Towards a suppositional inquisitive semantics*

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Abstract. Language is used, among other things, to request and to provide information. This can be done directly, as in *Will Susan sing? No, she won't*, but it is also often done in a less direct way, as in *If Pete plays the piano, will Susan sing? No, if Pete plays the piano, Susan won't sing.* In the latter type of exchange, both participants make a certain *supposition*, and exchange information under the assumption that this supposition holds. This paper develops a semantic framework for the analysis of this kind of information exchange. Building on earlier work in inquisitive semantics, it introduces a notion of meaning that captures informative, inquisitive, and suppositional content, and discusses how such meanings may be assigned in a natural way to sentences in a propositional language. The focus is on conditionals, which are the only kind of sentences in such a language that introduce non-trivial suppositional content.

1 Towards a more fine-grained notion of meaning

Traditionally, the meaning of a sentence is identified with its informative content, and the informative content of a sentence is taken to be determined by its truth conditions. Thus, the proposition expressed by a sentence is construed as a set of possible worlds, those worlds in which the sentence is true, and this set of worlds is taken to determine the effect that is achieved when the sentence is uttered in a conversation. Namely, when the sentence is uttered, the speaker is taken to *propose an update of the common ground* of the conversation (Stalnaker, 1978). The common ground of a conversation is the body of information that has been publicly established in the conversation so far. It is modeled as a set of possible worlds, namely those worlds that are compatible with the established information. When a speaker utters a sentence, she is taken to propose to update the common ground by restricting it to those worlds in which the uttered sentence is true, i.e., those worlds that are contained in the proposition expressed by the sentence. If accepted by the other conversational participants, this update ensures that the new common ground contains the information that the uttered sentence is true.

This basic picture of sentence meaning and the effect of an utterance in conversation has proven very useful, but it also has some inherent limitations.

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Perhaps most importantly, it is completely centred on informative content and truth conditions. Evidently, there are many meaningful sentences in natural language that cannot be said to be true or false, and that cannot be thought of as having any non-trivial informative content. Consider, for instance, the question in (1):

(1) Will Susan sing tonight?

Clearly, this question cannot be said to be true or false in any given situation, or to have any non-trivial informative content. So, in order to deal with such sentences, the basic picture sketched above needs to be generalized. One way to do this has been articulated in recent work on inquisitive semantics (e.g. Ciardelli, Groenendijk, and Roelofsen, 2012, 2013). The basic observation is that we could still think of a speaker who utters (1) as proposing to update the common ground of the conversation. However, she does not propose to update the common ground in one particular way, but rather, offers a choice: one way to comply with her proposal would be to restrict the common ground to worlds where Susan will sing, but another way to comply with her proposal, equally acceptable, would be to restrict the common ground to worlds where Susan won't sing. So the basic Stalnakerian picture of the effect of an utterance in terms of issuing a proposal to update the common ground of the conversation can be generalized appropriately, in such a way that it applies to declarative and interrogative sentences in a uniform way.

What about the basic truth conditional notion of sentence meaning? How could this be suitably generalized? The simplest approach that has been explored in inquisitive semantics is to move from truth conditions to *support conditions*. The idea is that the meaning of a sentence should determine precisely which pieces of information support the proposal that a speaker makes in uttering the sentence. This notion of meaning is adopted in the most basic implementation of inquisitive semantics, InqB (Groenendijk and Roelofsen, 2009; Ciardelli, 2009; Ciardelli and Roelofsen, 2011; Roelofsen, 2013; Ciardelli et al., 2013, a.o.).

Clearly, the support based notion of meaning adopted in InqB is directly tied to the idea that the effect of an utterance is to issue a proposal to update the common ground in one or more ways. The latter—let's call it the *proposal picture of conversation*—is one of the main tenets of the inquisitive semantics framework in general, not just of the particular system InqB. The support based notion of meaning, on the other hand, is specific to InqB. It ties in well with the proposal picture of conversation, but there may well be other notions of meaning that also tie in well with this picture.

The goal of this paper is to develop such a notion of meaning, which is more fine-grained than the InqB notion. Motivation for such a more fine-grained notion comes from the basic observation that proposals may not only be *supported* by a given piece of information; they may also be *rejected* or *dismissed*. To illustrate, consider (2), and the two responses to it in (3) and (4):

(2) If Pete plays the piano, Susan will sing.

(3) No, if Pete plays the piano, Susan won't sing.

(4) Pete won't play the piano.

Intuitively, both (3) and (4) are pertinent responses; they address the proposal that (2) expresses. However, they do not support the proposal. Rather, (3) rejects the proposal, while (4) dismisses a supposition of it and thereby renders it void.

We will consider what it means in general to reject a proposal or to dismiss a supposition of it, and how these notions are related to each other, as well as to support. We will define a semantics for a propositional language, which specifies recursively for every sentence (i) which information states (or equivalently, which pieces of information) support it, (ii) which information states reject it, and (iii) which information states dismiss a supposition of it. We refer to this system as InqS. We will argue that the more fine-grained notion of meaning adopted in InqS considerably broadens the empirical scope of InqB, especially in the domain of conditionals. In a separate paper it is shown that the framework developed here allows for a novel treatment of epistemic and deontic modals as well, exhibiting interesting connections with the treatment of conditionals presented here (Aher et al., 2014).[1]

The paper is organized as follows. Sect. 2 reviews the background and motivation for InqS in more detail; Sect. 3 presents the system itself, identifying its basic logical properties and discussing some illustrative examples; and finally, Sect. 4 summarizes and concludes.

2 Background and motivation

2.1 From truth to support

One way to obtain a notion of meaning that is suitable for both declarative and interrogative sentences is to move from truth conditions to *support conditions.* In a truth conditional setting, the idea is that one knows the meaning of a sentence,

[1] One way to reject the proposal expressed by (2), not listed above, is as follows:

(i) No, if Pete plays the piano, Susan might not sing.

This response involves the epistemic modal *might.* Accounting for such responses is beyond the scope of the current paper, but not beyond the scope of InqS in general. Indeed, the InqS analysis of epistemic modals presented in Aher et al. (2014) naturally characterizes (i) as a rejecting response to (2), and also clearly brings out the difference between (i) and (3). Namely, (i) rejects (2) in a *defeasible* way, subject to possible retraction when additional information becomes available, while (3) rejects (4a) indefeasibly. Or, phrased in terms of conversational attitudes, (i) signals that the addressee is *unwilling* to accept the proposal expressed by (2), while (3) signals that she is really *unable* to do so.

There is a rich literature on the denial of conditional statements (see, e.g., Handley et al., 2006; Espino and Byrne, 2012; Égré and Politzer, 2013, and references therein), but the distinction between defeasible and indefeasible rejection has, to the best of our knowledge, not been brought to attention previously.

or at least a core aspect thereof, if one knows under which circumstances the sentence is true and under which it is false. In a support based setting, the idea is that one knows the meaning of a sentence just in case one knows which information states—or equivalently, which pieces of information—support the given sentence, and which don't.

For instance, an information state s, modeled as a set of possible worlds, supports an atomic declarative sentence p just in case every world in s makes p true, it supports $\neg p$ if every world in s makes p false, and finally, it supports the interrogative sentence $?p$ just in case it supports either p or $\neg p$.

Given such a support-based semantics, we can think of a speaker who utters a sentence φ as proposing to enhance the common ground of the conversation, modeled as an information state, in such a way that it *comes to support* φ. Thus, in uttering p a speaker proposes to enhance the common ground in such a way that it comes to support p, and in uttering $?p$ a speaker proposes to enhance the common ground in such a way that it comes to support either p or $\neg p$.

Prima facie it is natural to assume that support is *persistent*, that is, if an information state s supports a sentence φ, then it is natural to assume that every more informed information state $t \subset s$ will also support φ. In other words, information growth cannot lead to retraction of support. This assumption is indeed made in InqB, and it determines to a large extent how the system behaves.

2.2 Support for conditionals

Let us now zoom in on conditional sentences, which is where we would like to argue that a more refined picture is ultimately needed. Consider again the conditional statement in (2), repeated in (5) below, and the corresponding conditional question in (6):

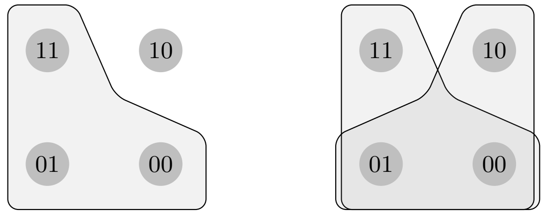
(5) If Pete plays the piano, Susan will sing. $p \to q$

(6) If Pete plays the piano, will Susan sing? $p \to ?q$

The meanings of these sentences in InqB are depicted in figures 1(a) and 1(b), respectively. In these figures, 11 is a world where p and q are both true, 10 a world where p is true but q is false, etcetera. We have only depicted the *maximal* states that support each sentence. Since support is persistent, all substates of these maximal supporting states also support the given sentences.

In general, in InqB a state s is taken to support a conditional sentence $\varphi \to \psi$ just in case every state $t \subseteq s$ that supports φ also supports ψ. For instance, the state $s = \{11, 01, 00\}$ supports $p \to q$, because any substate $t \subseteq s$ that supports p (there are only two such states, namely $\{11\}$ and $\emptyset$) also support q. Similarly, one can verify that the states $\{11, 01, 00\}$ and $\{11, 01, 00\}$ both support $p \to ?q$.

For convenience, we will henceforth use $|\varphi|$ to denote the state consisting of all worlds where φ is classically true. So the states $\{11, 01, 00\}$ and $\{10, 01, 00\}$ can be denoted more perspicuously as $|p \to q|$ and $|p \to \neg q|$, respectively.



(a) $[\![(5)]\!]$ (b) $[\![(6)]\!]$

Fig. 1. Support for declarative and interrogative conditionals.

2.3 Support and reject

The support conditions for a sentence φ capture an essential aspect of the proposal that is made in uttering φ, namely what is needed to compliantly settle this proposal. However, besides compliantly settling a given proposal, conversational participants may react in different ways as well. In particular, they may *reject* the proposal. What does it mean exactly to reject the proposal made in uttering φ? And can this, perhaps indirectly, be explicated in terms of the support conditions for φ as well?

At first sight, this seems quite feasible indeed. Suppose that a speaker A utters a sentence φ, and a responder B reacts with ψ. A proposes to enhance the common ground to a state that supports φ, while B proposes to enhance the common ground to a state that supports ψ. Then we could say that B *rejects* A's initial proposal just in case any state s that supports ψ is such that no consistent substate $t \subseteq s$ supports φ. After all, if this is the case, then any way of satisfying B's counterproposal leads to a common ground which does not support φ and which cannot be further enhanced in any way such that it comes to support φ while remaining consistent.

For many basic cases, this characterization of rejection in terms of support seems adequate. For instance, if A utters p and B responds with $\neg p$, then according to the given characterization, B rejects A's initial proposal, which accords with pre-theoretical intuitions.

However, in the case of conditionals, the given characterization is problematic. Intuitively, the proposal expressed by (5) above can be rejected with (7).

(7) No, if Pete plays the piano, Susan won't sing. $p \to \neg q$

However, there is a consistent state that supports both (5) and (7), namely $|\neg p|$. So according to the above characterization, (7) does not reject (5).

This example illustrates something quite fundamental: in general, reject conditions cannot be derived from support conditions. Thus, a semantics that aims to provide a comprehensive characterization of the proposals that speakers make when uttering sentences in conversation, needs to specify both support- and reject-conditions (and perhaps more).

InqB, which is only concerned with support, has been extended in previous work to a semantics that specifies reject conditions as well, with the aim to deal with the type of phenomena discussed here. The resulting framework is referred to as *radical inquisitive semantics*, InqR for short (Groenendijk and Roelofsen, 2010; Sano, 2012; Lojko, 2012; Aher, 2012, 2013).

Notice that the need to specify both support and reject conditions is independent from the need to have a notion of meaning that embodies both informative and inquisitive content. Depending on its given purposes, a logical framework may address either one of these needs, or both, or none. Inquisitive semantics is concerned with information exchange through conversation, which means that both considerations are relevant. But there are also several logical frameworks that address the first need while leaving inquisitive content out of consideration.[2] Some such frameworks, each quite closely related to InqR, are *data semantics* (Veltman, 1985), *game-theoretic semantics* and *independence friendly logic* (Hintikka and Sandu, 1997; Hodges, 1997), *dependence logic* (Väänänen, 2007), and *truth-maker semantics* (Van Fraassen, 1969; Fine, 2012).

2.4 Dismissing a supposition

The semantics to be developed in the present paper further extends the InqR framework, providing yet a more comprehensive characterization of the proposals that speakers make when uttering sentences in conversation. This further refinement is motivated by the observation that, besides compliant support and full-fledged rejection, there is, as we saw already in the introduction, yet another way to react to the conditionals in (5) and (6):

(8) Pete won't play the piano. $\neg p$

Suppose that A utters (5) and that B reacts with (8). One natural way to think about this response is as one that *dismisses a supposition* that A was making, namely the supposition 'that Pete will play the piano'. Similarly, if A utters the conditional question in (6), she can also be taken to make this supposition; and if B reacts with (8), she can again be taken to dismiss this supposition.

Clearly, the suppositions that a speaker makes in issuing a certain proposal, and responses that dismiss such suppositions, cannot be characterized purely in terms of the support conditions for that sentence.

2.5 From radical to suppositional

At first sight it may seem that suppositional phenomena *can* be captured if we have both support- and reject-conditions at our disposal. Indeed, an attempt to do so has been articulated in work on InqR (see in particular Groenendijk and Roelofsen, 2010, Sect. 3). There, states that dismiss a supposition of a sentence

[2] Instead of *support* and *reject*, the terms that are most commonly used in these frameworks are *verification* and *falsification*; however, these terms are not quite suitable when inquisitive content is taken into consideration.

φ are characterized as states that can be obtained by intersecting a state that supports φ with a state that rejects φ. Within the broader conceptual framework of InqR, such states can be thought of as ones that reject the *question behind* φ. If correct, this connection between support, rejection, and suppositional dismissal would show that there is no need to further refine the semantic machinery of InqR in order to deal with suppositional phenomena.

However, even though the given characterization works fine for simple cases like $p \rightarrow q$, it does not give satisfactory results for more complex cases. For instance, $\neg p$ is not predicted to dismiss a supposition of $(p \vee q) \rightarrow r$.

It is difficult to see how to avoid this and other problematic predictions by characterizing suppositional dismissal in terms of support and rejection in some other way. Thus, we will set out to develop a semantics in which the three notions are all characterized independently.

3 Suppositional inquisitive semantics

We will consider a propositional language $\mathcal{L}$, based on a finite set of atomic sentences $\mathcal{P}$. Complex sentences are built up using the usual connectives, $\neg$, $\wedge$, $\vee$, and $\rightarrow$, as well as an additional operator, ?. As in InqB, $?\varphi$ is defined as an abbreviation of $\varphi \vee \neg\varphi$ (the rationale behind this will become clear momentarily).

The basic ingredients of the semantics that we will develop for this language are *possible worlds*, which we take to be functions mapping every atomic sentence in $\mathcal{P}$ to a truth value, either 1 or 0, and *information states*, which we take to be sets of possible worlds. The set of all possible worlds is denoted as ω. For brevity, we will often simply talk about worlds and states instead of possible worlds and information states.

The semantics will consist in a simultaneous recursive definition of three notions:

$s \models^+ \varphi$ *s supports* φ
$s \models^- \varphi$ *s rejects* φ
$s \models^\circ \varphi$ *s dismisses a supposition of* φ

We will denote the set of all states that support a sentence φ as $[\varphi]^+$, and similarly for $[\varphi]^-$ and $[\varphi]^\circ$. The triple $\langle [\varphi]^+, [\varphi]^-, [\varphi]^\circ \rangle$ is called the *proposition* expressed by φ, and is denoted as $[\varphi]$. If two sentences φ and ψ express exactly the same proposition, they are said to be *equivalent*, notation $\varphi \equiv \psi$.

The rest of this section is organized as follows. In Sect. 3.1 we formulate four general postulates that propositions in InqS should satisfy. In Sect. 3.2-3.4 we discuss certain properties that propositions may have and some basic operations that may be performed on them. In Sect. 3.5 we provide a suppositional semantics for the Boolean fragment of our language, consisting of sentences that are built up using negation, disjunction, and conjunction. Finally, in Sect. 3.6 we present our suppositional treatment of implication, thereby extending the coverage of the semantics to the entire language $\mathcal{L}$. In a sense, this last subsection is where the real action takes place, but of course all the foregoing subsections are needed to prepare the ground.

3.1 General postulates

The first general postulate concerns persistence. Recall that in InqB support is taken to be persistent, which means that information growth cannot lead to retraction of support. However, as soon as suppositional dismissal is taken into account, this central feature of InqB is no longer defensible. For instance, we clearly want that the information state $|p \to q|$, consisting of all worlds where $p \to q$ is classically true, supports the sentence $p \to q$:

$$|p \to q| \models^{+} p \to q$$

However, we don't want that the information state $|\neg p|$ supports $p \to q$. Instead, this state should dismiss a supposition of the implication:

$$|\neg p| \models^{\circ} p \to q$$

But $|\neg p|$ is a substate of $|p \to q|$. So support cannot be persistent. Information growth can lead to suppositional dismissal, and thereby to retraction of support.

The same is true for rejection. For instance, $|p \to \neg q|$ should reject $p \to q$. However, we don't want that $|\neg p|$ rejects $p \to q$. Instead, as above, this state should dismiss a supposition of the implication. But $|\neg p|$ is a substate of $|p \to \neg q|$. So rejection cannot be persistent either. Since information growth can lead to suppositional dismissal, it can also lead to retraction of rejection.

Even though support and rejection cannot be assumed to be fully persistent in InqS, they can be assumed to be persistent *modulo dismissal of a supposition*. That is, if a state s supports a sentence φ, then any more informed state $t \subseteq s$ should either still support φ, or dismiss a supposition of φ. And similarly for rejection.

Finally, dismissal of a supposition should be fully persistent. If a state s dismisses a supposition of φ, then any more informed state $t \subseteq s$ should also dismiss a supposition of φ. There is no reason why information growth should lead to retraction of dismissal. These considerations lead to the following postulate.

Postulate 1

– *Support is persistent modulo dismissal of a supposition:*
  If $s \models^{+} \varphi$ *and* $t \subseteq s$*, then* $t \models^{+} \varphi$ *or* $t \models^{\circ} \varphi$
– *Rejection is persistent modulo dismissal of a supposition:*
  If $s \models^{-} \varphi$ *and* $t \subseteq s$*, then* $t \models^{-} \varphi$ *or* $t \models^{\circ} \varphi$
– *Dismissal of a supposition is fully persistent:*
  If $s \models^{\circ} \varphi$ *and* $t \subseteq s$*, then* $t \models^{\circ} \varphi$

The second postulate concerns the inconsistent state, $\emptyset$. It says that this state never supports or rejects a sentence, but always suppositionally dismisses it.

Postulate 2 *For any* φ*:* $\emptyset \not\models^{+} \varphi$ *and* $\emptyset \not\models^{-} \varphi$ *and* $\emptyset \models^{\circ} \varphi$

The third postulate says that support and rejection are mutually exclusive, i.e., a state can never support and reject a sentence at the same time.

Postulate 3 *For any* φ*:* $[\varphi]^+ \cap [\varphi]^- = \emptyset$

Note that we do not exclude the possibility that a state either supports or rejects a sentence and at the same time also dismisses a supposition of it. This option should indeed be left open. To see this, consider the following examples::

(9) a. Maria will go if Peter goes, or if Frank goes. $(p \to r) \vee (q \to r)$
b. Well, Peter isn't going, but indeed, if Frank goes, Maria will go as well. $\neg p \wedge (q \to r)$

(10) a. Maria will go if Peter goes, and if Frank goes. $(p \to r) \wedge (q \to r)$
b. Well no, Peter isn't going, and if Frank goes, Maria definitely won't. $\neg p \wedge (q \to \neg r)$

The response in (9b) supports (9a), but at the same time it also dismisses a supposition of it. Similarly, the response in (10b) rejects (10a), but again, it also dismisses a supposition of it.

Finally, the fourth postulate says that any consistent state of complete information, i.e., every information state consisting of a single possible world, should either support, or reject, or dismiss a supposition of any given sentence.

Postulate 4 *For any sentence* φ *and any world* w*:* $\{w\} \in ([\varphi]^+ \cup [\varphi]^- \cup [\varphi]^\circ)$

Together, the postulates imply that the three components of a proposition in InqS jointly always form a non-empty set of states S that is *downward closed*, i.e., for any $s \in S$ and $t \subseteq s$ we have that $t \in S$ as well.

Fact 1 *If all the postulates are satisfied, then for any* φ*,* $[\varphi]^+ \cup [\varphi]^- \cup [\varphi]^\circ$ *is a non-empty, downward closed set of states.*

In InqB, propositions are defined precisely as non-empty, downward closed sets of states. So InqS offers a more fine-grained notion of meaning than InqB in that it distinguishes three different meaning components; however, if we put these three meaning components together, we always obtain the same kind of semantic object that we had already in InqB. Thus, InqS is a refinement of InqB, but at the same time it retains one of its core features.

3.2 Informative content and alternatives

In uttering a sentence φ, a speaker proposes to establish a common ground that supports φ. Now suppose that w is a world that is not included in any state that supports φ. Then, any way of compliantly settling the given proposal will lead to a common ground that does not contain w. Thus, in uttering φ, a speaker proposes to exclude any world that is not in $\bigcup[\varphi]^+$ as a candidate for the actual world. In order words, she provides the information that the actual world must be contained in $\bigcup[\varphi]^+$. For this reason, we will refer to $\bigcup[\varphi]^+$ as the *informative content* of φ, and denote it as $\mathsf{info}(\varphi)$.

Definition 1 (Informative content). $\mathsf{info}(\varphi) := \bigcup[\varphi]^+$

Among the states that support a sentence φ, some are easier to reach, so to speak, than others. Suppose for instance, that s and t are two states that support φ, and that $t \subset s$. Establishing either s or t as the new common ground would be sufficient to compliantly settle the proposal expressed by φ. However, it is easier to establish s then it is to establish t, because this would require the elimination of fewer possible worlds, i.e., less information.

From this perspective, those states that support φ and are not contained in any other state that support φ have a special status. Namely, they are the *weakest* states supporting φ—states that support φ with a *minimal* amount of information. We will refer to such states as the *support-alternatives* for φ, and denote the set of all support-alternatives for φ as $\mathsf{alt}^+(\varphi)$. Similarly, we will refer to the weakest states that reject φ as the *reject-alternatives* for φ, and denote the sets of all these states as $\mathsf{alt}^-(\varphi)$.

Definition 2 (Alternatives).

- $\mathsf{alt}^+(\varphi) := \{s \mid s \models^+ \varphi$ *and there is no* $t \supset s$ *such that* $t \models^+ \varphi\}$
- $\mathsf{alt}^-(\varphi) := \{s \mid s \models^- \varphi$ *and there is no* $t \supset s$ *such that* $t \models^- \varphi\}$

In principle, it may be that there are no support-alternatives for a sentence φ, even if the sentence is supported by one or more states. To see this, consider an infinite sequence of states $s_0 \subseteq s_1 \subseteq s_2 \subseteq \ldots$, such that every state s_i is properly contained in the next, and all states in the sequence support φ. Then for every state s_i there are infinitely many weaker states that still support φ. So neither of these states counts as an alternative for φ.

However, in the current setting, where we consider a propositional language based on a finite set of atomic sentences, the set of all possible worlds is finite, and therefore the set of all states is also finite. This means that infinite sequences of the kind considered above do not exist. As a result, in this particular setting, every state that supports a sentence φ is included in an alternative for φ, and similarly for states that reject or dismiss a supposition of φ.

Fact 2 (Alternatives) *If the set of all possible worlds is finite, we have that:*

- *Every* $s \in [\varphi]^+$ *is contained in some* $\alpha \in \mathsf{alt}^+(\varphi)$
- *Every* $s \in [\varphi]^-$ *is contained in some* $\alpha \in \mathsf{alt}^-(\varphi)$

We will quite heavily rely on this fact in formulating and explaining the semantics, in particular the clause for implication, because certain notions become more transparent when explicated in terms of alternatives. However, we will of course also show how the semantics can be lifted to the more general setting where the existence of alternatives cannot be guaranteed.

3.3 Reversal and thematization

There are several natural operations that can be performed on propositions in InqS. One of these operations, which we refer to as *reversal*, is to swap the $+$ and $-$ components of a proposition around. We denote the reversal of a proposition $[\varphi]$ as $[\varphi]^*$. As we will see, for any sentence φ in our logical language, $[\varphi]^*$ is the proposition expressed by $\neg\varphi$.

Definition 3 (Reversal). *For any* φ: $[\varphi]^* := \langle [\varphi]^-, [\varphi]^+, [\varphi]^\circ \rangle$

Another natural operation, which we will refer to as *thematization*, is one that transfers every state in the $-$ component of a proposition to the $+$ component. We denote the thematization of a proposition $[\varphi]$ as $[\varphi]^?$. As we will see, for any sentence φ in our logical language, $[\varphi]^?$ is the proposition expressed by $?\varphi$.

Definition 4 (Thematization). *For any* φ: $[\varphi]^? := \langle [\varphi]^+ \cup [\varphi]^-, \emptyset, [\varphi]^\circ \rangle$

Notice that both reversal and thematization respect the postulates given above. That is, if $[\varphi]$ satisfies all the postulates, then $[\varphi]^*$ and $[\varphi]^?$ will do so as well.

3.4 Informative, inquisitive, and suppositional sentences

We will say that a sentence φ is *informative* just in case it has the potential to provide information, i.e., if $\mathsf{info}(\varphi) \neq \omega$. We will say that φ is *inquisitive* just in case (i) there is at least one state that supports φ, and (ii) in order to establish such a state as the new common ground it does not suffice for other conversational participants to simply accept $\mathsf{info}(\varphi)$. The latter holds if and only if $\mathsf{info}(\varphi)$ does not support φ, i.e., $\mathsf{info}(\varphi) \notin [\varphi]^+$. Finally, we will say that φ is *suppositional* just in case there is at least one consistent state that dismisses a supposition of φ, which means that $[\varphi]^\circ \neq \{\emptyset\}$.

Definition 5 (Informative, inquisitive and suppositional sentences).

- φ *is informative iff* $\mathsf{info}(\varphi) \neq \omega$
- φ *is inquisitive iff* $[\varphi]^+ \neq \emptyset$ *and* $\mathsf{info}(\varphi) \not\models^+ \varphi$
- φ *is suppositional iff* $[\varphi]^\circ \neq \{\emptyset\}$

If there are two or more alternatives for a sentence, then that sentence has to be inquisitive. After all, if φ is not inquisitive, then $\mathsf{info}(\varphi)$, which amounts to $\bigcup[\varphi]^+$, supports φ. But this means that $\bigcup[\varphi]^+$ is the unique alternative for φ, which contradicts the assumption that there are two or more alternatives for φ.

Fact 3 (Alternatives and inquisitiveness)

- *If* $\mathsf{alt}^+(\varphi)$ *has two or more elements, then* φ *is inquisitive.*

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Now consider the other direction. If φ is inquisitive, does that mean that $\mathsf{alt}^+(\varphi)$ has to contain two or more alternatives? Not necessarily. To see this, consider again an infinite sequence of states, $s_0 \subset s_1 \subset s_2 \subset \ldots$, and suppose that these are the only states that support φ. Then it is not the case that $\bigcup[\varphi]^+ \in [\varphi]^+$, so φ is inquisitive. And yet, there is not a single alternative for φ.

In our present setting, the set of all possible worlds is assumed to be finite, and no infinite state sequences can be constructed. This means that if φ is inquisitive, i.e., if $\bigcup[\varphi]^+ \notin [\varphi]^+$, there must be at least two states $s, t \in [\varphi]^+$ such that $s \cup t \notin [\varphi]^+$. But then, by Fact 2, there must be at least two alternatives for φ, one containing s, one containing t, and neither of them containing $s \cup t$. So in this particular setting, the converse of Fact 3 also holds.

Fact 4 (Alternatives and inquisitiveness in a finite setting)
If the set of all possible worlds is finite, then:

- *φ is inquisitive if and only if $\mathsf{alt}^+(\varphi)$ has two or more elements.*

With these basic notions and facts in place, we now turn to the clauses of InqS.

3.5 Atomic sentences and the Boolean connectives

We first consider the Boolean fragment of our language, which consists of all sentences that can be formed by means of the Boolean connectives, i.e., negation, conjunction, and disjunction. We denote this fragment of the language as $\mathcal{L}_B$. After considering $\mathcal{L}_B$, we will turn to implication. As the reader may expect, the clause for implication will be more intricate than those for the Boolean connectives, and several aspects of it will deserve some careful consideration.

The clauses for $\mathcal{L}_B$ are given in Def. 6 below. After laying out the definition, we will describe informally what each of the clauses amounts to.

Definition 6 (Atomic sentences and Boolean connectives).

1. $s \models^+ p$ *iff* $s \neq \emptyset$ *and* $\forall w \in s\colon w(p) = 1$
   $s \models^- p$ *iff* $s \neq \emptyset$ *and* $\forall w \in s\colon w(p) = 0$
   $s \models^\circ p$ *iff* $s = \emptyset$

2. $s \models^+ \neg\varphi$ *iff* $s \models^- \varphi$
   $s \models^- \neg\varphi$ *iff* $s \models^+ \varphi$
   $s \models^\circ \neg\varphi$ *iff* $s \models^\circ \varphi$

3. $s \models^+ \varphi \wedge \psi$ *iff* $s \models^+ \varphi$ *and* $s \models^+ \psi$
   $s \models^- \varphi \wedge \psi$ *iff* $s \models^- \varphi$ *or* $s \models^- \psi$
   $s \models^\circ \varphi \wedge \psi$ *iff* $s \models^\circ \varphi$ *or* $s \models^\circ \psi$

4. $s \models^+ \varphi \vee \psi$ *iff* $s \models^+ \varphi$ *or* $s \models^+ \psi$
   $s \models^- \varphi \vee \psi$ *iff* $s \models^- \varphi$ *and* $s \models^- \psi$
   $s \models^\circ \varphi \vee \psi$ *iff* $s \models^\circ \varphi$ *or* $s \models^\circ \psi$

Atomic sentences. A state s supports an atomic sentence p just in case s is consistent and p is true in all worlds in s. Similarly, s rejects p just in case s is consistent and p is false in all worlds in s. Finally, s dismisses a supposition of p if s is inconsistent. The idea behind the latter clause is that in uttering p, a speaker makes the trivial supposition that p may or may not be the case—a supposition that is dismissed only by the absurd, inconsistent state.

Negation. A state s supports $\neg\varphi$ just in case it rejects φ. Vice versa, it rejects $\neg\varphi$ just in case it supports φ. Finally, it dismisses a supposition of $\neg\varphi$ just in case it dismisses a supposition of φ. Thus, $\neg\varphi$ straightforwardly inherits the suppositional content of φ. Notice that, as anticipated above, $\neg\varphi$ always expresses the reversal of $[\varphi]$.

Fact 5 (Negation and reversal) *For any* φ, $[\neg\varphi] = [\varphi]^*$

As a consequence, $\neg\neg\varphi$ always expresses the same proposition as φ itself.

Fact 6 (Double negation) *For any* φ, $[\neg\neg\varphi] = [\varphi]$

Conjunction. A state s supports $\varphi \wedge \psi$ just in case it supports both φ and ψ, and it rejects $\varphi \wedge \psi$ just in case it rejects either φ or ψ. Finally, s dismisses a supposition of $\varphi \wedge \psi$ just in case it dismisses a supposition of φ or dismisses a supposition of ψ. Thus, $\varphi \wedge \psi$ inherits the suppositional content of φ and ψ in a straightforward, cumulative way.

Disjunction. A state s supports $\varphi \vee \psi$ just in case it supports either φ or ψ, and it rejects $\varphi \vee \psi$ just in case it rejects both φ and ψ. Finally, s dismisses a supposition of $\varphi \vee \psi$ just in case it dismisses a supposition of φ or dismisses a supposition of ψ. Thus, again, $\varphi \vee \psi$ inherits the suppositional content of φ and ψ in a straightforward, cumulative way.

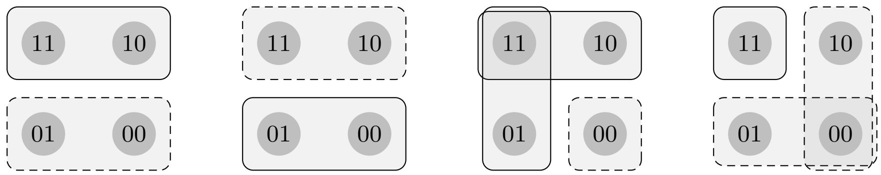
Notice that the Boolean connectives satisfy De Morgan's laws, which means that conjunction and disjunction are interdefinable by means of negation.

Fact 7 (De Morgan's laws) *For any* φ*:*

$$\varphi \wedge \psi \equiv \neg(\neg\varphi \vee \neg\psi)$$
$$\varphi \vee \psi \equiv \neg(\neg\varphi \wedge \neg\psi)$$

For every sentence $\varphi \in \mathcal{L}_B$, the informative content of φ in InqS, i.e., $\bigcup[\varphi]^+$, coincides precisely with the proposition that φ expresses in classical propositional logic (CPL). So, as far as its treatment of sentences in $\mathcal{L}_B$ is concerned, InqS is a conservative refinement of classical logic. That is, the two fully agree on the informative content of every sentence in $\mathcal{L}_B$. Only, classical logic *identifies* the meaning of a sentence with its informative content, whereas InqS has a more fine-grained notion of meaning.

Fact 8 (Conservative refinement of CPL) *For any* $\varphi \in \mathcal{L}_B$, $\mathsf{info}(\varphi) = |\varphi|$



(a) p (b) $\neg p$ (c) $p \vee q$ (d) $p \wedge q$

Fig. 2. The propositions expressed by some basic sentences.

Another conservative feature of InqS is that no sentence in $\mathcal{L}_B$ is suppositional.

Fact 9 (No suppositionality) *For any* $\varphi \in \mathcal{L}_B$, $[\varphi]^\circ = \{\emptyset\}$

Finally, it is not only the case that no sentence $\varphi \in \mathcal{L}_B$ is both supported and rejected by the same state, as is mandated by our general postulates, but also that for every $\varphi \in \mathcal{L}_B$, $\bigcup[\varphi]^+$ and $\bigcup[\varphi]^-$ are each other's set-theoretic complement, which means in particular that a state that supports φ can never have any overlap with a state that rejects φ.

Fact 10 (No overlap) *For any* $\varphi \in \mathcal{L}_B$, $\bigcup[\varphi]^+ = \omega - \bigcup[\varphi]^-$

The last two facts make it possible to visualize the propositions expressed by sentences in $\mathcal{L}_B$ in a particularly perspicuous way. This is done in Fig. 2 for some simple sentences. In this figure, as before, 11 is a world where both p and q are true, 10 a world where p is true and q is false, etcetera. The support- and reject-alternatives for each sentence are depicted with solid and dashed borders, respectively. Notice in particular that Fig. 2(c) immediately reveals that $p \vee q$ is inquisitive, since there are two support-alternatives for this sentence.

Recall that $?\varphi$ is defined as an abbreviation of $\varphi \vee \neg\varphi$. Thus, having spelled out the clauses for disjunction and negation, we can now derive the interpretation of $?\varphi$ as well. First, a state supports $?\varphi$ iff it supports either φ or $\neg\varphi$. So $[?\varphi]^+ = [\varphi]^+ \cup [\neg\varphi]^+ = [\varphi]^+ \cup [\varphi]^-$. Second, a state rejects $?\varphi$ iff it rejects both φ or $\neg\varphi$. But to reject $\neg\varphi$ is to support φ. Thus, in order to reject $?\varphi$, a state would have to support φ and reject φ at the same time. This is impossible, in view of one of our general postulates. So, for any φ, $[?\varphi]^-$ will be empty. Finally, a state dismisses a supposition of $?\varphi$ iff it dismisses a supposition of φ or of $\neg\varphi$, and the latter occurs just in case the state dimisses a supposition of φ itself. So, $[?\varphi]^\circ = [\varphi]^\circ$. This leads us to the conclusion that, for any φ, $[?\varphi] = \langle [\varphi]^+ \cup [\varphi]^-, \emptyset, [\varphi]^\circ \rangle$. But the latter amounts precisely to the thematization $[\varphi]^?$ of $[\varphi]$. So we have:

Fact 11 (Thematization) *For any* φ, $[?\varphi] = [\varphi]^?$

3.6 Implication

We now turn to implication, which typically introduces non-trivial suppositional content. The initial idea is that, for a state s to either support or reject an implication $\varphi \to \psi$, it is a necessary requirement that the antecedent φ be *supposable* in s. If this is not the case, then s suppositionally dismisses the implication, and does not support or reject it.

The key question, then, is what it means exactly for φ to be supposable in s. To answer this question, we will consider a number of concrete examples. We will start with the simplest case, and gradually consider more complex ones. As we proceed, our notion of supposability and the semantics for implication that is defined in terms of it will become more and more refined. Consider first an implication with an atomic antecedent and an atomic consequent:

(11) $\quad p \to r$

It would be natural to say that p is supposable in a state s iff the single support-alternative for p, $|p|$, is consistent with s, i.e., $s \cap |p| \neq \emptyset$. Furthermore, it would be natural to say that if this condition is met, s supports the implication iff $s \cap |p|$ supports r, and s rejects the implication iff $s \cap |p|$ rejects r. However, this characterization of supposability only applies if there is a *unique* support-alternative for the antecedent. To see how it may be generalized, let us consider an example in which there are *two* support-alternatives for the antecedent:

(12) $\quad (p \vee q) \to r$

To deal with such cases, as well as the simpler cases where there is a single support-alternative for the antecedent, it seems reasonable to say that, in general, the antecedent is supposable in s iff *every* support-alternative for it is consistent with s:

(13) $\quad \varphi$ is supposable in s, notation $s \lhd \varphi$, iff $\forall \alpha \in \mathsf{alt}^+(\varphi) : s \cap \alpha \neq \emptyset$

With this characterization of supposability in place, we may formulate the clauses for implication as follows:

$$s \models^+ \varphi \to \psi \quad \text{iff} \quad s \lhd \varphi \text{ and } \forall \alpha \in \mathsf{alt}^+(\varphi)\text{: } s \cap \alpha \models^+ \psi$$

$$s \models^- \varphi \to \psi \quad \text{iff} \quad s \lhd \varphi \text{ and } \exists \alpha \in \mathsf{alt}^+(\varphi)\text{: } s \cap \alpha \models^- \psi$$

$$s \models^\circ \varphi \to \psi \quad \text{iff} \quad s \not\lhd \varphi$$

However, this formulation of the clauses is problematic in several ways. One problem is that the given conditions for rejecting an implication are too stringent. To see this, consider the following state:

(14) $\quad s := |\neg p \wedge (q \to \neg r)|$

This state is inconsistent with one of the support-alternatives for the antecedent of (12), namely $|p|$. However, it is consistent with the other support-alternative, $|q|$, and if we intersect it with this alternative we get at the state $|\neg p \wedge \neg r|$, which rejects the consequent of the implication, r. So, on the one hand, not every support-alternative for the antecedent is consistent with s, and we want our semantics to capture this by characterizing s as dismissing a supposition of the implication; on the other hand, however, one of the support-alternatives for the antecedent *is* consistent with s, and restricting s to this alternative leads to rejection of the consequent. We want our semantics to capture this as well, by characterizing s as a state that rejects the implication as a whole (besides dismissing a supposition of it).

The general upshot of this example is that the idea that we started out with, namely that supposability of the antecedent as a whole is a necessary requirement for a state to support or reject an implication, is not exactly on the right track. In particular, it is too stringent in the case of rejection.

Rather than considering the supposability of the antecedent as a whole, it seems more suitable to consider the supposability of each support-alternative for the antecedent separately. Let us say, for now, that an alternative α is supposable in a state s just in case the two are consistent with each other:

(15) An alternative α is supposable in a state s, notation $s \lhd \alpha$, iff $s \cap \alpha \neq \emptyset$.

Then we arrive at the following revised formulation of the clauses for implication:

$$s \models^{+} \varphi \to \psi \quad \text{iff} \quad \forall \alpha \in \mathsf{alt}[\varphi]^{+}\text{: } s \lhd \alpha \text{ and } s \cap \alpha \models^{+} \psi$$

$$s \models^{-} \varphi \to \psi \quad \text{iff} \quad \exists \alpha \in \mathsf{alt}[\varphi]^{+}\text{: } s \lhd \alpha \text{ and } s \cap \alpha \models^{-} \psi$$

$$s \models^{\circ} \varphi \to \psi \quad \text{iff} \quad \exists \alpha \in \mathsf{alt}[\varphi]^{+}\text{: } s \ntriangleleft \alpha$$

This formulation, however, still needs some further refinement. First, consider a case in which there are no support-alternatives for the antecedent at all:

(16) $(p \wedge \neg p) \to r$

According to the clauses above, this implication is trivially supported by any state, because the clause for support quantifies universally over the support-alternatives for the antecedent, which in this case do not exist. On the other hand, according to the given clauses, there is no state that dismisses a supposition of the implication, because this requires inconsistency with some support-alternative for the antecedent, of which there are none. We want exactly the oppositive result: no state should support this implication, and every state should dismiss a supposition of it. Thus, the clauses should be adapted: support should require a non-empty set of support-alternatives for the antecedent, while dismissal of a supposition should occur if this set is empty. This leads us to the formulation below. For uniformity, we have adapted the rejection clause as well, although this is strictly speaking redundant; the new, redundant part of the clause is displayed in gray.

$$s \models^{+} \varphi \to \psi \quad \text{iff} \quad \mathsf{alt}^{+}(\varphi) \neq \emptyset \text{ and } \forall \alpha \in \mathsf{alt}^{+}(\varphi)\text{: } s \lhd \alpha \text{ and } s \cap \alpha \models^{+} \psi$$

$$s \models^{-} \varphi \to \psi \quad \text{iff} \quad \mathsf{alt}^{+}(\varphi) \neq \emptyset \text{ and } \exists \alpha \in \mathsf{alt}^{+}(\varphi)\text{: } s \lhd \alpha \text{ and } s \cap \alpha \models^{-} \psi$$

$$s \models^{\circ} \varphi \to \psi \quad \text{iff} \quad \mathsf{alt}^{+}(\varphi) = \emptyset \text{ or } \exists \alpha \in \mathsf{alt}^{+}(\varphi)\text{: } s \ntriangleleft \alpha$$

This formulation is appropriate as long as φ and ψ are non-suppositional, i.e., as long as they do not contain any implications themselves. However, to deal with nested implications, some further refinements are needed.

First consider a case where the consequent is suppositional, which will be relatively easy to accommodate.

(17) $p \to (q \to r)$

Consider the following state:

(18) $s := |p \to \neg q|$

The semantics should predict that this state dismisses a supposition of (17), because if we restrict it to the unique support-alternative for the antecedent, $|p|$, we arrive at the state $|\neg q|$, and this state dismisses a supposition of the consequent, $q \to r$. However, this is not captured by the clause for dismissal given above, which requires that there is a support-alternative for the antecedent that is inconsistent with s. This is clearly not the case here. So the clause needs to be adapted, and there is a natural way to do so: in order for s to dismiss a supposition of $\varphi \to \psi$ it should be the case that $\mathsf{alt}^+(\varphi)$ is empty, or that it contains an alternative that is not supposable in s, or that it contains an alternative α which is such that $s \cap \alpha$ dismisses a supposition of the consequent. Notice that, w.r.t. the previous formulation, the first two conditions are old, and the third one is newly added. Moreover, notice that whenever the consequent of the implication is non-suppositional, the second and the third requirement coincide, demanding that $s \cap \alpha$ be consistent. Leaving the support and reject clauses unchanged, we arrive at the following formulation:

$$s \models^+ \varphi \to \psi \quad \text{iff} \quad \mathsf{alt}^+(\varphi) \neq \emptyset \text{ and } \forall \alpha \in \mathsf{alt}^+(\varphi)\text{: } s \lhd \alpha \text{ and } s \cap \alpha \models^+ \psi$$

$$s \models^- \varphi \to \psi \quad \text{iff} \quad \mathsf{alt}^+(\varphi) \neq \emptyset \text{ and } \exists \alpha \in \mathsf{alt}^+(\varphi)\text{: } s \lhd \alpha \text{ and } s \cap \alpha \models^- \psi$$

$$s \models^\circ \varphi \to \psi \quad \text{iff} \quad \mathsf{alt}^+(\varphi) = \emptyset \text{ or } \quad \exists \alpha \in \mathsf{alt}^+(\varphi)\text{: } s \ntriangleleft \alpha \text{ or } \quad s \cap \alpha \models^\circ \psi$$

There is one more amendment to make, in order to deal with cases where the antecedent of the implication is itself suppositional. We will do this in two steps, again first considering the simplest case and then a more complex one. Consider first:

(19) $(p \to q) \to r$

Suppose that our state of evaluation is the following:

(20) $s := |\neg p \wedge r|$

According to the clauses as formulated above, this state supports the implication in (19), because there is a single support-alternative for the antecedent, $\alpha := |p \to q|$, which is consistent with s, and the intersection of s with α amounts to s itself, which supports the consequent, r. Moreover, the clauses do not characterize s

as a state that dismisses a supposition of (19), because $s \cap \alpha$ is consistent and does not dismiss a supposition of the consequent.

Again, we want precisely the opposite result: s should be characterized as dismissing a supposition of the implication, and not as supporting it. The culprit for this is our notion of supposability of support-alternatives. According to (15), a support-alternative α for a sentence φ is supposable in a state s iff $s \cap \alpha \neq \emptyset$. This is fine as long as φ is non-suppositional, which means that support is persistent and thus that for every support-alternative α, every consistent substate of α will still support φ. This will hold in particular for $s \cap \alpha$, given that it is consistent. However, if φ is suppositional this is no longer guaranteed, as witnessed by the example above where $\alpha := |p \to q|$ is the unique support-alternative for the antecedent, $p \to q$, but support does not persist in all substates of α; in particular, $s \cap \alpha$ no longer supports $p \to q$.

Our notion of supposability should be made sensitive to this, and the above example suggests how this should be done. Namely, we should not just require that $s \cap \alpha$ be consistent, but rather that in going from α to $s \cap \alpha$, support of φ be preserved:

(21) A support-alternative α for a sentence φ is supposable in a state s, notation $s \lhd \alpha$, iff $s \cap \alpha \models^{+} \varphi$.

With this refined notion of supposability in place, the clauses for implication can remain as they were formulated above. Examples like (19), with a suppositional, but non-inquisitive antecedent, are now suitably dealt with.

The final, most complex case to consider is one in which the antecedent is both suppositional and inquisitive, which means that it has multiple support-alternatives. Take the following example:

(22) $((p \to q) \vee l) \to r$

Notice that the antecedent is a disjunction, whose first disjunct is suppositional. Consider a state that dismisses the first disjunct, but supports the second, and moreover, supports the consequent of the implication:

(23) $s := |\neg p \wedge l \wedge r|$

According to the clauses as formulated above, this state supports the implication in (22). Let us see why this is the case. First, there are two support-alternatives for the antecedent, $|p \to q|$ and $|l|$. Intersecting s with either of these alternatives simply yields s, which supports the antecedent, so both support-alternatives for the antecedent are supposable. Moreover, the intersection of s with either of the support-alternatives for the antecedent also supports the consequent, r. Therefore, s supports the implication as a whole as well.

The clauses also characterize s as a state that does not dismiss any supposition of the implication in (22). This is because the intersection of s with either of the two support-alternatives for the antecedent is just s, and as we already saw, s supports the antecedent.

These are not the right results: we want the semantics to characterize s as a state that dismisses a supposition of the implication, and does not support it. The culprit for this is again our notion of supposability of support-alternatives. The idea is that a support-alternative α for φ is supposable in s iff in going from α to $s \cap \alpha$, *support of φ is preserved*. Formally, we require that $s \cap \alpha$ still supports φ.

But now look at the example again. There are two support-alternatives for the antecedent, corresponding to the two disjuncts, $|p \to q|$ and $|l|$. Let us focus on the first. Intersecting s with this alternative simply yields s, which supports the antecedent of the implication. Crucially, however, this is because it supports the *second* disjunct, l. It does not support the first disjunct, the one that corresponds to the support-alternative that we are considering. And, upon closer examination, there is a clear sense in which support is not fully *preserved* in going from $|p \to q|$ to s. Namely, there are states between $|p \to q|$ and s, such as $|\neg p|$, which do not support the antecedent. Only when we further strengthen these states in such a way that they come to support the second disjunct, do they come to support the antecedent as a whole. From this perspective, it is not right to say that support is preserved in going from $|p \to q|$ to s. It is true that we have support at s, but only after it was lost somewhere along the way. These considerations lead to the following, definitive, characterization of supposability of support-alternatives.

Definition 7 (Supposability of support-alternatives).
A support-alternative α for a sentence φ is supposable in a state s, notation $s \lhd \alpha$, iff for every state t between α and $s \cap \alpha$, i.e., every t such that $\alpha \supseteq t \supseteq (s \cap \alpha)$, we have that $t \models^+ \varphi$.

With this refined notion of supposability in place, the clauses for implication can remain as formulated above. For convenience we restate them here in an official definition.[3]

Definition 8 (Implication).

[3] Recall from our discussion in Sect. 3.2 that the fact that we are considering a propositional language based on a finite set of atomic sentences is crucial in ensuring that every state that supports a sentence is contained in an alternative for that sentence, which in turn justifies our formulation of the clauses for implication in terms of alternatives. However, this cannot always be ensured. For instance, if we consider a first-order language with an infinite domain of interpretation, the existence of alternatives can no longer be guaranteed (Ciardelli, 2010). Fortunately, there is a way to formulate the clauses for implication that does not make reference to alternatives, and which in the current setting, is equivalent to the clauses as formulated in Def. 8:

$$s \models^+ \varphi \to \psi \text{ iff } [\varphi]^+ \neq \emptyset \text{ and } \forall t \in [\varphi]^+ \exists u \supseteq t \in [\varphi]^+\text{: } s \lhd u \text{ and } s \cap u \models^+ \psi$$

$$s \models^- \varphi \to \psi \text{ iff } [\varphi]^+ \neq \emptyset \text{ and } \exists t \in [\varphi]^+ \forall u \supseteq t \in [\varphi]^+\text{: } s \lhd u \text{ and } s \cap u \models^- \psi$$

$$s \models^\circ \varphi \to \psi \text{ iff } [\varphi]^+ = \emptyset \text{ or } \exists t \in [\varphi]^+ \forall u \supseteq t \in [\varphi]^+\text{: } s \not\lhd u \text{ or } s \cap u \models^\circ \psi$$

$$s \models^{+} \varphi \to \psi \quad \textit{iff} \quad \mathsf{alt}^{+}(\varphi) \neq \emptyset \textit{ and } \forall \alpha \in \mathsf{alt}^{+}(\varphi)\textit{: } s \lhd \alpha \textit{ and } s \cap \alpha \models^{+} \psi$$

$$s \models^{-} \varphi \to \psi \quad \textit{iff} \quad \mathsf{alt}^{+}(\varphi) \neq \emptyset \textit{ and } \exists \alpha \in \mathsf{alt}^{+}(\varphi)\textit{: } s \lhd \alpha \textit{ and } s \cap \alpha \models^{-} \psi$$

$$s \models^{\circ} \varphi \to \psi \quad \textit{iff} \quad \mathsf{alt}^{+}(\varphi) = \emptyset \textit{ or } \exists \alpha \in \mathsf{alt}^{+}(\varphi)\textit{: } s \not\lhd \alpha \textit{ or } s \cap \alpha \models^{\circ} \psi$$

This completes our suppositional semantics for the full propositional language $\mathcal{L}$.

It can be shown by induction that this semantics respects the four general postulates that were formulated at the outset.

Fact 12 (Suitability of the semantics)
For any $\varphi \in \mathcal{L}$, $[\varphi]$ is a proposition that satisfies the four general postulates.

However, when taking implication into consideration, InqS diverges much more radically from CPL than it does when we restrict our attention to just the Boolean fragment. In particular, Facts 8 (conservative refinement), 9 (no suppositionality), and 10 (no overlap) no longer hold when implication is taken into account, which can all be shown with a single example, $\neg(p \to q)$. We have that $\mathsf{info}(\neg(p \to q)) = |p \to \neg q|$ which differs from the proposition expressed by $\neg(p \to q)$ in CPL. Furthermore, $\neg(p \to q)$ is suppositional, and it has supporting and rejecting states that overlap, for instance $|p \to \neg q|$ and $|p \to q|$, respectively.

One 'classical' property that InqS *does* preserve, even when implication is taken into consideration, is that whenever a state s supports a sentence φ, then no substate $t \subseteq s$ rejects φ, and vice versa, whenever s rejects φ, no substate $t \subseteq s$ supports φ. In the terminology of Veltman (1985), this means that every sentence in our language is *stable*.

Fact 13 (Stability) *For any $\varphi \in \mathcal{L}$ and any state s:*

- *If s supports φ then no $t \subseteq s$ rejects φ*
- *If s rejects φ then no $t \subseteq s$ supports φ*

Veltman introduced the notion of stability in his work on *data semantics*, which, like InqS, is concerned in particular with conditionals and epistemic modals. Veltman emphasizes that in data semantics, both conditionals and epistemic modals are typically unstable, unlike sentences that do not contain modals or conditionals. In InqS, it is still the case that sentences involving epistemic modals are typically unstable (see Aher et al., 2014). However, all sentences in the propositional language considered here, including conditionals, are stable.

Notice that the general postulates that we started out with do in principle allow for unstability. This is exactly because stability can no longer be guaranteed if we extend our logical language with epistemic modalities.

3.7 Reducing the clauses for implication

In motivating the clauses for implication, we started from simple examples, where the supposability of a support-alternative α for φ in s requires no more than that $\alpha \cap s \neq \emptyset$. Then we looked at examples with suppositional antecedents.

In these cases, we found that supposability of a support-alternative α for φ in s does not only require that $\alpha \cap s \neq \emptyset$, but something stronger, namely that $\alpha \cap s \models^+ \varphi$. Finally, we looked at even more involved cases, with antecedents that were both suppositional and inquisitive. These cases show that the requirement for supposability of a support-alternative α for φ in s has to be even stronger, namely that for all t between α and $\alpha \cap s$, it should be the case that $t \models^+ \varphi$.

In this section we move in the opposite direction. Starting out from the general notion of supposability as just described, we single out a specific semantic property such that if a sentence has that property, then for every support-alternative α for φ: $\alpha \lhd s$ iff $\alpha \cap s \models^+ \varphi$. Then we determine a second stronger property such that for every support-alternative α for φ: $\alpha \lhd s$ iff $\alpha \cap s \neq \emptyset$.

A property that obviously allows us to ignore states in between a support-alternative α for φ and its restriction to s, is *convexity* of $[\varphi]^+$.

Definition 9 (Convexity of support and rejection).

- φ *is* support-convex *iff* $\forall s, t, u$: *if* $s, t \in [\varphi]^+$ *and* $s \subset u \subset t$, *then* $u \in [\varphi]^+$
- φ *is* reject-convex *iff* $\forall s, t, u$: *if* $s, t \in [\varphi]^-$ *and* $s \subset u \subset t$, *then* $u \in [\varphi]^-$

The following fact tells us that when φ is support-convex, we can simplify the semantic clauses $\varphi \to \psi$, by unpacking $s \lhd \alpha$ as requiring $\alpha \cap s \models^+ \varphi$.

Fact 14 (Convexity reduction of supposability)

- *If φ is support-convex, and $\alpha \in \mathsf{alt}^+(\varphi)$, then $s \lhd \alpha$ iff $\alpha \cap s \models^+ \varphi$.*
- *If φ is rejection-convex, and $\alpha \in \mathsf{alt}^-(\varphi)$, then $s \lhd \alpha$ iff $\alpha \cap s \models^- \varphi$.*

The stronger property of *density* of $[\varphi]^+$ allows us to further reduce supposability to consistency. Support-density of φ means that there are no 'holes' in $[\varphi]^+$. And similarly for reject-density.

Definition 10 (Densitity of support and rejection).

- φ *is* support-dense *iff* $\forall s, t$: *if* $s \in [\varphi]^+$ *and* $t \subset s$ *and* $t \neq \emptyset$, *then* $t \in [\varphi]^+$
- φ *is* reject-dense *iff* $\forall s, t$: *if* $s \in [\varphi]^-$ *and* $t \subset s$ *and* $t \neq \emptyset$, *then* $t \in [\varphi]^-$

If φ is not suppositional, then φ is both support- and reject-dense.

Fact 15 (Not suppositional implies dense)

- *If φ is not suppositional, then φ is support-dense and reject-dense.*

However, the opposite does not hold. An example of a sentence that is support- and reject-dense but also suppositional is $\neg p \vee (p \to q)$.

The following fact tells us that when φ is support-dense we can rewrite the semantic clauses for $\varphi \to \psi$ in a way that they do not explicitly refer to supposability anymore.

Fact 16 (Density reduction of supposability)

– *If* φ *is support-dense and* $\alpha \in \mathsf{alt}^{+}(\varphi)$*, then* $s \lhd \alpha$ *iff* $\alpha \cap s \neq \emptyset$.

For instance, if φ is support-dense, the support clause of $\varphi \to \psi$ amounts to:

(24) $s \models^{+} \varphi \to \psi$ iff $\mathsf{alt}^{+}(\varphi) \neq \emptyset$ and $\forall \alpha \in \mathsf{alt}^{+}(\varphi)\colon\ \alpha \cap s \neq \emptyset$ and $\alpha \cap s \models^{+} \psi$

But since, generally, the inconsistent state does not support (or reject) any sentence φ, it is already implied by $\alpha \cap s \models^{+} \psi$ that $\alpha \cap s \neq \emptyset$. So, instead of (24) we can more economically write (25):

(25) $s \models^{+} \varphi \to \psi$ iff $\mathsf{alt}^{+}(\varphi) \neq \emptyset$ and $\forall \alpha \in \mathsf{alt}^{+}(\varphi)\colon\ \alpha \cap s \models^{+} \psi$

Independently of the reductions considered above, when φ is not inquisitive, we can eliminate quantification over support-alternatives α for φ, because in this case, if there is an alternative for φ at all, it is $\mathsf{info}(\varphi)$ and $\mathsf{info}(\varphi) \neq \emptyset$. So, if φ is not inquisitive, (25) boils down to the support clause in (26), where we also provide the two other clauses, for the case where φ is not inquisitive and support-dense.

(26)
$$s \models^{+} \varphi \to \psi \text{ iff } \mathsf{info}(\varphi) \neq \emptyset \text{ and } \mathsf{info}(\varphi) \cap s \models^{+} \psi$$
$$s \models^{-} \varphi \to \psi \text{ iff } \mathsf{info}(\varphi) \neq \emptyset \text{ and } \mathsf{info}(\varphi) \cap s \models^{-} \psi$$
$$s \models^{\circ} \varphi \to \psi \text{ iff } \mathsf{info}(\varphi) = \emptyset \text{ or } \mathsf{info}(\varphi) \cap s \models^{\circ} \psi$$

Depending on the properties of ψ, the dismissal clause may be further reduced. In particular, when ψ is support- and reject-dense, which is guaranteed to be the case when it is not suppositional, then the dismissal clause reduces to:

(27) $s \models^{\circ} \varphi \to \psi$ iff $s \cap \mathsf{info}(\varphi) = \emptyset$.

3.8 Derivative semantic relations and stability

InqS primarily characterizes when a state s supports, rejects, or dismisses a supposition of a sentence φ. However, based on these elementary notions, it is possible to define other notions as well. We will define a number of such notions here, which we think are particularly relevant for linguistic analysis.

First, if a state s is such that no substate of it supports φ, we say that it *excludes supportability* of φ, notation $s \models_{\lightning} \varphi$. Similarly, if no substate of s rejects φ, we say that s *excludes rejectability* of φ, notation $s \models_{\surd} \varphi$. If s excludes *both* supportability *and* rejectability of φ, we write $s \models_{\bullet} \varphi$; if s excludes *neither* supportability *nor* rejectability of φ, we write $s \models_{\diamond} \varphi$.

Definition 11 (Excluding supportability and rejectability).

– $s \models_{\lightning} \varphi$ *iff* $\forall t \subseteq s\colon t \not\models^{+} \varphi$
– $s \models_{\surd} \varphi$ *iff* $\forall t \subseteq s\colon t \not\models^{-} \varphi$
– $s \models_{\bullet} \varphi$ *iff* $s \models_{\lightning} \varphi$ *and* $s \models_{\surd} \varphi$

– $s \models_{\diamond} \varphi$ *iff* $s \not\models_{\lightning} \varphi$ *and* $s \not\models_{\surd} \varphi$

If s supports φ and also excludes rejectability of φ, then we say that it *indefeasibly supports* φ, notation $s \models^{+}_{\surd} \varphi$. Otherwise, we say that s *defeasibly supports* φ, notation $s \models^{+}_{\diamond} \varphi$. Similarly, if s rejects φ and also excludes supportability of φ, then we say that it *indefeasibly rejects* φ, notation $s \models^{-}_{\lightning} \varphi$. Otherwise, we say that s *defeasibly rejects* φ, notation $s \models^{-}_{\diamond} \varphi$.

Definition 12 (Defeasible and indefeasible support and rejection).

– $s \models^{+}_{\surd} \varphi$ *iff* $s \models^{+} \varphi$ *and* $s \models_{\surd} \varphi$
– $s \models^{+}_{\diamond} \varphi$ *iff* $s \models^{+} \varphi$ *and* $s \not\models_{\surd} \varphi$

– $s \models^{-}_{\lightning} \varphi$ *iff* $s \models^{-} \varphi$ *and* $s \models_{\lightning} \varphi$
– $s \models^{-}_{\diamond} \varphi$ *iff* $s \models^{-} \varphi$ *and* $s \not\models_{\lightning} \varphi$

If s dismisses a supposition of φ and moreover excludes supportability of φ without rejecting it, then we say that it *dismisses supportability* of φ, notation $s \models^{\circ}_{\lightning} \varphi$. Similarly, if s dismisses a supposition of φ and moreover excludes rejectability of φ without supporting it, then we say that it *dismisses rejectability* of φ, notation $s \models^{\circ}_{\surd} \varphi$.

Definition 13 (Dismissing supportability and rejectability).

– $s \models^{\circ}_{\lightning} \varphi$ *iff* $s \models^{\circ} \varphi$ *and* $s \models_{\lightning} \varphi$ *and* $s \not\models^{-} \varphi$
– $s \models^{\circ}_{\surd} \varphi$ *iff* $s \models^{\circ} \varphi$ *and* $s \models_{\surd} \varphi$ *and* $s \not\models^{+} \varphi$

If s dismisses a supposition of φ without supporting or rejecting it, and if moreover it does not exclude supportability or rejectability of φ, then we say that it *suppositionally flags* φ, notation $s \models^{\circ}_{\diamond} \varphi$. On the other hand, if s dismisses a supposition of φ and excludes both supportability and rejectability of φ, then we say that it *suppositionally dismisses* φ, notation $s \models^{\circ}_{\bullet} \varphi$.

Definition 14 (Suppositional flagging and dismissal).

– $s \models^{\circ}_{\diamond} \varphi$ *iff* $s \models^{\circ} \varphi$ *and* $s \not\models^{+} \varphi$ *and* $s \not\models^{-} \varphi$ *and* $s \not\models_{\lightning} \varphi$ *and* $s \not\models_{\surd} \varphi$
– $s \models^{\circ}_{\bullet} \varphi$ *iff* $s \models^{\circ}_{\lightning} \varphi$ *and* $s \models^{\circ}_{\surd} \varphi$

3.9 Logical answerhood relations

So far, we have been concerned with semantic relations between a state on the one hand and a sentence on the other. Based on these relations, we can also define logical relations between two sentences, characterizing different notions of *answerhood*. For instance, if φ is given as an answer to ψ, we could say that φ *supports* ψ just in case every state that supports φ also supports ψ. This means that any way of compliantly settling the proposal expressed by φ will lead to a state that supports ψ. Analogously, we could say that φ *rejects* ψ just in case every state that supports φ rejects ψ. And similarly for the other semantic relations. In defining these logical answerhood relations, we will assume that φ, the answer, is non-inquisitive. Below we provide a general scheme, from which a range of concrete answerhood relations can be obtained by filling in different semantic relations for †.

Definition 15 (Answerhood relations).
For any $\varphi, \psi \in \mathcal{L}$ where φ is non-inquisitive:

$$\varphi \models^{\dagger} \psi \quad \textit{iff} \quad \forall s : \textit{if } s \models^{+} \varphi, \textit{ then } s \models^{\dagger} \psi$$

With these answerhood relations in place, let us now return to our initial motivating examples, repeated below:

(28) a. If Pete plays the piano, will Susan sing? $p \to ?q$
b. Yes, if Pete plays the piano, Susan will sing. $p \to q$
c. No, if Pete plays the piano, Susan won't sing. $p \to \neg q$
d. Pete won't play the piano. $\neg p$

As desired, our semantics predicts that (28b) and (28c) both support (28a); that (28b) and (28c) reject each other; and that (28d) suppositionally dismisses (28a), (28b), and (28c). These examples are iconic for the issues that we set out to address in this paper. However, as we have seen along the way, the resulting semantics deals with many more complex cases as well.

4 Conclusion

Our starting point in this paper was the general perspective on meaning that is taken in inquisitive semantics, which is that sentences express proposals to update the common ground of the conversation in one or more ways. There are several ways in which a conversational participant may respond to such proposals, depending on her information state. The most basic inquisitive semantics framework, InqB, characterizes which states support a given proposal. Radical inquisitive semantics, InqR, also characterizes independently which states reject a given proposal. The suppositional inquisitive semantics developed in the present paper, InqS, further distinguishes states that dismiss a supposition of a given proposal. We have thus arrived at a more and more fine-grained formal characterization of proposals, and thereby at a more and more fine-grained characterization of meaning. We have argued that this is necessary for a better account of information exchange through conversation, in particular when the exchange involves conditional questions and assertions. Elsewhere, we argue that the framework developed here also offers new insights into the semantics of epistemic and deontic modals (Aher et al., 2014).

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