Foundational problems for attentive content

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The general aim of this essay is to address some foundational problems connected to the extension of inquisitive semantics to a framework accounting for attentive content. The focus will be on two main issues:

1. How the aim of speech-acts of drawing attention should be conceived of in order for inquisitive semantics' approach to meaning to still be understood as a weak dynamic approach;

2. What should be the meaning of the logical connectives in the extension of inquisitive semantics to attentiveness.

The essay can be divided in two parts, each dedicated to each of the problems mentioned above. I begin by introducing inquisitive semantics, doing so by inserting it on what I dub 'Weak Dynamic Approaches' to meaning, focusing on what is specific on the inquisitive approach. The notions of informative and inquisitive content will be introduced, as the recursive definition of a proposition.

I proceed to introduce attentive content, showing how a natural modification to the definition of a proposition allows for a treatment of attentiveness. Afterwards, the extension of the semantics to attentive content is put on the background of the semantics' weak dynamic approach to meaning. I will argue that speech-acts of drawing attention should be understood as having as their primary aim to affect the shared epistemic state of the participants in a conversation in a particular way, and that this constitutes a departure from the mainstream inquisitive framework where the relevant speech-acts aim at affecting the common ground, which is but one part of the shared epistemic state. An account of how a shared epistemic state should be represented will thus be provided, and of how it gets updated by the performance of speech-acts.

In the last section of the first part of the essay I will argue that the background provides the tools for the rejection of Seth Yalcin's claim to the effect that the explanation for epistemic contradictions is not pragmatic, but semantic. This will provide an example of how the theoretical background being proposed has 'philosophical bite'.

In the second part I begin by introducing the problem of how to account for the meaning of the logical connectives once inquisitive semantics is extended to deal with attentive content. The algebraic approach to the meaning of the logical connectives will be introduced, and it will be shown how it works when inquisitive semantics restricts itself to informative and

inquisitive content. In order to use the algebraic approach to ground the meaning of the connectives in the unrestricted setting dealing with attentiveness I propose a prima facie plausible attentiveness order. I show that this order does not vindicate none of the definitions of conjunction that have been equated with what conjunction stands for.

Finally, I present a result by Floris Roelofsen, to the effect that the attentiveness order proposed is not appropriate for providing a principled account of the meaning of the logical constants.

What is Inquisitive Semantics?

Stalnaker's work on the interaction between context and what is said (e.g. [Stalnaker, 1978]) has highlighted the importance of more than truth-conditional semantics in order to characterize what is said. A major feature of his theory lies in that acts of assertion have as their primary aim to affect the *common ground*, that which the participants in a conversation take to be shared knowledge at a particular context. This view has inspired two different, *dynamic* approaches to meaning:

Weak Dynamic Approach In characterizing the meaning of a sentence, what is said, one must take into account what speech-acts involving it aim at; in particular, in which distinctive way do they aim at enhancing the common ground;

Strong Dynamic Approach The meaning of a sentence consists in the way a particular kind of use of that sentence leads to enhancing the common ground of the context in one or another way.

The notion of knowledge is fundamental for both approaches, as is the notion of an *update*, where an updated common ground is the result of certain speech-acts, and also of what was shared knowledge between the participants before the use of the sentence. To give but an example, the assertion of the sentence 'John is married' will produce an update to the common ground, where the updated common ground will be such that the participants in the conversation will share the knowledge that John is married. In this essay I will associate inquisitive semantics with the first of these two approaches, even though it is arguable whether it falls under the first or the second approach.

A distinguishing trait of inquisitive semantics lies in its focus on the commonalities between acts of assertion and of questioning. The theory understands both assertions and questions as *proposals* to update the common ground, and endorses the common-sense view that whether such updates take effect is something which depends on some kinds of actions that the participants might perform in reaction to those speech-acts. By focusing on what is shared by assertions and questions, the semantics is able to ground a unified conception of meaning, one encompassing both informative and inquisitive content. This contrasts with both static and other dynamic approaches to meaning. Static approaches usually account solely for informative content, which is also a common trait of most dynamic theories, possibly due to the

fact that, either explicitly or implicitly, they primarily focus on what is conveyed by acts of assertion.

Informative and Inquisitive Content

Inquisitive semantics is thus concerned with the kind of speech-acts that constitute proposals to update the common ground. *Qua* proposal, a speech-act may propose more than one way of updating the common ground. Each of the ways being proposed corresponds to a representation of how the world is, and its intended effect on the common ground is for the participants to acquire the knowledge that the world is as represented by the representation being proposed. This way of understanding proposals and their effect on the common ground leads to a conception of propositions as possessing different representations of how the world is.

The informative content of a proposition is equated with what the different representations of how the world is have in common. This is the information provided by a proposal, prior to any reaction with respect to the updates being proposed. As for a sentence's inquisitive content, it consists in the different representations of how the world is contained in the proposition it expresses, and also in the way the world is represented as not being. The upshot is that this is the information required by the participants in the conversation in order to realize one of the updates, or reject the proposal. It is in this sense that a sentence's meaning encompasses both informative and inquisitive content.

The representation of the common ground assumed by inquisitive semantics is as a set of possible worlds, those worlds consistent with the shared knowledge of the participants. Each representation of how the world is consists also in a set of possible worlds, the possible worlds that are that way, and thus propositions are conceived as sets of sets of possible worlds (henceforth I will also be using the expressions 'index' and 'possibility' to refer, respectively, to possible worlds and sets of possible worlds, nomenclatures standardly adopted in the literature on inquisitive semantics). A sentence expresses a proposition, the proposal performed by the use of the sentence proposing one or more ways to update the common ground, and the resulting possible updates consisting in the indexes in the common ground compatible with the representations of how the world is. An update is proposed with the purpose of increasing the shared knowledge with the knowledge that the world is the way represented by the possibility yielding the updated common ground. Enhancing the common ground is here understood as eliminating possible worlds, those that are not consistent with the representation of the world accepted by the participants, since this captures the increase in shared knowledge.

On this view of propositions as sets of possibilities, the informative content of a proposition X, info(X), consists in the union of all the possibilities belonging to X, for this is the representation of how the world is that is shared by all the representations in X. Informative content can also be defined with respect to a sentence φ. It consists in the union of all the representations

belonging to the proposition expressed by φ, $[\![\varphi]\!]$.[1]

- info(φ) $=_{def} \bigcup [\![\varphi]\!]$.

Let a set x be maximal with respect to a set X to which it belongs just in case x is not a proper subset of any set y belonging to X. The inquisitive content of a sentence φ, inq(φ) consists on the set of maximal possibilities in the union of $[\![\varphi]\!]$ with the set of worlds which do not belong to info(φ):

- inq(φ) $=_{def}$ ALT($[\![\varphi]\!] \cup \overline{\text{info}(\varphi)}$), where ALT is a function such that ALT(X) = $\{x \in X | \neg \exists y \in X : x \subset y\}$.

The set $\overline{\text{info}(\varphi)}$ accounts for the information required in order to exclude the proposal made by the use of the sentence, since what is required is the information to the effect that the world is not as represented by any of the possibilities in the proposition. Each of the sets on $[\![\varphi]\!]$ account for the different information one could give in order for the proposal to be accepted. The function ALT takes out of consideration those sets in $[\![\varphi]\!] \cup \overline{\text{info}(\varphi)}$ that are subsets of some other set in $[\![\varphi]\!] \cup \overline{\text{info}(\varphi)}$ (that are not maximal). These are the only ones required to settle the proposal made by the use of φ: to ask for information enough to settle at least one of α or β is to ask for information enough to establish β to be provided. Hence, non-maximal possibilities do not reflect the information required in order to settle the proposal being made, and therefore do not figure in an account of inquisitive content.

The inquisitive approach to meaning allows for a categorization of sentences into informative and inquisitive ones which is based on the semantics' dynamic flavor. A sentence φ is informative with respect to a common ground S if the proposal performed by its use yields new knowledge (that is, if it shrinks the common ground). This is the case just in case the intersection of info(φ) with S is different that S itself.

- φ is informative with respect to a common ground S if and only if (info(φ) $\cap S$) $\neq S$.

A sentence φ is inquisitive with respect to a common ground S just in case the representation of how the world is that results from adding the informative content of φ to S is not included in any possibility for φ. The reason is that if this is the case, then the result of adding the informative content of φ to the shared knowledge of the participants in the conversation does not suffice to determine a possibility with which to update the common ground, and therefore the participants in the conversation are required to provide more information in order to determine with which possibility to perform the update.

- φ is inquisitive with respect to the common ground S if and only if $\neg \exists x \in [\![\varphi]\!] : $ (info(φ) $\cap$ S) $\subseteq x$.

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[1] In what follows I will define several notions related to informative and inquisitive content with respect to sentences. However, these definitions could also be given with respect to propositions. It is important to bear this in my mind, for later on these notions will be directly applied to propositions.

A sentence φ can be also be said to be informative and inquisitive *tout court*, if one considers the special case where there is no shared knowledge between the speakers, the case where S consists in the set of all indexes:

- φ is informative if and only if info(φ) $\neq \omega$, where ω stands for the set of all indices;

- φ is inquisitive if and only if $\neg \exists x \in [\![\varphi]\!]$: info(φ) $\subseteq x$; that is, if and only if info(φ) $\notin [\![\varphi]\!]$.

One of the things that is interesting once informativeness and inquisitiveness are conceived in such terms is that some sentences turn out to be both informative and inquisitive. I will proceed by presenting the recursive definition of propositions that comes out from the ideas mentioned above. It will be salient that most disjunctions will be both informative and inquisitive.

Propositions

As mentioned before, a proposition consists in a set of possibilities. However, there are the provisos that:

1. No proposition is empty;

2. All possibilities in a proposition are maximal (*alternative*) possibilities. That is, for any proposition P, $\neg \exists x, y \in P : x \subset y$.

Some of the motivation for the requirement that possibilities be maximal was already provided when introducing the notion of inquisitive content. Furthermore, there is a sense in which a non-maximal possibility $\alpha \subset \beta$, where β is a maximal possibility, does not help represent the information provided or requested by a speech-act: if one says that at least one of a α or β obtains, this is just as informative as saying that β obtains. And if one says that β obtains, then this is as informative as saying that at least one of α or β obtains.

The proposition expressed by a sentence of a propositional language is recursively defined as follows:

1. $[\![p]\!] = \{w | w(p) = 1\}$, if p is an atomic sentence;

2. $[\![\neg\varphi]\!] = \{\overline{\bigcup [\![\varphi]\!]}\}$

3. $[\![\varphi \vee \psi]\!] = \text{ALT}([\![\varphi]\!] \cup [\![\psi]\!])$

4. $[\![\varphi \wedge \psi]\!] = \text{ALT}(\{\alpha \cap \beta | \alpha \in [\![\varphi]\!] \text{ and } \beta \in [\![\psi]\!]\})$

5. $[\![\varphi \to \psi]\!] = \text{ALT}(\{\gamma_f | f \in [\![\psi]\!]^{[\![\varphi]\!]}\})$, where $\gamma_f = \bigcap_{\alpha \in [\![\varphi]\!]} (\overline{\alpha} \cup f(\alpha))$

One can now see an example of a sentence that is both informative and inquisitive. Consider the sentence $p \vee q$. It is informative, since its informative content, the set of indexes w such that $w(p) = 1$ or $w(q) = 1$, is not equal to ω. Furthermore, it is also inquisitive, for info(p) $\notin [\![p \vee q]\!]$.

(a) The possibilities in $[\![p \lor q]\!]$

(b) The informative content of $p \lor q$
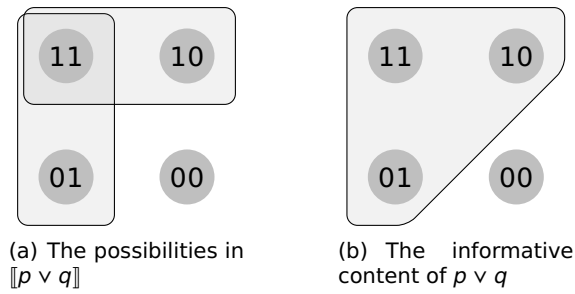
Figure 1: The possibilities in $[\![p \lor q]\!]$ and the sentence's informative content.

Attentive content

Consider the following sentence:

(1)   Michael might be at home.

Uses of the sentence typically do not purport to increase the shared knowledge of the participants in any way (if there are such uses, these are not the ones of interest for the notion of content presently being considered). This means that the sentence is neither informative nor inquisitive. Still, its use is not without motivation. Prima facie, the sentence is being used to *draw attention* to the possibility that Michael is at home. Furthermore, the semantic contribution of 'might' appears to be such that 'might φ' draws attention to the possibility that φ. The upshot of these considerations is that the sentence (1) possesses yet another notion of content, *attentive content*, and that even though the sentence is neither informative nor inquisitive, it is in fact attentive.

A remarkable feature of inquisitive semantics' framework is that it seems to predict the existence of such kind of content. Propositions, as mentioned before, are conceived as sets of alternative possibilities. But if a different, more natural conception of propositions as just sets of possibilities is adopted, the framework provides the tools to account for attentiveness. As an example, consider again sentence (1). Since the sentence is neither informative nor inquisitive, it expresses a proposition containing ω. Furthermore, since the sentence draws attention to the possibility that John is at home, the proposition should contain this latter possibility in addition to ω. That is, adoption of inquisitive semantics makes it natural to add the following clause in the recursive definition of a proposition for a propositional language added with a $\lozenge$ operator, intuitively standing for 'might':

6. $[\![\lozenge\varphi]\!] = \{\omega\} \cup [\![\varphi]\!]$

The notion of attentive content that arises from this extended conception of a proposition consists in the set of all possibilities contained in a proposition. These are the possibilities that uses of sentences, proposals or otherwise, draw attention to.

- att$(\varphi) =_{def} [\![\varphi]\!]$

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Given any set Y, RES(Y) consists in the set of non-maximal sets belonging to it. A sentence is *attentive* just in case it contains non-maximal possibilities:

- A sentence φ is attentive if and only if RES($[\![\varphi]\!]$) $\neq \emptyset$.

Attentive content and the 'dynamic turn'

Stalnaker's original motivation for taking into account the common ground when characterizing the meaning of sentences was based on the fact that assertions have as their primary aim to affect the common ground. For this reason, one could recover what was said by the performance of an assertion by considering how the assertion affects the common ground. As mentioned, an important insight behind inquisitive semantics is that assertions are but one of a kind of speech-acts which constitute proposals to update the common ground (questions constituting another kind of proposals). The weak dynamic nature of inquisitive semantics thus relies on this observation in order to ground a conception of meaning unifying both informative and inquisitive content, with particular importance being given to how those speech-acts falling under the general kind of proposals aim at enhancing the common ground.

Extending inquisitive semantics to attentive content constitutes, I believe, an even more radical approach to meaning than the one in which inquisitive semantics was initially based. Attentive content cannot be recovered from the way a use of a sentence affects or could affect the shared knowledge of the participants in a conversation, since uses of sentences such as (1) just don't seem to consist in any kind of proposal on how to update the common ground. To be clear, such uses do seem to have some dependence on the common ground: for instance, in Veltman's update semantics [Veltman, 1996], an update with a sentence 'might φ' is allowed just in case there is some index belonging to the worlds compatible with the knowledge of the agent where the sentence 'φ' is true. My point is that this dependence seems to be a consequence of the aim of uses of sentences such as (1), not the aim of such uses (for lack of a better term, from now on I will be calling speech-acts such as those typically involving utterances of sentences like (1) acts of drawing attention). For this reason, it seems to me that accounting for the meaning of the sentences being used in acts of drawing attention requires that inquisitive semantics adopts a broader background theory than the one adopted in order to account for proposals.

I propose that, for the purposes of a theory of informativeness, inquisitiveness and attentiveness, the relevant kinds of speech-acts (those of proposing and of drawing attention) aim at enhancing the *shared epistemic state* of the participants in a conversation. However, what constitutes an epistemic state, and how to model it, seems something quite difficult to pinpoint. For the present purposes I believe that a representation of the shared epistemic state seems to require both a representation of the common ground, and of the possibilities to which attention has been drawn to. Two ways of representing the shared epistemic state that satisfy these requirements suggest themselves:

1. A shared epistemic state S consists in a set of set of indexes $S = \{CG, \ldots\}$, where CG

stands for a set of indexes representing the common ground, and all the other possibilities in S are proper subsets of the common ground and represent the possibilities to which attention has been drawn to, restricted to CG;

2. A shared epistemic state S consists in a pair $S = \langle CG, DA \rangle$, where CG, the common ground, consists in a set of indexes, and DA is a set of possibilities, those possibilities to which attention has been drawn to.

My position is that the second option is more appropriate as a representation of the shared epistemic state. The motivation I find for such representation stems from considering other acts of drawing attention besides those resulting from utterances of sentences 'might φ'. Consider, for instance, the case of Michael and Ann, which are trying to see with whom they can leave Ace, their dog, during the weekend. At some point, Michael utters sentence (2):

(2)   Hey, my parents will be at home.

Assume that the context is one where Michael and Ann know that Michael's parents will be at home during the weekend. On such context, sentence (2) does not constitute an act of assertion, but of drawing attention: Michael did not want to update the common ground with the possibility that his parents will be at home, but solely draw attention to that possibility (this, I take it, shows that speech-acts of drawing attention need not restrict to drawing attention to possibilities with respect to which it is unknown whether they hold or not). Suppose furthermore that in the follow-up Ann utters

(3)   And they love Ace,

something that is common knowledge between Michael and Ann.

If one wishes to represent the shared epistemic state in such a way that it exhibits the fact that both of these possibilities have been drawn attention to, one cannot endorse the first option. For both the possibility that Michael's parents will be at home and the possibility that Michael's parents love Ace, when restricted to the common ground, will be identical with it. However, the second option is able to capture such acts of drawing attention. For this reason, I find it to be a better representation of the shared epistemic state for the purposes of a theory incorporating attentive content.

In order to get a hold on how sentences of the form 'might φ' are intuitively taken to interact with the shared epistemic state, consider sentence (4):

(4)   It might be raining

uttered by John's mother just before he leaves home. In those circumstances, we can suppose, John knows that it either is raining or it isn't, and his utterance is not intended to affect John's knowledge in any way. As was the case with sentence (1), the utterance of (4) is intended to make John aware that his state of knowledge is not one where the possibility that it is

raining has been discarded, so that he can act accordingly. Letting p = 'it is raining', $[\![\Diamond p]\!]$ is the following proposition,
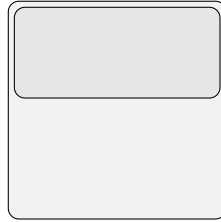

Figure 2: The possibilities in $[\![\Diamond p]\!]$.

where the outer possibility is equal to ω and the inner possibility consists in the set $\{w|w(p) = 1\}$ of indexes where p is true. The shared epistemic state that results from accepting John's mother's utterance is just like the previous shared epistemic state, with the difference that at least the possibility corresponding to the set $\{w|w(p) = 1\}$ is added to the set DA. In general, the effect of accepting attentive utterances is to at least update the shared epistemic state in this way. In the following section we will focus on how exactly should speech-acts be understood as updating the shared epistemic state.

Attentive updates

The way in which both assertions and questions update the shared epistemic state is given by the way they update the common ground: an assertion, if accepted, leads to an updated common ground consisting in the intersection of the previous common ground and the informative content of the proposition expressed by the assertion; with respect to questions, participants are requested to choose between the different possibilities being proposed, and the updated common ground will consists on the intersection of the chosen possibility with the previous common ground.

A first option concerning how the shared epistemic state $S = \langle CG, DA \rangle$ should be updated by actions of 'drawing attention' is that the updated shared epistemic state $U(S)$ consists in a pair $\langle U(CG), U(DA) \rangle$, where $U(CG) = CG$, and $U(DA)$ consists in the union of DA with $\text{RES}([\![\varphi]\!])$, where φ is the sentence being used in the act of drawing attention. I will refer to this option as 'APV', for the 'Adding Possibilities View'.

A different view might be extracted from the work of Yalcin on the semantics of epistemic modals. In [Yalcin, 2008], one of Yalcin's concerns is to provide a model of the state of mind one is in when believing that $\triangle\varphi$ (where $\triangle$ stands for an epistemic modal, such as 'possibly' or 'might'). Yalcin contrasts his views with a model proposed by Veltman [Veltman, 1996], according to which one believes that $\triangle\varphi$ just in case his belief state is compatible with the possibility that φ, and proposes that believing that $\triangle\varphi$ is not just a question of the epistemic state being compatible with the possibility, but that the possibility must also be marked as an open possibility according to that state of mind.

Consider again the case of John being about to leave home, and his mother's utterance of (4). One can suppose that John's belief state before his mother's utterance was already compatible with the possibility that it is raining. Still, it seems that after his mother's utterance John got to be in a different epistemic state. Yalcin proposes that the effect of his mother's utterance was that John's state of mind became one taking note of a distinction between it being raining and it not being raining, and that the distinctions a state of mind takes note of can be represented by a partition of the logical space (the set of all indexes). For instance, the distinction that John's state of mind takes note of is that between the possibility that it is raining and the possibility that it is not raining. Thus, the drawing of such distinction can be represented by the drawing of a line in logical space, a line distinguishing the between the possibility that it is raining and the possibility that it is not raining:
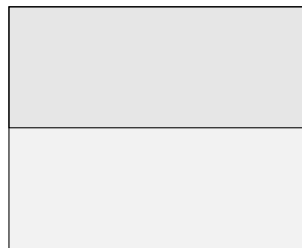

Figure 3: Distinguishing between the possibilities that is is raining and that it is not raining.

Furthermore, if John's mother had also uttered (5):

(5)   And I might go pick you up at school

then a further line should be drawn in the logical space, one distinguishing between the possibility in which John's mother will pick him up at school, and the possibility where this is not the case.
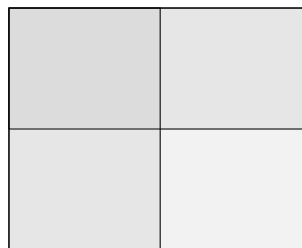

Figure 4: Distinguishing between the possibilities that is is raining and that it is not raining, and the possibilities that John's mother will pick him up at school and that she won't.

The distinctions that an agent takes note are, therefore, the ones that 'carve according to the lines' drawn in the logical space. That is, they are the ones corresponding to the union of some cells in the partition of the logical space. Yalcin proceeds to contend that beliefs are resolution-sensitive: the belief state of an agent can be seen as given by a subset of a resolution (a partition of the logical space). This gives Yalcin the elements required for an

explanation of the cases where one believes that $\Delta\varphi$: roughly, an agent believes that $\Delta\varphi$ if there is a cell in the belief state of the agent such that φ is true in every one of the cell's indexes.

What matters for the present purposes is that Yalcin's position on the resolution-sensitivity of belief states could be adapted to constitute an alternative on how to conceive updates to the shared epistemic state of the participants on a conversation. The idea would be that an act of drawing attention imposes a partition on the logical space, and the set $U(DA)$ should contain the cells belonging to the partition. The possibilities which are being taken under consideration by the participants in the conversation are those that 'carve according to the lines' drawn in the logical space, the ones corresponding to unions of subsets of $U(DA)$. I will refer to this option as 'PaV', for the 'Partition View'.

For instance, assuming now that John and his mother's previous set DA contained no possibility, the effect of John's mother's utterances of both (4) and (5) would be to add the four possibilities depicted in figure 4 to DA. The possibilities that John and his mother would be taking into consideration at that shared epistemic state would be all the possible unions of cells in figure 4.

I am uncertain as to whether one should adopt APV or PaV. On the one hand, when one utters a sentence with a non-maximal possibility, it seems to be the case that one also draws attention to the complement of that possibility. For instance, it is not uncommon to have utterances such as that of (4) be accompanied by

(6)   Just as it might not

which seems to indicate that when attention is drawn to a possibility, it is also drawn to its complement. Since PaV predicts that this will be the case, this fact could be seen as a reason to prefer the latter view on how to update the shared epistemic state. On the other hand, perhaps PaV predicts that more possibilities are drawn attention to than those with respect to which this should be the case. If John's mother utters to him:

(7)   It might be raining, I might go pick you up at school and we might be having dinner at a restaurant

PAV predicts that attention might be drawn to 8 possibilities plus possibilities resulting from unions of these.[2] This might just be too much. Furthermore, the fact that when attention is drawn to a possibility, then it is also drawn to its complement might not depend on essential features of the interaction between shared epistemic states and acts of drawing attention, but

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[2]The possibilities are: the possibility that it is raining, John's mother will pick him up at school and they will go to have dinner at a restaurant; the possibility that it is raining, John's mother will pick him up at school and they will not have dinner at a restaurant; the possibility that it is raining, John's mother will not pick him up at school and they will go to have dinner at a restaurant; the possibility that it is raining, John's mother will not pick him up at school and they will not have dinner at a restaurant; the possibility that it is not raining, John's mother will pick him up at school and they will go to have dinner at a restaurant; the possibility that it is not raining, John's mother will pick him up at school and they will not have dinner at a restaurant; the possibility that it is not raining, John's mother will not pick him up at school and they will go to have dinner at a restaurant; the possibility that it is not raining, John's mother will not pick him up at school and they will not have dinner at a restaurant; and unions of all these possibilities.

instead on the reasoning abilities of the participants on the conversation and how they might feel prompted to perform them, given the occurrence of the act of drawing attention. When some utters that it might be raining, at the same time attention is drawn to the possibility that it is raining, it is also indicated (perhaps presupposed) that the common ground is solely compatible with this possibility, that the possibility is not known to be the case. But then, one is prompted to consider the opposite representation of the world.

Nevertheless, the aim of the speaker in performing an act of drawing attention by uttering $\Diamond\varphi$ was not for the participants to consider both the possibility that φ and the complement of that possibility. Clearly, John's mother's aim when uttering sentence (4) was not to draw attention to the possibility that it is not raining. Stalnaker's account of how acts of assertion perform updates to the common ground/epistemic state is based on what he takes as being the main aim of performing an assertion. If one follows the same strategy with respect to how acts of drawing attention update the common ground, PaV constitutes a mistaken position, and APV should be adopted instead, since the aim of uttering $\Diamond\varphi$ seems to be drawing attention solely to the possibility that φ, instead of both to the possibility that φ and its complement. For this reason, I will adopt APV during the rest of the essay.

If either a proposal or an act of drawing attention using a sentence φ is performed and accepted in a context determining an epistemic state S with common ground CG, then I propose that the updated epistemic state $U(S) = \langle U(CG), U(DA)\rangle$ is as follows:

- $U(CG) = CG \cap \mathrm{info}(\varphi)$

- $U(DA) = DA \cup \mathrm{RES}(\llbracket\varphi\rrbracket)$

Furthermore, if the speech act also requested a choice between possibilities, then choosing a possibility P has the effect that the shared epistemic state S will be updated in such a way that $U(CG) = CG \cap P$.

Epistemic contradictions

Consider the following sentences:

(8)   It is raining and it might not be raining;

(9)   It is not raining and it might be raining.

These sentences sound defective, or even contradictory. The **classical explanation** for this fact is that the above sentences entail, in a way that is obvious to speakers, respectively, the Moore sentences (10) and (11):

(10)   It is raining and I do not know that it is raining;

(11)   It is not raining and I know that it is raining.

The point is that Moore sentences also sound defective, and for this fact there is a straightforward pragmatic explanation: when a speaker makes an assertion he represents himself as knowing that what he is asserting is the case, given that the aim of assertions is to increase the shared knowledge of the participants; this has the effect that to assert sentences (10) or (11) is to do something incoherent, since then one represents oneself as both knowing that a possibility holds/does not hold, and not knowing that it holds/knowing that it holds. However, semantically, the sentences are unproblematic. Their linguistic meaning is such that they do not stand for any contradiction whatsoever.

Yalcin rejects the **classical explanation**. His point is that epistemic contradictions and Moore sentences behave differently under the scope of 'suppose', and for this reason, contrary to what is the case with respect to Moore sentences, they are semantically defective.

(12)   Suppose it is raining and I do not know that it is raining;

(13)   Suppose it is not raining and I know that it is raining;

(14)   Suppose it is raining and it might not be raining;

(15)   Suppose it is not raining and it might be raining.

Sentences (12) and (13) are unproblematic, and Yalcin takes this fact as vindicating the thesis that sentences (12) and (13) are not semantically defective, since the possibility that what they state is the case is at least entertainable. However, sentences (14) and (15) do sound defective. That is, the possibility that what the sentences state is the case is not even entertainable. This leads Yalcin to characterize the phenomenon as being semantic, since the possibility that what they state is the case is not even entertainable.

In this section I will argue that the representation of the shared epistemic state proposed in the two previous sections provides the tools to reject Yalcin's argument against the **classical explanation**. My argument consists in showing that sentences such as (8) and (9) are not embeddable under 'suppose' not because they are semantically contradictory, but due to the fact that attentive sentences are not embeddable under 'suppose' (and such fact does not make attentive sentences, in general, semantically defective).

For my argument I will assume a strongly dynamic perspective of how 'suppose' works in natural language. That is, I will assume that the meaning of 'suppose' is fully given by the contribution it gives to the way suppositions using sentences containing the expression update the shared epistemic state. Recall that I have previously identified inquisitive semantics with a weak dynamic approach to meaning. Perhaps others will see some tension here. I do not. Some expressions will play a part in how the sentences containing them represent the world; however, given the aim of speech-acts as consisting in updating the shared epistemic state, other expressions will just be concerned with performing changes on the shared epistemic state, without aiming at representing the world in any particular way. It seems reasonable to suppose that natural language possesses the resources to perform both kinds of tasks.

I submit that using a sentence of the form 'suppose φ' has the effect of, at least for some time, updating a shared epistemic state $S = \langle CG, DA \rangle$ to an updated epistemic state $U(S) = \langle U(CG), U(DA) \rangle$, where $U(DA) = DA$ and $CG \subseteq \text{info}(\varphi)$. I am not able to provide the details of how $U(CG)$ looks like. If someone utters 'suppose it is not raining', and at that point I know that John is wet due to the rain, then not only will $U(CG)$ be a subset of the possibility where it is not raining, but furthermore, prima facie, there will be indexes in the updated common ground where it is false that John is wet. However, finding a general recipe for such update is hard (as people working on the semantics of counterfactuals have discovered a long time ago).

The view on 'suppose' seems natural: what one wants with the use of the expression is to momentarily provide the information in the clause embedded under 'suppose', as if it constituted new knowledge, allowing for that putative knowledge to be used in reasoning tasks. But this perspective on 'suppose' has the effect that the following sentence should be quite odd, given our representation of the common ground:

(16)   Suppose that it might be raining

and, in general, sentences instantiating the following schema should all sound defective:

(17)   Suppose that might φ

Why should this be so? Two elements play a crucial role:

1. No update to the epistemic state is, in practice, being performed: 'it might be raining' is uninformative. This means that no information is added to the updated common ground;

2. The embedded sentence is attentive.

The participants should therefore feel puzzled: the act is one of supposition, which is semantically indicated by 'suppose', but nothing is being in fact supposed. Furthermore, the embedded clause is an attentive one. But acts of supposition act on CG, not on DA. Hence, there is no reason for an attentive sentence to be supposed. The outcome is that no sense can be made of sentence (16), nor of instantiations of (17).

If this is correct, then one has the elements to reject Yalcin's argument to the effect that the reason why sentences (8) and (9) sound contradictory is semantic. Consider the intuitively true principle:

Distributivity of 'suppose' If an agent x supposes that φ and ψ, then x supposes that φ and x supposes that ψ.

If x supposes that it is not raining and it might be raining, then x supposes that it is raining and x supposes that it might be raining. But x cannot suppose that it might be raining, since attentive sentences are not entertainable, for the reasons presented before. Therefore, sentence (15) must sound contradictory. By the same reasoning, sentence (14) must also sound contradictory. But this does not show that there is some semantic defect in sentences

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(9) and (8), only something specific concerning the interaction between attentive sentences and 'suppose': attentive sentences cannot be embedded under 'suppose'. This is made salient by the fact that the shared epistemic state consists not only of a common ground, but of a set of possibilities to which attention has been drawn to. Supposing affects the shared epistemic state only by affecting its first coordinate, not by affecting the set of possibilities drawn attention to. If there was no general conception of a shared epistemic state, but only of a common ground, one would not be able to see what is specific in the way 'suppose' functions, and why its behavior conflicts with that of attentive sentences.

Perhaps something has gone wrong with my whole argument, since English speakers do not find that sentence (16) sounds odd. The first thing I can say on behalf of the argument is that intuitions are not straightforward here: some native English speakers do find that there is something strange with the sentence. I also can provide an explanation for the cases where the sentence sounds perfectly normal: in such cases, it is being interpreted as meaning the same as 'it is possible that φ', a sentence which is not attentive, but informative, and that states something that can be represented by a proposition containing a unique possibility, which contains the indexes w that bear a particular relation to indexes w' such that $w'(\varphi) = 1$. Furthermore, and crucially, when this kind of interpretation is at play, speakers can also make sense of (15), which means that Yalcin's argument for a semantic defect does not apply when 'might' is interpreted in this way.

My point can be brought home if one can find a language with an expression whose meaning is more purely attentive, and sentences of the form '*sup might φ*' usually sound defective, where '*sup*' stands for the translation of 'suppose' into that language, '*might*' for the attentive expression, 'φ' for any sentence of that language, '*might*' has wide scope over 'φ', and '*sup*' has wide scope over '*might φ*'.

Fortunately, Portuguese seems to be one such language. The following is a sentence which constitutes the most direct translation of (9), where 'talvez', corresponds to '*might*' and 'supõe' to '*sup*':

(18)   Supõe que talvez esteja chover.

This sentence sounds defective. Also, a different, non-attentive interpretation is be even more difficult than what is the case with respect to (16) (if a different, non-attentive interpretation is possible at all). For this reason, the case is the one worth considering, since the background assumption is that we are dealing with the attentive analysis of 'might' (otherwise my argument would not go through). Hence, I find the objection that (16) does not sound odd not to be as cogent as it might have initially seemed to be, and that my initial argument retains its force.

The meaning of the logical connectives

Even though extending inquisitive semantics in such a way that the framework is able to account for attentive content is a natural move, it is not one without difficulties. A particularly acute problem has to do with what should be the contribution of the logical connectives to the attentive content of the sentences in which they occur. The most straightforward option to pursue would be to just keep the original clauses as previously defined, except for the fact that the ALT function will not occur in them, since non-alternative possibilities are also allowed. The recursive clauses would thus be the following:

1. $\llbracket p \rrbracket = \{w \mid w(p) = 1\}$, if p is an atomic sentence;

2. $\llbracket \neg \varphi \rrbracket = \{\overline{\bigcup \llbracket \varphi \rrbracket}\}$

3. $\llbracket \varphi \vee \psi \rrbracket = \llbracket \varphi \rrbracket \cup \llbracket \psi \rrbracket$

4. $\llbracket \varphi \wedge \psi \rrbracket = \{\alpha \cap \beta \mid \alpha \in \llbracket \varphi \rrbracket \text{ and } \beta \in \llbracket \psi \rrbracket\}$

5. $\llbracket \varphi \to \psi \rrbracket = \{\gamma_f \mid f \in \llbracket \psi \rrbracket^{\llbracket \varphi \rrbracket}\}$, where $\gamma_f = \bigcap_{\alpha \in \llbracket \varphi \rrbracket}(\overline{\alpha} \cup f(\alpha))$

Unfortunately, this option does not work. A natural constraint for the semantics to obey is idempotence of meaning for conjunction, the principle according to which the meaning of a sentence should be the same as the meaning of the conjunction of that sentence with itself:

$\wedge$**-Idempotence** $\llbracket \varphi \wedge \varphi \rrbracket = \llbracket \varphi \rrbracket$

However, the tweak proposed to the clause for conjunction has the undesired effect that an extra, non-alternative possibility - the possibility resulting from the intersection of the possibility that p with the possibility that q - is added to $\llbracket p \vee q \rrbracket$:

(a) The possibilities in $\llbracket p \vee q \rrbracket$

(b) The possibilities in $\llbracket (p \vee q) \wedge (p \vee q) \rrbracket$
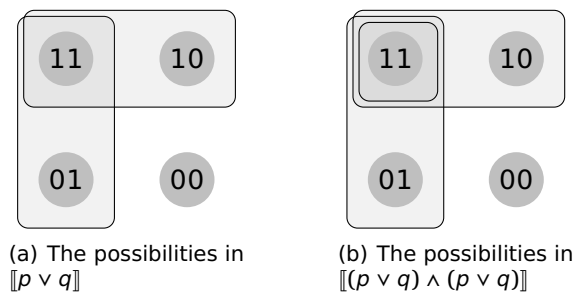
Figure 5: $\llbracket p \vee q \rrbracket$ and $\llbracket (p \vee q) \wedge (p \vee q) \rrbracket$

Problems occur also with respect to the meaning of implication. Prima facie, $(p \vee \neg p) \rightarrow (p \vee \neg p)$ brings attention to no inconsistency. Nonetheless, this is a consequence of the tweak to the recursive clause for $\rightarrow$:
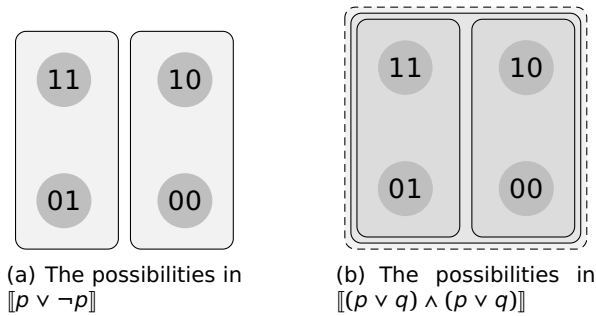

(a) The possibilities in $[\![p \vee \neg p]\!]$

(b) The possibilities in $[\![(p \vee q) \wedge (p \vee q)]\!]$

Figure 6: $[\![p \vee \neg p]\!]$ and $[\![(p \vee \neg p) \rightarrow (p \vee \neg p)]\!]$

The outer, dashed possibility in $[\![(p \vee \neg p) \rightarrow (p \vee \neg p)]\!]$ stands for the empty set, and is a product of the function that assigns the possibility that p to the possibility that $\neg p$, and the possibility that $\neg p$ to the possibility that p, for the union of the complement of the possibility that $\neg p$ with the possibility that p yields the possibility that p, and the the union of the complement of the possibility that p with the possibility that $\neg p$ yields the possibility that $\neg p$, and their intersection corresponds to the empty set.

These difficulties show the need to provide a principled account of the meaning of the connectives. One such account can be given by the adoption of an algebraic approach.

The algebraic approach

The algebraic approach for defining the meaning of the connectives is already present in the classic approach to meaning, where the proposition expressed by a sentence is equated with the set of indexes where that sentence is true. The idea is that, given such understanding of propositions, these can be ordered in terms of how informative they are, and from that order one can recover the meaning of the logical constants. An ordering for informativeness is quite straightforward: if a proposition X increases the shared knowledge of the participants at least as much as another proposition Y, X is as least as informative as Y. This will be the case when X is a subset of Y, and therefore one can straightforwardly define the informativeness order of classical propositions in terms of the subset relation. On this setting, conjunction should be seen as standing for an operation J such that, given a pair of propositions $\{X, Y\}$, $J(\{X, Y\})$ is a proposition at least as informative as both X and Y, and not more informative than any of the propositions which are at least as informative as both X and Y. That is, given any pair of classical propositions $\{X, Y\}$, conjunction should stand for the *join* of X and Y, that is, for the least upper bound of the pair with respect to the informativeness ordering. As for disjunction, it should stand for an operation M such that, given a pair of propositions $\{X, Y\}$, the operation yields a proposition $M(\{X, Y\})$ such that both X and Y are at least as attentive

as $M(\{X, Y\})$, and there is no proposition Z such that X and Y are at least as attentive as Z, and Z is more attentive than $M(\{X, Y\})$ (this is the greatest lower bound of $\{X, Y\}$ with respect to the ordering, also designated as 'meet'). The other classical connectives can be defined in a similar fashion, the main idea being that each connective should stand for a particular operation on a pair of classical propositions (or unit set, in the case of negation).

The approach has been successfully extended for the case of inquisitive semantics where propositions stand for sets of alternative possibilities. The logical connectives are now taken as standing for operations on sets of propositions based on both an ordering for informativeness and an ordering for inquisitiveness. The ordering for informativeness is inherited from the classical case, the difference being that now it is defined with respect to the informative content of a proposition. That is:

- A proposition X is at least as informative as a proposition Y if and only if $\text{info}(X) \subseteq \text{info}(Y)$

The ordering for inquisitiveness is based on the idea that a proposition X is at least as inquisitive as a proposition Y just in case X requests at least as much information as Y. That is:

- A proposition X is at least as inquisitive as a proposition Y if and only if $\forall x \in X \exists y \in Y : x \cap \text{info}(Y) \subseteq y$

The conjunction of sentences φ and ψ is understood as standing for the join of $[\![\varphi]\!]$ and $[\![\psi]\!]$ with respect to the ordering $\geq_{inf;inq}$ on the set of propositions, an ordering defined in the following way:

- $X \geq_{inf;inq} Y$ if and only if X is at least as informative as Y and at least as inquisitive as Y.

The disjunction of φ and ψ is defined as the meet of $[\![\varphi]\!]$ and $[\![\psi]\!]$ with respect to the ordering, and the meaning of the other logical connectives is taken as consisting in operations on sets of propositions defined in terms of $\geq_{inf;inq}$.

Attentiveness Order

The same approach seems to recommend itself with respect to the extension of inquisitive semantics to attentive content. The point is, for instance, to have conjunction stand for the operation such that, given any pair of propositions $\{X, Y\}$, yields their join with respect to the ordering $\geq_{inf;inq;att}$, where this ordering is defined as follows:

- $X \geq_{inf;inq;att} Y$ if and only if X is at least as informative, at least as inquisitive, and at least as attentive as Y.

What is required, therefore, is an ordering of propositions that captures attentive content and yields the right predictions. Unfortunately, providing an appropriate ordering of attentiveness is not a trivial task. In what follows I will present a possible candidate for the ordering.

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My results will, however, be wholly negative. As it will be shown, the order is not appropriate for the task of providing an algebraic foundation for the meaning of the logical connectives, despite its plausibility as an attentiveness ordering. For this reason, the importance of what follows is to show that plausible path for solving the problem of the meaning of the logical connectives is not one to be pursued.

Attentiveness as detail

Comparison of detail between maps provides an interesting analogy with the attentiveness order to be proposed. Suppose we are presented with two maps, map A and map B, and intend to compare them with respect to detail. In map B one can distinguish the existence of a region of mountains (say, the Himalayas). As for map A, one gets to distinguish, in that region, Mount Everest, Manaslu and Annapurna, something which is not the case with respect to map B. However, map A does not cover the Indian Himalayas, whereas map B covers the whole Himalayan region. If someone wishes to know whether map A has at least as much detail as map B, it makes sense not to impose that A depicts with at least as much detail as B the region of the Indian Himalayas, since A is not a map of that region. What must be ascertained is whether one can discern in B more than what one can discern in A, with respect to the regions covered by map B that are also covered by map A. If this is not the case, then A is a map with at least as much detail as B.

The **attentiveness as detail** proposal on how to order the space of propositions with respect to attentiveness is based on the idea that a proposition X is at least as attentive as a proposition Y just in case it provides at least as much 'detail' into the logical space as that provided by each of the possibilities in Y. That will be the case if for every possibility y in Y there is a subset of X whose union is equal to y. This in agreement with the idea that map A is at least as attentive as map B just in case each of the regions that B maps are mapped by A with more detail.

The only thing that one must be aware of is that the logical space covered by the two propositions might itself be different. When this is the case, the requirement must be that, for each possibility in Y, its *intersection* with the informative content of X is equal to the union of some subset of X. On the map analogy this corresponds to the fact that the regions covered by map B to be compared with respect to detail must be those also covered by A. The proposal is as follows:

AaD (Attentiveness as Detail) $X \geq_{att} Y$ if and only if $\forall y \in Y \exists Z \subseteq X : y \cap \mathrm{info}(X) = \bigcup Z$.

The ordering does seem to have some intuitive plausibility, and yields the positive result that one of the operations on pairs of propositions $\{X, Y\}$ which are candidates for being the meaning of conjunction yields an upper bounds for $\{X, Y\}$ with respect to $\geq_{att}$. The following operations on pairs of propositions have been taken into consideration as plausible accounts of conjunction:

Conj1 Conjunction stands for an operation g such that, given a pair $\{X, Y\}$ of propositions,
$g(\{X, Y\}) = X_Y \cup Y_X$, where

$X_Y = \bigcup_{x \in X} \text{ALT}(\{x \cap y | y \in Y\})$ and

$Y_X = \bigcup_{y \in Y} \text{ALT}(\{y \cap x | x \in X\});$

Conj2 Conjunction stands for an operation h such that, given a pair $\{X, Y\}$ of propositions,
$h(\{X, Y\}) = X_{|Y|} \cup Y_{|X|}$, where

$X_{|Y|} = \{x \cap \text{info}(Y) | x \in X\}$, and

$Y_{|X|} = \{y \cap \text{info}(X) | y \in Y\}.$

$h(\{X, Y\})$ does not constitute an upper bound of X and Y with respect to the ordering $\geq_{inf;inq;att}$. The problem is that there are choices of X and Y such that $h(\{X, Y\})$ is not at least as *inquisitive* as Y (nor as X). Consider the following counterexample:
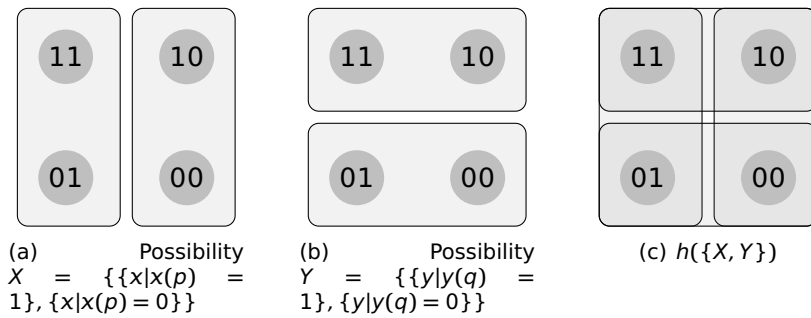

(a) Possibility
$X = \{\{x | x(p) = 1\}, \{x | x(p) = 0\}\}$

(b) Possibility
$Y = \{\{y | y(q) = 1\}, \{y | y(q) = 0\}\}$

(c) $h(\{X, Y\})$

Figure 7: The operation h on the set of propositions $\{X, Y\}$

The possibility $\{x | x(p) = 1\}$ belongs to $h(\{X, Y\})$. However, $\{x | x(p) = 1\} \cap \omega \not\subseteq \{x | x(q) = 1\}$, and $\{x | x(p) = 1\} \cap \omega \not\subseteq \{x | x(q) = 0\}$. Therefore, $h(\{X, Y\})$ is not at least as inquisitive as Y.

The operation $g(\{X, Y\})$ does constitute an upper bound of $\{X, Y\}$ with respect to the ordering $\geq_{inf;inq;att}$.

Proof. It is easy to see that $g(\{X, Y\})$ is both at least as informative and at least as inquisitive as X and Y, for any choices of propositions X, Y.

We will prove the following claim: $\forall x \in X : x \cap \text{info}(g(\{X, Y\})) = \bigcup\{x \cap z | z$ is a possibility in $Y\}$.

Assume $w \in \bigcup\{x \cap y | y$ is a possibility in $Y\}$, for arbitrary w and x. Clearly, if $w \in \bigcup\{x \cap y | y$ is a possibility in $Y\}$, then $w \in x$. Furthermore, if $w \in \bigcup\{x \cap y | y$ is a possibility in $Y\}$, then $w \in x \cap y$ for some possibility $y \in Y$.

Assume $(x \cap y) \in g(\{X, Y\})$. In such case, $w \in \text{info}(g(\{X, Y\}))$. So, assume otherwise. This means that $\exists u \in Y$ such that $(x \cap y) \subset (x \cap u)$, and thus, $w \in \text{info}(g(\{X, Y\}))$. Either way, it follows that $w \in \text{info}(g(\{X, Y\}))$. From this it follows that $\bigcup\{x \cap y | y$ is a possibility in $Y\} \subseteq x \cap \text{info}(g(\{X, Y\}))$.

Assume $w \in x \cap info(g(\{X, Y\}))$, for an arbitrary w.

This means that $w \in x \cap u$ for some possibility $u \in g(\{X, Y\})$. If $u = x \cap y$ for some possibility $y \in Y$, then $w \in \bigcup \{x \cap y|$ is a possibility in $Y\}$, and therefore:

$x \cap info(g(\{X, Y\})) \subseteq \bigcup \{x \cap y|y$ is a possibility in $Y\}$.

Assume otherwise. In such case, $u = (s \cap y)$ for some possibilities $s \in X$ and $y \in Y$. From this it follows that $(x \cap u) \subseteq (x \cap y)$ for some possibility $y \in Y$, and thus:

$w \in \bigcup \{x \cap y|y$ is a possibility in $Y\}$. Hence, $x \cap info(g(\{X, Y\})) \subseteq \bigcup \{x \cap y|y$ is a possibility in $Y\}$.

Therefore, $x \cap info(g(\{X, Y\})) = \bigcup \{x \cap y|y$ is a possibility in $Y\}$. This proves that $g(\{X, Y\}) \geq_{att} X$. By similar reasoning it can be proved that $g(\{X, Y\}) \geq_{att} Y$, and thus that $g(\{X, Y\})$ is an upper bound for $\{X, Y\}$ with respect to the ordering $\geq_{att}$.

$\square$

This result sounds promising. In order for $g(\{X, Y\})$ to be a least upper bound of $g(\{X, Y\})$ with respect to $\geq_{inf;inq;att}$, the following two conditions must hold:

1. $g(\{X, Y\}) \geq_{inf;inq;att} X$ and $g(\{X, Y\}) \geq_{inf;inq;att} Y$;

2. For any propositions Z such that $Z \geq_{inf;inq;att} X$ and $Z \geq_{inf;inq;att} Y$, $Z \geq_{inf;inq;att} g(\{X, Y\})$

I have proved that the first condition holds. However, the second condition does not hold. The problem lies in that there are choices of X and Y such that there are propositions $Z \geq_{inf;inq;att} X$ and $Z \geq_{inf;inq;att} Y$, and $Z \ngeq_{att} g(\{X, Y\})$, from where it follows that $Z \ngeq_{inf;inq;att} g(\{X, Y\})$. The counterexample is provided by the propositions $X = \{\{3, 5\}, \{2, 6\}, \{2, 4, 5, 7, 8\}\}$, $Y = \{\{2, 4\}, \{5, 7\}, \{2, 3, 5, 6, 8\}\}$, $Z = \{\{2, 4\}, \{5, 7\}, \{3, 5\}, \{2, 6\}\}$, where indexes are named by natural numbers from 2 to 8.

Proof. $info(Z) = \{2, 3, 4, 5, 6, 7\}$, while $info(X) = \{2, 3, 4, 5, 6, 7, 8\} = info(Y) = \{2, 3, 4, 5, 6, 7, 8\}$. Thus, $info(Z) \subseteq info(X)$ and $info(Z) \subseteq info(Y)$. Hence, $Z \geq_{inf} X$ and $Z \geq_{inf} Y$.

$\{2, 4\} \subseteq \{2, 4, 5, 7, 8\}$, $\{5, 7\} \subseteq \{2, 4, 5, 7, 8\}$, $\{3, 5\} \subseteq \{3, 5\}$, $\{2, 6\} \subseteq \{2, 6\}$. Hence, $Z \geq_{inq} X$.

$\{2, 4\} \subseteq \{2, 4\}$, $\{5, 7\} \subseteq \{5, 7\}$, $\{3, 5\} \subseteq \{2, 3, 5, 6, 8\}$, $\{2, 6\} \subseteq \{2, 3, 5, 6, 8\}$. Hence, $Z \geq_{inq} Y$.

$\{3, 5\} \cap info(Z) = \{3, 5\} = \bigcup \{\{3, 5\}\}$, $\{2, 6\} \cap info(Z) = \{2, 6\} = \bigcup \{\{2, 6\}\}$ $\{2, 4, 5, 7, 8\} \cap info(Z) = \{2, 4, 5, 7\} = \bigcup \{\{2, 4\}, \{5, 7\}\}$. Hence, $Z \geq_{att} X$.

$\{2, 4\} \cap info(Z) = \{2, 4\} = \bigcup \{\{2, 4\}\}$, $\{5, 7\} \cap info(Z) = \{5, 7\} = \bigcup \{\{5, 7\}\}$, $\{2, 3, 5, 6, 8\} \cap info(Z) = \{2, 3, 5, 6\} = \bigcup \{\{3, 5\}, \{2, 6\}$. Hence, $Z \geq_{att} Y$.

The set $\{2, 5, 8\} \in g(\{X, Y\})$. However, $\{2, 5, 8\} \cap info(Z) = \{2, 5\}$, and there is no $U \subseteq Z$ s.t. $\bigcup U = \{2, 5\}$. Thus, $Z \ngeq_{att} g(\{X, Y\})$.

$\square$

Floris Roelofsen pointed out to me that this result might not immediately exclude the operation g as a candidate for being the meaning of conjunction. His proposal is that attentiveness is, in a sense, determined in function of the informativeness and inquisitiveness of propositions. With respect to conjunction, the idea is that the meaning of conjunction consists on the least upper bound of a pair $\{X, Y\}$ with respect to the ordering of attentiveness, where the set of propositions under consideration is not the set of all propositions, but the set of propositions which constitute least upper bounds with respect to the ordering $\geq_{inf;inq}$.

The proposal seems to provide a quite plausible conception of the meaning of conjunction. However, assuming $\geq_{att}$ as the attentiveness order, the operation g does not provide the meaning of conjunction according to this proposal. The following is a counterexample:

Proof. Let $X = \{\{3, 5\}, \{2, 6\}, \{2, 4, 5, 7\}\}$, $Y = \{\{2, 4\}, \{2, 5, 7\}, \{2, 3, 5, 6\}\}$,

$Z = \{\{3, 5\}, \{2, 6\}, \{2, 4\}, \{2, 5, 7\}$. $g(\{X, Y\}) = \{\{3, 5\}, \{2, 6\}, \{2, 4\}, \{2, 5, 7\}, \{2, 5\}\}$.

Since $\text{info}(X) = \text{info}(Y) = \text{info}(g(\{X, Y\})) = \text{info}(Z)$, it follows that $Z =_{inf} X$, $Z =_{inf} Y$, $Z =_{inf} g(\{X, Y\})$. Furthermore, it is easy to see that $Z \geq_{inq} X$, $Z \geq_{inq} Y$, and $Z =_{inq} g(\{X, Y\})$. Hence, Z is a least upper bound of $\{X, Y\}$ with respect to the ordering $\geq_{inf;inq}$ just in case $g(\{X, Y\})$. If Z is not a least upper bound of $\{X, Y\}$ with respect to the ordering $\geq_{inf;inq}$, then $g(\{X, Y\})$ isn't as well, and therefore g does not correspond to the meaning of conjunction according to the proposal being assumed. Assume therefore that Z is a least upper bound of $\{X, Y\}$ with respect to $\geq_{inf;inq}$.

$Z \geq_{att} X$, and $Z \geq_{att} Y$. Hence, Z is an upper bound of $\{X, Y\}$ with respect to the ordering $\geq_{att}$ on the set of propositions which are least upper bounds of $\{X, Y\}$ with respect to the ordering $\geq_{inf;inq}$ on the set of all propositions. One can check that the same holds for $g(\{X, Y\})$. However, $g(\{X, Y\}) \geq_{att} Z$, but $Z \not\geq_{att} g(\{X, Y\})$, since $\{2, 5\} \cap \{2, 3, 4, 5, 6, 7\} = \{2, 5\} \neq U$, for all $\bigcup U \subseteq Z$. Hence, $g(\{X, Y\})$ is not the least upper bound of $\{X, Y\}$ with respect to the ordering $\geq_{att}$ on the set of least upper bounds of $\{X, Y\}$ with respect to the ordering $\geq_{inf;inq}$, which shows that g does not provide the required operation. □

$\geq_{att}$ is not appropriate

Prima facie, the above result just shows that the meaning of conjunction is not given by the operation g. Recently, Roelofsen has given the example of two propositions X and Y for which: i) there is no least upper of $\{X, Y\}$ with respect to the order $\geq_{inf;inq;att}$; ii) there is no least upper bound of $\{X, Y\}$ with respect to the order $\geq_{att}$, restricted to the set of least upper bounds of $\{X, Y\}$ with respect to the order $\geq_{inf;inq}$. Furthermore, the point of the example is that there are two propositions which are equally plausible candidates for being, respectively, a least upper bound of $\{X, Y\}$ with respect to the order $\geq_{inf;inq;att}$ and a least upper bound of $\{X, Y\}$ with respect to the order $\geq_{att}$, restricted to the set of least upper bounds of $\{X, Y\}$ with respect to the order $\geq_{inf;inq}$, and none of those propositions is at least as attentive as the other.

The upshot of the example is thus that $\geq_{att}$ is not an appropriate order for attentiveness for the purposes of grounding the meaning of the logical constants, since it leaves undecided which of two different propositions should be the one corresponding to the conjunction of a set of propositions $\{X, Y\}$.

Consider propositions X and Y:

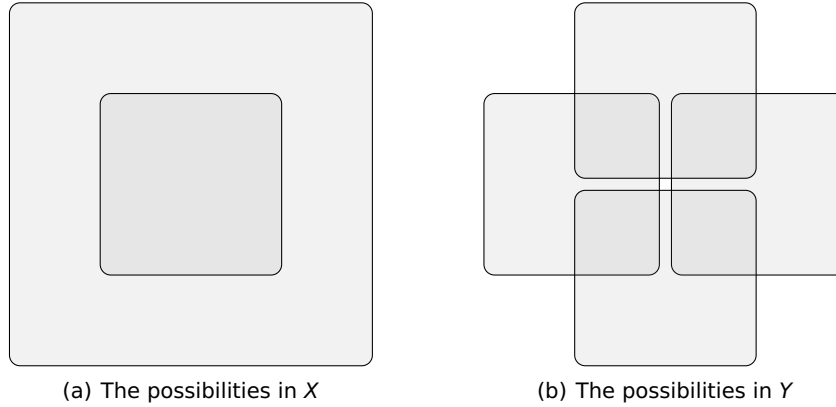(a) The possibilities in X (b) The possibilities in Y

Figure 8: Propositions X and Y

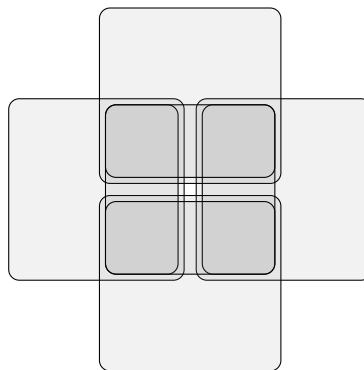The proposition $g(\{X, Y\})$ is as follows:

Figure 9: Proposition $g(\{X, Y\})$

The following propositions W and Z belong to the set of upper bounds of $\{X, Y\}$ with respect to $\geq_{inf;inq;att}$, and to the set of least upper bounds of $\{X, Y\}$ with respect to $\geq_{inf;inq}$, and are strictly more attentive than $g(\{X, Y\})$:

Furthermore, W (Z) is not at least as attentive as Z (W), and if Z (W) was not an element of the set of upper bounds of $\{X, Y\}$ with respect to $\geq_{inf;inq;att}$/the set of upper bounds of $\{X, Y\}$ with respect to $\geq_{att}$, restricted to the set of least upper bounds of $\{X, Y\}$ with respect to the order $\geq_{inf;inq}$, then W (Z) would be a least upper bound of $\{X, Y\}$ with respect to $\geq_{inf;inq;att}$/ a least upper bound of $\{X, Y\}$ with respect to the order $\geq_{att}$ restricted to the set of least upper bounds of $\{X, Y\}$ with respect to the order $\geq_{inf;inq}$. W and Z are, therefore, equally plausible candidates for being the meaning of conjunction, and the order is not 'strong' enough as to
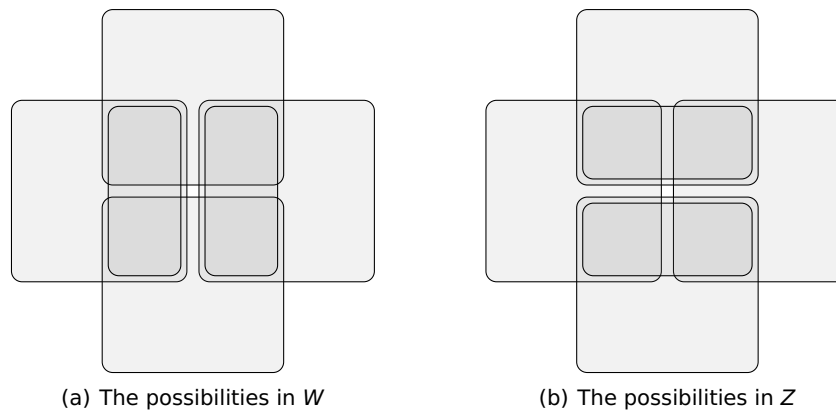
(a) The possibilities in *W*   (b) The possibilities in *Z*

Figure 10: Propositions *W* and *Z*

show that only one of those propositions should constitute the meaning of conjunction. Hence, the order is not appropriate for this purpose.

Conclusion

The main aim of this essay was to address some foundational problems in inquisitive semantics that arise once the framework is extended to deal with attentive content. Two main problems were addressed: i) how the aim of speech-acts of drawing attention should be conceived of in order for inquisitive semantics' approach to meaning to still be understood as a weak dynamic approach; ii) what should be the meaning of the logical constants in the extension of inquisitive semantics to attentiveness.

In answer to the first problem speech-acts of drawing attention were seen as having as their purpose to enhance the shared epistemic state. This has required some reformulation of the object in which speech-acts are intended to act, with a shift from the shared knowledge of the speakers to the shared epistemic state of the speakers being necessary. It has also made salient the need to provide an appropriate representation of a shared epistemic state, and an account of how the relevant speech-acts (those of proposing and of drawing attention) should update shared epistemic states.

It was proposed that, for this purpose, an appropriate representation of the shared epistemic state consists in a pair where the first coordinate is the participants common ground and the second coordinate is the set of propositions to which attention has been drawn to. Two different views on how the shared epistemic state is updated by acts of drawing attention were distinguished, APV and PaV, and PaV was argued to be a more appropriate view.

Still in relation to the first problem, I have argued that the framework devised in order to account for how speech-acts of drawing attention enhance the shared epistemic state allows for an objection to Seth Yalcin's argument that an explanation of what is defective with epistemic contradictions cannot be pragmatic.

Concerning the second problem, in this essay I have adopted an algebraic approach as

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grounding the meaning of the logical connectives. An intuitively plausible attentiveness order, **attentiveness as detail**, was explored, with the prospects that it could provide the needed rationale. I have showed that such ordering does not justify none of the two candidate operations for being the appropriate meaning of conjunction. Furthermore, I also presented an argument by Roelofsen to the effect that **attentiveness as detail** does not constitute an attentiveness order capable of grounding the meaning of the logical connectives.

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