as in classical logic. This gives rise to a hybrid system, where sentences can be both informative and inquisitive at the same time, and there is no clearcut division between declaratives and interrogatives.

It may seem that these two general approaches in the existing literature are quite incompatible. The main aim of this paper is to show that this is *not* the case. We develop an inquisitive semantics for a logical language that has a clearcut division between declaratives and interrogatives. We show that this language coincides in expressive power with the hybrid language that is standardly assumed in inquisitive semantics, we establish a sound and complete axiomatization for the associated logic, and we consider a natural enrichment of the system with presuppositional interrogatives.

Keywords Logics of questions · inquisitive semantics · partition semantics

I.A. Ciardelli, J. Groenendijk, F. Roelofsen
Institute for Logic, Language, and Computation, University of Amsterdam
E-mail: i.a.ciardelli@uva.nl / j.a.g.groenendijk@uva.nl / f.roelofsen@uva.nl

1 Introduction

In many natural languages, there are clear syntactic and/or intonational differences between *declarative* sentences, which are primarily used to provide information, and *interrogative* sentences, which are primarily used to request information. Most logical frameworks, both in the linguistic and in the philosophical tradition, restrict their attention to the former. Those that are concerned with both (Hamblin, 1973; Karttunen, 1977; Hintikka, 1981, 1983, 1999, 2007; Groenendijk and Stokhof, 1984, 1997; Wiśniewski, 1996, 2001, among others) usually assume a logical language that makes a clear syntactic distinction between declaratives and interrogatives. We will say that such analyses are *syntactically dichotomous*.

Most of these analyses do not only assume a syntactic distinction between declaratives and interrogatives, but also assign different types of semantic values to these two sentence types. We will say that such analyses are not only syntactically dichotomous, but also *semantically dichotomous*.

A concrete example of an approach that is both syntactically and semantically dichotomous is the *partition semantics* of Groenendijk and Stokhof (1984, 1997). For simplicity, let us consider only the propositional fragment of the system, as presented in Groenendijk and Stokhof (1997, §4). This system assumes a logical language that contains (i) a standard propositional language $\mathcal{L}$, and (ii) sentences of the form $?\varphi$, where $\varphi \in \mathcal{L}$. Sentences in the basic propositional language $\mathcal{L}$ are called declaratives, while sentences of the form $?\varphi$ are called interrogatives. This means that every sentence in the language is either declarative or interrogative. Thus, the system is syntactically dichotomous.

Semantically, the basic picture that Groenendijk and Stokhof argue for is that declaratives express *propositions*, viewed as sets of possible worlds, while interrogatives express *equivalence relations* over the set of possible worlds. Since equivalence relations correspond to *partitions*, interrogatives can be seen as partitioning the set of all possible worlds, while declaratives can be seen as carving out a particular region in the set of all possible worlds. In uttering a declarative φ, a speaker provides the information that the actual world is located in the region carved out by φ, while in uttering an interrogative $?\varphi$, a speaker requests enough information to locate the actual world in one of the cells of the partition induced by $?\varphi$. Thus, declaratives and interrogatives receive different types of semantic values, which means that the system is not only syntactically dichotomous, but also semantically dichotomous.

A different approach has been pursued in recent work on *inquisitive semantics* (Groenendijk and Roelofsen, 2009; Ciardelli, 2009; Ciardelli and Roelofsen, 2011, among others). This approach does not take the syntactic distinction between declaratives and interrogatives as its starting point, but rather a new notion of *meaning* that captures both informative and inquisitive content in an integrated way. The standard way to treat the logical connectives in this approach is to associate them with the basic algebraic operations on these new types of meanings (Roelofsen, 2011, 2013; Ciardelli et al., 2012). For instance, conjunction and disjunction are treated as *meet* and *join* operators, respectively, just as in classical logic. This treatment of the logical connectives gives rise to a *hybrid* system, where sentences may be both informative and inquisitive at the same time, and there is no clearcut division between declaratives and interrogatives. This system is referred to as InqB, where B stands for basic. InqB is not syntactically dichotomous,

and since semantic dichotomy presupposes syntactic dichotomy, it is not semantically dichotomous either. The logic that InqB gives rise to has been investigated in Ciardelli (2009); Ciardelli and Roelofsen (2009, 2011).

It may seem that the two approaches are quite incompatible, since their architectures diverge in such a fundamental way. The main goal of this paper is to show that this is in fact *not* the case. Even though inquisitive semantics, viewed as a general approach to meaning, does not *require* a clearcut syntactic distinction between declaratives and interrogatives, it is perfectly *compatible* with such a distinction. To demonstrate this, we will show how the new type of meanings assumed in inquisitive semantics can be assigned in a natural way to sentences in a language that has a clearcut distinction between declaratives and interrogatives. The resulting system will be referred to as InqD, where D stands for dichotomous. Evidently, InqD is syntactically dichotomous. Strictly speaking, it is not semantically dichotomous, since all sentences are assigned the same type of meanings, capturing both their informative and their inquisitive content. However, in the case of declarative sentences, inquisitive content will always be trivial, while in the case of interrogative sentences, informative content will always be trivial. Thus, even though the system is not semantically dichotomous in the strict sense of the term, it does clearly capture the crucial semantic difference between declaratives and interrogatives.

We provide meaning preserving translations between InqD and InqB, as well as a sound and complete proof system for the logic that InqD gives rise to. We show that the expressive power of InqD crucially extends that of partition semantics. In particular, InqD allows for a basic analysis of disjunctive and conditional questions, which are notoriously beyond the expressive reach of partition semantics (see, e.g., Mascarenhas, 2009; Ciardelli et al., 2013a).

We also consider an extension of InqD with *presuppositional* interrogatives. We refer to this system as InqD$_\pi$, where π stands for presuppositional. We show that the proof system developed for InqD is also sound and complete for InqD$_\pi$, provided that its inference rules are taken to apply to the wider range of interrogatives available in InqD$_\pi$. From the perspective of comparing different erotetic logics, an important feature of InqD$_\pi$ is that it exhibits some fundamental similarities with some of the most widely-studied existing systems, in particular the *inferential erotetic logic* (IEL) of Wiśniewski (1996, 2001) and the *interrogative model of inquiry* (IMI) of Hintikka (1981, 1983, 1999, 2007). Just like InqD$_\pi$, these systems assume a dichotomous language with presuppositional interrogatives. This similarity makes it easier to compare inquisitive semantics with IEL and IMI, and to transfer insights between the different approaches. For instance, it seems that the notion of entailment considered in inquisitive semantics is meaningful and relevant in the context of IEL and IMI as well, and that the axiomatization result established in this paper can be exported straightforwardly.

The system InqD achieves division of semantic labor at the cost of sacrificing the algebraic treatment of the logical constants embodied by InqB. We conclude by sketching a way to reconcile the tenets of the two approaches by building division of labor not directly *into* the core system, but rather *on top* of it.

The paper is organized as follows. We start in section 2 with a brief review of InqB. Then, in section 3, we present InqD, and meaning preserving translations between InqD and InqB. In section 4 we provide a proof system for the logic that InqD gives rise to, and in section 5 we compare the expressive power of InqD with

that of partition semantics. In section 6, we present the system InqD_π, and show that the proof system for InqD is also sound and complete for InqD_π, provided that we take its inference rules to apply to the wider range of interrogatives available in InqD_π. In section 7 we briefly sketch an alternative way of achieving division of labor than the one offered by InqD, and indicate its potential relevance for the semantic analysis of natural language. Section 8 concludes.

2 Basic inquisitive semantics

We start with a brief review of InqB. Throughout the paper we will restrict our attention to the propositional case. The language of InqB, $\mathcal{L}_{\mathsf{InqB},\mathcal{P}}$, is a standard propositional language based on a set of atomic sentences $\mathcal{P}$, with $\bot$, $\wedge$, $\vee$, and $\to$ as its basic connectives. We also make use of three abbreviations.

Definition 1 (Abbreviations)

1. $\neg\varphi$ abbreviates $\varphi \to \bot$
2. $!\varphi$ abbreviates $\neg\neg\varphi$
3. $?\varphi$ abbreviates $\varphi \vee \neg\varphi$

The ingredients for the semantics of InqB are worlds and states. A $\mathcal{P}$-world is simply a valuation function for $\mathcal{P}$, assigning a truth value to each atomic sentence $p \in \mathcal{P}$. A $\mathcal{P}$-*state* is a set of $\mathcal{P}$-worlds. Reference to the set of atomic sentences $\mathcal{P}$ will be dropped whenever possible. The semantics of InqB is defined not in terms of truth at worlds, but rather in terms of *support* at states.

Definition 2 (Support for InqB)

1. $s \models p$ iff $\forall w \in s : w(p) = 1$
2. $s \models \bot$ iff $s = \emptyset$
3. $s \models \varphi \vee \psi$ iff $s \models \varphi$ or $s \models \psi$
4. $s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$
5. $s \models \varphi \to \psi$ iff $\forall t \subseteq s : $ if $t \models \varphi$ then $t \models \psi$

Support is *persistent*: if a state s supports a sentence φ, then every substate $t \subseteq s$ also supports φ. Moreover, the empty state supports every sentence, which means that every sentence is supported by at least one state. Thus, the set of states that supports a given sentence is always *non-empty* and *downward closed*. Non-empty, downward closed sets of states are referred to as *propositions* in inquisitive semantics, and the set of states that support a sentence φ is referred to as the *proposition expressed by* φ.

Definition 3 (Propositions in inquisitive semantics)
A $\mathcal{P}$-proposition is a non-empty, downward closed set of $\mathcal{P}$-states.

Definition 4 (Propositions expressed by sentences in InqB)
The proposition expressed by a sentence φ in InqB, $[\varphi]_{\mathsf{InqB}}$, is the set of all states that support φ in InqB.

We will usually simply write $[\varphi]$ instead of $[\varphi]_{\mathsf{InqB}}$. Given the properties of support mentioned above, the proposition $[\varphi]$ expressed by a sentence in InqB is indeed always a proposition in the sense of definition 3. Moreover, the language of InqB is *expressively complete*, in the sense that for any finite set $\mathcal{P}$ of atomic sentences, any $\mathcal{P}$-proposition is the proposition expressed by some sentence in $\mathcal{L}_{\mathsf{InqB},\mathcal{P}}$. The proof of this fact can be found in Ciardelli (2009).

Proposition 1 (Expressive completeness of InqB)
Let $\mathcal{P}$ be a finite set of atomic sentences. Then for any $\mathcal{P}$-proposition A there is a sentence $\varphi \in \mathcal{L}_{\mathsf{InqB},\mathcal{P}}$ such that $[\varphi] = A$.

In uttering a sentence φ, a speaker is taken to provide the information that the actual world is included in one of the states that supports φ, and to request enough information from other participants to establish a specific state that supports φ. Thus, the proposition expressed by φ captures both its informative and its inquisitive content.

Definition 5 (Entailment and equivalence)

1. $\varphi \models \psi$ iff $[\varphi] \subseteq [\psi]$
2. $\varphi \equiv \psi$ iff $\varphi \models \psi$ and $\psi \models \varphi$

One sentence φ entails another sentence ψ just in case every state that supports φ also supports ψ. This means that whenever we accept the information provided by φ and supply the information requested by φ, establishing a state that supports φ, we also accept the information provided by ψ and supply the information requested by ψ.

The set of all propositions in inquisitive semantics, together with the inquisitive entailment order, forms a Heyting algebra, just like the set of all classical propositions together with the classical entailment order. Moreover, disjunction, conjunction, negation, and implication behave semantically as *join*, *meet*, and *(relative) pseudo-complement* operators in this algebra, just like in classical logic (Roelofsen, 2011). Thus, InqB can be seen as the equivalent of classical propositional logic (CPL) in the inquisitive setting. For this reason, it is regarded as the most basic inquisitive semantics.

Since in uttering φ, a speaker provides the information that the actual world is located in one of the states in $[\varphi]$, the informative content of φ is characterized by $\bigcup[\varphi]$.

Definition 6 (Informative content) $\mathsf{info}(\varphi) \coloneqq \bigcup[\varphi]$

In classical propositional logic, the informative content of a sentence φ is embodied by the set of all worlds in which that sentence is true, which we will denote as $|\varphi|$. It can be shown that for every $\varphi \in \mathcal{L}_{\mathsf{InqB}}$, $\mathsf{info}(\varphi) = |\varphi|$. This means that, while inquisitive semantics adds an inquisitive dimension to the classical notion of meaning, InqB does not diverge from CPL as far as informative content goes.

A sentence φ is called informative iff its informative content is non-trivial, i.e., iff $\mathsf{info}(\varphi) \neq \omega$, where ω denotes the set of all worlds. On the other hand, φ is called inquisitive iff in order to establish a state that supports φ it is not enough to just accept the informative content of φ, i.e., iff $\mathsf{info}(\varphi) \notin [\varphi]$. This means that in order to establish a state that supports φ, additional information beyond $\mathsf{info}(\varphi)$ is needed.
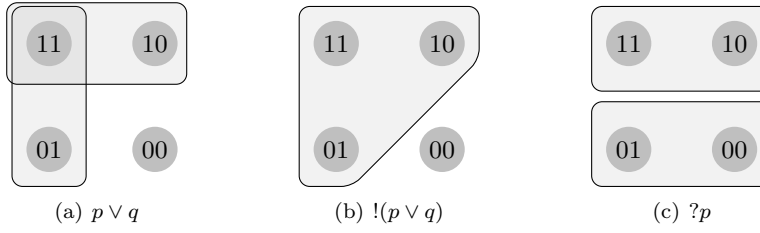
Fig. 1 An example of a hybrid (a), an assertion (b), and a question (c).

Definition 7 (Informativeness and inquisitiveness)

- φ is informative iff $\mathsf{info}(\varphi) \neq \omega$
- φ is inquisitive iff $\mathsf{info}(\varphi) \notin [\varphi]$

In terms of informativeness and inquisitiveness, the following semantic categories can be distinguished.

Definition 8 (Assertions, questions, hybrids, and tautologies)

- φ is an assertion iff it is non-inquisitive
- φ is a question iff it is non-informative
- φ is a hybrid iff it is both informative and inquisitive
- φ is a tautology iff it is neither informative nor inquisitive

It can be shown that, as long as we restrict ourselves to the propositional case, every state that supports a given sentence φ is included in a *maximal* state supporting φ (see, e.g., Ciardelli, 2009; Ciardelli and Roelofsen, 2011).[1] Together with the fact that support is persistent, this means that the proposition expressed by φ is completely determined by the set of maximal states that support φ. These states are referred to as the *possibilities for* φ.

Definition 9 (Possibilities for a sentence)
The maximal states that support a sentence φ are called the *possibilities* for φ.

Fact 1 (Possibilities and propositions)
For every state s and every sentence φ, $s \in [\varphi]$ iff s is included in a possibility for φ.

This allows for an alternative characterization of inquisitiveness and assertions.

Fact 2 (Inquisitiveness in terms of possibilities)

1. *φ is inquisitive iff there are at least two possibilities for φ*
2. *φ is an assertion iff there is exactly one possibility for φ*

The characterization of propositions in terms of possibilities also allows for a perspicuous visual representation of propositions. Figure 1 depicts the propositions expressed by some simple sentences in a language that has just two atomic sentences, p and q. Given such a language, there are just four possible worlds to

---

[1] This fact does not hold anymore in the first-order version of InqB (Ciardelli, 2009, 2010; Ciardelli et al., 2013b).

consider: a world where both p and q are true, one where p is true and q is false, one where p is false and q is true, and one where both p and q are false. In figure 1 these worlds are marked 11, 10, 01, and 00, respectively. Figure 1(a) depicts the proposition expressed by $p \vee q$. Since the proposition expressed by a sentence is fully determined by the possibilities for that sentence, we only depict these possibilities. In the case of $p \vee q$, there are two possibilities: one consisting of all worlds where p is true, 11 and 10, and one consisting of all worlds where q is true, 11 and 01. Notice that $p \vee q$ is *informative*, since $\mathsf{info}(p \vee q) \neq \omega$. It provides the information that the actual world is one in which at least one of p and q is true. It is also *inquisitive*, since it has more than one possibility. Since $p \vee q$ is both informative and inquisitive, it is a *hybrid*.

Figure 1(b) depicts the proposition expressed by $!(p \vee q)$. This sentence has the same informative content as $p \vee q$, but it is not inquisitive. Thus, it is an *assertion*. This illustrates a more general fact. Namely, for any sentence φ, $!\varphi$ has precisely the same informative content as φ. Moreover, $!\varphi$ is always an assertion, i.e., it is never inquisitive. For this reason, ! is referred to as the *non-inquisitive projection operator* in InqB.

Finally, figure 1(c) depicts the proposition expressed by $?p$. This sentence is not informative, since $\mathsf{info}(?p) = \omega$. However, it does request information. Specifically, it requests enough information to either establish a state that supports p or a state that supports $\neg p$. Thus, $?p$ is a *question*. Again, this illustrates a more general fact. Namely, for any φ, $?\varphi$ requests enough information to establish a state that supports either φ or $\neg\varphi$. Moreover, $?\varphi$ is always a question, i.e., it is never informative. For this reason, ? is referred to as the *non-informative projection operator* in InqB.

A sentence φ is always equivalent to the conjunction of its two projections, $?\varphi$ and $!\varphi$. This fact, which will play an important role later on, is known as 'division' (see, e.g. Ciardelli, 2009; Groenendijk and Roelofsen, 2009).

Fact 3 (Division) *For any sentence φ, $\varphi \equiv ?\varphi \wedge !\varphi$*

Given these notions, it is natural to think of sentences in InqB as inhabiting a two-dimensional space, as depicted in figure 2 (see Mascarenhas, 2009; Ciardelli, 2009; Roelofsen, 2013). One of the axes is inhabited by questions, which are always non-informative; the other axis is inhabited by assertions, which are always non-inquisitive; the 'zero-point' of the space is inhabited by tautologies, which are neither informative nor inquisitive; and the rest of the space is inhabited by hybrids, which are both informative and inquisitive. Every hybrid sentence φ has a projection onto the horizontal axis, $!\varphi$, and a projection onto the vertical axis, $?\varphi$. The former is always an assertion, the latter is always a question, and the conjunction of the two is always equivalent with φ itself.

This concludes our brief review of InqB. Notice that the system is not syntactically dichotomous: there is no clearcut syntactic division between declaratives and interrogatives. It may be suitable to think of sentences of the form $!\varphi$ and $?\varphi$ as corresponding to declaratives and interrogatives in natural language, respectively (see Roelofsen, 2013). However, even so, the logical language also contains many sentences that are not of this form, and can as such not be classified as being either declarative or interrogative.

Note also that questions and assertions are defined in InqB as *semantic* categories. That is, whether a given sentence counts as an assertion or a question is not
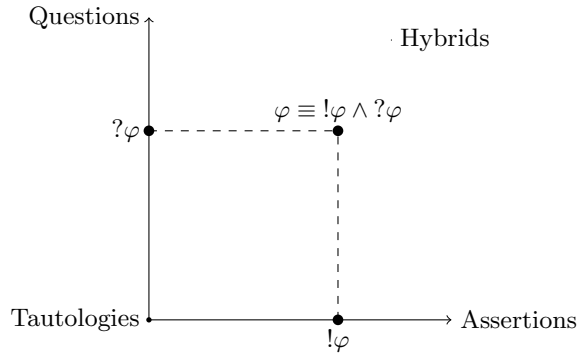
Fig. 2 Projection and division.

defined in terms of syntactic features, but rather in terms of semantic features: the proposition that the sentence expresses, and its informative content. Thus, these categories are quite different from the categories of declaratives and interrogatives, which are syntactic in nature.

Finally, note that InqB is not semantically dichotomous either. All sentences are assigned the same type of semantic value: a non-empty, downward closed set of states. These semantic values capture both informative and inquisitive content at the same time. In some cases, the informative content or the inquisitive content of a sentence is trivial. However, the system does not make use of two distinct types of semantic values, one to capture informative content and the other to capture inquisitive content, as is characteristic of semantically dichotomous approaches.

3 Inquisitive semantics for declaratives and interrogatives

We now turn to the heart of the paper, which aims to show that, even though inquisitive semantics does not *require* a clearcut syntactic division between declaratives and interrogatives, it is certainly *compatible* with such a division. We will specify an alternative system, InqD, which uses exactly the same inquisitive semantic machinery as InqB, but applies this machinery to a language that makes a clear division between declaratives and interrogatives.

3.1 Language

The logical language that we will consider, $\mathcal{L}_{\mathsf{InqD}}$, consists of declaratives, interrogatives, and sequences of sentences that may be either declarative or interrogative. The latter are included to allow for meaning-preserving translations back and forth between InqD and InqB. In defining $\mathcal{L}_{\mathsf{InqD}}$ we will use α and β as meta-variables ranging over declaratives, μ and ν as meta-variables ranging over interrogatives, and φ and ψ as meta-variables ranging over arbitrary sentences. We start with a definition of the declarative fragment of the language, $\mathcal{L}_!$.

Definition 10 (Declaratives) $\mathcal{L}_!$ is the smallest set containing a given set $\mathcal{P}$ of atomic sentences, as well as $\bot$, and is closed under conjunction and implication. That is:

- If $\alpha, \beta \in \mathcal{L}_!$, then $\alpha \wedge \beta \in \mathcal{L}_!$ as well
- If $\alpha, \beta \in \mathcal{L}_!$, then $\alpha \rightarrow \beta \in \mathcal{L}_!$ as well

Negation and disjunction are defined as abbreviations, as follows.

Definition 11 (Abbreviations) For any $\alpha, \beta \in \mathcal{L}_!$:

- $\alpha \rightarrow \bot$ is abbreviated as $\neg \alpha$
- $\neg(\neg \alpha \wedge \neg \beta)$ is abbreviated as $\alpha \vee \beta$

Now let us turn to interrogatives. As is common practice in a number of logical approaches to questions, in particular the *inferential erotetic logic* (IEL) of Wiśniewski (1996, 2001) and the *interrogative model of inquiry* (IMI) of Hintikka (1981, 1983, 1999, 2007), we take basic interrogatives to be of the form $?\{\alpha_1, \ldots, \alpha_n\}$, where $\alpha_1, \ldots, \alpha_n$ are declarative sentences.[2] Intuitively, the declaratives $\alpha_1, \ldots, \alpha_n$ determine what is needed to resolve the issue that is raised by $?\{\alpha_1, \ldots, \alpha_n\}$. This intuition will be reflected by the semantic clauses given below: a state will support $?\{\alpha_1, \ldots, \alpha_n\}$ just in case it supports at least one of $\alpha_1, \ldots, \alpha_n$.

We place one restriction on the formation of basic interrogatives. Namely, the set $\{\alpha_1, \ldots, \alpha_n\}$ must be such that $\alpha_1 \vee \ldots \vee \alpha_n$ constitutes a classical tautology. In other words, for every possible world w, at least one of $\alpha_1, \ldots, \alpha_n$ must be true in w. Intuitively, this means that it must always be possible to truthfully resolve a basic interrogative in any possible world.[3]

Unlike in IEL and IMI, besides basic interrogatives, our language will also contain more complex interrogatives. These are built from basic ones using conjunction and implication. In the case of conjunction, both conjuncts must be interrogatives. In the case of implication, the antecedent must be declarative, and the consequent interrogative.[4]

---

[2] Like IEL, we take the symbols '{' and '}' to be part of the object language. This means that, e.g., $?\{p, \neg p\}$, $?\{\neg p, p\}$, and $?\{p, p, \neg p\}$ are three distinct interrogative formulas. Unlike IEL, we impose no requirement that $n \geq 2$ or that the α's should be syntactically distinct. However, different choices in this respect would be just as compatible with the framework that we are going to propose, and would not impinge on the results that will be established here.

[3] Instead of imposing this restriction, we could also think of a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ as *presupposing* that the actual world is one where the interrogative can be truthfully resolved, i.e., a world where at least one of $\alpha_1, \ldots, \alpha_n$ is true (see Hintikka, 1981, 1983, 1999, 2007; Wiśniewski, 1996, 2001). This alternative will be explored in section 6. Yet another strategy would be to assume that a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ cannot only be resolved by establishing that one of $\alpha_1, \ldots, \alpha_n$ is true, but also by establishing that all of $\alpha_1, \ldots, \alpha_n$ are false. This also guarantees that basic interrogatives can be truthfully resolved in every world (see Groenendijk, 2011).

[4] The constraint that the antecedent of a conditional interrogative must be a declarative, is a bit arbitrary from a purely semantic perspective. In InqB, an implication is bound to be a question (i.e., non-informative) as soon as its consequent is. So, unlike the constraint on conjunction that both conjuncts must be interrogative, which is needed to guarantee that the conjunction as a whole expresses a question, the constraint on implication is not semantically motivated (see Groenendijk (2011) for more detailed discussion of this point).

Definition 12 (Interrogatives) $\mathcal{L}_?$ is the smallest set such that:

– If $\alpha_1, \ldots, \alpha_n \in \mathcal{L}_!$ and $\alpha_1 \vee \cdots \vee \alpha_n$ is a classical tautology,
  then $?\{\alpha_1, \ldots, \alpha_n\} \in \mathcal{L}_?$
– If $\mu, \nu \in \mathcal{L}_?$, then $\mu \wedge \nu \in \mathcal{L}_?$
– If $\alpha \in \mathcal{L}_!$ and $\mu \in \mathcal{L}_?$, then $\alpha \to \mu \in \mathcal{L}_?$

Finally, we define $\mathcal{L}_{\mathsf{InqD}}$ as the language containing both $\mathcal{L}_!$ and $\mathcal{L}_?$, as well as finite sequences of sentences from either $\mathcal{L}_!$ or $\mathcal{L}_?$.

Definition 13 ($\mathcal{L}_{\mathsf{InqD}}$) $\mathcal{L}_{\mathsf{InqD}}$ is the smallest set such that:

– If $\varphi \in \mathcal{L}_! \cup \mathcal{L}_?$, then $\varphi \in \mathcal{L}_{\mathsf{InqD}}$
– If $\varphi_1, \ldots, \varphi_n \in \mathcal{L}_! \cup \mathcal{L}_?$, then $\langle \varphi_1, \ldots, \varphi_n \rangle \in \mathcal{L}_{\mathsf{InqD}}$

A sequence $\langle \varphi_1, \ldots, \varphi_n \rangle$ may be thought of as a discourse or text consisting of multiple consecutive sentences.

3.2 Semantics

In defining the semantics of InqD, we follow exactly the same pattern as we did in defining the semantics for InqB above. We first specify recursively when a sentence is supported by a given state.

Definition 14 (Support for InqD)

1. $s \models p$ iff $\forall w \in s : w(p) = 1$
2. $s \models \bot$ iff $s = \emptyset$
3. $s \models ?\{\alpha_1, \ldots, \alpha_n\}$ iff $s \models \alpha_1$ or $\ldots$ or $s \models \alpha_n$
4. $s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$
5. $s \models \alpha \to \psi$ iff $\forall t \subseteq s : $ if $t \models \alpha$ then $t \models \psi$
6. $s \models \langle \varphi_1, \ldots, \varphi_n \rangle$ iff $s \models \varphi_1$ and $\ldots$ and $s \models \varphi_n$

The proposition expressed by φ, denoted $[\varphi]_{\mathsf{InqD}}$, is the set of all states supporting φ. Just like for InqB, support for InqD is persistent, and moreover the empty state supports every sentence. This ensures that the proposition $[\varphi]_{\mathsf{InqD}}$ expressed by a sentence is always a proposition in the sense of inquisitive semantics (definition 3). All other semantic notions—entailment, equivalence, the informative content of a sentence, the possibilities for a sentence, informative and inquisitive sentences, questions, assertion, hybrids, and tautologies—carry over directly from InqB to InqD as well.

When comparing the definition of support for InqD and InqB, we find two differences, which concern the third and the sixth clause. The sixth clause in the support definition for InqD is concerned with *sequences* of sentences. Such sequences were not part of the language in InqB, so this clause is not present in the support definition for InqB. In InqD, sequences are treated just like conjunctions: a state supports a sequence just in case it supports every element of the sequence.

Now let us consider the third clause of the support definition. In InqB, the third clause was concerned with *disjunction*. In InqD, disjunction is not a basic connective, rather $\alpha \vee \beta$ is defined as an abbreviation of $\neg(\neg\alpha \wedge \neg\beta)$. Thus, the support definition for InqD does not include a clause for disjunction. Rather, the

third clause of the definition is concerned with basic interrogatives. However, notice that the clause is still very similar to the clause for disjunction in InqB: a state supports $?\{\alpha_1, \ldots, \alpha_n\}$ just in case it supports at least one of $\alpha_1, \ldots, \alpha_n$, while in InqB, a state supports a disjunction just in case it supports at least one of the disjuncts. Thus, from a semantic point of view, basic interrogatives in InqD behave just like disjunctions in InqB. The only difference is that the 'disjuncts' of a basic interrogative in InqD cannot be chosen arbitrarily. In order to guarantee that the issue raised by a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ can be truthfully resolved in every world, the 'disjuncts' $\alpha_1, \ldots, \alpha_n$ have to be chosen in such a way that in every world, at least one of them is true.[5]

The fifth clause of the support definition, concerning conditionals, is the same as in InqB. However, it is easy to see that, in the context of InqD, the clause can be given a simpler and more intuitive formulation.

Fact 4 (Alternative clause for conditionals in InqD)
For any conditional $\alpha \to \varphi$ in InqD and any state s:

$$s \models \alpha \to \varphi \iff s \cap |\alpha| \models \varphi$$

A final observation to make about the support definition for InqD is that the clauses for conjunction and implication apply uniformly to both declaratives and interrogatives. There is no need to specify separate clauses for the two types of sentences. This is made possible by the fact that, even though InqD is syntactically dichotomous, it is not *semantically* dichotomous. Just as in InqB, all sentences are assigned the same type of semantic value: a non-empty, downward closed set of states.

One striking difference between InqB and InqD is that the latter does not contain any hybrid sentences. Declaratives are never inquisitive, and interrogatives are never informative. In other words, every declarative is an assertion, and every interrogative is a question. The labor of providing and requesting information is strictly divided between the two sentence types.

Fact 5 (Questions, assertions, and hybrids in InqD)

- *Every declarative in InqD is an assertion.*
- *Every interrogative in InqD is a question.*
- *No single sentence in InqD is a hybrid.*

Of course, a sequence of sentences in InqD may very well be hybrid. For instance, the sequence $\langle p, ?\{q, \neg q\}\rangle$ consisting of the declarative p and the interrogative $?\{q, \neg q\}$, provides the information that p is the case, and requests further information to determine whether q is the case (see figure 3(h) and the discussion below).

Recall that in InqB, the informative content of a sentence φ always amounts to the set of worlds in which it is classically true: $\mathsf{info}(\varphi) = |\varphi|$. This means that InqB does not diverge from the classical treatment of informative content, it just adds an inquisitive dimension to the notion of meaning. An analogous result holds for InqD. Namely, the informative content of every declarative sentence α in InqD

---

[5] This restriction will be lifted in section 6, where we will consider interrogatives that can only be truthfully resolved in *some* worlds.
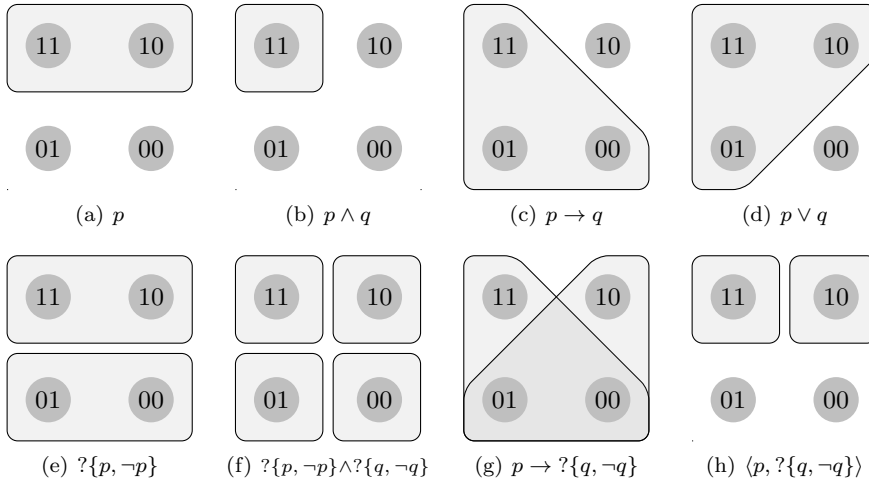
Fig. 3 Propositions expressed by some simple sentences in InqD.

amounts to the set of worlds in which it is classically true: $\mathsf{info}(\alpha) = |\alpha|$. Moreover, since every declarative α is an assertion in InqD, there is always a unique possibility for α, which coincides with $|\alpha|$. Thus, for all intents and purposes, the meaning of a declarative in InqD can be identified with its classical meaning.

Fact 6 (Declaratives behave classically in InqD)
For every declarative α in InqD, $s \models \alpha \iff s \subseteq |\alpha|$. In particular, a declarative α has a unique possibility, which coincides with $|\alpha|$.

As we will see, this fact entails that the logic of the declarative fragment of InqD simply coincides with classical propositional logic (CPL). Thus, InqD is a *conservative extension* of CPL, and in this sense it extends CPL in a less drastic way than InqB. However, there is also a sense in which InqD extends CPL in a more drastic way than InqB. Namely, besides enriching the semantic machinery, it also enriches the syntax. It does not only add an inquisitive dimension to the notion of meaning, but also a new syntactic category—the category of interrogatives—to the logical language.

3.3 Examples

To illustrate the semantics of InqD, we have depicted the propositions expressed by some simple sentences in figure 3. As before, we assume that our language contains just two atomic sentences, p and q, which means that there are just four possible worlds, 11, 10, 01, and 00. Also as before, we only depict possibilities. As we saw above, for every declarative α there is a single possibility, which coincides with the proposition expressed by α in CPL. This is illustrated for some simple declarative sentences in figure 3(a)–3(d).

Now let us turn to interrogatives. First consider the basic interrogative $?\{p, \neg p\}$. As depicted in figure 3(e), there are two possibilities for this interrogative, one

consisting of all worlds where p is true, and one consisting of all worlds where $\neg p$ is true. Thus, this interrogative may be taken to correspond to the polar question whether p is true or false. Polar questions in English usually only make one of the two 'disjuncts' explicit, as illustrated in (1).

(1)   Is John going to the party?

It is also possible to make both disjuncts explicit, as in (2), and in many natural languages other than English, such as Mandarin Chinese, this is in fact the standard way to formulate polar questions.

(2)   Is John going to the party or not?

In terms of inquisitive content, (1) and (2) are equivalent. Both are used to raise the issue whether John is going to the party or not, and both are resolved if and only if one of the two options is established. This is suitably captured both in InqB and in InqD. It has been noted that in English questions like (2) usually carry a sense of urgency that is not necessarily conveyed by standard polar questions like (1) (Bolinger, 1978; Biezma, 2009). In order to capture this fine-grained difference, it would be necessary to further refine the basic inquisitive semantic machinery of InqB and InqD.

Turning back to the logical language of InqD, recall that there are two ways to construct complex interrogatives, using either conjunction or implication. Let us consider one example of both. As depicted in figure 3(f), the conjunctive interrogative $?\{p, \neg p\} \wedge ?\{q, \neg q\}$ has four possibilities. Each of these possibilities contains enough information to determine whether p is true and also whether q is true.

Now let us consider a conditional interrogative, $p \rightarrow ?\{q, \neg q\}$. As depicted in figure 3(g), there are two possibilities for this sentence, $|p \rightarrow q|$ and $|p \rightarrow \neg q|$. Thus, the sentence has the same resolution conditions as a simple conditional question in English, exemplified in (3).

(3)   If John is going to the party, will Mary go as well?

The two possibilities correspond to the two basic resolving answers to this question:

(4)   a.   Yes, if John is going, Mary is going as well.
        b.   No, if John is going, Mary won't go.

Finally, let us consider a simple sequence of two sentences, $\langle p, ?\{q, \neg q\}\rangle$. Notice that the first element of the sequence is declarative and the second interrogative. As depicted in figure 3(h), there are two possibilities for $\langle p, ?\{q, \neg q\}\rangle$. Both of these support p. In addition, one of them supports q and the other supports $\neg q$. Thus, both possibilities contain the information that p is true, as well as sufficient information to determine whether q is true or not.

Notice that all the interrogative sentences we considered are such that their possibilities together cover the set of all possible worlds, as depicted in figures 3(e)–3(g). This means that none of them is informative, they are all questions, in accordance with fact 5. The sequence $\langle p, ?\{q, \neg q\}\rangle$ has two possibilities, which means that it is inquisitive, but these states do not cover the set of all possible worlds, which means that the sequence is informative as well. Thus, as noted

earlier, this example illustrates that sequences can be hybrid, unlike individual sentences in InqD.

3.4 Translations

We will now show that there is a straightforward translation procedure that transforms any sequence of sentences in InqD into an equivalent conjunction of sentences in InqB, and conversely any sentence φ in InqB can be turned into an equivalent sequence of two sentences $\langle \alpha_\varphi, \mu_\varphi \rangle$ in InqD, where α_φ is a declarative equivalent with $!\varphi$, and μ_φ an interrogative equivalent with $?\varphi$.[6]

A corollary of this result is that InqD and InqB have exactly the same expressive power. Recall that InqB is expressively complete (see proposition 1 above). Thus, the fact that it is possible to translate every sentence in $\mathcal{L}_{\mathsf{InqB}}$ into an equivalent sequence of sentences in $\mathcal{L}_{\mathsf{InqD}}$ implies that InqD is also expressively complete.

Let us first consider the translation from $\mathcal{L}_{\mathsf{InqD}}$ to $\mathcal{L}_{\mathsf{InqB}}$, which is straightforward. Note that the translation procedure given below only applies to sentences that do not contain the non-basic connectives $\neg$ and $\vee$. If we want to translate a sentence that does contain $\neg$ or $\vee$, we first have to rewrite it in terms of the basic connectives, and then translate it into $\mathcal{L}_{\mathsf{InqB}}$ using the procedure given here.

Definition 15 (Translation from $\mathcal{L}_{\mathsf{InqD}}$ to $\mathcal{L}_{\mathsf{InqB}}$)

1. $(p)^\dagger = p$ for all $p \in \mathcal{P}$
2. $(\bot)^\dagger = \bot$
3. $(?\{\alpha_1, \ldots, \alpha_n\})^\dagger = (\alpha_1 \vee \ldots \vee \alpha_n)$
4. $(\alpha \to \psi)^\dagger = \alpha \to (\psi)^\dagger$
5. $(\varphi \wedge \psi)^\dagger = (\varphi)^\dagger \wedge (\psi)^\dagger$
6. $(\langle \varphi_1, \ldots, \varphi_n \rangle)^\dagger = (\varphi)^\dagger \wedge \ldots \wedge (\varphi_n)^\dagger$

This translation procedure is meaning preserving.

Fact 7 ($(.)^\dagger$ is meaning preserving) *For all $\varphi \in \mathcal{L}_{\mathit{InqD}}$: $[\varphi]_{\mathit{InqD}} = [(\varphi)^\dagger]_{\mathit{InqB}}$.*

As may be expected, translation in the other direction is less straightforward. For a start, given that $\mathcal{L}_{\mathsf{InqD}}$ does not contain any single sentences that are hybrid, it will not be possible to translate every sentence in $\mathcal{L}_{\mathsf{InqB}}$ into a *single sentence* in $\mathcal{L}_{\mathsf{InqD}}$ in a meaning preserving way. It is possible, however, to translate each sentence in $\mathcal{L}_{\mathsf{InqB}}$ into a *pair of sentences* in $\mathcal{L}_{\mathsf{InqD}}$.

Recall from section 2 that in InqB, every sentence φ is equivalent with the conjunction of its two projections, $!\varphi \wedge ?\varphi$. We referred to this fact as the *division* fact: the informative and the inquisitive content of a sentence φ can always be divided over one sentence that is an assertion, $!\varphi$, and another sentence that is a question, $?\varphi$. We will use this fact to establish the desired meaning preserving translation from $\mathcal{L}_{\mathsf{InqB}}$ to $\mathcal{L}_{\mathsf{InqD}}$. Namely, we will first show how to translate sentences of the form $!\varphi$, and then sentences of the form $?\varphi$. Together with the division fact, this will yield a meaning-preserving translation procedure for the entire language.

---

[6] Throughout this section we will assume that both $\mathcal{L}_{\mathsf{InqD}}$ and $\mathcal{L}_{\mathsf{InqB}}$ are based on the same set of atomic sentences $\mathcal{P}$.

First, recall that the informative content of every sentence φ in InqB coincides with the set of worlds in which this sentence is classically true, $|\varphi|$, and the same holds for every declarative in InqD. It follows that whenever φ is an *assertion* in InqB, it expresses exactly the same proposition in InqB as in InqD, namely $\wp(|\varphi|)$.

Fact 8 *If φ is an assertion in **InqB**, then $[\varphi]_{InqD} = [\varphi]_{InqB} = \wp(|\varphi|)$.*

Recall that every sentence of the form $!\varphi$ is an assertion in InqB. So $!\varphi$ expresses the proposition $\wp(|!\varphi|)$. But since $!\varphi$ abbreviates $\neg\neg\varphi$, $|!\varphi|$ amounts to $|\varphi|$, which means that $[!\varphi]_{InqB} = \wp(|\varphi|) = [\varphi]_{InqD}$. Thus, for sentences of the form $!\varphi$, there is a straightforward meaning preserving translation from InqB to InqD.

Now let us turn to sentences in InqB of the form $?\varphi$. In order to deal with these sentences, we need to build on the fact that every sentence φ in $\mathcal{L}_{InqB}$ can be turned into an equivalent sentence of the form $\neg\varphi_1 \vee \ldots \vee \neg\varphi_n$, a disjunction of negated sentences. One way of achieving this is given by the following *disjunctive negative translation*.

Definition 16 (Disjunctive negative translation)

1. $\text{DNF}(p) = \neg\neg p$
2. $\text{DNF}(\bot) = \neg\neg\bot$
3. $\text{DNF}(\psi \vee \chi) = \text{DNF}(\psi) \vee \text{DNF}(\chi)$
4. $\text{DNF}(\psi \wedge \chi) = \bigvee\{\neg\neg(\psi_i \wedge \chi_j) \,|\, 1 \leq i \leq n, 1 \leq j \leq m\}$ [7]

   where:
   - $\text{DNF}(\psi) = \psi_1 \vee \ldots \vee \psi_n$
   - $\text{DNF}(\chi) = \chi_1 \vee \ldots \vee \chi_m$
5. $\text{DNF}(\psi \to \chi) = \bigvee\{\neg\neg \bigwedge_{1 \leq i \leq n}(\psi_i \to \chi_{f(i)}) \,|\, f : \{1, \ldots, n\} \to \{1, \ldots, m\}\}$

   where:
   - $\text{DNF}(\psi) = \psi_1 \vee \ldots \vee \psi_n$
   - $\text{DNF}(\chi) = \chi_1 \vee \ldots \vee \chi_m$

The sentence $\text{DNF}(\varphi)$ is called the *disjunctive negative form* of φ. As shown in (Ciardelli, 2009; Ciardelli and Roelofsen, 2011), a sentence is always equivalent with its disjunctive negative form.

Fact 9 *For any $\varphi \in \mathcal{L}_{InqB}$, $[\varphi]_{InqB} = [\text{DNF}(\varphi)]_{InqB}$.*

Every negated sentence is an assertion in InqB. So the DNF of a sentence is always a disjunction of assertions, and by fact 8 all these assertions express exactly the same proposition in InqD as they do in InqB. Now consider a sentence of the form $?\varphi$ in InqB, and its disjunctive negative form $\text{DNF}(?\varphi)$. Since $?\varphi$ is a question, $\text{DNF}(?\varphi)$ is also a question, which means that $|\text{DNF}(?\varphi)| = \omega$. In other words, for every possible world $w \in \omega$, there is at least one disjunct of $\text{DNF}(?\varphi)$ that is classically true in w. This means that:

$$?\{\psi \,|\, \psi \text{ is a disjunct of } \text{DNF}(?\varphi)\}$$

is a well-formed basic interrogative in InqD. Moreover, it is clear that this basic interrogative expresses exactly the same proposition in InqD as $\text{DNF}(?\varphi)$ does in

---

[7] If Φ is a finite set of formulas, we write $\bigvee \Phi$ to denote the disjunction $\varphi_1 \vee \cdots \vee \varphi_n$, where $\varphi_1, \ldots, \varphi_n$ is an arbitrary enumeration of the elements of Φ. Similarly, later on we shall write $?\Phi$ for the interrogative $?\{\varphi_1, \ldots, \varphi_n\}$, where $\varphi_1, \ldots, \varphi_n$ is an arbitrary enumeration of Φ.

InqB, which in turn is the same as the proposition expressed by $?\varphi$ in InqB. This, then, establishes the desired meaning preserving translation.

As before, the translation procedure given below is intended to be applied only to sentences in InqB that do not contain any non-basic connectives. If we want to translate a sentence φ that does contain non-basic connectives, then we first have to rewrite it in terms of the basic connectives in InqB, and then translate it using the procedure given below. Moreover, recall that disjunction is a basic connective in InqB but not in InqD; if the translation of φ contains a disjunction $\beta \vee \gamma$, this is to be treated in InqD as an abbreviation of $\neg(\neg\beta \wedge \neg\gamma)$.

Definition 17 (Translation from $\mathcal{L}_{\mathsf{InqB}}$ to $\mathcal{L}_{\mathsf{InqD}}$)
For every $\varphi \in \mathcal{L}_{\mathsf{InqB}}$: $(\varphi)^{\sharp} = \langle \varphi, ?\{\psi \mid \psi \text{ is a disjunct of } \mathrm{DNF}(?\varphi)\}\rangle$.

Fact 10 $((.)^{\sharp}$ is meaning preserving) *For every $\varphi \in \mathcal{L}_{\mathit{InqB}}$: $[(\varphi)^{\sharp}]_{\mathit{InqD}} = [\varphi]_{\mathit{InqB}}$*

Thus, despite the considerable differences in their syntax, InqD and InqB are equivalent in terms of expressive power. Moreover, it can be seen from the translation in definition 17 that removing conjunctive and conditional interrogatives from InqD would not reduce the expressive power of the system. All propositions can be expressed by declaratives, basic interrogatives, and sequences consisting of declaratives and basic interrogatives.

In section 5 we will consider two natural ways to restrict the expressive power of InqD. The most radical way of doing so will yield a system that essentially amounts to the propositional fragment of the partition semantics of Groenendijk and Stokhof (1984, 1997), discussed in the beginning of the paper. But, before coming to that, we will first consider the logic that InqD gives rise to.

4 Entailment and validity in InqD

InqD comes with a notion of entailment and validity that applies uniformly to declaratives and interrogatives.

Definition 18 (Entailment and validity)

- We say that a set of sentences Φ *entails* a sentence ψ in InqD, notation $\Phi \models_{\mathsf{InqD}} \psi$, just in case ψ is supported by any state that supports all sentences in Φ.
- We say that φ is *valid* in InqD just in case is it is supported by all states.
- The set of all validities in InqD is called the *logic* of InqD, and is denoted L_{InqD}.

Before investigating the formal properties of entailment and validity in InqD, let us first consider what these notions amount to at an intuitive level, depending on the syntactic category of the sentences involved. First consider a sentence φ that is valid in InqD. There are two cases to consider, the case where φ is a declarative and the case in which it is an interrogative. If φ is a declarative, then it is valid just in case its informative content is trivial. This means that it cannot be used to provide non-trivial information. If φ is an interrogative, then it is valid just in case the issue that it raises is trivially resolved. This means that it cannot be used to request non-trivial information. Thus, from a conversational point of view, valid sentences, be they declarative or interrogative, are sentences that cannot be used to make any non-trivial contribution to the conversation. Moreover, it follows from

fact 6 that a declarative is valid in InqD if and only if it is a tautology in classical logic.

Now consider entailment. Let us restrict ourselves to cases where the antecedent is a single sentence rather than a set containing multiple sentences. Then there are four cases to consider, the case where both antecedent and consequent are declarative, the case where both are interrogative, and two cases where one is declarative and the other interrogative. We will continue to use α and β as meta-variables ranging over declaratives, and μ and ν as meta-variables ranging over interrogatives, and we will use $\models_{CPL}$ to denote entailment in classical propositional logic.

Case 1: declarative entails declarative

If α and β are declaratives, then it follows from fact 6 that $\alpha \models_{InqD} \beta \iff |\alpha| \subseteq |\beta| \iff \alpha \models_{CPL} \beta$. Thus, a declarative α entails another declarative β just in case α provides as least as much information as β, that is, iff α classically entails β.

Case 2: interrogative entails interrogative

Intuitively, a state supports an interrogative just in case it contains enough information to resolve the issue expressed by the interrogative. Thus, an interrogative μ entails another interrogative ν just in case any piece of information that resolves the issue expressed by μ also resolves the issue expressed by ν; in other words, just in case μ requests at least as much information as ν does. Thus, in this case entailment does not compare informative strength, as it does in the case of declaratives, but rather inquisitive strength.

Case 3: declarative entails interrogative

It follows from fact 6 that for any declarative α and any interrogative μ, $\alpha \models_{InqD} \mu \iff |\alpha| \models \mu$. So, α entails μ just in case the information provided by α is sufficient to resolve the issue expressed by μ. In this case, entailment is thus related to *answerhood*.

Case 4: interrogative entails declarative

Suppose now an interrogative μ entails a declarative α. This implies that the informative content of μ entails the informative content of α, $\mathsf{info}(\mu) \subseteq \mathsf{info}(\alpha)$. But since μ is an interrogative, its informative content is trivial, i.e., coincides with the set ω of all worlds. But then, $\mathsf{info}(\alpha)$ must be trivial as well, which means that α must be a tautology. Thus, an interrogative can never entail a declarative unless the latter is a tautology (in which case, it is entailed by *any* sentence).

Now that it is clear what validity and entailment amount to at an intuitive level, let us turn to the formal properties of these notions. We will focus on entailment, since validity is a special case thereof. We first note that the deduction theorem holds for declaratives.

Proposition 2 (Deduction theorem)
For any set of sentences Φ, any declarative α and any sentence ψ,

$$\Phi, \alpha \models_{InqD} \psi \iff \Phi \models_{InqD} \alpha \to \psi$$

Next we note that the logic is compact. The proof of this fact follows the same line of reasoning as the proof of the corresponding fact for InqB, which can be found in Ciardelli (2009, p. 24–25).

Proposition 3 (Compactness)
For any Φ and ψ, if $\Phi \models_{InqD} \psi$ then there is a finite $\Phi' \subseteq \Phi$ such that $\Phi' \models_{InqD} \psi$.

The interrogative operator is a syntactically constrained form of constructive disjunction. This is witnessed by the fact that it has the disjunction property.

Proposition 4 (Disjunction property for ?) *For any declaratives $\alpha_1, \ldots, \alpha_n$:*

$$\models_{InqD} ?\{\alpha_1, \ldots, \alpha_n\} \iff \models_{InqD} \alpha_i \text{ for some } 1 \leq i \leq n$$

In fact, this is a particular case of a more general and important fact.

Proposition 5 *For any declaratives $\alpha_1, \ldots, \alpha_n$ and any set of declaratives Γ:*

$$\Gamma \models_{InqD} ?\{\alpha_1, \ldots, \alpha_n\} \iff \Gamma \models_{InqD} \alpha_i \text{ for some } 1 \leq i \leq n$$

We now come to the task of axiomatizing $\models_{InqD}$. Since we've already seen that the declarative fragment of InqD behaves classically, we will have to enrich a system for classical logic with special rules to deal with interrogatives. We choose to build on a natural deduction system for classical logic.

The rules of the system are listed in Figure 4 on the next page, using α, β, γ for declaratives, μ, ν, λ for interrogatives, and φ, ψ for generic sentences, which may belong to either category. We write $\Phi \vdash_{InqD} \psi$ if there is a proof of ψ whose undischarged assumptions are all included in Φ. Throughout the rest of this section we will drop the subscript InqD and simply write $\models$ and $\vdash$ for $\models_{InqD}$ and $\vdash_{InqD}$.

A couple of remarks are in place here. First, notice that the introduction and elimination rules for the interrogative operator have exactly the same shape as the (standard) rules for disjunction. The crucial difference between the two operators lies in the generality of the elimination rule: in the case of ?, the conclusion of the elimination rule can be any formula, declarative or interrogative, whereas for $\vee$, the conclusion must be a declarative. This simple restriction prevents obviously unsound derivations, such as the one from the tautology $\alpha \vee \neg\alpha$ to the polar interrogative $?\{\alpha, \neg\alpha\}$.

The Kreisel-Putnam rule is named after a similar rule proposed and investigated by Kreisel and Putnam (1957) in the context of intuitionistic logic. The original rule is concerned with implications that have a negative antecedent and a disjunctive consequent. It distributes the disjuncts of the consequent over the implication as a whole. Similarly, our inference rule distributes the 'disjuncts' of a basic interrogative that forms the consequent of an implication over the implication as a whole. An analogous axiom also plays a crucial role in the axiomatization of InqB (Ciardelli, 2009; Ciardelli and Roelofsen, 2011).

One can check that each of the rules is sound with respect to the semantics, which means that the deduction system as a whole is sound as well.

Proposition 6 (Soundness) *For any set of sentences Φ and any sentence ψ:*

$$\Phi \vdash \psi \implies \Phi \models \psi$$

As far as declaratives are concerned, the system coincides with the usual natural deduction system for CPL, and therefore it is complete for our semantics.
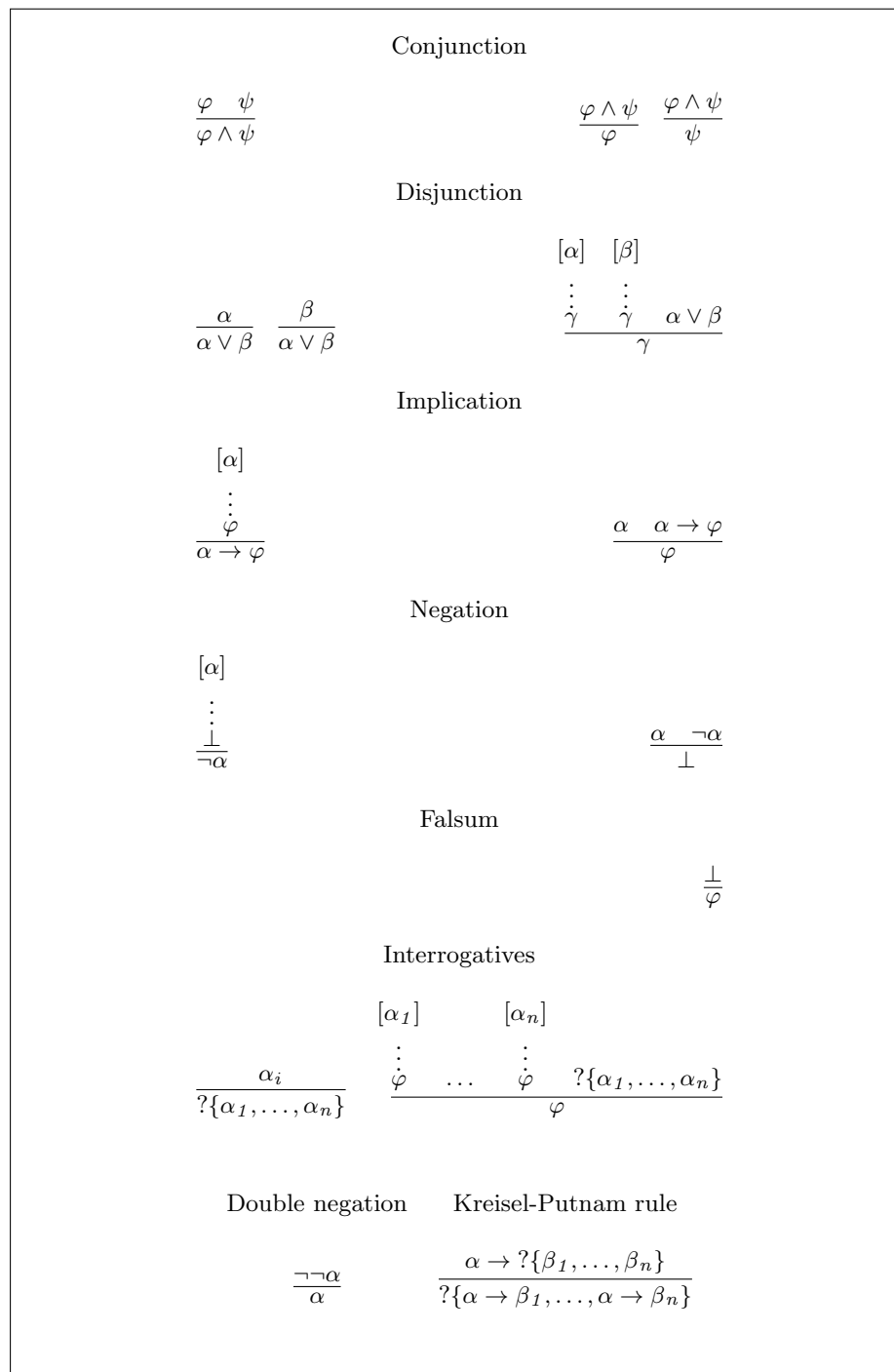
<div style="border:1px solid">

Conjunction

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \qquad\qquad\qquad \frac{\varphi \wedge \psi}{\varphi} \quad \frac{\varphi \wedge \psi}{\psi}$$

Disjunction

$$\frac{\alpha}{\alpha \vee \beta} \quad \frac{\beta}{\alpha \vee \beta} \qquad\qquad \begin{array}{cc} [\alpha] & [\beta] \\ \vdots & \vdots \\ \dot\gamma & \dot\gamma \quad \alpha \vee \beta \\ \hline \gamma \end{array}$$

Implication

$$\begin{array}{c} [\alpha] \\ \vdots \\ \dot\varphi \\ \hline \alpha \rightarrow \varphi \end{array} \qquad\qquad \frac{\alpha \quad \alpha \rightarrow \varphi}{\varphi}$$

Negation

$$\begin{array}{c} [\alpha] \\ \vdots \\ \bot \\ \hline \neg\alpha \end{array} \qquad\qquad \frac{\alpha \quad \neg\alpha}{\bot}$$

Falsum

$$\frac{\bot}{\varphi}$$

Interrogatives

$$\frac{\alpha_i}{?\{\alpha_1,\ldots,\alpha_n\}} \qquad \begin{array}{ccc} [\alpha_1] & & [\alpha_n] \\ \vdots & & \vdots \\ \dot\varphi & \ldots & \dot\varphi \quad ?\{\alpha_1,\ldots,\alpha_n\} \\ \hline & \varphi & \end{array}$$

Double negation   Kreisel-Putnam rule

$$\frac{\neg\neg\alpha}{\alpha} \qquad\qquad \frac{\alpha \rightarrow ?\{\beta_1,\ldots,\beta_n\}}{?\{\alpha \rightarrow \beta_1,\ldots,\alpha \rightarrow \beta_n\}}$$

</div>

Fig. 4 A derivation system for InqD.

Proposition 7 *For any set of declaratives Γ and any declarative α:*

$$\Gamma \models \alpha \;\Rightarrow\; \Gamma \vdash \alpha$$

Next consider the case in which the premises are declaratives and the conclusion is a basic interrogative.

Proposition 8 *For any set of declaratives Γ and any basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$:*

$$\Gamma \models ?\{\alpha_1, \ldots, \alpha_n\} \;\Rightarrow\; \Gamma \vdash ?\{\alpha_1, \ldots, \alpha_n\}$$

Proof Suppose $\Gamma \models ?\{\alpha_1, \ldots, \alpha_n\}$. Then, by proposition 5, we must have $\Gamma \models \alpha_i$ for some $1 \leq i \leq n$. But then it follows from the completeness of our system for declaratives, proposition 7, that $\Gamma \vdash \alpha_i$, whence by using the introduction rule for ? we can conclude that $\Gamma \vdash ?\{\alpha_1, \ldots, \alpha_n\}$.   □

The next step towards completeness is to show that in fact, any interrogative is provably equivalent to a basic interrogative, so that the previous proposition generalizes to arbitrary interrogatives.

Definition 19 (Provable equivalence) We say that two sentences φ and ψ are *provably equivalent*, notation $\varphi \equiv_P \psi$, just in case $\varphi \vdash \psi$ and $\psi \vdash \varphi$.

Lemma 1 *Any interrogative is provably equivalent to a basic one.*

Proof The proof goes by induction on the complexity of the interrogative μ under consideration. The claim is trivially true for a basic interrogative, so we just have to consider the induction step for conjunctive and conditional interrogatives.

1. Consider a conjunctive interrogative $\mu \wedge \nu$ and suppose the induction hypothesis holds for μ and ν, that is, suppose there are declaratives $\alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_m$ such that
   - $\mu \equiv_P\; ?\{\alpha_1, \ldots, \alpha_n\}$
   - $\nu \equiv_P\; ?\{\beta_1, \ldots, \beta_m\}$
   We claim that $\mu \wedge \nu$ is provably equivalent to the basic interrogative

   $$\lambda := ?\{\alpha_i \wedge \beta_j \mid 1 \leq i \leq n, 1 \leq j \leq m\}$$

   We are going to show that this is the case by indicating how these two sentences may be derived from each other in our system.
   (a) Assume $\mu \wedge \nu$. Eliminating the conjunction we obtain both μ and ν, whence by induction hypothesis we obtain both $?\{\alpha_1, \ldots, \alpha_n\}$ and $?\{\beta_1, \ldots, \beta_m\}$. Now assume α_i for some $1 \leq i \leq n$. Whatever β_j we assume, for $1 \leq j \leq m$, we will be able to derive $\alpha_i \wedge \beta_j$, whence, by introduction of the ? operator, we obtain λ.
   Since λ may be obtained from β_j for all j and since we have the interrogative $?\{\beta_1, \ldots, \beta_m\}$, by the rule of ?-elimination we can discharge all the assumptions β_j and obtain λ.
   This proof of λ can be carried out under the assumption α_i for any i. Since we have the interrogative $?\{\alpha_1, \ldots, \alpha_n\}$, we can apply again the ?-elimination rule, discharge all the hypotheses α_i and conclude λ.

(b) Conversely, assume λ. Now, if we assume any item $\alpha_i \wedge \beta_j$ we can conclude α_i and thus, by ?-introduction, we obtain $?\{\alpha_1, \ldots, \alpha_n\}$. Then from $?\{\alpha_1, \ldots, \alpha_n\}$ we can obtain the provably equivalent interrogative μ. Now, since μ can be obtained under the assumption $\alpha_i \wedge \beta_j$ for any $1 \leq i \leq n$ and $1 \leq j \leq m$, and since we have the interrogative $\lambda = ?\{\alpha_i \wedge \beta_j \mid 1 \leq i \leq n, 1 \leq j \leq m\}$, the ?-elimination rule applies: all the assumptions $\alpha_i \wedge \beta_j$ can be discharged and μ can be concluded.

With a totally analogous strategy we proceed to obtain ν, and finally by the introduction of conjunction we conclude $\mu \wedge \nu$.

2. Now consider a conditional interrogative $\alpha \to \mu$ and assume by the induction hypothesis that μ is provably equivalent to a basic interrogative $?\{\beta_1, \ldots, \beta_m\}$. We claim that $\alpha \to \mu$ is provably equivalent to the basic interrogative $?\{\alpha \to \beta_1, \ldots, \alpha \to \beta_n\}$. Again we will indicate how these sentences are interderivable in the system.

(a) Assume $\alpha \to \mu$. Now assume α: then we can conclude μ and therefore, by assumption, also $?\{\beta_1, \ldots, \beta_m\}$. Now we discharge the hypothesis α and conclude $\alpha \to ?\{\beta_1, \ldots, \beta_m\}$. At this point we have to resort to our Kreisel-putnam rule, which yields $?\{\alpha \to \beta_1, \ldots, \alpha \to \beta_n\}$. The role of the Kreisel-Putnam rule in the system is precisely to guarantee this last inference, which would not be possible otherwise.

(b) Conversely, assume $?\{\alpha \to \beta_1, \ldots, \alpha \to \beta_n\}$. Also, assume α. Now, whichever of the items $\alpha \to \beta_j$ we assume, eliminating the implication we will be able to conclude β_j; then we can obtain $?\{\beta_1, \ldots, \beta_m\}$ by ?-introduction, and therefore also the provably equivalent sentence μ.

Hence, μ may be concluded from any of the assumptions $\alpha \to \beta_j$ for any $1 \leq j \leq n$, and since we have the interrogative $?\{\alpha \to \beta_1, \ldots, \alpha \to \beta_n\}$, we can apply the ?-elimination rule, discharge all the hypotheses $\alpha \to \beta_j$ and conclude μ.

Finally, we discharge the hypothesis α and conclude $\alpha \to \mu$. The only remaining undischarged assumption is $?\{\alpha \to \beta_1, \ldots, \alpha \to \beta_n\}$. This concludes the proof of the lemma. □

Proposition 8 can now be generalized to arbitrary interrogatives.

Proposition 9 *For any set of declaratives Γ and any interrogative μ:*

$$\Gamma \models \mu \;\Rightarrow\; \Gamma \vdash \mu$$

Proof Suppose that $\Gamma \models \mu$. By the previous lemma, μ is provably equivalent to a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$. In particular, $\mu \vdash ?\{\alpha_1, \ldots, \alpha_n\}$. Since the proof system is sound we have that $\mu \models ?\{\alpha_1, \ldots, \alpha_n\}$, and since $\Gamma \models \mu$ we also have that $\Gamma \models ?\{\alpha_1, \ldots, \alpha_n\}$. It follows then from proposition 8 that $\Gamma \vdash ?\{\alpha_1, \ldots, \alpha_n\}$. Finally, since $?\{\alpha_1, \ldots, \alpha_n\} \vdash \mu$, we can conclude that $\Gamma \vdash \mu$. □

Putting together propositions 7 and 9 yields a completeness result for the case in which all premises are declaratives.

Proposition 10 *For any set of declaratives Γ and any arbitrary sentence φ:*

$$\Gamma \models \varphi \;\Rightarrow\; \Gamma \vdash \varphi$$

Now let us examine the case where we have an interrogative premise.

Proposition 11 *Let μ be an interrogative and φ an arbitrary sentence.*

$$\mu \models \varphi \;\Rightarrow\; \mu \vdash \varphi$$

Proof Suppose that $\mu \models \varphi$. According to lemma 1, μ is provably equivalent to a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$. In particular, $?\{\alpha_1, \ldots, \alpha_n\} \vdash \mu$. But then, since the proof system is sound, we have that $?\{\alpha_1, \ldots, \alpha_n\} \models \mu$, and since $\mu \models \varphi$, we also have that $?\{\alpha_1, \ldots, \alpha_n\} \models \varphi$. Now consider any α_i, $1 \leq i \leq n$. Any state that supports α_i also supports $?\{\alpha_1, \ldots, \alpha_n\}$, and therefore also φ. This means that $\alpha_i \models \varphi$. But then, since α_i is a declarative, proposition 10 tells us that $\alpha_i \vdash \varphi$. So φ can be derived from any α_i, $1 \leq i \leq n$. But then, using the ?-elimination rule, it can be derived from $?\{\alpha_1, \ldots, \alpha_n\}$, and therefore also from the provably equivalent interrogative μ. Thus, we can conclude that $\mu \vdash \varphi$. □

So, the case in which there is a single interrogative premise is fine too. We now turn to the general case, where we have an arbitrary set of premises.

Theorem 11 (Completeness theorem)
For any set of sentences Φ and any sentence ψ:

$$\Phi \models \psi \;\Rightarrow\; \Phi \vdash \psi$$

Proof Suppose that $\Phi \models \psi$. First, since the system is compact (proposition 3), there is a finite $\Phi' \subseteq \Phi$ with $\Phi' \models \psi$. Let us divide this finite set Φ' into a set of declaratives Γ and a set of interrogatives Λ. We then have that $\Lambda, \Gamma \models \psi$.

Now, a state supports all the interrogatives in Λ if and only if it supports the conjunctive interrogative $\lambda := \bigwedge \Lambda$. Analogously, a state supports all the declaratives in Γ if and only if it supports the conjunction $\gamma := \bigwedge \Gamma$.

Therefore, $\Lambda, \Gamma \models \psi$ is equivalent to $\lambda, \gamma \models \psi$. In turn, by the deduction theorem, this is equivalent to $\lambda \models \gamma \rightarrow \psi$. Now we are reduced to having only one interrogative as our assumption, and proposition 11 ensures completeness for this case, yielding $\lambda \vdash \gamma \rightarrow \psi$. But if $\lambda \vdash \gamma \rightarrow \psi$, then $\lambda, \gamma \vdash \psi$.

Finally, notice that both λ and γ are conjunctions of sentences in Φ: therefore, both are derivable from Φ. So from Φ one can derive both λ and γ, whence in turn one can derive ψ. Thus, we have shown that $\Phi \vdash \psi$, and the completeness result is established. □

5 Restricting the expressive power of InqD

In this section, we consider two ways of restricting the expressive power of InqD, and argue that the full expressive power is needed for a suitable analysis of natural language. We also address the issue whether, and to what extent, a semantics of interrogatives is to be intensional, which is related to the issue of expressive power.

5.1 Simpler basic interrogatives

We saw in section 3.4 that the expressive power of InqD would not be reduced if we removed conjunctive and conditional interrogatives from the language. That is, every proposition that is expressed by a conjunctive or a conditional interrogative can just as well be expressed by a basic interrogative in the language.

However, if we restrict the syntax of basic interrogatives themselves, then the expressive power of the system does significantly decrease. Given the usual form of polar interrogatives in natural languages like English, there is one particularly natural way to restrict the syntax of basic interrogatives in our logical language. Namely, rather than assuming that the interrogative operator applies to a finite *set* of declaratives, we could assume that it applies to a *single* declarative. Rather than basic interrogatives of the form $?\{\alpha_1, \ldots, \alpha_n\}$, for any $n \geq 1$, we would then only have basic interrogatives of the form $?\alpha$. Let us call the system that results from this adjustment InqC.[8]

Definition 20 (Syntax of InqC) The syntax of InqC is just like that of InqD, except for the formation of basic interrogatives. For any declarative α, $?\alpha$ is a basic interrogative, and nothing else is a basic interrogative in InqC.

Semantically, a basic interrogative of the form $?\alpha$ is most naturally treated as raising the issue whether α is true or not. In terms of support, this means that a state supports $?\alpha$ just in case it supports either α or $\neg\alpha$.

Definition 21 (Semantics of InqC) The semantics of InqC is just like that of InqD, except for the support clause for basic interrogatives, which is as follows:

- $s \models ?\alpha$ iff $s \models \alpha$ or $s \models \neg\alpha$

One may expect that, given the possibility to form conjunctive and conditional interrogatives, restricting the syntax of basic interrogatives in the way just described does not reduce the expressive power of the overall system. But it does. To demonstrate this, we will show that all sentences of InqC are characterized by a particular property, that is not a general feature of sentences in InqD and InqB.

Proposition 12 (InqC is pair-distributive)
For every sentence φ in InqC and every state s:

$$s \models \varphi \iff \forall w, v \in s \colon \{w, v\} \models \varphi$$

Proof The left-to-right direction of the equivalence holds by the persistence of support. For the converse, we will prove the contrapositive implication: if $s \not\models \varphi$, then there are $w, v \in s$ such that $\{w, v\} \not\models \varphi$. We proceed by induction on the complexity of φ.

- φ is a declarative α. Recall that a state s supports a declarative α iff $s \subseteq |\alpha|$ (fact 6). So, if $s \not\models \alpha$, then there is a world $w \in s$ which is not in $|\alpha|$. But then, again by the same property of declaratives, $\{w\} \not\models \alpha$, and notice that $\{w\}$ is of the form $\{w, v\}$ with $w = v \in s$.

---

[8] It can be shown that this system coincides with the propositional fragment of the system presented in Velissaratou (2000), which amounts to an enrichment of partition semantics with conditional questions. This explains the C in InqC.

- φ is a polar interrogative $?\alpha$. Suppose $s \not\models ?\alpha$. This means that s is not included in either $|\alpha|$ or $|\neg\alpha|$, and thus must contain both a world w where α is true and a world v where α is false. But then the state $\{w,v\}$ is also not included in either $|\alpha|$ or $|\neg\alpha|$, whence $\{w,v\} \not\models ?\alpha$.
- φ is a conjunctive interrogative $\mu \wedge \nu$. If $s \not\models \mu \wedge \nu$, then either $s \not\models \mu$, or $s \not\models \nu$. Suppose $s \not\models \mu$. Then by the induction hypothesis there are $w,v \in s$ such that $\{w,v\} \not\models \mu$. But then also $\{w,v\} \not\models \mu \wedge \nu$. Similarly if $s \not\models \nu$.
- φ is a conditional interrogative $\alpha \to \mu$. Suppose $s \not\models \alpha \to \mu$. According to fact 4, this happens if and only if $s \cap |\alpha| \not\models \mu$. Now by the induction hypothesis there are two worlds $w,v \in s \cap |\alpha|$ such that $\{w,v\} \not\models \mu$. But since w and v are in α, $\{w,v\} \cap |\alpha| = \{w,v\}$, so we also have $\{w,v\} \cap |\alpha| \not\models \mu$. Using again fact 4 we get $\{w,v\} \not\models \alpha \to \mu$, which is what we wanted, since both worlds w and v are in s. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

The fact that InqC is pair-distributive means that the semantics can be based on a notion of support that is only concerned with pairs of worlds (more precisely, with states containing at most two worlds) rather than with arbitrary sets of worlds. In such a setup, then, the proposition expressed by a sentence can be encoded as a *binary relation* on the set of all possible worlds, namely $\sim_\varphi = \{\langle w,v\rangle \,|\, \{w,v\} \models \varphi\}$.

This is reminiscent of partition semantics, where interrogatives also express a relation on the set of all possible worlds. In the case of partition semantics, the relation expressed by an interrogative is always an *equivalence relation*, which corresponds with a partition. In the case of InqC, viewed as a relational semantics, the proposition expressed by a sentence is not always an equivalence relation. For instance, a conditional interrogative like $p \to ?q$ is supported by the pair of worlds $\{11, 00\}$ and also by the pair $\{00, 01\}$ but not by the pair $\{11, 01\}$ (where the labels of the worlds are as in our earlier examples). We will show below that if we further restrict the syntax of InqC by taking away conditional interrogatives, we obtain a system that has exactly the same expressive power as partition semantics.

The earliest version of inquisitive semantics, presented in Groenendijk (2009) and Mascarenhas (2009), was also defined as a relational semantics, satisfying pair-distributivity. However, it has been argued in Ciardelli (2009); Ciardelli and Roelofsen (2011) that pair-distributivity is *not* a desirable feature for inquisitive semantics, and the main systems discussed in this paper, InqD and InqB, therefore purposefully lack this feature.

The gist of the problem that pair-distributivity gives rise to can be illustrated with a simple example. Consider a language with three atomic sentences, p, q, and r, and consider an issue that can be resolved by establishing that either one of these sentences is true, or that they are all false. In InqB this issue is expressed by the sentence $?(p \vee q \vee r)$, in InqD it is expressed by the sentence $?\{p, q, r, \neg(p \vee q \vee r)\}$. In natural languages like English issues of this kind are expressed by disjunctive interrogatives with rising intonation on all disjuncts, indicated in the example below with upward pointing arrows.[9]

---

[9] Disjunctive questions like (5), with rising intonation on all disjuncts, are called *open disjunctive questions* (Roelofsen and van Gool, 2010). Open disjunctive questions are to be distinguished from *alternative questions*, which come with *falling* intonation on the final disjunct (Bartels, 1999; Pruitt and Roelofsen, 2013), and have different semantic characteristics as well. We will return to alternative questions momentarily.

(5)   Is Peter going to Italy↑ this summer, or to France↑, or to Spain↑?

    a.   To Italy.

    b.   To France.

    c.   To Spain.

    d.   No, he is not going anywhere.

The issue raised by (5) can be resolved by answering that Peter is going to one of the three countries, or that he is not going to any of them.

Consider the proposition expressed by $?\{p, q, r, \neg(p \vee q \vee r)\}$ in InqD. Since there are only three atomic sentences in our language, there are eight possible worlds: 111, 110, 101, 011, 100, 001, 010, and 000. The proposition expressed by $?\{p, q, r, \neg(p \vee q \vee r)\}$ has four maximal elements: $|p|$, $|q|$, $|r|$, and $|\neg(p \vee q \vee r)|$. We will show that *this proposition cannot be expressed in any system that is pair-distributive*. Towards a contradiction, suppose that φ is a sentence that expresses the given proposition in a system that is pair-distributive. Then, φ is supported by all pairs in $|p| = \{111, 110, 101, 100\}$, all pairs in $|q| = \{111, 110, 011, 010\}$, and all pais in $|r| = \{111, 101, 011, 001\}$. But then it is also supported by all pairs in $\{111, 101, 011, 101\}$, the set of all worlds in which at least two of the atomic sentences are true. But then, again by pair-distributivity, the state $\{111, 101, 011, 110\}$ as a whole also supports φ. But this state does *not* support $?\{p, q, r, \neg(p \vee q \vee r)\}$ in InqD, because it does not support any of the 'disjuncts' of the interrogative. So φ does not express the proposition under consideration, which contradicts our initial assumption.

The upshot of this example is that any pair-distributive system, including InqC and the relational inquisitive semantics of Groenendijk (2009) and Mascarenhas (2009), has trouble dealing with disjunctive questions with three or more disjuncts. In InqD and InqB, this problem does not arise. Thus, the difference in expressive power between InqD and InqB on the one hand and InqC on the other, is crucial from the point of view of natural language semantics.

5.2 No conditional interrogatives

Now, let us consider a further restriction of InqC, leaving out conditional interrogatives from the syntax of the language. We saw that in the case of InqD, leaving out conditional interrogatives did not have a real impact on the expressive power of the system. But in the case of InqC it does. Namely, it essentially leads us back to the propositional fragment of the partition theory that we discussed in the beginning of the paper as one of the most elementary semantic theories dealing with both declaratives and interrogatives.

To see this, first consider the proposition expressed by a basic interrogative $?\alpha$ in InqC. There are two cases to consider. First, if α is a contradiction or a tautology, then there is a single possibility for $?\alpha$, namely the set of all possible worlds. This possibility, then, forms a (trivial) partition of the set of all possible worlds, consisting of only one cell. Second, if α is not a contradiction or a tautology, then there are exactly two possibilities for $?\alpha$. One of these is the unique possibility for α, and the other is the unique possibility for $\neg\alpha$. These possibilities are disjoint, and together they cover the set of all possible worlds. Thus, they form a partition of the set of all possible worlds consisting of two cells.

It is easy to see that if μ and ν are two interrogatives whose possibilities form a partition of the set of all possible worlds, then the conjunction of μ and ν has this property as well. But this means that, as long as we do not allow for conditional interrogatives, all interrogatives in InqC express partition-like propositions. Thus, in terms of expressive power, we have the following hierarchy:

$$\text{partition semantics} < \mathsf{InqC} < \mathsf{InqD} = \mathsf{InqB}$$

Notoriously, disjunctive and conditional questions are beyond the reach of a basic partition semantics.[10] If we move one step up, to InqC, a basic account of conditional questions comes within reach, though disjunctive questions remain problematic. If we move one more step up, to InqD/InqB, a basic account of disjunctive questions becomes available as well.

5.3 The debate on the intensionality of interrogatives

In Groenendijk and Stokhof (1997) it is extensively argued, for the most basic case of *yes/no*-interrogatives in a propositional language, that it is impossible to construe an *extensional* semantics for interrogatives that gives rise to suitable logical notions of answerhood and entailment. According to Groenendijk and Stokhof, the semantic evaluation of interrogatives has to be intensional in the sense that it needs to relate to *more than one* possible world.

This claim has been challenged by Nelken and Francez (2002), who develop what they call an extensional semantics for interrogatives, which assigns to each interrogative one of 5 truth values, organized as a bilattice.[11] Unfortunately, as is noticed in the paper itself, the interpretation is inadequate in that it does not validate, e.g., $\models ?(\alpha \vee \neg\alpha)$, where the disjunction is classical, and hence, tautological. Nelken and Francez try to overcome this inadequacy by calling upon an underlying *intuitionistic* logic. But once such a move is made, it can no longer be claimed that the resulting semantics for interrogatives is extensional.

A new attempt to provide an extensional semantics for interrogatives was launched in Nelken and Shan (2006). Remarkably, Nelken and Shan operate in the framework of *modal logic*. They equate a basic interrogative of the form $?\alpha$ with a modal statement $\Box\alpha \vee \Box\neg\alpha$, where $\Box\alpha$ is taken to mean that "α is known" or "α is in the common ground". The idea of using modal logic for the analysis of interrogatives goes back to Åqvist (1965) and Hintikka (1976), who interpret a question as a *request for knowledge*. The statement $\Box\alpha \vee \Box\neg\alpha$ can be thought of as capturing the knowledge that is needed to completely satisfy the request expressed by $?\alpha$. As Nelken and Shan put it: "We interpret a question as the *knowledge condition* required to answer it completely."

---

[10] Although see Isaacs and Rawlins (2008) for an analysis of conditional questions in a dynamic partition semantics that allows for hypothetical updates of the context of evaluation.

[11] In the extended version of Groenendijk (2009) (url provided in the references), it is shown that the relational inquisitive semantics developed there allows for an alternative formulation that also assigns one of 5 values to each sentence in the language. A brief comparison with the 5-valued system of Nelken and Francez (2002) is also made. The two systems are closely related, but there is one value in each system that splits into two values in the other. It is this difference that causes the problem for Nelken and Francez noted in the main text, a problem that does not occur in the system of Groenendijk (2009). See also Groenendijk (2008, §6.5) for more extensive discussion of this point.

Nelken and Shan claim that the resulting treatment of interrogatives is extensional. More precisely, they claim (and prove in the appendix of their paper) that under their analysis "any entailment between questions is satisfiable iff it is satisfiable with two worlds, and falsifiable iff it is falsifiable with two worlds" (p.256) and subsequently note that "a two-world structure can be easily simulated by a non-modal first-order model that assigns to each atomic formula one of 4 truth values: {FF, TF, FT, TT}. The truth value of a more complex formula is then computed by applying the regular truth tables of the logical operators pointwise; for example, the disjunction of TF and FT is TT. The ? operator checks whether the two worlds agree: it maps TT and FF to TT; and TF and FT to FF" (p.257). Strictly speaking, this 4-valued system is indeed extensional, in the sense that it no longer makes explicit reference to multiple possible worlds. However, as Nelken and Shan note, it still clearly "has an intensional flavor, in that the 4 truth values simply encode 2 possible worlds" (p.257).

This result needs to be interpreted with some care. First of all, while it could be seen as refuting the intensionality claim made by Groenendijk and Stokhof, which says that a semantic treatment of interrogatives needs to make reference to more than one possible world, it can also be seen as making this claim more precise, showing that two worlds are in fact enough, at least for the kind of interrogatives considered by Nelken and Shan.

With respect to the latter qualification, it is important to note that in the proof of their claim, Nelken and Shan actually explicitly restrict themselves to interrogatives which are *conjunctions of basic interrogatives*. This means, in particular, that the claim does not apply to conditional and disjunctive interrogatives; it only applies to interrogatives that correspond to partitions.[12] Thus, in our terms, Nelken and Shan essentially show that *partition semantics is pair-distributive*. In the present setting, this is an immediate consequence of Proposition 12, which says that not only partition semantics but also InqC, which is strictly more expressive in that it also allows for conditional interrogatives which do not correspond to partitions, is still pair-distributive.

However, we have argued above that the expressive power of partition semantics, and even of InqC, is not sufficient for a suitable semantic analysis of interrogatives in natural language. Such an analysis requires us, at the very least, to move to a system with the expressive power of InqD and InqB.

Within the modal approach of Nelken and Shan, such a move can be made by allowing for basic interrogatives of the form $?\{\alpha_1, \ldots, \alpha_n\}$ (rather than just $?\alpha$), corresponding to the modal formula $\Box\alpha_1 \vee \ldots \vee \Box\alpha_n$. However, the moment we extend the expressive power of the interrogative fragment of the language in this way, we know from Ciardelli (2009), Ciardelli and Roelofsen (2011), and the discussion above that the number of possible worlds needed for the evaluation of interrogatives is no longer bounded. In particular, two worlds are not sufficient anymore. This holds irrespectively of whether one adopts a support semantics relative to information states, or a semantics à la Nelken and Shan (2006) in terms of knowledge conditions expressed by modal statements.

---

[12] This may also explain the fact why for Nelken and Shan 4 values are sufficient, whereas Groenendijk (2009) needs 5. The latter, unlike the former, does allow for disjunctive and conditional interrogatives, although we have seen above that its treatment of disjunctive interrogatives is problematic.

This allows us to make Groenendijk and Stokhof's intensionality claim again more precise: as long as we restrict the expressive power of the interrogative fragment of our formal language to that of partition semantics, as Nelken and Shan do, or even to that of InqC, our semantics need not make reference to more than two possible worlds. However, the moment we move to a system with the expressive power of InqD and InqB, two worlds are no longer sufficient — indeed, our semantics needs to make reference to sets of possible worlds with arbitrary cardinality.[13]

5.4 Two loose ends

We have shown that InqD is a natural, conservative extension of CPL, equivalent in expressive power with the standard inquisitive system InqB, and that the full expressive power of InqD and InqB is needed for the analysis of declaratives and interrogatives in natural language. However, along the way we brought up two issues that should still be considered in further depth.

First, a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ in InqD expresses an issue that is settled precisely when one of $\alpha_1, \ldots, \alpha_n$ is established. As mentioned, this notion of basic interrogatives is familiar from other erotetic logics, in particular Wiśniewski's IEL and Hintikka's IMI. However, InqD constrains the formation of basic interrogatives in a particular way: a formula $?\{\alpha_1, \ldots, \alpha_n\}$ only counts as a basic interrogative if the disjunction $\alpha_1 \vee \cdots \vee \alpha_n$ is a classical tautology. The role of this condition is to ensure that the proposition expressed by an interrogative always covers the whole logical space ω, so that, semantically, the interrogative is a question. We mentioned in footnote 3 that instead of imposing this restriction, we could also think of a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ as *presupposing* that the actual world is one where the interrogative can be truthfully resolved, i.e., a world where at least one of $\alpha_1, \ldots, \alpha_n$ is true. This is indeed how basic interrogatives are construed in IEL and IMI. We will explore this alternative in section 6.

The second issue is that, even though the expressive power of InqD is sufficient to capture the meaning of disjunctive questions like (5), with rising intonation on all disjuncts, it is not sufficient to suitably capture the meaning of disjunctive questions like (6) below, with *falling* intonation on the final disjunct:

(6)   Is Peter going to Italy↑ or France↓?

---

[13] To avoid confusion, it should be noted that the syntax of Nelken and Shan's system does allow for formulas of the form $p \rightarrow ?q$, corresponding to $p \rightarrow (\Box q \vee \Box \neg q)$. Since Nelken and Shan explicitly talk about such formulas as "conditional questions" it may be puzzling that we present them as restricting the expressive power of the interrogative fragment of their formal language to that of partition semantics. However, as remarked above, in proving their extensionality claim, Nelken and Shan do explicitly restrict themselves to conjunctions of basic interrogatives. This means that formulas like $p \rightarrow ?q$, at least for the purpose of Nelken and Shan's central result, do not count as interrogatives. Furthermore, in our view, formulas like $p \rightarrow ?q$, as interpreted by Nelken and Shan, do not suitably capture the knowledge conditions of conditional interrogatives in natural language. In effect, the prediction is that a complete answer to $p \rightarrow ?q$ is known just in case p happens to be false, or a complete answer to $?q$ is known. The right analysis, in our view, is that a complete answer to $p \rightarrow ?q$ is known just in case a complete answer to $?q$ is known under the assumption that p holds. But this is impossible to express by a modal formula in the interrogative fragment of Nelken and Shan's formal language, even if we extend this fragment with conditionals of the form $\varphi \rightarrow \mu$, where φ is an arbitrary formula and μ a basic interrogative. Rather, a new abbreviation would be needed for $\Box(p \rightarrow q) \vee \Box(p \rightarrow \neg q)$.

As mentioned in footnote 9, disjunctive questions with rising intonation on all disjuncts are called *open questions*, while disjunctive questions with falling intonation on the final disjunct are called *alternative questions*. Intuitively, there is a clear semantic difference between the two. Namely, alternative questions, unlike open questions, imply that *exactly one* of the disjuncts holds.[14]

Crucially, this exclusive implication has a different status than the information provided by the corresponding disjunctive assertion. As illustrated in (7) and (8) below, in the case of the assertion, a denial of the exclusive implication is typically marked with the direct disagreement particle *no*, whereas in the case of the question, a denial of the exclusive implication should rather be marked with less direct disagreement particles like *actually* or *in fact*.

(7)   A: Peter is going to Italy or France.
        B: No, he is staying home.

(8)   A: Is Peter going to Italy↑ or France↓?
        B: #No / In fact, / Actually, he is staying home.

Disagreement particles like *actually* and *in fact* are typically used to mark denials of *non-at-issue* implications, in particular *presuppositions*, as illustrated in (9).

(9)   A: John has quit smoking.
        B: #No / In fact, / Actually, he *never* smoked.

For this reason, the exclusive implication of an alternative question is generally regarded as a non-at-issue implication (see, e.g., Aloni et al., 2009; Haida, 2010; Roelofsen and van Gool, 2010; AnderBois, 2011; Biezma and Rawlins, 2012). None of the systems discussed in this paper so far are designed to deal with non-at-issue implications, so none of them can be expected to suitably deal with alternative questions. In other words, while we argued above that the full expressive power of InqD and InqB is *necessary* for the analysis of questions in natural language, the case of alternative questions shows that it is not yet *sufficient*.

6 Extending InqD with presuppositional interrogatives

In order to address the issues laid out above, we will consider a natural extension of InqD that includes presuppositional interrogatives. We will refer to this extended system as $InqD_\pi$, where π stands for presuppositional. The section is structured as follows: we will first illustrate the changes that need to be made to include presuppositions in our semantic picture, then we will introduce the system $InqD_\pi$, and finally we will consider two natural notions of entailment in this enriched

---

[14] It should be noted that, although this description of the semantic difference between open and alternative questions is good enough for our purposes here, it needs to be refined in view of cases like (i) below.

(i)   Is Peter going to Italy↑, or France↑, or both↓?

If (i) were to imply that exactly one of its disjuncts holds, then it would be contradictory, which is clearly not the case. For discussion of this issue, we refer to Roelofsen and van Gool (2010).
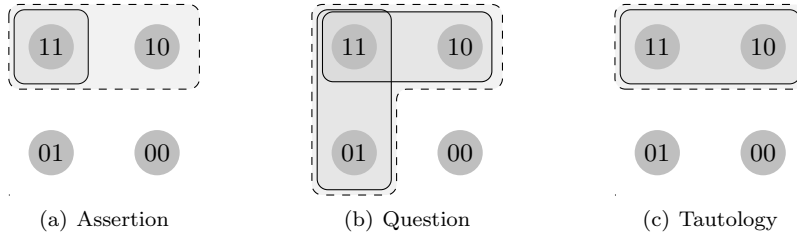
(a) Assertion   (b) Question   (c) Tautology

Fig. 5 Assertion, question, and tautology with presuppositions.

setting, showing that the completeness result established earlier for InqD can be extended to InqD_π.

6.1 Presuppositions in inquisitive semantics

In presuppositional inquisitive semantics (Ciardelli et al., 2012), the meaning of a sentence φ is determined by a pair $\langle \pi(\varphi), [\varphi] \rangle$, where $\pi(\varphi)$ is a state—the presupposition of φ—and $[\varphi]$ is the proposition expressed by φ, with the condition that $\mathsf{info}(\varphi) \subseteq \pi(\varphi)$. In uttering φ, a speaker is taken to presuppose that the actual world is included in $\pi(\varphi)$. Furthermore, as before, she is taken to provide the information that the actual world is included in $\mathsf{info}(\varphi)$, and to request enough information from other participants to establish a specific state in $[\varphi]$. The condition $\mathsf{info}(\varphi) \subseteq \pi(\varphi)$ ensures that the information provided is a (possibly trivial) enhancement of the information presupposed.

In a presuppositional setting, the notion of *informativeness* given above needs to be reformulated to take presuppositions into account: a sentence φ is called *informative* just in case it provides strictly more information than it presupposes, that is, $\mathsf{info}(\varphi) \subset \pi(\varphi)$. As before, φ is called a *question* iff it is non-informative. Given the new, more general definition of informativeness, this now means that $\mathsf{info}(\varphi) = \pi(\varphi)$. Thus, a question may be characterized as a sentence whose proposition $[\varphi]$ covers the presupposition $\pi(\varphi)$. As before, a *tautology* is defined as a sentence that is neither informative nor inquisitive. However, notice that tautologies may now have non-trivial presuppositions.

Non-presuppositional systems can be regarded as special cases of presuppositional systems where the presupposition of every sentence is trivial, i.e., $\pi(\varphi) = \omega$ for every φ. The systems InqB and InqD discussed above can be regarded as two such systems. Notice that in a non-presuppositional system, the notion of informativeness boils down to the one given in section 2. As a consequence, a question can be characterized as a sentence φ whose informative content coincides with ω.

Figure 5 depicts three examples of meanings in a presuppositional inquisitive semantics. In each of the figures, the state drawn with dashed borders is the presupposition $\pi(\varphi)$, whereas the states drawn with solid borders are the possibilities that make up the proposition $[\varphi]$. If a sentence expresses the meaning depicted in figure 5(a), it has a non-trivial presupposition, namely, it presupposes that p. Moreover, it is informative, since its informative content is strictly included in the presupposition. And finally, it is an assertion, since it has only one possibility. If a sentence expresses the meaning depicted in picture 5(b), it again has a

non-trivial presupposition, namely, it presupposes that at least one of p and q is true. Moreover, it is a question, since its informative content coincides with its presupposition; and it is inquisitive, since it has two different possibilities, one corresponding to the information that p, and the other corresponding to the information that q. Finally, if a sentence expresses the meaning depicted in 5(c), it is a tautology, since it is neither informative—its informative content coincides with its presupposition—nor inquisitive—it has just one possibility; however, it does have a non-trivial presupposition, namely, it presupposes that p.

6.2 The system InqD_π

InqD is designed to embody a strict division of labor between the two syntactic categories: declaratives only provide information, while interrogatives only request information. That is, every declarative is an assertion, and every interrogative is a question. In a non-presuppositional setting, the desire to ensure this division of labor forces us to restrict the syntactic rule for forming basic interrogatives.

To see this, suppose we want a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ to be a question. The semantic clause for the interrogative operator in InqD implies that $[?\{\alpha_1, \ldots, \alpha_n\}] = [\alpha_1] \cup \cdots \cup [\alpha_n]$, so that $\mathsf{info}(?\{\alpha_1, \ldots, \alpha_n\}) = \mathsf{info}(\alpha_1) \cup \cdots \cup \mathsf{info}(\alpha_n)$. Now, recall that InqD is a conservative extension of classical propositional logic, that is, for any declarative α we have that $\mathsf{info}(\alpha) = |\alpha|$. So the above informative content amounts to $|\alpha_1| \cup \cdots \cup |\alpha_n| = |\alpha_1 \vee \cdots \vee \alpha_n|$. Now, since InqD is a non-presuppositional system, something is a question iff its informative content amounts to ω. So, if we want a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ to come out as a question, we need to ensure that $|\alpha_1 \vee \cdots \vee \alpha_n|$ amounts to ω, which means that $\alpha_1 \vee \cdots \vee \alpha_n$ should be a classical tautology. Hence the requirement that is placed on basic interrogatives in InqD. If $\alpha_1 \vee \cdots \vee \alpha_n$ is *not* a classical tautology, then $?\{\alpha_1, \ldots, \alpha_n\}$ would be *informative*, and thus the division of labor between declaratives and interrogatives would be violated.

But notice that we are only compelled to this conclusion because InqD is a non-presuppositional system. If we bring presuppositions into the picture, it can very well be that $?\{\alpha_1, \ldots, \alpha_n\}$ is a question while $\mathsf{info}(?\{\alpha_1, \ldots, \alpha_n\})$ is different from ω. What is required is just that $\mathsf{info}(?\{\alpha_1, \ldots, \alpha_n\})$ coincides with the presupposition $\pi(?\{\alpha_1, \ldots, \alpha_n\})$, which may be non-trivial. So, by bringing presuppositions into the picture we can allow the formation of a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ for an arbitrary sequence of declaratives $\alpha_1, \ldots, \alpha_n$, while still retaining a strict division of labor between declaratives and interrogatives. This idea is implemented in the system InqD_π.

The syntax of InqD_π coincides with the syntax of InqD, except that a basic interrogative $?\{\alpha_1, \ldots, \alpha_n\}$ may now be formed out of *any* finite sequence of declaratives $\alpha_1, \ldots, \alpha_n$. Since presuppositions are now part of the picture as well, the semantics of InqD_π needs to specify two maps, a map π assigning a presupposition to each sentence, and a map $[\,]$ assigning a proposition to each sentence. We let the latter be defined simply by the support clauses given for InqD in definition 14; only, clause 3 now applies to a broader class of interrogatives.

As for the map π, we will proceed as follows: we will set aside presuppositions of declaratives for simplicity, assuming the declarative fragment behaves just like in InqD; as for presuppositions of interrogatives, the constraint that interrogatives
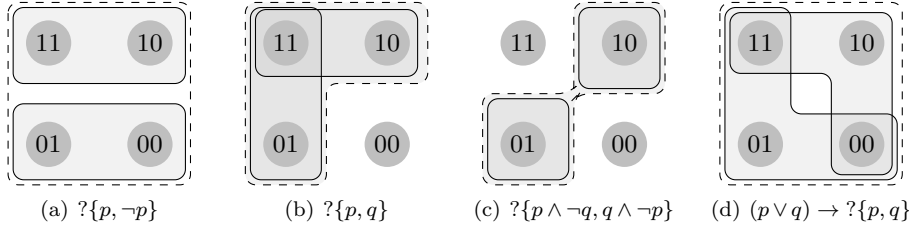
Fig. 6 Four interrogatives in InqD_π.

should always be questions does not leave us much choice: the presupposition $\pi(\mu)$ of an interrogative μ must always coincide with its informative content $\mathsf{info}(\mu)$, which is determined by the proposition $[\mu]$. So, for any interrogative μ, $\pi(\mu)$ is fully determined by $[\mu]$ and the requirement that, semantically, interrogatives should be questions. Some calculation shows that the resulting inductive clauses for π should be the following.

Definition 22 (Presuppositions)

- $\pi(\alpha) = \omega$ for any declarative α
- $\pi(?\{\alpha_1, \ldots, \alpha_n\}) = |\alpha_1 \vee \cdots \vee \alpha_n|$
- $\pi(\mu \wedge \nu) = \pi(\mu) \cap \pi(\nu)$
- $\pi(\alpha \to \mu) = \{w \mid w \notin |\alpha| \text{ or } w \in \pi(\mu)\}$.

These definitions guarantee that the division of labor is preserved in this system.

Fact 12 (Division of semantic labor in InqD_π)

- *any declarative α is an assertion*
- *any interrogative μ is a question*

Moreover, it is easy to see from the definitions that InqD_π is a conservative extension of InqD: for any sentence φ which belongs to the language of InqD we have that $\pi(\varphi) = \omega$ and (trivially) $[\varphi]_{\mathsf{InqD}_\pi} = [\varphi]_{\mathsf{InqD}}$. So, InqD_π simply extends InqD, allowing the treatment of a less constrained class of interrogative sentences, which includes basic interrogatives with non-trivial presuppositions, and complex interrogatives having such basic interrogatives as components.

Figure 6 shows the meanings of some interrogatives in InqD_π. Figure 6(a) depicts the meaning of $?\{p, \neg p\}$: this interrogative is also a sentence in InqD, and indeed, as expected, it receives the same interpretation in InqD_π as in InqD: it expresses the polar question whether p and it has a trivial presupposition. Figure 6(b) depicts the meaning of $?\{p, q\}$: since $p \vee q$ is not a classical tautology, this interrogative is not a sentence of InqD; in InqD_π, it is a question that presupposes that at least one of p and q is true, and requests enough information to establish either p or q. Figure 6(c) depicts the meaning of $?\{p \wedge \neg q, q \wedge \neg p\}$: again, since $(p \wedge \neg q) \vee (q \wedge \neg p)$ is not a classical tautology, this interrogative is not a sentence of InqD; in InqD_π, it is a question that presupposes that exactly one of p and q is true, and requests enough information to establish either p or q. This is precisely the type of meaning that we need to suitably deal with *alternative questions* in

natural languages, as exemplified in (6) above.[15] Finally, figure 6(d) depicts the conditional interrogative $(p \vee q) \to ?\{p, q\}$. The consequent of this interrogative is the basic interrogative $?\{p, q\}$ of figure 6(b) which, as we have just seen, carries a presupposition; however, it follows from the last clause of definition 22 that this presupposition is 'cancelled' by the antecedent of the conditional, that is, the sentence as a whole carries no presupposition.[16] As the figure shows, the resulting interrogative is a question that requests enough information to establish at least one of $(p \vee q) \to p$ and $(p \vee q) \to q$.

6.3 Entailment in InqD_π

There are two different natural ways to generalize the notion of entailment we considered for InqD to InqD_π. Both ways are sensible, but they capture different intuitions. Let us say that a sentence φ *makes sense* in a state s in case the information presupposed by φ is available in s, that is, $s \subseteq \pi(\varphi)$. In the first, stronger sense, a sentence φ entails a sentence ψ in case whenever φ is supported, ψ makes sense and it is supported as well.[17] In the second, weaker sense, a sentence φ entails a sentence ψ in case whenever φ is supported and ψ makes sense, ψ is supported as well . Intuitively, the latter condition means that ψ cannot be used to make any non-trivial contribution to the conversation after φ is settled, and thus can be read as "ψ is redundant after φ". In the general definition of these notions, the antecedent may be a set of sentences rather than a single sentence.

Definition 23 (Entailment relations in InqD_π)

- $\Phi \models_{\mathsf{InqD}_\pi}{}^s \psi$ iff for any state s, if $s \models \varphi$ for all $\varphi \in \Phi$ then $s \models \psi$.
- $\Phi \models_{\mathsf{InqD}_\pi}{}^w \psi$ iff for any state s, if $s \models \varphi$ for all $\varphi \in \Phi$ and $s \subseteq \pi(\psi)$, then $s \models \psi$.

In the following, we will mostly drop the subscript InqD_π and simply write $\models^s$ and $\models^w$. To illustrate the difference between the two notions, consider the declarative p and the interrogative $?\{p \wedge \neg q, q \wedge \neg p\}$, which, as we saw above, can be taken to represent the alternative question whether p or q. Suppose p is supported in s. Then, if the interrogative $?\{p \wedge \neg q, q \wedge \neg p\}$ makes sense in s, s supports $(p \wedge \neg q) \vee (q \wedge \neg p)$. Since s supports p, it follows that s must support $p \wedge \neg q$, and thus also the interrogative $?\{p \wedge \neg q, q \wedge \neg p\}$. Thus, p weakly entails $?\{p \wedge \neg q, q \wedge \neg p\}$, which captures the fact that $?\{p \wedge \neg q, q \wedge \neg p\}$ cannot be used to make any non-trivial contribution to a conversation after p is established. However,

---

[15] Notice that we are not concerned here with how this type of meaning is constructed compositionally in natural languages. This is of course a very important issue, but it is beyond the scope of the present paper. All we want to show by means of this example is that, in terms of expressive power, the system InqD_π is rich enough to deal with alternative questions.

[16] It is a distinctive feature of presuppositions that in compound sentences a presupposition of one of the components can be cancelled by the *informative content* of another component. It has proved to be notoriously difficult to accurately account for such cancellation phenomena, which is known as the *projection problem* for presuppositions. Since we only deal here with presuppositions of interrogatives, and since the only compound sentences in which interrogatives can combine with informative sentences are conditional interrogatives, projection poses no problem here.

[17] Notice that, since $\bigcup [\varphi] \subseteq \pi(\varphi)$, a formula φ can only be supported in states in which it makes sense in the first place. Thus, the condition $s \subseteq \pi(\varphi)$ need not appear explicitly in the definition of strong entailment, as it is implied by the condition $s \models \varphi$.

the mere fact that p is established in s does not by itself ensure that the question $?\{p \wedge \neg q, q \wedge \neg p\}$ makes sense in s, and so p does not strongly entail $?\{p \wedge \neg q, q \wedge \neg p\}$.

Fact 13

 – $p \not\models {}^s ?\{p \wedge \neg q, q \wedge \neg p\}$
 – $p \models {}^w ?\{p \wedge \neg q, q \wedge \neg p\}$

This example illustrates a general difference between weak and strong entailment from declaratives to interrogatives. The declaratives that strongly entail an interrogative μ are the ones that, *by themselves*, provide sufficient information to establish a state that supports μ. The declaratives that weakly entail μ, on the other hand, are those that, *in response to μ* –thus, assuming the information presupposed by μ– provide sufficient information to establish a state that supports μ. As the name suggests, strong entailment implies weak entailment.

Fact 14 *If $\Phi \models {}^s \psi$, then $\Phi \models {}^w \psi$.*

It is immediate from the definition that weak and strong entailment coincide when the conclusion ψ has a trivial presupposition. Thus, in particular, they deliver the same results when the conclusion is a declarative. Unlike in InqD, where an interrogative could only entail a declarative if the declarative is a tautology, entailment from interrogatives to declaratives may now hold non-trivially: an interrogative μ entails a declarative α iff μ *presupposes* α.

Fact 15 $\mu \models {}^s \alpha \iff \mu \models {}^w \alpha \iff \pi(\mu) \subseteq |\alpha|$

When restricted to sentences in InqD, both notions simply coincide with entailment in InqD.

Fact 16 *If $\Phi \subseteq \mathcal{L}_{InqD}$ and $\psi \in \mathcal{L}_{InqD}$, then*

$$\Phi \models_{InqD_\pi} {}^s \psi \iff \Phi \models_{InqD_\pi} {}^w \psi \iff \Phi \models_{InqD} \psi$$

This means in particular that, just like entailment in InqD, both weak and strong entailment are conservative extensions of classical propositional logic. Finally, we are now going to see that weak entailment can actually be *reduced* to strong entailment. For, a sentence ψ is weakly entailed by Φ if and only if ψ is strongly entailed by Φ with the addition of a declarative γ_ψ that captures the presupposition of ψ. For any sentence φ, the declarative γ_φ is given by the following recursive definition.

Definition 24

 – $\gamma_\alpha = \top$ for α declarative
 – $\gamma_{?\{\alpha_1, \ldots, \alpha_n\}} = \alpha_1 \vee \cdots \vee \alpha_n$
 – $\gamma_{(\mu \wedge \nu)} = \gamma_\mu \wedge \gamma_\nu$
 – $\gamma_{(\alpha \rightarrow \mu)} = \alpha \rightarrow \gamma_\mu$

The declarative γ_φ expresses the presupposition of φ, in the following sense.

Fact 17 *For any sentence φ and any state s, $s \models \gamma_\varphi \iff s \subseteq \pi(\varphi)$.*

Now the following connection between weak and strong entailment follows from the definitions together with the previous fact.

Fact 18 (Weak entailment reduces to strong entailment)
For any set Φ of sentences and any sentence ψ,

$$\Phi \models {}^w\psi \iff \Phi, \gamma_\psi \models {}^s\psi$$

In section 4 we presented a sound and complete aziomatization of the logic of InqD. Since the language of $InqD_\pi$ makes use of exactly the same connectives as the language of InqD, the deduction system for InqD presented in section 4 includes rules for all connectives of $InqD_\pi$. It turns out that that very deduction system, when applied to the extended language of $InqD_\pi$, yields a sound and complete axiomatization of strong entailment in $InqD_\pi$ (which is defined just like entailment in InqD). To convince oneself of this, it suffices to go through the completeness proof presented in section 4: nowhere does the argument appeal to the syntactic restriction on basic interrogatives, or to the semantic fact that the proposition expressed by an interrogative covers the logical space. All it relies on are the support clauses, which $InqD_\pi$ inherits unchanged from InqD. Thus, the very same argument establishes the following completeness theorem for strong entailment in $InqD_\pi$.

Theorem 19 (Soundness and completeness theorem for strong entailment)
For any set of sentences Φ and any sentence ψ,

$$\Phi \models {}^s\psi \iff \Phi \vdash \psi$$

Notice that, since weak entailment reduces in a simple way to strong entailment, indirectly we obtain an axiomatic characterization of weak entailment as well.

7 Division of labor without dichotomy

We started this paper with the observation that many natural languages are characterized by a *division of labor* between two categories of sentences, distinguished by syntactic and/or intonational features: declarative sentences are used primarily to provide information, while interrogative sentences are used primarily to request information.

In this paper we have been concerned with the construction of a formal system InqD that reflects such a division of labor, all the while being based on the uniform notion of meaning provided by inquisitive semantics.[18] However, it is important to note that, in InqD, the division of labor was achieved at the cost of giving up one important feature of the basic inquisitive semantics system InqB, namely the uniform algebraic treatment of the logical constants.

This trade-off is unavoidable given the dichotomous nature of the system InqD. After all, we have seen that, as soon as disjunction is associated with the join operation on the space of inquisitive propositions, simple disjunctions of assertions express hybrid propositions. Therefore, if all sentences are to be either questions or assertions, disjunction cannot simply express the join operation. Rather, in InqD, the join operation has to be adapted in such a way that it always delivers either

---

[18] We take it that as far as the division of labor is concerned InqD and $InqD_\pi$ do not significantly differ, and that what we say in this section about InqD, by and large also applies to its presuppositional extension $InqD_\pi$.

an assertion or a question. Indeed, ∨ is associated in InqD with a variant of the join operation that always yields an assertion, and ? with a variant of the join operation that always yields a question.

However, the strict dichotomy embodied by InqD is not the only way that division of labor may come about in a semantic system. We would like to suggest here a different, more subtle strategy that a system based on the inquisitive notion of meaning may employ to attain division of labor while fully retaining the algebraic treatment of the logical constants.

The proposal is to assume a structured system with two layers. The more fundamental layer consists of an InqB-like system, where the logical constants correspond semantically with the basic algebraic operations on the space of meanings. Call sentences of this system *proto-sentences*. A second, surface layer is then obtained by heading a proto-sentence with one of a certain set of projection operators. Semantically, all the projection operators available in the system have the property of turning a proto-sentence into something which is either an assertion, or a question (two such operators could be the projections ! and ? that we encountered in section 2). Call sentences of this second layer *projected sentences*.

Unlike the dichotomous InqD, such a system fully preserves the algebraic treatment of the logical constants, and only *on top of that* it implements the division of labor, by heading each sentence with a projection operator that turns it into an assertion or a question.

With regard to natural language, the idea underlying this approach is that the function of certain syntactic and intonational features is precisely to signal that certain projection operators need to be applied, resulting in a projected sentence, and thus ensuring a clear division of labor.

Now, how can we determine which of these two strategies, if any, is actually employed by natural languages? One way this may be assessed is to consider disjunctive sentences that are embedded in larger constructions, for instance in the antecedent of a conditional, or in the scope of a modal operator. If such embedded disjunctions are best treated as being purely informative, as in InqD, this speaks in favor of employing the dichotomous strategy. However, if such embedded disjunctions are best treated as hybrid, alternative-generating sentences, as in InqB, it is rather the non-dichotomous strategy sketched here that is called for.

A range of recent linguistic investigations of conditionals, modals, imperatives, comparatives and other constructions (Kratzer and Shimoyama, 2002; Simons, 2005; Alonso-Ovalle, 2006; Aloni, 2007; Aloni and Roelofsen, 2012, among others) indicates that, in English and a number of other languages, embedded disjunctions behave in such a way that not only their informative content matters, but also their potential to generate alternatives. This suggests that those natural languages are not based on a strict InqD-style dichotomy, but implement division of labor on top of a non-dichotomous, InqB-style semantics.

8 Conclusions and further work

In this paper we have seen how the notion of meaning that forms the cornerstone of inquisitive semantics can be taken as the basis for a dichotomous propositional system InqD in which the tasks of providing and requesting information are rigidly divided between declaratives and interrogatives. We connected this system with

the standard inquisitive system InqB by means of meaning preserving translations, establishing that InqD, just like InqB, is expressively complete for the relevant notion of meaning.

InqD gives rise to a uniform, cross-categorical notion of entailment, that generalizes classical entailment between declaratives to a combined logic involving both declaratives and interrogatives. For this logic we provided a simple proof system, showing that the logical properties of the interrogative operator of InqD are those of a disjunction with a particularly strong elimination rule, and that the usual rules for conjunction and implication also govern the logic of conjunctive and conditional interrogatives.

We considered certain restrictions on the syntax of InqD, showing that they lead to a reduction in expressive power that affects the ability of the system to deal with certain natural types of issues, and brought this in relation to the debate on whether and to what extent a semantics for interrogatives needs to be intensional. Finally, we extended our semantic apparatus to deal with presuppositional interrogatives. This allowed us to lift the unnatural restriction placed on the formation of basic interrogatives in InqD, yielding a system $InqD_\pi$ which is able to deal with a wider range of interrogatives. While semantically richer, $InqD_\pi$ inherits the well-behaved notion of combined entailment of InqD, as well as the associated completeness result.

The systems InqD and $InqD_\pi$ are of interest in several ways. First, they concretely demonstrate that the notion of meaning proposed by inquisitive semantics is not inherently linked to the particular treatment of the logical constants embodied by the system InqB. In particular, although this notion of meaning does not *require* a dichotomous language, it is *compatible* with one.

Second, InqD and $InqD_\pi$ may be particularly useful in applications where questions are relevant, but the alternative-generating character of disjunction is not (a case in point is the *inquisitive dynamic epistemic logic* developed in Ciardelli and Roelofsen, 2012). In such contexts, the use of a dichotomous language often helps to keep intuitions clearer. For these applications, the availability of a simple and perspicuous deduction system is a fundamental feature.

Third, several existing erotetic logics, most notably Wiśniewski's IEL and Hintikka's IMI, assume dichotomous languages that share many features with $InqD_\pi$. This similarity facilitates comparisons and transfer of insights to and from these traditions. For instance, it seems that the notion of entailment considered in this paper is meaningful and relevant in the context of IEL and IMI as well, and that the completeness result established here may be exported to those systems.

Needless to say, the analysis conducted here leaves many issues open. For one thing, in this paper we focused exclusively on propositional systems. Many interesting phenomena concerning questions can only be analyzed appropriately in a first-order setting. Our expectation is that issues of translatability and axiomatization will be vastly more complicated in that context.

Furthermore, even though we remarked in several places on the potential relevance of our logical investigations for the analysis of natural language, this connection remains to be explored in much greater detail. In particular, while we pointed out that, in terms of expressive power, the systems developed here are in principle capable of dealing with a considerable range of declarative and interrogative constructions in natural language, we have only been concerned with compositionality issues at a very high level of abstraction. Much further work is needed to spell

out precisely how the relevant meanings may be constructed compositionally in natural languages.

References

Maria Aloni. Free choice, modals and imperatives. *Natural Language Semantics*, 15:65–94, 2007.

Maria Aloni and Floris Roelofsen. Indefinites in comparatives. Submitted to *Natural Language Semantics*, 2012.

Maria Aloni, Paul Égré, and Tikitu de Jager. Knowing whether A or B. *Synthese*, 2009.

Luis Alonso-Ovalle. *Disjunction in Alternative Semantics*. PhD thesis, University of Massachusetts, Amherst, 2006.

Scott AnderBois. *Issues and alternatives*. PhD thesis, University of California Santa Cruz, 2011.

L. Åqvist. *A new approach to the logical theory of interrogatives*. University of Uppsala, 1965.

Christine Bartels. *The intonation of English statements and questions: a compositional interpretation*. Routledge, 1999.

Maria Biezma. Alternative vs polar questions: the cornering effect. In Satoshi Ito and Ed Cormany, editors, *Proceedings of Semantics and Linguistic Theory (SALT XIX)*. CLC Publications, 2009.

Maria Biezma and Kyle Rawlins. Responding to alternative and polar questions. *Linguistics and Philosophy*, 35(5):361–406, 2012. doi: 10.1007/s10988-012-9123-z.

Dwight Bolinger. Yes-no questions are not alternative questions. In H. Hiz, editor, *Questions*, pages 87–105. Reidel Publishing Company, Dordrecht, Holland, 1978.

Ivano Ciardelli. Inquisitive semantics and intermediate logics. Master Thesis, University of Amsterdam, 2009.

Ivano Ciardelli. A first-order inquisitive semantics. In Maria Aloni, Harald Bastiaanse, Tikitu de Jager, and Katrin Schulz, editors, *Logic, Language, and Meaning: Selected Papers from the Seventeenth Amsterdam Colloquium*. Springer, 2010.

Ivano Ciardelli and Floris Roelofsen. Generalized inquisitive logic: Completeness via intuitionistic Kripke models. Proceedings of Theoretical Aspects of Rationality and Knowledge, 2009.

Ivano Ciardelli and Floris Roelofsen. Inquisitive logic. *Journal of Philosophical Logic*, 40(1):55–94, 2011.

Ivano Ciardelli and Floris Roelofsen. Inquisitive dynamic epistemic logic. Submitted to *Synthese*, 2012. URL http://tinyurl.com/bxo7mud.

Ivano Ciardelli, Jeroen Groenendijk, and Floris Roelofsen. Inquisitive semantics. NASSLLI lecture notes, 2012. URL www.illc.uva.nl/inquisitive-semantics.

Ivano Ciardelli, Jeroen Groenendijk, and Floris Roelofsen. Inquisitive semantics: a new notion of meaning. Forthcoming in *Language and Linguistics Compass.*, 2013a.

Ivano Ciardelli, Jeroen Groenendijk, and Floris Roelofsen. Towards a logic of information exchange: an inquisitive witness semantics. In Guram Bezhanishvili, Vincenzo Marra, Sebastian Löbner, and Frank Richter, editors, *Logic, Language, and Computation: revised selected papers from the Ninth International Tbilisi Symposium on Logic, Language, and Computation*, pages 51–72. Springer, 2013b.

Jeroen Groenendijk. Inquisitive semantics and dialogue management. ESSLLI course notes, www.illc.uva.nl/inquisitive-semantics, 2008.

Jeroen Groenendijk. Inquisitive semantics: Two possibilities for disjunction. In Peter Bosch, David Gabelaia, and Jérôme Lang, editors, *Seventh International Tbilisi Symposium on Language, Logic, and Computation*. Springer-Verlag, 2009. Extended version: http://www.illc.uva.nl/Research/Reports/PP-2008-26.text.pdf.

Jeroen Groenendijk. Erotetic languages and the inquisitive hierarchy. In a Festschrift for Martin Stokhof, 2011. URL http://dare.uva.nl/document/487828.

Jeroen Groenendijk and Floris Roelofsen. Inquisitive semantics and pragmatics. Presented at the Workshop on Language, Communication, and Rational Agency at Stanford, May 2009, 2009. URL www.illc.uva.nl/inquisitive-semantics.

Jeroen Groenendijk and Martin Stokhof. *Studies on the Semantics of Questions and the Pragmatics of Answers*. PhD thesis, University of Amsterdam, 1984.

Jeroen Groenendijk and Martin Stokhof. Questions. In Johan van Benthem and Alice ter Meulen, editors, *Handbook of Logic and Language*, pages 1055–1124. Elsevier, 1997.

Andreas Haida. On the semantics and pragmatics of yes/no-questions, yes/no-question disjunctions, and alternative questions: evidence from Chadic. Manuscript, Humboldt University of Berlin, draft version of October 8, 2010, 2010.

Charles L. Hamblin. Questions in Montague English. *Foundations of Language*, 10:41–53, 1973.

Jaako Hintikka. The semantics of questions and the questions of semantics. *Acta Philosophica Fennica*, 1976.

Jaako Hintikka. On the logic of an interrogative model of scientific inquiry. *Synthese*, 47(1):69–83, 1981.

Jaako Hintikka. New foundations for a theory of questions and answers. In *Questions and Answers*, pages 159–190. Reidel, Dordrecht, 1983.

Jaako Hintikka. *Inquiry as inquiry: A logic of scientific discovery*. Kluwer Academic Publishers, 1999.

Jaako Hintikka. *Socratic epistemology: explorations of knowledge-seeking by questioning*. Cambridge University Press, 2007.

J. Isaacs and K. Rawlins. Conditional questions. *Journal of Semantics*, 25:269–319, 2008.

Lauri Karttunen. Syntax and semantics of questions. *Linguistics and Philosophy*, 1:3–44, 1977.

Angelika Kratzer and Junko Shimoyama. Indeterminate pronouns: The view from Japanese. In Y. Otsu, editor, *The Proceedings of the Third Tokyo Conference on Psycholinguistics*, pages 1–25, 2002.

G. Kreisel and H. Putnam. Eine Unableitbarkeitsbeweismethode für den intuitionistischen Aussagenkalkül. *Archiv für Mathematische Logik und Grundlagenforschung*, 3:74–78, 1957.

Salvador Mascarenhas. Inquisitive semantics and logic. Master Thesis, University of Amsterdam, 2009.

Rani Nelken and Nissim Francez. Bilattices and the semantics of natural language questions. *Linguistics and Philosophy*, 25:37–64, 2002.

Rani Nelken and Chung-Chieh Shan. A modal interpretation of the logic of interrogation. *Journal of Logic, Language, and Information*, 15:251–271, 2006.

Kathryn Pruitt and Floris Roelofsen. The interpretation of prosody in disjunctive questions. Forthcoming in *Linguistic Inquiry*, 2013.

Floris Roelofsen. Algebraic foundations for inquisitive semantics. In Hans van Ditmarsch, Jerome Lang, and Ju Shier, editors, *Proceedings of the Third International Conference on Logic, Rationality, and Interaction*, pages 233–243. Springer-Verlag, 2011.

Floris Roelofsen. Algebraic foundations for the semantic treatment of inquisitive content. *Synthese*, 2013. doi: 10.1007/s11229-013-0282-4. http://link.springer.com/article/10.1007%2Fs11229-013-0282-4.

Floris Roelofsen and Sam van Gool. Disjunctive questions, intonation, and highlighting. In Maria Aloni, Harald Bastiaanse, Tikitu de Jager, and Katrin Schulz, editors, *Logic, Language, and Meaning: Selected Papers from the Seventeenth Amsterdam Colloquium*, pages 384–394. Springer, 2010.

Mandy Simons. Dividing things up: the semantics of *or* and the modal/*or* interaction. *Natural Language Semantics*, 13(3):271–316, 2005.

Sophia Velissaratou. Conditional questions and which-interrogatives. Master Thesis, University of Amsterdam, 2000.

Andrzej Wiśniewski. The logic of questions as a theory of erotetic arguments. *Synthese*, 109(1):1–25, 1996.

Andrzej Wiśniewski. Questions and inferences. *Logique et analyse*, 173(175):5–43, 2001.