Towards a suppositional inquisitive semantics*

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Abstract. One of the primary usages of language is to exchange information. This can be done directly, as in *Will Susan sing? No, she won't*, but it is also often done in a less direct way, as in *If Pete plays the piano, will Susan sing? No, if Pete plays the piano, Susan won't sing.* In the latter type of exchange, both participants make a certain *supposition*, and exchange information under the assumption that this supposition holds. This paper develops a semantic framework for the analysis of this kind of information exchange. Building on earlier work in inquisitive semantics, it introduces a notion of meaning that captures informative, inquisitive, and suppositional content, and discusses how such meanings may be assigned in a natural way to sentences in a propositional language. The focus is on conditionals, which are the only kind of sentences in a propositional language that introduce non-trivial suppositional content.

1 Towards a more fine-grained notion of meaning

Traditionally, the meaning of a sentence is identified with its informative content, and the informative content of a sentence is taken to be determined by its truth conditions. Thus, the proposition expressed by a sentence is construed as a set of possible worlds, those worlds in which the sentence is true, and this set of worlds is taken to determine the effect that is achieved when the sentence is uttered in a conversation. Namely, when the sentence is uttered, the speaker is taken to *propose an update of the common ground* of the conversation (Stalnaker, 1978). The common ground of a conversation is the body of information that has been publicly established in the conversation so far. It is modeled as a set of possible worlds, namely those worlds that are compatible with the established information. When a speaker utters a sentence, she is taken to propose to update the common ground by restricting it to those worlds in which the uttered sentence is true, i.e., those worlds that are contained in the proposition expressed by the sentence. If accepted by the other conversational participants, this update ensures that the new common ground contains the information that the uttered sentence is true.

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This basic picture of sentence meaning and the effect of an utterance in conversation has proven very useful, but it also has some inherent limitations. Perhaps most importantly, it is completely centred on informative content and truth conditions. Evidently, there are many meaningful sentences in natural language that cannot be thought of as being true or false. Questions are a prominent case in point. For instance, the meaning of the question *Is Susan singing?* clearly cannot be taken to reside in its truth conditions. So in order to deal with such sentences, the basic picture sketched above needs to be generalized. One way to do this has been articulated in recent work on inquisitive semantics (e.g. Ciardelli et al., 2012, 2013). The basic idea is that a speaker who utters *Is Susan singing?* still proposes to update the common ground of the conversation; however, she does not propose one particular update, but rather offers a choice: one way to comply with her proposal would be to restrict the common ground to worlds where Susan is singing, but another way to comply with her proposal, equally acceptable, would be to restrict the common ground to worlds where Susan is not singing. So the basic Stalnakerian picture of the effect of an utterance in terms of issuing a proposal to update the common ground of the conversation can be generalized appropriately, in such a way that it applies to declarative and interrogative sentences in a uniform way.

What about the basic truth conditional notion of sentence meaning? How could this be suitably generalized? The simplest approach that has been explored in inquisitive semantics is to move from truth conditions to *support conditions*. The idea is that the meaning of a sentence should determine precisely which pieces of information support the proposal that a speaker makes in uttering the sentence. This notion of meaning is adopted in the most basic implementation of inquisitive semantics, InqB.[1] Clearly, the support based notion of meaning is directly tied to the idea that the conversational effect of an utterance is a proposal to update the common ground in one or more ways. The latter—let's call it the *proposal picture of conversation*—is one of the main tenets of the inquisitive semantics framework in general, not just of the particular system InqB. The support based notion of meaning, on the other hand, is specific to InqB. It ties in well with the proposal picture of conversation, but there may well be other notions of meaning that also tie in well with this picture.

The goal of this paper is to develop such a notion of meaning, which is more fine-grained than the InqB notion. Motivation for such a more fine-grained notion comes from the basic observation that proposals may not only be *supported* by a given piece of information; they may also be *rejected* or *dismissed*. To illustrate, consider (1a), and the two responses to it in (1b) and (1c):

(1) a. If Pete plays the piano, Susan will sing.
b. No, if Pete plays the piano, Susan won't sing.
c. Pete won't play the piano.

[1] See Ciardelli (2009); Groenendijk and Roelofsen (2009); Ciardelli and Roelofsen (2011) for early expositions of InqB, and Roelofsen (2013); Ciardelli et al. (2013) for a more recent perspective and comparison with earlier work on the semantics of questions (e.g., Hamblin, 1973; Karttunen, 1977; Groenendijk and Stokhof, 1984).

Intuitively, both (1b) and (1c) are pertinent responses; they address the proposal that (1a) expresses. However, rather than supporting the proposal, (1b) rejects it, while (1c) dismisses a supposition of it and thereby renders it void.

We will consider what it means in general to reject a proposal or to dismiss a supposition of it, and how these notions are related to each other, as well as to support. We will define a semantics for a propositional language, which specifies recursively for every sentence (i) which information states (or equivalently, which pieces of information) support it, (ii) which information states reject it, and (iii) which information states dismiss a supposition of it. We refer to this system as InqS. We will argue that the more fine-grained notion of meaning adopted in InqS considerably broadens the empirical scope of InqB, especially in the domain of conditionals. In Aher et al. (2014) it is shown that the framework developed here allows for a novel treatment of epistemic and deontic modals as well, with interesting connections to the treatment of conditionals presented here.[2]

The paper is organized as follows. Sect. 2 reviews the background and motivation for InqS in more detail; Sect. 3 presents the system itself, identifying its basic logical properties and discussing some illustrative examples; and finally, Sect. 4 summarizes and concludes.

2 Background and motivation

2.1 Support and persistence

In a support based semantics, the basic idea is that one knows the meaning of a sentence just in case one knows which information states—or equivalently, which pieces of information—support the given sentence, and which don't. For instance, an information state s, modeled as a set of possible worlds, supports an atomic declarative sentence p just in case every world in s makes p true; it supports $\neg p$ if every world in s makes p false; and finally, it supports the interrogative sentence $?p$ just in case it supports either p or $\neg p$.

[2] One way to reject the proposal expressed by (1a), not listed above, is as follows:

(i) No, if Pete plays the piano, Susan might not sing.

This response involves the epistemic modal *might*. Accounting for such responses is beyond the scope of the current paper, but not beyond the scope of InqS in general. Indeed, the InqS analysis of epistemic modals presented in Aher et al. (2014) naturally characterizes (i) as a rejecting response to (1a), and also clearly brings out the difference between (i) and (1b). Namely, (i) rejects (1a) in a *defeasible* way, subject to possible retraction when additional information becomes available, while (1b) rejects (1a) indefeasibly. Or, phrased in terms of conversational attitudes, (i) signals that the addressee is *unwilling* to accept the proposal expressed by (1a), while (1b) signals that she is really *unable* to do so.

There is a rich psycholinguistic literature on the denial of conditional statements (see, e.g., Handley et al., 2006; Espino and Byrne, 2012; Égré and Politzer, 2013, and references therein), but the distinction between defeasible and indefeasible rejection has, to the best of our knowledge, not been brought to attention previously.

Given such a support-based semantics, we can think of a speaker who utters a sentence φ as proposing to enhance the common ground of the conversation, modeled as an information state, in such a way that it *comes to support* φ. Thus, in uttering p a speaker proposes to enhance the common ground in such a way that it comes to support p, and in uttering $?p$ a speaker proposes to enhance the common ground in such a way that it comes to support either p or $\neg p$.

Prima facie it is natural to assume that support is *persistent*, that is, if an information state s supports a sentence φ, then it is natural to assume that every more informed information state $t \subset s$ will also support φ. In other words, information growth cannot lead to retraction of support. This assumption is indeed made in InqB, and it determines to a large extent how the system behaves.

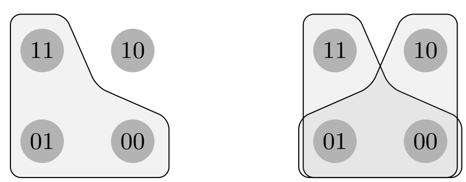
2.2 Support for conditionals

Let us now zoom in on conditional sentences, which is where we would like to argue that a more refined picture is ultimately needed. Consider again the conditional statement in (1a), repeated in (2) below, and the corresponding conditional question in (3):

(2) If Pete plays the piano, Susan will sing. $p \to q$

(3) If Pete plays the piano, will Susan sing? $p \to ?q$

The meanings of these sentences in InqB can be depicted as follows:



(a) $p \to q$ (b) $p \to ?q$

In these figures, 11 is a world where p and q are both true, 10 a world where p is true but q is false, etcetera. We have only depicted the *maximal* states that support each sentence. Since support is persistent, all substates of these maximal supporting states also support the given sentences.

In general, in InqB a state s is taken to support a conditional sentence $\varphi \to \psi$ just in case every state $t \subseteq s$ that supports φ also supports ψ. For instance, the state $s = \{11, 01, 00\}$ supports $p \to q$, because any substate $t \subseteq s$ that supports p (there are only two such states, namely $\{11\}$ and $\emptyset$) also support q. Similarly, one can verify that the states $\{11, 01, 00\}$ and $\{11, 01, 00\}$ both support $p \to ?q$.

For convenience, we will henceforth use $|\varphi|$ to denote the state consisting of all worlds where φ is classically true. So the states $\{11, 01, 00\}$ and $\{10, 01, 00\}$ can be denoted more perspicuously as $|p \to q|$ and $|p \to \neg q|$, respectively.

2.3 Support and reject

The support conditions for a sentence φ capture an essential aspect of the proposal that is made in uttering φ, namely what is needed to compliantly settle this proposal. However, besides compliantly settling a given proposal, conversational participants may react in different ways as well. In particular, they may *reject* the proposal. What does it mean exactly to reject the proposal made in uttering φ? And can this, perhaps indirectly, be explicated in terms of the support conditions for φ as well?

At first sight, this seems quite feasible indeed. Suppose that a speaker A utters a sentence φ, and a responder B reacts with ψ. A proposes to enhance the common ground to a state that supports φ, while B proposes to enhance the common ground to a state that supports ψ. Then we could say that B *rejects* A's initial proposal just in case any state s that supports ψ is such that no consistent substate $t \subseteq s$ supports φ. After all, if this is the case, then any way of satisfying B's counterproposal leads to a common ground which does not support φ and which cannot be further enhanced in any way such that it comes to support φ while remaining consistent.

For many basic cases, this characterization of rejection in terms of support seems adequate. For instance, if A utters an atomic sentence p and B responds with $\neg p$, then according to the given characterization, B rejects A's initial proposal, which accords with pre-theoretical intuitions.

However, in the case of conditionals, the given characterization is problematic. Intuitively, the proposal expressed by (2) above can be rejected with (4).

(4) No, if Pete plays the piano, Susan won't sing. $\quad p \to \neg q$

However, there is a consistent state that supports both (2) and (4), namely $|\neg p|$. So according to the above characterization, (4) does not reject (2).

This example illustrates something quite fundamental: in general, reject conditions cannot be derived from support conditions. Thus, a semantics that aims to provide a comprehensive characterization of the proposals that speakers make when uttering sentences in conversation, needs to specify both support- and reject-conditions (and perhaps more). InqB, which is only concerned with support, has been extended in previous work to a semantics that specifies reject conditions as well, with the aim to deal with the type of phenomena discussed here. The resulting framework is referred to as *radical inquisitive semantics*, InqR for short (Groenendijk and Roelofsen, 2010; Sano, 2012; Aher, 2013).[3]

[3] The need to specify both support and reject conditions is independent from the need to have a notion of meaning that embodies inquisitive content. There is a lot of work addressing the first issue while leaving inquisitive content out of the picture, e.g., work on *data semantics* (Veltman, 1985), *game-theoretic semantics* and *independence friendly logic* (Hintikka and Sandu, 1997; Hodges, 1997), *dependence logic* (Väänänen, 2007), and *truth-maker semantics* (Van Fraassen, 1969; Fine, 2012).

2.4 Dismissing a supposition

The semantics to be developed in the present paper further extends the InqR framework, providing yet a more comprehensive characterization of the proposals that speakers make when uttering sentences in conversation. This further refinement is motivated by the observation that, besides compliant support and full-fledged rejection, there is, as we saw already in the introduction, yet another way to react to the conditionals in (2) and (3):

(5) Pete won't play the piano. $\neg p$

Suppose that A utters (2) or (3) and that B reacts with (5). One natural way to think about this response is as one that *dismisses a supposition* that A was making, namely the supposition 'that Pete will play the piano'.

Clearly, the suppositions that a speaker makes in issuing a certain proposal, and responses that dismiss such suppositions, cannot be characterized purely in terms of the support conditions for that sentence.

2.5 From radical to suppositional

At first sight it may seem that suppositional phenomena *can* be captured if we have both support- and reject-conditions at our disposal. Indeed, an attempt to do so has been articulated in work on InqR (see in particular Groenendijk and Roelofsen, 2010, Sect. 3). There, states that dismiss a supposition of a sentence φ are characterized as states that can be obtained by intersecting a state that supports φ with a state that rejects φ. Within the broader conceptual framework of InqR, such states can be thought of as ones that reject the *question behind* φ. If correct, this connection between support, rejection, and suppositional dismissal would show that there is no need to further refine the semantic machinery of InqR in order to deal with suppositional phenomena.

This characterization works fine for simple cases like $\neg p$ in response to $p \to q$. Namely, $|\neg p|$ can be obtained as the intersection of $|p \to q|$ and $|p \to \neg q|$, which support and reject $p \to q$, respectively. So $\neg p$ is correctly predicted to dismiss a supposition of the conditional. However, the predictions for more complex cases are not always satisfactory. Consider, for instance, a conditional with a disjunctive antecedent, and a response rejecting just one of the disjuncts:

(6) a. If Pete or Bill plays the piano, Susan will sing. $(p \vee q) \to r$
    b. Well, Pete won't play the piano. $\neg p$

Under the strategy under consideration, InqR fails to predict that (6b) dismisses a supposition of (6a). Any state s that supports $(p \vee q) \to r$ has to support both $p \to r$ and $q \to r$. Then s cannot contain a world w where q holds, and p and r do not hold. But this world is included in $|\neg p|$. So, $|\neg p|$ cannot be obtained as the intersection of a state that supports $(p \vee q) \to r$ and a state that rejects it.

In order to avoid this and other problematic predictions when characterizing suppositional dismissal in terms of support and rejection, we will develop a semantics in which all three notions are characterized separately.

3 Suppositional inquisitive semantics

We will consider a propositional language $\mathcal{L}$, based on a finite set of atomic sentences $\mathcal{P}$. Complex sentences are built up using the usual connectives, $\neg$, $\wedge$, $\vee$, and $\rightarrow$, as well as an additional operator, ?. As in InqB, $?\varphi$ is defined as an abbreviation of $\varphi \vee \neg\varphi$ (the rationale behind this will become clear later).

The basic ingredients of the semantics that we will develop for this language are *possible worlds*, which we take to be functions mapping every atomic sentence in $\mathcal{P}$ to a truth value, 1 or 0, and *information states*, which are sets of possible worlds. For brevity, we will often simply talk about worlds and states instead of possible worlds and information states. ω will denote the set of all worlds.

The semantics consists in a simultaneous recursive definition of three notions:

$s \models^{+} \varphi$	*s supports* φ
$s \models^{-} \varphi$	*s rejects* φ
$s \models^{\circ} \varphi$	*s dismisses a supposition of* φ

In terms of these three semantic notions we can define corresponding *logical relations of responsehood* along the following lines:[4]

> ψ *supports (rejects, dismisses a supposition of)* φ iff every state that supports ψ, supports (rejects, dismisses a supposition of) φ.

We will denote the set of all states that support a sentence φ as $[\varphi]^{+}$, and similarly for $[\varphi]^{-}$ and $[\varphi]^{\circ}$. The triple $\langle[\varphi]^{+}, [\varphi]^{-}, [\varphi]^{\circ}\rangle$ is called the *proposition* expressed by φ, and is denoted as $[\varphi]$. If two sentences φ and ψ express exactly the same proposition, they are said to be *equivalent*, notation $\varphi \equiv \psi$.

Before turning to the the semantics proper, we first introduce some auxiliary notions which will be helpful in articulating and explaining the system.

3.1 Informative content and alternatives

In uttering a sentence φ, a speaker proposes to establish a common ground that supports φ. Now suppose that w is a world that is not included in any state that supports φ. Then, any way of compliantly settling the given proposal will lead to a common ground that does not contain w. Thus, in uttering φ, a speaker proposes to exclude any world that is not in $\bigcup[\varphi]^{+}$ as a candidate for the actual world. In other words, she provides the information that the actual world must be contained in $\bigcup[\varphi]^{+}$. For this reason, we will refer to $\bigcup[\varphi]^{+}$ as the *informative content* of φ, and denote it as $\mathsf{info}(\varphi)$.

Definition 1 (Informative content). $\mathsf{info}(\varphi) \coloneqq \bigcup[\varphi]^{+}$

[4] In terms of the three basic semantic notions, a whole range of derived semantic notions can be defined, which can be used in the same way to define additional logical responsehood relations. One case in point is the notion of a state *s suppositionally dismissing a sentence* φ, which holds when s dismisses a supposition of φ, and no substate of s supports or rejects φ. For lack of space, a proper discussion of these logical responsehood relations has to be left for another occcasion.

Among the states that support a sentence φ, some are easier to reach than others. Suppose for instance, that s and t are two states that support φ, and that $t \subset s$. Establishing either s or t as the new common ground would be sufficient to compliantly settle the proposal expressed by φ. However, it is easier to establish s then it is to establish t, because this would require the elimination of fewer possible worlds, i.e., it would require less information.

From this perspective, those states that support φ and are not contained in any other state that supports φ have a special status. They are the *weakest*, least informed states supporting φ. We will refer to such states as the *support-alternatives* for φ, and denote the set of all support-alternatives for φ as $\mathsf{alt}^+(\varphi)$. Similarly, we will refer to the weakest states that reject φ as the *reject-alternatives* for φ, and denote the sets of all these states as $\mathsf{alt}^-(\varphi)$. We will sometimes refer to support-alternatives simply as *alternatives*.

Definition 2 (Alternatives).

- $\mathsf{alt}^+(\varphi) := \{s \mid s \models^+ \varphi$ *and there is no* $t \supset s$ *such that* $t \models^+ \varphi\}$
- $\mathsf{alt}^-(\varphi) := \{s \mid s \models^- \varphi$ *and there is no* $t \supset s$ *such that* $t \models^- \varphi\}$

In our current setting, where we consider a propositional language based on a finite set of atomic sentences, the set of all possible worlds is finite, and therefore the set of all states is also finite. This means that infinite sequences of states $s_0 \subset s_1 \subset s_2 \subset \ldots$ supporting a certain sentence do not exist. As a result, every state that supports a sentence φ is included in a support-alternative for φ, and similarly for states that reject φ.

Fact 1 (Alternatives)

- *Every* $s \in [\varphi]^+$ *is contained in some* $\alpha \in \mathsf{alt}^+(\varphi)$
- *Every* $s \in [\varphi]^-$ *is contained in some* $\alpha \in \mathsf{alt}^-(\varphi)$

We will rely on this fact in formulating and explaining the semantics, in particular the clause for implication, because certain notions become more transparent when explicated in terms of alternatives. We will also show how the semantics can be lifted to the more general setting where the set of possible worlds is infinite and the existence of alternatives cannot be guaranteed.

3.2 Informative, inquisitive, and suppositional sentences

We will say that a sentence φ is *informative* just in case (i) it has the potential to provide information, i.e., $\mathsf{info}(\varphi) \neq \omega$, and (ii) it can be rejected, i.e., $[\varphi]^- \neq \emptyset$. We will say that φ is *inquisitive* just in case (i) there is at least one state that supports φ, and (ii) in order to establish such a state as the new common ground it does not suffice for other conversational participants to simply accept $\mathsf{info}(\varphi)$. The latter holds if and only if $\mathsf{info}(\varphi)$ does not support φ, i.e., $\mathsf{info}(\varphi) \notin [\varphi]^+$. Finally, we will say that φ is *suppositional* just in case there is at least one consistent state that dismisses a supposition of φ, which means that $[\varphi]^\circ \neq \{\emptyset\}$.

Definition 3 (Informative, inquisitive and suppositional sentences).

– *φ is informative iff $[\varphi]^- \neq \emptyset$ and $\mathsf{info}(\varphi) \neq \omega$*
– *φ is inquisitive iff $[\varphi]^+ \neq \emptyset$ and $\mathsf{info}(\varphi) \notin [\varphi]^+$*
– *φ is suppositional iff $[\varphi]^\circ \neq \{\emptyset\}$*

If there are two or more alternatives for a sentence, then that sentence has to be inquisitive. After all, if φ is not inquisitive, then $\mathsf{info}(\varphi)$, which amounts to $\bigcup[\varphi]^+$, supports φ. But this means that $\bigcup[\varphi]^+$ is the unique alternative for φ, which contradicts the assumption that there are two or more alternatives for φ.

Vice versa, if φ is inquisitive, i.e., if $\bigcup[\varphi]^+ \notin [\varphi]^+$, then, given our assumption that there are finitely many possible worlds, there must be at least two states $s, t \in [\varphi]^+$ such that $s \cup t \notin [\varphi]^+$. But then, by Fact 1, there must be at least two support-alternatives for φ, one containing s, one containing t, and neither of them containing $s \cup t$. So there is a straightforward connection between inquisitiveness and the number of support-alternatives for a sentence.

Fact 2 (Alternatives and inquisitiveness)

– *φ is inquisitive iff $\mathsf{alt}^+(\varphi)$ has two or more elements.*

With these basic notions and facts in place, we now turn to the clauses of InqS.

3.3 InqS: the Boolean fragment

We first consider the Boolean fragment of our language, which we denote as $\mathcal{L}_B$. After considering $\mathcal{L}_B$, we will turn to implication. As the reader may expect, the clause for implication will be more intricate than those for the Boolean connectives, and several aspects of it will deserve some careful consideration.

The clauses for $\mathcal{L}_B$ are given in Def. 4 below. After laying out the definition, we will describe informally what each of the clauses amounts to.

Definition 4 (Atomic sentences and Boolean connectives).

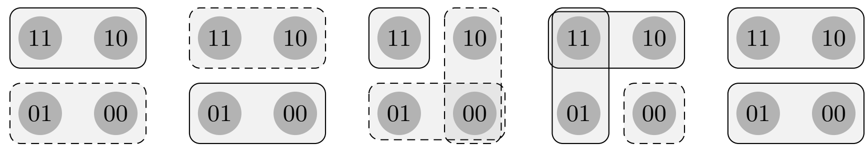
1. $s \models^+ p$ *iff* $s \neq \emptyset$ *and* $s \subseteq |p|$
$s \models^- p$ *iff* $s \neq \emptyset$ *and* $s \cap |p| = \emptyset$
$s \models^\circ p$ *iff* $s = \emptyset$

2. $s \models^+ \neg\varphi$ *iff* $s \models^- \varphi$
$s \models^- \neg\varphi$ *iff* $s \models^+ \varphi$
$s \models^\circ \neg\varphi$ *iff* $s \models^\circ \varphi$

3. $s \models^+ \varphi \wedge \psi$ *iff* $s \models^+ \varphi$ *and* $s \models^+ \psi$
$s \models^- \varphi \wedge \psi$ *iff* $s \models^- \varphi$ *or* $s \models^- \psi$
$s \models^\circ \varphi \wedge \psi$ *iff* $s \models^\circ \varphi$ *or* $s \models^\circ \psi$

4. $s \models^+ \varphi \vee \psi$ *iff* $s \models^+ \varphi$ *or* $s \models^+ \psi$
$s \models^- \varphi \vee \psi$ *iff* $s \models^- \varphi$ *and* $s \models^- \psi$
$s \models^\circ \varphi \vee \psi$ *iff* $s \models^\circ \varphi$ *or* $s \models^\circ \psi$

Atomic sentences. A state s supports an atomic sentence p just in case s is consistent and p is true in all worlds in s. Similarly, s rejects p just in case s is consistent and p is false in all worlds in s. Finally, s dismisses a supposition of p if s is inconsistent. The idea behind the latter clause is that in uttering p, a speaker makes the trivial supposition that p may or may not be the case—a supposition that is dismissed only by the absurd, inconsistent state.



(a) p (b) $\neg p$ (c) $p \wedge q$ (d) $p \vee q$ (e) $?p := p \vee \neg p$

Fig. 1. The propositions expressed by some basic sentences.

Negation. A state s supports $\neg\varphi$ just in case it rejects φ. Vice versa, it rejects $\neg\varphi$ just in case it supports φ. Finally, it dismisses a supposition of $\neg\varphi$ just in case it dismisses a supposition of φ. Thus, $\neg\varphi$ straightforwardly inherits the suppositional content of φ. Notice that, as in classical logic, $\neg\neg\varphi \equiv \varphi$ for any φ.

Conjunction. A state s supports $\varphi \wedge \psi$ just in case it supports both φ and ψ, and it rejects $\varphi \wedge \psi$ just in case it rejects either φ or ψ. Finally, s dismisses a supposition of $\varphi \wedge \psi$ just in case it dismisses a supposition of φ or dismisses a supposition of ψ. Thus, $\varphi \wedge \psi$ inherits the suppositional content of φ and ψ in a straightforward, cumulative way.

Disjunction. A state s supports $\varphi \vee \psi$ just in case it supports either φ or ψ, and it rejects $\varphi \vee \psi$ just in case it rejects both φ and ψ. Finally, s dismisses a supposition of $\varphi \vee \psi$ just in case it dismisses a supposition of φ or dismisses a supposition of ψ. Thus, again, $\varphi \vee \psi$ inherits the suppositional content of φ and ψ in a straightforward, cumulative way.

Propositions expressed by sentences in $\mathcal{L}_B$ can be visualized in a perspicuous way. This is done in Fig. 1 for some simple sentences. In this figure, as before, 11 is a world where both p and q are true, 10 a world where p is true and q is false, etcetera. The support- and reject-alternatives for each sentence are depicted with solid and dashed borders, respectively. Notice in particular that Fig. 1(d) and Fig. 1(e) immediately reveal that $p \vee q$ and $?p$ are inquisitive, since there are two support-alternatives for these sentences.

Some logical properties. The Boolean connectives satisfy De Morgan's laws:

$$\begin{aligned} \varphi \wedge \psi &\equiv \neg(\neg\varphi \vee \neg\psi) \\ \varphi \vee \psi &\equiv \neg(\neg\varphi \wedge \neg\psi) \end{aligned}$$

Moreover, it can be shown that for every sentence $\varphi \in \mathcal{L}_B$, the informative content of φ in InqS, i.e., $\bigcup[\varphi]^+$, coincides precisely with the proposition that φ expresses in classical propositional logic (CPL). So, as far as $\mathcal{L}_B$ is concerned, InqS is a conservative refinement of classical logic. That is, the two fully agree on the informative content of every sentence in the language; only, while classical logic *identifies* the meaning of a sentence with its informative content, InqS has a more fine-grained notion of meaning.

Fact 3 (Conservative refinement of CPL) *For any $\varphi \in \mathcal{L}_B$,* $\mathsf{info}(\varphi) = |\varphi|$

The *inconsistent* state, $\emptyset$, never supports or rejects a sentence in $\mathcal{L}_B$, but always suppositionally dismisses it.

Fact 4 (Inconsistency) *For any $\varphi \in \mathcal{L}_B$: $\emptyset \not\models^+ \varphi$, $\emptyset \not\models^- \varphi$, and $\emptyset \models^\circ \varphi$.*

Moreover, the inconsistent state is the *only* state that suppositionally dismisses any sentence in $\mathcal{L}_B$. In other words, no sentence in $\mathcal{L}_B$ is suppositional.

Fact 5 (No suppositionality) *For any $\varphi \in \mathcal{L}_B$, $[\varphi]^\circ = \{\emptyset\}$.*

Recall that in InqB support is persistent, i.e., information grows never leads to retraction of support. It follows from Fact 4 that in InqS support and rejection are not fully persistent: any state that supports or rejects a sentence φ has a substate, namely $\emptyset$, which no longer supports/rejects φ. However, in the Boolean fragment of InqS, support and rejection *are* persistent *modulo inconsistency*. That is, if s supports φ then any *consistent* substate of s still supports φ. And similarly for rejection.

Fact 6 (Persistence modulo inconsistency)
For any $\varphi \in \mathcal{L}_B$, $\star \in \{+, -\}$, if $s \models^\star \varphi$ and $s \supseteq t \neq \emptyset$, then $t \models^\star \varphi$.

A state never supports and rejects a sentence at the same time.

Fact 7 (Support and rejection are mutually exclusive)
For any $\varphi \in \mathcal{L}_B$, $[\varphi]^+ \cap [\varphi]^- = \emptyset$.

Finally, it follows from Facts 6 and 7 that in the Boolean fragment support and rejection are incompatible in a stronger sense as well: a state that supports a sentence φ can never have any *overlap* with a state that rejects φ.

Fact 8 (Support and rejection do not overlap)
For any $\varphi \in \mathcal{L}_B$, if $s \models^+ \varphi$ and $t \models^- \varphi$, then $s \cap t = \emptyset$.

3.4 Implication

We now turn to implication, which typically introduces non-trivial suppositional content. The initial idea is that, for a state s to either support or reject an implication $\varphi \to \psi$, it is a necessary requirement that the antecedent φ be *supposable* in s. If this is not the case, then s suppositionally dismisses the implication, and does not support or reject it.

The key question, then, is what it means exactly for φ to be supposable in s. To answer this question, we will consider a number of concrete examples. We will start with the simplest case, and gradually consider more complex ones. As we proceed, our notion of supposability and the semantics for implication that is defined in terms of it will become more and more refined. Consider first an implication with an atomic antecedent and an atomic consequent:

(7) $\quad p \to r$

It would be natural to say that p is supposable in a state s iff the single support-alternative for p, $|p|$, is consistent with s, i.e., $s \cap |p| \neq \emptyset$. Furthermore, it would be natural to say that if this condition is met, s supports the implication iff $s \cap |p|$ supports r, and s rejects the implication iff $s \cap |p|$ rejects r. However, this characterization of supposability only applies if there is a *unique* support-alternative for the antecedent. To see how it may be generalized, let us consider an example in which there are *two* support-alternatives for the antecedent:

(8) $\quad (p \vee q) \to r$

To deal with such cases, as well as the simpler cases where there is a single support-alternative for the antecedent, it seems reasonable to say that, in general, the antecedent is supposable in s iff *every* support-alternative for it is consistent with s:

(9) $\quad \varphi$ is supposable in s, notation $s \lhd \varphi$, iff $\forall \alpha \in \mathsf{alt}^+(\varphi) : s \cap \alpha \neq \emptyset$

With this characterization of supposability in place, we may formulate the clauses for implication as follows:

$$s \models^+ \varphi \to \psi \quad \text{iff} \quad s \lhd \varphi \text{ and } \forall \alpha \in \mathsf{alt}^+(\varphi)\text{: } s \cap \alpha \models^+ \psi$$

$$s \models^- \varphi \to \psi \quad \text{iff} \quad s \lhd \varphi \text{ and } \exists \alpha \in \mathsf{alt}^+(\varphi)\text{: } s \cap \alpha \models^- \psi$$

$$s \models^\circ \varphi \to \psi \quad \text{iff} \quad s \not\lhd \varphi$$

However, this formulation of the clauses is problematic in several ways. One problem is that the given conditions for rejecting an implication are too stringent. To see this, consider the following state:

(10) $\quad s := |\neg p \wedge (q \to \neg r)|$

This state is inconsistent with one of the support-alternatives for the antecedent of (8), namely $|p|$. However, it is consistent with the other support-alternative, $|q|$, and if we intersect it with this alternative we get at the state $|\neg p \wedge \neg r|$, which rejects the consequent of the implication, r. So, on the one hand, not every support-alternative for the antecedent is consistent with s, and we want our semantics to capture this by characterizing s as dismissing a supposition of the implication; on the other hand, however, one of the support-alternatives for the antecedent *is* consistent with s, and restricting s to this alternative leads to rejection of the consequent. We want our semantics to capture this as well, by characterizing s as a state that rejects the implication as a whole (besides dismissing a supposition of it).

The general upshot of this example is that the idea that we started out with, namely that supposability of the antecedent as a whole is a necessary requirement for a state to support or reject an implication, is not exactly on the right track. In particular, it is too stringent in the case of rejection.

Rather than considering the supposability of the antecedent as a whole, it seems more suitable to consider the supposability of each support-alternative for the antecedent separately. Let us say, for now, that an alternative α is supposable in a state s just in case the two are consistent with each other:

(11) An alternative α is supposable in a state s, notation $s \lhd \alpha$, iff $s \cap \alpha \neq \emptyset$.

Then we arrive at the following revised formulation of the clauses for implication:

$$
\begin{array}{lll}
s \models^{+} \varphi \to \psi & \text{iff} & \forall \alpha \in \mathsf{alt}[\varphi]^{+}\text{: } s \lhd \alpha \text{ and } s \cap \alpha \models^{+} \psi \\
s \models^{-} \varphi \to \psi & \text{iff} & \exists \alpha \in \mathsf{alt}[\varphi]^{+}\text{: } s \lhd \alpha \text{ and } s \cap \alpha \models^{-} \psi \\
s \models^{\circ} \varphi \to \psi & \text{iff} & \exists \alpha \in \mathsf{alt}[\varphi]^{+}\text{: } s \not\lhd \alpha
\end{array}
$$

This formulation, however, still needs further refinement. First, consider a case in which there are no support-alternatives for the antecedent at all:

(12) $(p \wedge \neg p) \to r$

According to the clauses above, this implication is trivially supported by any state, because the clause for support quantifies universally over the support-alternatives for the antecedent, which in this case do not exist. On the other hand, according to the given clauses, there is no state that dismisses a supposition of the implication, because this requires inconsistency with some support-alternative for the antecedent, of which there are none. We want exactly the opposite result: no state should support this implication, and every state should dismiss a supposition of it. Thus, the clauses should be adapted: support should require a non-empty set of support-alternatives for the antecedent, while dismissal of a supposition should occur if this set is empty. This leads us to the formulation below. For uniformity, we have adapted the rejection clause as well, although this is strictly speaking redundant; the new, redundant part of the clause is displayed in gray.

$$
\begin{array}{lll}
s \models^{+} \varphi \to \psi & \text{iff} & \mathsf{alt}^{+}(\varphi) \neq \emptyset \text{ and } \forall \alpha \in \mathsf{alt}^{+}(\varphi)\text{: } s \lhd \alpha \text{ and } s \cap \alpha \models^{+} \psi \\
s \models^{-} \varphi \to \psi & \text{iff} & \mathsf{alt}^{+}(\varphi) \neq \emptyset \text{ and } \exists \alpha \in \mathsf{alt}^{+}(\varphi)\text{: } s \lhd \alpha \text{ and } s \cap \alpha \models^{-} \psi \\
s \models^{\circ} \varphi \to \psi & \text{iff} & \mathsf{alt}^{+}(\varphi) = \emptyset \text{ or } \exists \alpha \in \mathsf{alt}^{+}(\varphi)\text{: } s \not\lhd \alpha
\end{array}
$$

This formulation is appropriate as long as φ and ψ are non-suppositional, i.e., as long as they do not contain any implications themselves. However, to deal with nested implications, some further refinements are needed.

First consider a case where the *consequent* is suppositional, which will be relatively easy to accommodate.

(13) $p \to (q \to r)$

Consider the following state:

(14) $s := |p \to \neg q|$

The semantics should predict that this state dismisses a supposition of (13), because if we restrict it to the unique support-alternative for the antecedent, $|p|$, we arrive at the state $|\neg q|$, and this state dismisses a supposition of the consequent, $q \to r$. However, this is not captured by the clause for dismissal given above, which requires that there is a support-alternative for the antecedent that is inconsistent with s. This is clearly not the case here. So the clause needs to be adapted, and there is a natural way to do so: in order for s to dismiss a supposition of $\varphi \to \psi$ it should be the case that $\mathsf{alt}^+(\varphi)$ is empty, or that it contains an alternative that is not supposable in s, or that it contains an alternative α which is such that $s \cap \alpha$ dismisses a supposition of the consequent. Notice that, w.r.t. the previous formulation, the first two conditions are old, and the third one is newly added. Moreover, notice that whenever the consequent of the implication is non-suppositional, the second and the third requirement coincide, demanding that $s \cap \alpha$ be consistent. Leaving the support and reject clauses unchanged, we arrive at the following formulation:

$$
\begin{array}{lll}
s \models^+ \varphi \to \psi & \text{iff} & \mathsf{alt}^+(\varphi) \neq \emptyset \text{ and } \forall \alpha \in \mathsf{alt}^+(\varphi)\text{: } s \lhd \alpha \text{ and } s \cap \alpha \models^+ \psi \\
s \models^- \varphi \to \psi & \text{iff} & \mathsf{alt}^+(\varphi) \neq \emptyset \text{ and } \exists \alpha \in \mathsf{alt}^+(\varphi)\text{: } s \lhd \alpha \text{ and } s \cap \alpha \models^- \psi \\
s \models^\circ \varphi \to \psi & \text{iff} & \mathsf{alt}^+(\varphi) = \emptyset \text{ or } \exists \alpha \in \mathsf{alt}^+(\varphi)\text{: } s \not\lhd \alpha \text{ or } s \cap \alpha \models^\circ \psi
\end{array}
$$

There is one more amendment to make, in order to deal with cases where the antecedent of the implication is itself suppositional. We will do this in two steps, again first considering the simplest case and then a more complex one. Consider first:

(15) $(p \to q) \to r$

Suppose that our state of evaluation is the following:

(16) $s := |\neg p \wedge r|$

According to the clauses as formulated above, this state supports the implication in (15), because there is a single support-alternative for the antecedent, $\alpha := |p \to q|$, which is consistent with s, and the intersection of s with α amounts to s itself, which supports the consequent, r. Moreover, the clauses do not characterize s as a state that dismisses a supposition of (15), because $s \cap \alpha$ is consistent and does not dismiss a supposition of the consequent.

Again, we want precisely the opposite result: s should be characterized as dismissing a supposition of the implication, and not as supporting it. The culprit for this is our notion of supposability of support-alternatives. According to (11), a support-alternative α for a sentence φ is supposable in a state s iff $s \cap \alpha \neq \emptyset$. However, even if $s \cap \alpha \neq \emptyset$, it may be the case that $s \cap \alpha$ no longer supports φ. This is indeed the case in the example above where $\alpha := |p \to q|$ is the unique support-alternative for the antecedent, $p \to q$, and $s \cap \alpha$ no longer supports $p \to q$. For this reason, α should not be characterized as supposable in s.

Our notion of supposability should be made sensitive to this. That is, we should not just require that $s \cap \alpha$ is consistent, but rather that in going from α to $s \cap \alpha$, support of φ is preserved:

(17) A support-alternative α for a sentence φ is supposable in a state s, notation $s \lhd \alpha$, iff $s \cap \alpha \models^+ \varphi$.

With this refined notion of supposability in place, the clauses for implication can remain as they were formulated above. Examples like (15), with a suppositional, but non-inquisitive antecedent, are now suitably dealt with.

The final, most complex case to consider is one in which the antecedent is both suppositional and inquisitive, which means that it has multiple support-alternatives. Take the following example:

(18) $((p \to q) \vee l) \to r$

Notice that the antecedent is a disjunction, whose first disjunct is suppositional. Consider a state that dismisses the first disjunct, but supports the second, and moreover, supports the consequent of the implication:

(19) $s := |\neg p \wedge l \wedge r|$

According to the clauses as formulated above, this state supports the implication in (18). Let us see why this is the case. First, there are two support-alternatives for the antecedent, $|p \to q|$ and $|l|$. Intersecting s with either of these alternatives simply yields s, which supports the antecedent, so both support-alternatives for the antecedent are supposable. Moreover, the intersection of s with either of the support-alternatives for the antecedent also supports the consequent, r. Therefore, s supports the implication as a whole as well.

The clauses also characterize s as a state that does not dismiss any supposition of the implication in (18). This is because the intersection of s with either of the two support-alternatives for the antecedent is just s, and as we already saw, s supports the antecedent.

These are not the right results: we want the semantics to characterize s as a state that dismisses a supposition of the implication, and does not support it. The culprit for this is again our notion of supposability of support-alternatives. The idea was that a support-alternative α for φ is supposable in s iff in going from α to $s \cap \alpha$, *support of φ is preserved*. Formally, we require that $s \cap \alpha$ should still support φ.

But now look at the example again. There are two support-alternatives for the antecedent, corresponding to the two disjuncts, $|p \to q|$ and $|l|$. Let us focus on the first. Intersecting s with this alternative simply yields s, which supports the antecedent of the implication. Crucially, however, this is because it supports the *second* disjunct, l. It does not support the first disjunct, the one that corresponds to the support-alternative that we are considering. And, upon closer examination, there is a clear sense in which support is not fully *preserved* in going from $|p \to q|$ to s. Namely, there are states between $|p \to q|$ and s, such as $|\neg p|$, which do not support the antecedent. Only when we further strengthen these states in such a way that they come to support the second disjunct, do they come to support the antecedent as a whole. From this perspective, it is not right to say that support is preserved in going from $|p \to q|$ to s. It is true that we have support at s, but only after it was lost somewhere along the way. These considerations lead to the following, definitive, characterization of supposability of support-alternatives.[5]

Definition 5 (Supposability of support-alternatives).
A support-alternative α for a sentence φ is supposable in a state s, notation $s \lhd \alpha$, iff for every state t between α and $s \cap \alpha$, i.e., every t such that $\alpha \supseteq t \supseteq (s \cap \alpha)$, we have that $t \models^{+} \varphi$.

With this refined notion of supposability in place, the clauses for implication can remain as formulated above. We restate them here in an official definition.[6]

Definition 6 (Implication).

$$s \models^{+} \varphi \to \psi \quad \textit{iff} \quad \mathsf{alt}^{+}(\varphi) \neq \emptyset \textit{ and } \forall \alpha \in \mathsf{alt}^{+}(\varphi)\textit{: } s \lhd \alpha \textit{ and } s \cap \alpha \models^{+} \psi$$
$$s \models^{-} \varphi \to \psi \quad \textit{iff} \quad \mathsf{alt}^{+}(\varphi) \neq \emptyset \textit{ and } \exists \alpha \in \mathsf{alt}^{+}(\varphi)\textit{: } s \lhd \alpha \textit{ and } s \cap \alpha \models^{-} \psi$$
$$s \models^{\circ} \varphi \to \psi \quad \textit{iff} \quad \mathsf{alt}^{+}(\varphi) = \emptyset \textit{ or } \quad \exists \alpha \in \mathsf{alt}^{+}(\varphi)\textit{: } s \not\lhd \alpha \textit{ or } \quad s \cap \alpha \models^{\circ} \psi$$

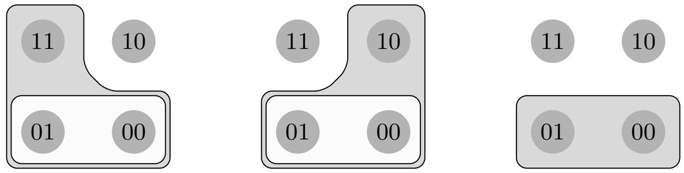
This completes our suppositional semantics for the full propositional language $\mathcal{L}$.

Depicting propositions. The propositions expressed by simple conditional sentences can again be visualized. Fig. 2 does this for the most basic case, $p \to q$. Fig. 2(a) depicts the maximal state supporting $p \to q$, i.e., $|p \to q|$, as well as its maximal substate that *no longer* supports $p \to q$, i.e., $|\neg p|$. Any substate of $|p \to q|$ that is not completely contained in $|\neg p|$ still supports $p \to q$. Similarly, Fig. 2(b) depicts the maximal state rejecting $p \to q$, i.e., $|p \to \neg q|$, as well as its maximal substate that no longer rejects $p \to q$, i.e., $|\neg p|$. Finally, Fig. 2(c) depicts the maximal state that dismisses a supposition of $p \to q$, i.e., again $|\neg p|$.

[5] This notion of supposability preserves a key property of the simple notion of supposability in terms of consistency, namely that for every support-alternative α of any sentence φ, there is a unique 'turning point' state s, such that α is supposable in any superstate of s and no longer supposable in any substate of s. For this to obtain, it is crucial that the notion requires support to be preserved in all states between α and $s \cap \alpha$, and not just in $s \cap \alpha$.

[6] Recall from our discussion in Sect. 3.1 that the fact that we are considering a propositional language based on a finite set of atomic sentences is crucial in ensuring that every state that supports a sentence is contained in an alternative for that sentence, which in turn justifies our formulation of the clauses for implication in terms of alternatives. However, this cannot always be ensured. For instance, if we consider a first-order language with an infinite domain of interpretation, the existence of alternatives can no longer be guaranteed (Ciardelli, 2009). Fortunately, there is a way to formulate the clauses for implication that does not make reference to alternatives, and which in the current setting is equivalent to the clauses as formulated in Def. 6:

$$s \models^{+} \varphi \to \psi \text{ iff } [\varphi]^{+} \neq \emptyset \text{ and } \forall t \in [\varphi]^{+} \exists u \supseteq t \in [\varphi]^{+}\text{: } s \lhd u \text{ and } s \cap u \models^{+} \psi$$
$$s \models^{-} \varphi \to \psi \text{ iff } [\varphi]^{+} \neq \emptyset \text{ and } \exists t \in [\varphi]^{+} \forall u \supseteq t \in [\varphi]^{+}\text{: } s \lhd u \text{ and } s \cap u \models^{-} \psi$$
$$s \models^{\circ} \varphi \to \psi \text{ iff } [\varphi]^{+} = \emptyset \text{ or } \quad \exists t \in [\varphi]^{+} \forall u \supseteq t \in [\varphi]^{+}\text{: } s \not\lhd u \text{ or } \quad s \cap u \models^{\circ} \psi$$



(a) support (b) reject (c) dismiss

Fig. 2. States that support, reject, and dismiss a supposition of $p \to q$.

Some logical properties. Recall again that in InqB support is fully persistent, and that in the Boolean fragment of InqS support and reject are persistent modulo inconsistency (Fact 6). In the full fragment of InqS this feature is lost. For instance, the information state $|p \to q|$ supports the sentence $p \to q$, but the state $|\neg p|$, which is a substate of $|p \to q|$, does not support $p \to q$; rather, it dismisses a supposition of it. Similarly for rejection: $|p \to \neg q|$ rejects $p \to q$, but $|\neg p|$, which is a substate of $|p \to \neg q|$, does not reject $p \to q$. Instead, as noted above, $|\neg p|$ dismisses a supposition of the implication. Thus, unlike in InqB, information growth can lead to suppositional dismissal, and thereby also to retraction of support or rejection, even if consistency is preserved.

However, a weaker form of persistency is still maintained in InqS, namely persistency *modulo suppositional dismissal*. That is, if a state s supports a sentence φ, then any more informed state $t \subseteq s$ either still supports φ, or dismisses a supposition of φ. And similarly for rejection. Finally, dismissal of a supposition is fully persistent. If a state s dismisses a supposition of φ, then so does any more informed state $t \subseteq s$. Information growth cannot lead to retraction of dismissal.

Fact 9 (Persistence modulo suppositional dismissal)
For any φ, $\star \in \{+, -, \circ\}$, if $s \models^\star \varphi$ and $t \subseteq s$, then $t \models^\star \varphi$ or $t \models^\circ \varphi$.

Fact 4 extends from the Boolean fragment of InqS to the full system: the inconsistent state never supports or rejects a sentence but always suppositionally dismisses it. The same goes for Fact 7: a state never supports and rejects a sentence at the same time. However, the full system does not exclude the possibility that a state either supports or rejects a sentence and at the same time also dismisses a supposition of it. To see that this option should indeed be left open, consider the following examples:

(20) a. Maria will go if Peter goes, or if Frank goes. $(p \to r) \vee (q \to r)$
b. Well, Peter isn't going, but indeed,
if Frank goes, Maria will go as well. $\neg p \wedge (q \to r)$

(21) a. Maria will go if Peter goes, and if Frank goes. $(p \to r) \wedge (q \to r)$
b. Well no, Peter isn't going, and if Frank goes,
Maria definitely won't. $\neg p \wedge (q \to \neg r)$

The response in (20b) supports (20a), but at the same time it also dismisses a supposition of it. Similarly, the response in (21b) rejects (21a), but again, it also dismisses a supposition of it.

Facts 4 and 9 together imply that the three components of a proposition in InqS jointly form a non-empty set of states S that is *downward closed*, i.e., for any $s \in S$ and $t \subseteq s$ we have that $t \in S$ as well.

Fact 10 *For any φ, $[\varphi]^+ \cup [\varphi]^- \cup [\varphi]^\circ$ is non-empty and downward closed.*

In InqB, propositions are defined precisely as non-empty, downward closed sets of states. So, while InqS offers a more fine-grained notion of meaning than InqB in that it distinguishes three different meaning components, if we put these three meaning components together, we always obtain the same kind of semantic object that we had already in InqB. Thus, InqS is a refinement of InqB, but at the same time it retains one of its core features.

We saw in Sect. 3.3 that the Boolean fragment of InqS preserves many central features of CPL. As soon as implication is taken into consideration, however, InqS diverges more radically from CPL. In particular, Facts 3 (conservative refinement), 5 (no suppositionality), and 8 (no overlap) no longer hold, which can all be shown with a single example: $\neg(p \to q)$. We have that $\mathsf{info}(\neg(p \to q)) = |p \to \neg q|$ which differs from the proposition expressed by $\neg(p \to q)$ in CPL. Furthermore, $\neg(p \to q)$ is suppositional, and it has supporting and rejecting states that overlap, for instance $|p \to \neg q|$ and $|p \to q|$, respectively.

One 'classical' property that InqS *does* preserve, even when implication is taken into consideration, is that whenever a state s supports a sentence φ, then no substate $t \subseteq s$ rejects φ, and vice versa, whenever s rejects φ, no substate $t \subseteq s$ supports φ. In the terminology of Veltman (1985), this means that every sentence in our language is *stable*.

Fact 11 (Stability) *For any $\varphi \in \mathcal{L}$ and any state s:*

- *If s supports φ then no $t \subseteq s$ rejects φ*
- *If s rejects φ then no $t \subseteq s$ supports φ*

Veltman introduced the notion of stability in his work on *data semantics*, which, like InqS, is concerned in particular with conditionals and epistemic modals. Veltman emphasizes that in data semantics, both conditionals and epistemic modals are typically unstable, unlike sentences that do not contain modals or conditionals. In InqS, it is still the case that sentences involving epistemic modals are typically unstable (see Aher et al., 2014). However, all sentences in the propositional language considered here, including conditionals, are stable.

Finally, recall that we defined $?\varphi$ as an abbreviation of $\varphi \vee \neg\varphi$. Having spelled out the clauses for all the basic connectives in our system, we can now derive the interpretation of $?\varphi$ as well. First, a state supports $?\varphi$ iff it supports either φ or $\neg\varphi$. So $[?\varphi]^+ = [\varphi]^+ \cup [\neg\varphi]^+ = [\varphi]^+ \cup [\varphi]^-$. Second, a state rejects $?\varphi$ iff it rejects both φ and $\neg\varphi$. But to reject $\neg\varphi$ is to support φ. Thus, in order to reject $?\varphi$, a state would have to support φ and reject φ at the same time, which is impossible. So, for any φ, $[?\varphi]^-$ will be empty. Finally, a state dismisses a supposition of $?\varphi$ iff it dismisses a supposition of φ or of $\neg\varphi$, and the latter occurs just in case the state dimisses a supposition of φ itself. So, $[?\varphi]^\circ = [\varphi]^\circ$.

Fact 12 *For any φ,* $[?\varphi] = \langle [\varphi]^+ \cup [\varphi]^-, \emptyset, [\varphi]^\circ \rangle$

Now let us return to our initial motivating examples, repeated below:

(22)
a. If Pete plays the piano, will Susan sing? $p \to ?q$
b. Yes, if Pete plays the piano, Susan will sing. $p \to q$
c. No, if Pete plays the piano, Susan won't sing. $p \to \neg q$
d. Pete won't play the piano. $\neg p$

As desired, our semantics predicts that (22b) and (22c) support (22a); that (22b) and (22c) reject each other; and that (22d) neither supports nor rejects any of (22a), (22b), and (22c), but dismisses a supposition of all three of them.[7] These examples are iconic for the issues that we set out to address. But, as we saw along the way, the semantics deals with many more complex cases as well.

4 Conclusion

Our starting point in this paper was the general perspective on meaning that is taken in inquisitive semantics, which is that sentences express proposals to update the common ground of the conversation in one or more ways. There are several ways in which a conversational participant may respond to such proposals, depending on her information state. The most basic inquisitive semantics framework, InqB, characterizes which states support a given proposal. Radical inquisitive semantics, InqR, also characterizes independently which states reject a given proposal. The suppositional inquisitive semantics developed in the present paper, InqS, further distinguishes states that dismiss a supposition of a given proposal. We have thus arrived at a more and more fine-grained formal characterization of proposals, and thereby at a more and more fine-grained characterization of meaning. We have argued that this is necessary for a better account of information exchange through conversation, in particular when the exchange involves conditional questions and assertions. Elsewhere, we argue that the framework developed here also offers new insights into the semantics of epistemic and deontic modals (Aher et al., 2014).

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[7] Sentence (22d) not only dismisses a supposition of the other three sentences, but it *suppositionally dismisses* them, given the way this notion was defined in footnote 4.

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