Inquisitive Semantics goes Type Theory

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Contents

1 Introduction

Inquisitive semantics is a semantic framework centred around an enriched notion of sentence meaning: under the inquisitive view, the utterance of a sentence in a discourse is a proposal to change the common ground in one of possibly *many different ways*. Compared to the standard picture in dynamic semantics, which equates semantic content with update potential, the inquisitive conception of sentence meaning is thus more differentiated: it does not reduce this meaning to only *one* unique update, but allows one and the same sentence to express *several alternative* updates. For this reason, the concept of semantic alternatives (Hamblin 1973), which has been fruitfully explored in formal semantics (among others, Karttunen 1977; Rooth 1985; Kratzer and Shimoyama 2002; Menéndez-Benito 2005), is quasi *built into* the inquisitive notion of semantic content.

We will show how this fact can be exploited and how a compositional framework for alternative semantics can be constructed based on the inquisitive conception of sentence meaning. This framework will be somewhat akin to those in the spirit of Hamblin (1973), but—or so we claim—conceptually more solidly founded and technically less troubled. In particular, our approach circumvents problems that have previously been noted to arise from the interplay of alternative semantics and variable binding (Shan 2004; Romero and Novel 2013).

The rest of the paper is organised as follows. Section 2 introduces the central ideas of inquisitive semantics and presents an inquisitive semantics for first-order logic as an illustration. Section 3 translates these ideas into a type-theoretical setting and demonstrates how this system can handle a range of empirical phenomena. Section 4 compares the inquisitive framework with systems in the tradition of Hamblin semantics. Section 5 concludes.

2 The inquisitive conception of sentence meaning

In formal semantics, the meaning of a sentence is classically modelled as a set of possible worlds—namely those worlds that are compatible with the statement made by the sentence. In a dynamic framework, this conception directly relates to updating the common ground, which itself is modelled as a set of worlds—namely those worlds compatible with what is commonly known among the discourse participants. Adding new information to the common ground then amounts to eliminating from it all worlds that are not contained in the proposition[1] expressed by the sentence.

[1]The way in which the term *proposition* is used in inquisitive semantics deviates from the standard usage of this term. To preserve clarity, throughout this text, *proposition* will only refer to the classical understanding of a proposition as a set of worlds. In an inquisitive context, this term will be avoided altogether. Instead, we will just talk about *sentence meaning* or about the *denotation* of a sentence. Forestalling a bit, the sets of worlds contained in an inquisitive sentence meaning will be called *states*. The maximal elements among these states will be referred to interchangeably as *possibilities* or *alternatives*. Additionally, however, *alternatives* will be used in the sense of *Hamblin alternatives*. Which sense applies, should either emerge from the context or be negligible.

Inquisitive semantics departs from this picture: The meaning of a sentence is perceived not as a direct update of the common ground, but as a *proposal* to do so in one of possibly many different ways. Under this view, a sentence denotes a set of alternative *states*, which are themselves sets of worlds. Each of these states represents one possible way of changing the common ground. Utterances are then conceived as having a two-fold effect: the speaker can use them both to *convey* information and/or to *request* information. As in the classical setting, conveying information amounts to locating the actual world w_0 in a subset of all possible worlds. We will return to the corresponding notion of a sentence's *informative content* in section 2.2; what interests us at this point is the *requesting* of information.

If a speaker requests information, this means he asks the other discourse participants to locate the actual world more precisely within the already established common ground. More specifically, through an utterance of φ, he invites a reply that locates w_0 in one of the states in the denotation of φ. Any answer meeting this request will be said to *resolve* or *settle* the issue raised by φ. It is clear that, if an answer resolves an issue, then a more informative answer will also resolve this issue. Translated into the inquisitive setting—if locating w_0 in a state s in the meaning of φ resolves the issue raised by φ with sufficient precision, then locating w_0 in a subset $t \subseteq s$ will be precise enough as well. For this reason, inquisitive sentence meanings are not just sets of states, but *downward closed* sets of states: if a state is contained in a meaning, then all its subsets will be, too. The maximal elements among the states are called *possibilities* or *alternatives*; they correspond to the information that is *minimally* required to settle the issue.

In this sense, possibilities can be taken to constitute something like *basic answers*—a concept also found in frameworks for alternative semantics in the spirit of Hamblin (1973). In Hamblin systems, questions denote sets of alternative *basic answers* to the question. There hence seems to be a close connection between inquisitive and Hamblin frameworks. But although their respective notions of what constitutes an answer can be made to coincide, both systems approach the concept of answerhood from different angles. In inquisitive semantics, it is the notion of resolution which is conceptually prior and from which the notion of answerhood is derived. This setup lends a certain flexibility to the system: it can cater to basic answers in the Hamblin sense as well as to conceptions of answerhood deviating from this (e. g. exhaustive answers). In contrast, Hamblin semantics takes the notion of basic answers as conceptually prior—where basic answers are simply the propositions contained in the denotation of a question. We will return to these issues when comparing our type-theoretical inquisitive framework to Hamblin semantics in section 4.

2.1 An inquisitive semantics for predicate logic

We start by looking at a basic inquisitive semantics (the so-called system InqB) for propositional logic. In due course, it will be extended to the first-order setting. Importantly, note that this semantics merely serves as an *illustration* of the ideas underlying inquisitive frameworks. It will *not* be used in the eventual type-theoretical system which we will proceed to develop in the following sections. However, the setup of that type-logical system will be directly influenced by the same conception of sentence meaning that is also the basis for the InqB-semantics. Many of the type-theoretical lexical entries we devise in section 3 will therefore be strongly reminiscent of the clauses below.

In (1), the inquisitive meaning $[\varphi]$ of a propositional sentence φ is defined recursively, making use of basic algebraic operations. We will leave most clauses uncommented and mostly focus on the atomic case and the semantics for disjunction. In particular, we will not expand on the clause for implication.[2] For a detailed exposition of InqB, consult Ciardelli et al. (2012).

(1) Inquisitive semantics for a propositional language

1. $[p] := \wp(|p|)$
2. $[\bot] := \{\emptyset\}$
3. $[\varphi \wedge \psi] := [\varphi] \cap [\psi]$
4. $[\varphi \vee \psi] := [\varphi] \cup [\psi]$
5. $[\varphi \to \psi] := [\varphi] \Rightarrow [\psi]$
6. $[\neg\varphi] := [\varphi]^* = \wp(\overline{\bigcup[\varphi]})$

There are two things to say about the **atomic case**. Firstly, note that in order for a set of worlds to be contained in $[p]$, p has to be true at every world in that set. We will use this insight extensively when defining type-theoretical translations in section 3. Secondly, the denotation of p comprises the truth set for p *as well as* all subsets of this truth set. Through the recursive definition of the non-atomic cases, this *downward closedness* pertains to inquisitive sentence meanings in general. It is an important design feature of certain inquisitive systems—having repercussions on the treatment of many sentence connectives. We will discuss this in detail when comparing Hamblin and inquisitive frameworks in section 4.

The treatment of **disjunction** is the decisive feature of inquisitive semantics that gives rise to the formation of alternative states. We obtain the denotation of a disjunction $\varphi \vee \psi$ simply by taking the union of the denotations $[\varphi]$ and $[\psi]$. Since these are sets of world-sets, their union will be, too. To see how this treatment differs from that in classical logic, consider the classical notion of the *truth set* of a sentence φ: it is the set of all worlds in which φ is classically true. If worlds are represented as propositional valuation functions v, then this amounts to the following definition.

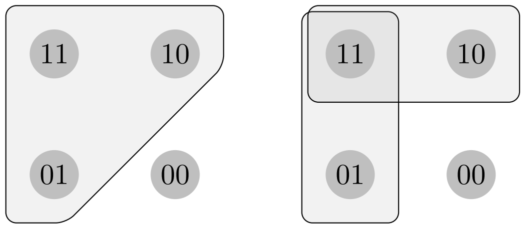
(2) $|p| = \{v \mid v(p) = 1\}$

Now, in order to form the classical truth set $|\varphi \vee \psi|$ of a disjunction, we take the union $|\varphi| \cup |\psi|$ of the disjuncts' truth sets. Hence, $|\varphi| \cup |\psi|$ is plainly a set of worlds

[2] The $\Rightarrow$-operation used in that clause denotes *relative pseudo-complementation*, which in our setting can be defined the following way: $A \Rightarrow B = \{\, s \mid \text{for every } t \subseteq s, \text{ if } t \in A \text{ then } t \in B\}$.

without any further structure imposed on it. In contrast, the union $[\varphi] \cup [\psi]$ of two denotations in inquisitive semantics is a set of *sets* of worlds. It has an internal structure with (usually) at least two alternatives.

This difference becomes clear from figure 1, where both a classical truth set $|p \vee q|$ and an inquisitive sentence meaning $[p \vee q]$ are depicted. To keep pictures like (1b) simpler, all states are left out from the picture that are properly contained in another state; only the possibilities, i. e. the maximal states, are depicted. Comparing 1(a-b), notice that the truth set has no internal structure, while the inquisitive meaning contains two separate world-sets: one such that in all contained worlds p holds, and likewise one for q.



(a) $|p \vee q|$ (b) $[p \vee q]$

Figure 1: Classical and inquisitive treatment of disjunction.

Turning to the clause for **negation**, however, it is this same internal structure of inquisitive meanings which complicates matters somewhat. While, in the classical setting, negation simply amounts to taking the complement set of the original proposition, here, this does not yield the desired results: $\overline{[\varphi]}$, of course, contains all world-sets not in $[\varphi]$—even those that have worlds in common with some state in $[\varphi]$. What we are after for the meaning of $\neg\varphi$ instead, is the set of only those states that do not have *any* overlap with states in $[\varphi]$. This set can be obtained through the algebraic operation of *pseudo-complementation*: $[\neg\varphi] = [\varphi]^* = \wp(\overline{\bigcup[\varphi]})$. Also note that, given this definition, a negated sentence will always only contain a single possibility.

Making only a few modifications, the semantics in (1) can be lifted to suit a first-order language: Possible worlds are no longer valuations, but FOL-models consisting of a domain D and an interpretation function I. The definition of a truth set changes accordingly. The atomic clause is substituted by the analogous one below, and clauses for the quantifiers are added. For us, it is important to observe that the existential quantifier is treated in terms of a large disjunction—with each disjunct corresponding to one way of instantiating the variable with an individual from the domain. Due to this disjunctive semantics, the meaning of existentially quantified sentences often contains more than one possibility.[3] Analogously, the universal quantifier is treated in terms of a large conjunction. Whether the denotation of a universally quantified sentence $\forall x \varphi(x)$ contains more than one possibility, depends

[3]This is not *always* the case, though: if the domain contains only one individual, there will clearly be only one possibility; and, more interestingly, if there are no maximal states in the sentence denotation, there will not be any possibilities at all (see Ciardelli 2010a).

on the nature of φ; in contrast to existential quantification, universal quantification itself does not create different states in the denotation of the quantified sentence.

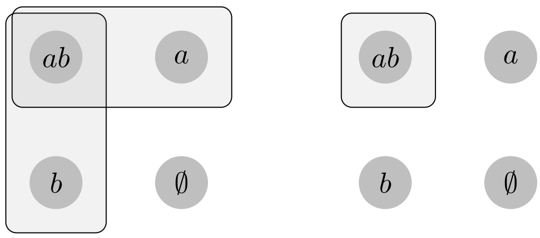
(3) Inquisitive semantics for a first-order language

1'. $[R(t_1,\ldots,t_n)] := \wp(|R(t_1,\ldots,t_n)|)$

7. $[\forall x\varphi(x)] := \bigcap_{d\in D}[\varphi(\overline{d})]$

8. $[\exists x\varphi(x)] := \bigcup_{d\in D}[\varphi(\overline{d})]$

To see the connection even more clearly, take each world to be a FOL-model $\mathcal{M}_{D'} = \langle D, I_{D'}\rangle$ where $D' \subseteq D$. Further let the domain $D = \{a, b\}$ be shared by all such models. Then, $[\exists xP(x)]$ can be depicted in just the same way as $[p\vee q]$ in figure (1b) above (repeated here in figure 2(a)): in world $\mathcal{M}_{ab}$, both $a \in I_{ab}(P)$ and $b \in I_{ab}(P)$; in world $\mathcal{M}_a$, only $a \in I_a(P)$, but $b \notin I_a(P)$; and so on. Thus, the two possibilites for $\exists xP(x)$ directly correspond to two ways of instantiating the existential statement. The denotation of $\forall xP(x)$ on the other hand contains only one possibility, namely the intersection of all possibilities for $\exists xP(x)$. We will make use of this view on quantification when defining translations for various natural language expressions in the next section.



(a) $[\exists xP(x)] = [Pa \vee Pb]$ (b) $[\forall xP(x)] = [Pa \wedge Pb]$

Figure 2: Existential and universal quantification

2.2 Informativeness and inquisitiveness

As already outlined above, we conceive utterances in a discourse as having a two-fold effect: on the one hand, the speaker can *convey* information; on the other hand, she can *request* information. With notions like sentence meaning and alternatives now in place, it is easy to formally describe this double sidedness.

Conveying information amounts to locating the actual world w_0 in a subset of all possible worlds. By uttering φ, a speaker expresses that w_0 is located in at least one of the states in $[\varphi]$, that is, within $\bigcup[\varphi]$. We call this union the *informative content* $\mathsf{info}(\varphi)$ of φ.

(4) $\mathsf{info}(\varphi) = \bigcup[\varphi]$

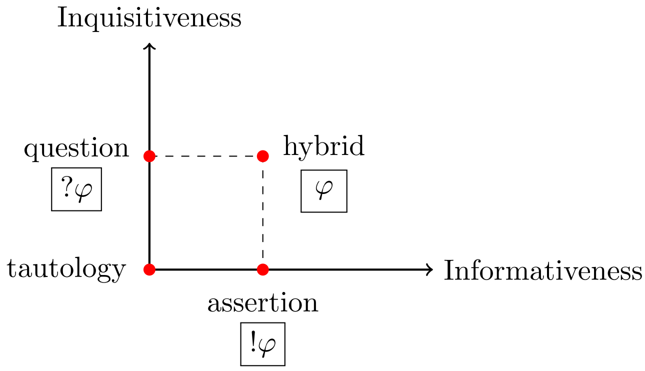


Figure 3: The different sentence types in a two-dimensional space

In contrast, if a speaker requests information, he asks the other discourse participants to locate the actual world more precisely within $\mathsf{info}(\varphi)$, namely, to locate it in one of the states in $[\varphi]$.

This conception suggests a natural way to characterise sentences along two dimensions: *inquisitiveness* and *informativeness*. We call a sentence φ *inquisitive* if its informative content $\mathsf{info}(\varphi)$ is not contained in $[\varphi]$. Intuitively, such a sentence requests information, but does not provide enough information itself to satisfy this request. Any sentence whose meaning contains at least two possibilities is inquisitive.[4] Along with inquisitiveness comes the related notion of *informativeness*: intuitively, an informative sentence is one that conveys new information. Formally, this means it has the potential to eliminate worlds from the common ground. For a sentence φ to have this potential, $\mathsf{info}(\varphi)$ must be a proper subset of the set of all possible worlds ω.

(5) φ is *inquisitive* iff $\mathsf{info}(\varphi) \notin [\varphi]$.
φ is *informative* iff $\mathsf{info}(\varphi) \neq \omega$.

It is important to note that this distinction is just a terminological one and does not determine any specific discourse-theoretical interpretation of inquisitiveness. Here, we will endorse what has been coined the *strong* perspective on inquisitiveness (Ciardelli et al. 2012 : 42): in uttering a sentence φ, a speaker *always* requests a response which contains sufficient information in order to locate the actual world in one of the states in $[\varphi]$. If φ is not inquisitive to begin with, this locating-task is trivial, and the utterance does not "actually" request information.

2.3 Declarative and interrogative projection

As illustrated in figure 3, the binary properties inquisitiveness and informativeness span a two-dimensional space. Four different types of sentences can be distinguished

[4] However, the converse does not hold (Ciardelli 2009, 2010b).

in this space: *questions*, which are purely inquisitive, *assertions*, which are purely informative, *hybrids*, which are both informative and inquisitive, and finally *tautologies*, which are neither. Our introductory example, $\varphi := p \vee q$, is a hybrid. Turning it into a question can be thought of as projecting it onto the inquisitiveness axis; turning it into an assertion analogously as a projection onto the informativeness axis. We add operators ? and ! to our logical language, and denote the non-informative (*interrogative*) projection as $?\varphi$, the non-inquisitive (*declarative*) projection as $!\varphi$.

We already know a way to obtain the declarative projection $!\varphi$: the informative content $\mathsf{info}(\varphi)$ contains exactly those worlds in which φ holds. The powerset $\wp(\mathsf{info}(\varphi))$ therefore conveys exactly the same information as $[\varphi]$—without however being inquisitive. This makes it straightforward to add the required clause to the existing semantics in (3).

(6) Semantics of the declarative projection:

9. $[!\varphi] := \wp(\mathsf{info}(\varphi))$

For the interrogative projection, we need to (i) turn a sentence φ into a question, that is, a non-informative sentence—while (ii) preserving its inquisitive content *as much as possible.* To accomplish (i), we have to ensure that $\mathsf{info}(\varphi)$ and ω coincide:

(7) φ is a question iff $\mathsf{info}(\varphi) = \omega$.

This also means, however, that the interrogative projection cannot just leave the inquisitive content completely unaltered. Recall that the informative content is defined as $\mathsf{info}(\varphi) = \bigcup[\varphi]$. Thus, if we augment the informative content, the inquisitive content will necessarily change with it. What we *can* do, though, is to keep intact the *decision set* of φ, that is, the set of those pieces of information which *decide* on the issue raised by φ. A piece of information is said to decide on an issue if it either resolves the issue (by locating the actual world in one of the states in $[\varphi]$) or *dismisses* it (by locating the actual world outside of any state in $[\varphi]$). Hence, we need to define the interrogative projection $?\varphi$ in such a way that a piece of information decides on the issue raised by $?\varphi$ just in case it also decides on the issue raised by φ.

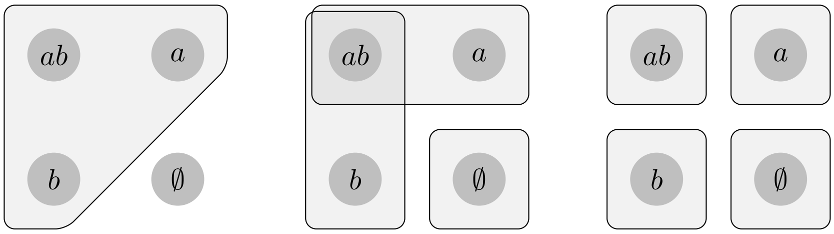
Locating the actual world outside the possibilities in $[\varphi]$ means locating it in one of the states in $[\neg\varphi] = [\varphi]^*$. Taking the union of $[\varphi]$ and $[\varphi]^*$ therefore allows us to obtain a set of possibilities which exhaustively covers ω while also preserving the decision set of φ.

(8) Semantics of the interrogative projection:

9. $[?\varphi] := [\varphi \vee \neg\varphi] = [\varphi] \cup [\varphi]^*$

There is a natural way to strengthen the interrogative projection, namely by universal quantification: the sentence $\forall x ?\varphi(x)$ denotes a partition on ω such that, within each block of this partition, exactly the same individuals have property φ. We can hence understand $\forall x?\varphi(x)$ as a more demanding question than $?\varphi$—it asks for an exhaustive specification of the property φ.

Wrapping up this brief introduction to inquisitive semantics, the interplay of different projection operators and quantifiers is exemplified in figure 4. We will encounter type-theoretical counterparts of these constructions when computing the meaning of declarative and interrogative natural language sentences in the next section.



(a) $[!\exists x P(x)]$ (b) $[?\exists x P(x)]$ (c) $[\forall x?P(x)]$

Figure 4: Informative and interrogative projections in combination with quantifiers

3 A type-theoretic inquisitive grammar fragment

3.1 Framework

In this section, we will spell out a two-step approach towards a compositional semantic treatment of natural language: first, English sentences are *translated* to an intensional type-theoretic language; then, the expressions in this language receive a model-theoretic *interpretation*. It is only the first step whose implementation reflects the inquisitive notion of meaning; the model-theoretic interpretation in the second step proceeds classically.[5] This two-step approach is in line with work in the tradition of Montague (1973), but differs from the *direct* method adopted in Heim and Kratzer (1998) and a large body of generatively oriented work. We chose to adopt this setup for two reasons: firstly, we hope that, without merging two steps into one, the connection between the translation step and the inquisitive conception of meaning will be more transparent. Secondly, being able to rely on the semantics of a well-defined logical language will make it easier to specify translation rules and grammar entries with a sufficient level of formal precision.

The syntactic structures driving the translation process, however, are broadly generative. In particular, we derive different scope configurations based on syntactic movement: the displaced constituent is assumed to have an index λ_i sitting directly underneath its landing site and to leave a co-indexed trace t_i at its original position. Traces and pronouns are translated as variables in the type-theoretic language.

We further assume that declaratives contain a covert declarative marker $\textsc{m}_!$, *wh*-interrogatives a covert interrogative marker $\textsc{m}_?$ and polar interrogatives a covert

[5]For syntax and semantics of the intensional theory of types, see appendix A.

polar interrogative marker $\textsc{m}_{?!}$, situated at the top of their syntactic structure respectively. To make sure that *wh*-question-words can only occur in *wh*-interrogatives, they are assumed to carry a feature which needs to be checked with $\textsc{m}_{?}$. Likewise, polar question morphology is only licensed in sentences headed by $\textsc{m}_{?!}$.

3.2 Lexicon

We start in the thick of things and directly specify the grammar fragment (that is, the translation function *Tr* from natural language to type-theoretical logical language) to be used in the rest of the paper. We then spell out the rationale behind it and explain some of the translations in detail. The grammar will be able to handle the following range of constructions and phenomena:

1. declaratives (*John smiled*, *John saw Mary*)
2. negated declaratives (*John did not smile*, *John did not see Mary*)
3. *wh*-questions (*Who smiled?*, *Who saw Mary?*), including in-situ *wh*-questions (*Mary saw whom?*) and multiple *wh*-questions (*Who saw whom?*)
4. polar questions (*Did John call?*)
5. inverse quantifier scope (*Some students were assigned to every project*)
6. bound variable pronouns (*Everybody phoned his mother*)

Lexical entries for all the relevant syntactic categories are listed in table 1. Although we will see that the system at hand produces results in the spirit of an alternative semantics, no special rules are needed to compute the meaning of a sentence; the derivation is driven by the ordinary rules for *functional application* and *predicate abstraction* in (9). Functional application can be considered the default case of semantic composition: The translation $Tr(\beta)$ of a subtree is applied to the translation $Tr(\alpha)$ of its sister subtree. Which subtree acts as the function and which as the argument is determined by their types. In contrast, predicate abstraction is triggered by the presence of an index λ_i in the syntactic structure: all free occurrences of the variable x_i within $Tr(\beta)$ are λ-bound.

(9) a. **Functional application (FA):**
If α is a branching node and $\{\beta, \gamma\}$ the set of its daughters, then $Tr(\alpha)$ is defined if $Tr(\beta)$ and $Tr(\gamma)$ are defined and $Tr(\beta)$ is of type $\langle\sigma, \tau\rangle$ and $Tr(\gamma)$ is of type σ. In this case, $Tr(\alpha) = Tr(\beta)(Tr(\gamma))$.

$$Tr(\alpha) = Tr(\beta)(Tr(\gamma)) :: \tau$$

$$Tr(\beta) :: \langle\sigma, \tau\rangle \qquad\qquad Tr(\gamma) :: \sigma$$

b. **Predicate abstraction (PA):**
If α is a branching node whose daughters are the movement index λ_i and β, then $Tr(\alpha)$ is defined if $Tr(\beta)$ is defined. In this case, $Tr(\alpha) = \lambda x_i.Tr(\beta)$.

$$\begin{array}{c} Tr(\alpha) = \lambda x_i.Tr(\beta) :: \langle e, \tau\rangle \\ \begin{array}{cc} \lambda_i & \quad Tr(\beta) :: \tau \end{array} \end{array}$$

3.3 Sentences

As we have seen above, in inquisitive semantics the meaning of a sentence is represented as a set of states, i. e. a set of sets of worlds. In terms of semantic types, this means that sentences have type $\langle\langle s,t\rangle,t\rangle$. We will abbreviate this type as T.

While this notion of alternativehood at sentence level is roughly shared by both inquisitive and Hamblin semantics, we will see that *compositionally* it comes about somewhat differently in either system. In Hamblin semantics, all expressions denote sets, most of which are singleton sets. It is only certain quantification-like elements such as *wh*-phrases that translate as multi-membered sets (in the case of *wh*-phrases, as sets of individuals). Through a special alternative-friendly version of functional application, these sets are combined to compute the entire sentence-meaning. This way, the alternatives percolate upwards in the tree.

[6] The $\boxed{\neg}$ denotes a type-theoretical version of inquisitive negation, which will be introduced in section 3.5.

cat.	α	$Tr(\alpha)$	type
PN	John	$\mathbf{j}$	e
DP	he$_i$/t_i	x_i	e
CN	man	$\lambda x_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to man(w)(x))$	$\langle e,T\rangle$
IV	smiled	$\lambda x_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to smile(w)(x))$	$\langle e,T\rangle$
TV	saw	$\lambda x_e.\lambda y_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to see(w)(x)(y)))$	$\langle e,\langle e,T\rangle\rangle$
DP	who	$\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists xP(x)(p)$	$\langle\langle e,T\rangle,T\rangle$
DP	someone	$\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists xP(x)(p)$	$\langle\langle e,T\rangle,T\rangle$
DP	everybody	$\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\forall xP(x)(p)$	$\langle\langle e,T\rangle,T\rangle$
DP	nobody	$\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\boxed{\neg}\exists xP(x)(p)$[6]	$\langle\langle e,T\rangle,T\rangle$
D	a	$\lambda P_{\langle e,T\rangle}.\lambda P'_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists x(P(x)(p) \wedge P'(x)(p))$	$\langle\langle e,T\rangle,\langle\langle e,T\rangle,T\rangle\rangle$
D	which	$\lambda P_{\langle e,T\rangle}.\lambda P'_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists x(P(x)(p) \wedge P'(x)(p))$	$\langle\langle e,T\rangle,\langle\langle e,T\rangle,T\rangle\rangle$

Table 1: Exemplary translations with their types

In inquisitive semantics, on the other hand, the conception of sentence meanings as sets of alternative states lies at the very foundation of the logical framework, predetermining the way in which we will capture the meaning of a sentence. For the type-theoretic system laid out here, one might even say that the only “meaningful” alternativehood exists at sentence level. The denotations of all subexpressions derive from that sentence-meaning. But let us now see what this means in practice.

Recall from section 2 that the denotation of a sentence $R(t_1, \ldots, t_n)$ contains all subsets of $|R(t_1, \ldots, t_n)|$. Accordingly, we will now let a sentence denote all those states p in which the predication expressed by the sentence holds in every world in p. For example, the sentence *John smiled* denotes the set of all states p such that John smiled in every world contained in p. Once we have pinpointed this denotation, the translations of the subexpressions fall into place, too.

(10) $Tr(\text{John smiled}) = \lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \mathit{smile}(w)(\mathbf{j}))$

3.4 Verbs

Intransitive verbs like *smile* need to combine with an individual to form a sentence. This means they have type $\langle e, T\rangle$. Their denotation differs from that of the entire sentence only in so far that the subject variable is lambda-bound:

(11) $Tr(\text{smiled}) = \lambda x_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \mathit{smile}(w)(x))$

Analogously, transitive verbs, expecting two individuals, are of type $\langle e, \langle e, T\rangle\rangle$:

(12) $Tr(\text{saw}) = \lambda x_e.\lambda y_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \mathit{see}(w)(x)(y))$

The type conflict that would ensue through quantified DPs in object position is circumvented by assuming that these DPs move out of the VP to a landing site just above the subject. Their movement index triggers a predicate abstraction, ensuring that the displaced constituent fills in the object argument slot (see section 3.6).

3.5 DP-like phrases

The translations of all DP-like constituents fall into two categories, depending on their semantic type. While proper names and pronouns are directly translated as logical constants and variables of type e, quantifiers, *wh*-phrases and actual determiner phrases have denotations of type $\langle\langle e, T\rangle, T\rangle$.

Proper names, pronouns and traces

In order to see how the grammar up to now can handle simple declaratives with e-type DPs, consider the following derivation. As expected, the sentence meaning is of type T. Note however, that it only contains a single possibility, namely the classical truth set of the sentence.

(13) John saw her.

- T — $\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \textit{see}(w)(x_1)(\mathbf{j}))$
  - John :: e — j
  - $\langle e, T\rangle$ — $\lambda y_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \textit{see}(w)(x_1)(y))$
    - saw :: $\langle e, \langle e, T\rangle\rangle$ — $\lambda x_e.\lambda y_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \textit{see}(w)(x)(y))$
    - her :: e — x_1

Turning to *negated* declaratives, matters are a little more complicated. Uttering (14), a speaker provides the information that the actual world is not located in any state such that John smiled in every world in that state. The speaker does not, however, request any information beyond that. Hence, a negated sentence is not inquisitive.

(14) John did not smile.

Recall that in InqB negation cannot simply be treated as the complement set of the original denotation. Analogously, we cannot naively translate sentence level negation as $\lambda P_T.\lambda p_{\langle s,t\rangle}.\neg P(p)$, since this term would also yield world-sets overlapping with states in P, while what we want is only those world-sets which are completely disjoint from any set in P. As in InqB, we hence need to form the pseudo-complement of P, that is, the set $\wp(\overline{\bigcup P})$. This is formulated in type-theoretic terms in (15).

In order to keep the translations more readable, we will employ a number of inquisitive operators and connectives in our type-logical language that are not part of the syntax of that language, but only serve to abbreviate longer λ-expressions. The first such operator is $\boxed{\neg}$ in (15), denoted by a boxed version of its analogue in InqB. We will stick to this notational convention with the following operators as well.

(15) $\boxed{\neg} := \lambda P_T.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \neg\exists p'(P(p') \wedge p'(w)))$

Finally, the example derivation illustrates how this operator is put to use. To get accustomed to the notation, here, both the $\boxed{\neg}$-abbreviated and the unabbreviated variant of the translations are spelled out. In later examples, we will settle with only the abbreviated notation. Also note that, indeed, the translation of (16) only contains a single possibility (this might be easier to observe when thinking of $\boxed{\neg}$ in terms of $\wp(\overline{\bigcup P})$). Here, this is hardly worth mentioning, in fact, since the corresponding non-negated sentence would not be inquisitive, either; but the observation carries over to sentences with more than one possibility, too.

(16) John did not smile.

T

$$\boxed{\neg}\,(\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \mathit{smile}(w)(\mathbf{j})))$$
$$= \lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \neg\exists p'(\forall w'(p'(w') \rightarrow \mathit{smile}(w')(\mathbf{j})) \wedge p'(w)))$$

not :: $\langle T, T\rangle$

$$\boxed{\neg} = \lambda P_T.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \neg\exists p'(P(p') \wedge p'(w)))$$

T

$$\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \mathit{smile}(w)(\mathbf{j}))$$

John :: e

$\mathbf{j}$

smile :: $\langle e, T\rangle$

$$\lambda x_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \mathit{smile}(w)(x))$$

Quantified DPs and the declarative marker

Moving on to the other types of DPs, we notice that *wh*-phrases (*who*) as well as certain indefinite (*a man*) and quantificational (*someone*) DPs share a common characteristic: their existential semantics. On some level, they all express that some (further specified) individual exists. This meaning is naturally captured by existential quantification:

(17)
$$\begin{aligned} \mathit{Tr}(\text{who/someone}) &= \lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists x P(x)(p) \\ \mathit{Tr}(\text{a/which man}) &= \lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists x(\forall w(p(w) \rightarrow \mathit{man}(w)(x)) \wedge P(x)(p)) \end{aligned}$$

Recall that, in InqB, the disjunctive semantics of existential quantifiers gives rise to alternative possibilities. In some cases, most prominently in questions, this is desired: by asking *Who smiled?*, a discourse participant requests information as to which individuals smiled. The issue raised by her question could be settled in several different ways—with *Mary smiled* or with *John smiled*, but also with *Mary and John smiled*. Each resolving answer should correspond to a state in the translation of *Who smiled?*. On the other hand, sentences like *A man smiled* with only simple non-*wh* determiner phrases do not seem to request information. Since we adopted the strong perspective on inquisitiveness (see section 2.2), we do not want the meanings of those sentences to come out as inquisitive.

Hence, we have to make sure that different denotations are assigned to sentences like *Who smiled?* on the one hand and *Someone smiled* on the other hand. However, the lexical entries in (17) can remain unchanged.[7] Recall that we assumed declarative

[7]This synonymy might have independent motivations, too: cross-linguistically, there is a strong tendency for *wh*- and indefinite pronouns to be morphologically related. On the basis of this, Haida (2007) proposes to assume the same (existential) denotation for both kinds of pronouns.

and interrogative markers to be present in the syntactic structures. It is these markers which will take care of establishing the above distinction.

Syntactically, we assume that the declarative syntactic marker $\textsc{M}_!$ licenses certain lexical elements such as declarative complementisers (*that*). Semantically, its translation is based on the notion of the declarative projection operator ! in InqB. Recall that this operator has the effect of turning the meaning of φ into $\wp(\mathsf{info}(\varphi))$. The same can be expressed in our typed language: $\textsc{M}_!$ denotes a function that takes an inquisitive sentence meaning P and also returns an inquisitive sentence meaning. The latter of these two meanings, however, is non-inquisitive: it simply contains all subsets of $\bigcup P$.

(18) $\quad Tr(\textsc{M}_!) = \lambda P_T.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \exists p'(P(p') \wedge p'(w))) =: \boxed{!}$

To see how the computation of declaratives works out in practice, consider the examples below. In (19), it can be nicely observed how the application of $\boxed{!}$ changes the widest-scoping quantifier from $\exists x$ to $\forall w$—hence folding all states from $\lambda p_{\langle s,t\rangle}.\exists x \forall w(p(w) \to \mathit{smile}(w)(x))$ into one single possibility.

(19) Someone smiled.

T
$$\boxed{!}(\lambda p_{\langle s,t\rangle}.\exists x \forall w(p(w) \to \mathit{smile}(w)(x)))$$
$$= \lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \exists p'(\exists x \forall w'(p'(w') \to \mathit{smile}(w')(x)) \wedge p'(w)))$$

$\textsc{M}_! :: \langle T, T\rangle$
$$\boxed{!} = \lambda P_T.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \exists p'(P(p') \wedge p'(w)))$$

T
$$\lambda p_{\langle s,t\rangle}.\exists x \forall w(p(w) \to \mathit{smile}(w)(x))$$

someone :: $\langle\langle e,T\rangle,T\rangle$
$$\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists x P(x)(p)$$

smiled :: $\langle e,T\rangle$
$$\lambda x_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \mathit{smile}(w)(x))$$

(20) Everybody smiled.

$$T$$
$$\boxed{!}\,(\lambda p_{\langle s,t\rangle}.\forall x \forall w(p(w) \rightarrow \mathit{smile}(w)(x)))$$
$$= \lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \exists p'(\forall x \forall w'(p'(w') \rightarrow \mathit{smile}(w')(x)) \wedge p'(w)))$$

$$\text{M}_! :: \langle T, T\rangle$$
$$\boxed{!} = \lambda P_T.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \exists p'(P(p') \wedge p'(w)))$$

$$T$$
$$\lambda p_{\langle s,t\rangle}.\exists x \forall w(p(w) \rightarrow \mathit{smile}(w)(x))$$

$$\text{everybody} :: \langle\langle e, T\rangle, T\rangle$$
$$\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\forall x P(x)(p)$$

$$\text{smiled} :: \langle e, T\rangle$$
$$\lambda x_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \mathit{smile}(w)(x))$$

Wh-phrases, polar questions and interrogative markers

We want to be able to derive the meaning of *wh*-questions like (21) and polar questions like (22)—both of which we will treat as purely inquisitive, non-informative sentences. The main objective is thus to have their translations reflect the respective set of resolving answers: we need to make sure that there is a one-to-one correspondence between those pieces of information that settle the issue raised by the question and the states contained in the interpretation of the question.

(21) a. Who failed the exam?
b. (Only) John/(Only) Mary/(Only) John and Mary/Everybody/Nobody.

(22) a. Was the exam difficult?
b. Yes (it was difficult)/No (it was not difficult).

Wh-questions, to begin with, can be understood in two different ways: interpreted *exhaustively*, they ask for a complete specification of which individuals have a certain property and which do not; interpreted as *mention-some questions* on the other hand, they only ask for a salient subset of those individuals that do have the property. Under either of these interpretations, people tend to have clear intuitions about which answers resolve the issue raised by the question. Imagine a situation in which a group of students, including John, Mary and others, have taken an exam. Both John and Mary failed it, all the others passed. Under an exhaustive reading of (21a) the only true resolving answer from (21b) will be *Only John and Mary*, whereas under a mention-some reading also either of *John* and *Mary* will truthfully settle the issue.

What resolving answers under both interpretations have in common, however, is that they specify possible instantiations of the existential statement expressed by the *wh*-question. Additionally it is possible under both readings to negate this existential statement (*Nobody*). On these grounds, the already familiar interrogative projection operator ? (recall that in InqB $[?\varphi] = [\varphi \vee \neg\varphi]$) appears well suited for the mention-some case, and the translation of the syntactic interrogative marker $\textsc{m}_?$ can be modelled after the semantics of ?.

(23) $Tr(\textsc{m}_?) = \lambda P_T.\lambda p_{\langle s,t\rangle}.P(p) \vee \boxed{\neg}\, P(p) \quad =: \; \boxed{?}$[8]

(24) Who smiled? [mention-some interpretation]

$$T$$
$$\boxed{?}\,(\lambda p_{\langle s,t\rangle}.\exists x \forall w(p(w) \to \mathit{smile}(w)(x)))$$
$$= \lambda p_{\langle s,t\rangle}.\exists x \forall w(p(w) \to \mathit{smile}(w)(x)) \vee \forall w(p(w) \to \neg\exists p'(p'(w) \wedge \exists x \forall w'(p'(w') \to \mathit{smile}(w')(x))))$$

$\textsc{m}_?$ :: $\langle T, T\rangle$

$\boxed{?} = \lambda P_T.\lambda p_{\langle s,t\rangle}.P(p) \vee \boxed{\neg}\, P(p)$

T

$\lambda p_{\langle s,t\rangle}.\exists x \forall w(p(w) \to \mathit{smile}(w)(x))$

who :: $\langle\langle e, T\rangle, T\rangle$

$\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists x P(x)(p)$

smiled :: $\langle e, T\rangle$

$\lambda x_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \mathit{smile}(w)(x))$

In contrast, the exhaustive interpretation corresponds to partitioning the set of worlds relative to the predication expressed by the question: in our example, this means that worlds in the same partition block would agree exactly on the extension of *smile*. In the context of InqB, we have already seen that such a partition can be induced by combining universal quantification and interrogative projection ($\forall x ? \varphi(x)$). Here, we will use a similar strategy and derive the partition reading from the mention-some interpretation. To this end, we define an exhaustivity marker $\textsc{m}_{\text{exh}}$, which we assume to sit atop the syntactic structure of exhaustive *wh*-questions.

We take the mention-some interpretation to be the default reading of *wh*-questions, since, in our framework, the exhaustive question meaning is easily derivable from the non-exhaustive denotation—but not the other way around: the non-exhaustive meaning has a richer internal structure, and going from non-exhaustive to exhaustive

[8] To make $\boxed{\neg}$ resemble the classical negation symbol more closely, a few brackets have been omitted. With meticulous bracketing, $m_?$ translates as $\lambda P_T.\lambda p_{\langle s,t\rangle}.P(p) \vee (\boxed{\neg}(P))(p)$.

interpretation entails loosing the information about this structure. Some languages, however, both have overt markers for exhaustivity (e. g. *allemaal* in Dutch and *alles* in German; Rullmann and Beck 1996) and for non-exhaustivity (e. g. *zoal* in Dutch and *so* in German; ibid.), which explicitly select for the exhaustive or mention-some reading respectively. In a language with non-exhaustivity markers, the exhaustive interpretation cannot be the default one in our framework. For if it was, we would have no way to derive the non-exhaustive reading of sentences like (25a) which contain a non-exhaustivity marker. A further point in our favour seems to be that adding a non-exhaustivity marker often—such as in (25a)—does *not* seem to change the set of resolving answers, whereas adding an exhaustivity marker often does: in (25b) for example, the unmarked question seems to allow partial answers, but not so the *allemaal*-question.

(25) a. Wie komt er (zoal) langs?
b. Wie komt er (allemaal) langs?

Now, before devising a lexical entry for the exhaustivity operator, let us reflect for a moment on what exactly we require of the partition that $\textsc{m}_{\text{exh}}$ is supposed to induce. One way to think of an exhaustive question like *Who smiled?* is as a conjunction of polar questions, one for each individual: *Did Mary smile, and did John smile, and did Carol smile, and... ?* An exhaustive answer provides an answer to each single one of these polar questions. We hence need to split up ω in such a way that worlds in the same partition block coincide in the answers they give to the polar questions. This condition can be rephrased in terms of what we will call *(true) basic answers*: given a question φ and world w, the (true) basic answers to φ (at w) are the minimally informative states that resolve φ (and contain w). If there are alternatives in the denotation of φ, then these are the basic answers. Obviously, this is the case for *Who smiled?*, where each alternative can be viewed as affirmatively answering one of the polar questions above. What our requirement amounts to for *Who smiled?* is thus the following. Two worlds are in the same partition cell just in case they have the same set of true basic answers. This idea is expressed in the following preliminary lexical entry for $\textsc{m}_{\text{exh}}$. However, the situation is not always that simple, and we will soon move on to a more general setting.

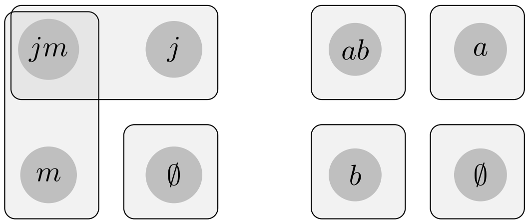
(26)
$$\begin{aligned} & Tr(\textsc{m}_{\text{exh}}) \\ \stackrel{prelim.}{=} \ & \lambda P_T.\lambda p_{\langle s,t\rangle}.\forall w(\forall w'((p(w) \wedge p(w')) \to \textsc{basic}(P)(w) = \textsc{basic}(P)(w'))) \\ \stackrel{prelim.}{=:} \ & \boxed{\text{EXH}} \end{aligned}$$

This translation makes use of the auxiliary predicate $\textsc{basic}$, defined in (27): given a question φ with denotation $Tr(\varphi) = P$ and a world w, $\textsc{basic}(P)(w)$ denotes the set of those basic answers to φ which are true in w. The first two outer conjuncts in the λ-term check whether a state p is contained in P and whether it contains w. The last conjunct takes care of the maximality requirement.

(27) BASIC

$$= \lambda P_T.\lambda w_s.\lambda p_{\langle s,t\rangle}.P(p) \wedge p(w) \wedge \neg\exists p'(P(p') \wedge p \neq p' \wedge \forall w'(p(w') \rightarrow p'(w')))$$

To see how—at least for all examples up to now—this results in the desired interpretation, it might be helpful looking at visualisations of the same kind we used for the sentences in InqB. Consider again a set of worlds $\{\mathcal{M}_{mj}, \mathcal{M}_m, \mathcal{M}_j, \mathcal{M}_\emptyset\}$ such that in $\mathcal{M}_{mj}$ both Mary and John smile, in $\mathcal{M}_m$ only Mary smiles, in $\mathcal{M}_j$ only John smiles and in $\mathcal{M}_\emptyset$ none of them smiles. Under the mention-some reading, (24) can then be depicted as figure 5(a). Applying the $\boxed{\text{EXH}}$-operator to this state set yields the partition in 5(b). Note that in our example the partition blocks are singleton sets since no two distinct worlds share the exact same set of basic answers. This need not always be the case, though.



(a) Who smiled? (b) Who smiled? (exh.)

Figure 5: Visualisation of *wh*-questions under a mention-some and an exhaustive interpretation.

There are two reasons why this version of the exhaustivity operator falls short. Firstly, it does not preserve the informative content of a sentence: consider an informative sentence φ, that is, a sentence φ such that there are worlds $w \notin \mathsf{info}(\varphi)$. Then these same worlds w *will* be contained in $\mathsf{info}(\boxed{\text{EXH}}\,\varphi)$, meaning that $\mathsf{info}(\varphi) \subsetneq \mathsf{info}(\boxed{\text{EXH}}\,\varphi)$. While for questions (which are non-informative sentences) this does not matter, we would in principle want to devise an operator that can exhaustify interrogatives as well as declaratives. One conceivable application of such an operator would be the free choice reading of indefinites (cp. the *exclusiveness operator* in Menéndez-Benito 2005).

The second shortcoming of the preliminary exhaustivity operator has to do with the fact that, by using the concept of minimally informative answers, the operator relies on the existence of maximal states in a sentence denotation. However, it has been brought to attention by Ciardelli (2010a) that, making certain assumptions, there are in fact sentences whose denotations do *not* contain maximal elements. A prominent example of such problematic sentences is the *boundedness formula* $\varphi = \exists x B(x)$. It is true in a model $\mathcal{M}$ precisely if there exists an upper bound for a natural number n, where n is the extension of a predicate N in $\mathcal{M}$. The problems arising from this sentence have to do with the fact that, *assuming standard arithmetic*, if e. g. 1 is a bound for n, then so are 2, 3, 4 and so on. Hence, each of the states that resolve φ by providing an instantiation of the existence statement is properly contained in infinitely many other states which also resolve φ. Owing to this infinite inclusion hierarchy, there are no maximal states and hence no minimally informative answers. The picture emerging from this situation is sketched in figure 6. We can however avoid a structure like this by letting the laws of arithmetic vary from world to world—just as other facts about the world do as well. The crucial implication that, for all natural numbers n, if $B(n)$, then also $B(m)$ for all $m > n$, would no longer hold across worlds. Unfortunately, this might avoid the infinite hierarchy in the case of the boundedness formula; but we cannot be certain that there is no other sentence in our logical language giving rise to a similar structure. Hence, we need to find a more general definition for $\boxed{\textsc{exh}}$, one that is independent of the existence of maximal states.

In particular, we would like to be able to cope with sentences whose denotation has the structure (very roughly) sketched in figure 7: there is not only one infinite state hierarchy, but there are at least two separate ones, each of which we would like to regard as a distinct exhaustive answer. The preliminary exhaustivity operator is not up to this task: it would simply pool together the worlds from both hierarchies, since they all share the common trait of not having any true basic answers. The condition that all worlds in the same partition block need to fulfil thus has be stronger than sharing the same set of true basic answers—those worlds also need to agree on the "very large", but non-maximal states they are contained in. To be more explicit, for two worlds w and w' to be in the same partition cell, the following has to hold. If either of v or w is contained in a state p from the non-exhaustified denotation P, then there has to exist a state $q \in P$ such that $p \subseteq q$ and q contains both w and w'. Clearly, if the denotation has maximal states, this boils down to the condition that $\textsc{basic}(P)(w) = \textsc{basic}(P)(w')$. For the case without maximal states, one could think of it as demanding that there has to be a level (or rather, a state q) in the inclusion hierarchy from which on both w and w' are contained in all "superstates" of q. In a way, this behaviour seems to emulate the concept of maximal states for the case when there are no maximal states. Returning to the denotation in figure 7, we now see that the new exhaustivity operator will partition the state set into exactly two blocks, where each block is the (infinite) union of one of the infinite hierarchies. Correspondingly, we define the translation of $\boxed{\textsc{exh}}$ as in (28).

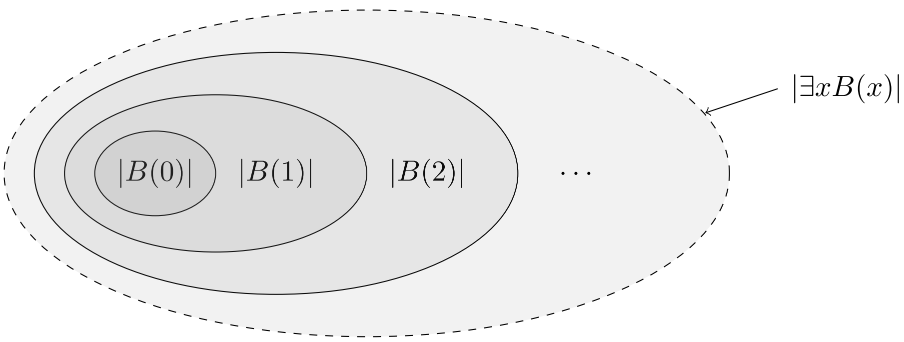


Figure 6: Visualisation of the boundedness formula

(28) $\quad Tr(\text{M}_{\text{exh}})$

$$= \lambda P_T.\lambda p_{\langle s,t\rangle}.\forall w(\forall w'((p(w) \wedge p(w')) \to \forall q((P(q) \wedge (q(w) \vee q(w'))) \to \exists q'(P(q') \wedge q'(w) \wedge q'(w') \wedge \forall v(q(v) \to q'(v))))))$$

$$=: \boxed{\text{EXH}}$$

For sentences like *Who smiles?* with maximal states in their denotation, this operator yields just the same exhaustive interpretation. For sentences without maximal states, it is difficult to motivate on an intuitive level why the meaning we obtain by applying $\boxed{\text{EXH}}$ is the correct exhaustive reading. For the boundedness formula, for example, subsequent application of the interrogative projection and the exhaustivity operator yield a set with exactly two alternatives: one for the negative reply $\neg\exists x B(x)$ and one for the affirmative reply $\exists x B(x)$. In comparison, the resolving states in $[\exists x B(x)]$ give a specific instantiation of x and could therefore be viewed as more demanding answers than the unspecific existential statements which we predict as "exhaustive" answers. However, since there is no way for a finite sentence to list infinitely many instantiations of x, the closed form of $\exists x B(x)$ might in fact be the closest we can get to an exhaustive answer.

Turning to polar interrogatives, the situation becomes less complex: they allow two possible answers only (*Yes* and *No*). We can obtain the possibilities corresponding to these answers by first taking the declarative and subsequently the interrogative projection of the sentence. To this end, we assume that the polar interrogative marker $\text{M}_{?!}$ is present in polar questions. It provides the necessary semantics (that is, the composition $\boxed{?} \circ \boxed{!}$ of declarative and interrogative projection). Additionally, it blocks the occurrence of *wh*-pronouns, and takes care of polar question morphology (*whether*/*if* in embedded questions, *do*-constructions) and of subject-verb inversion.

(29) $\quad Tr(\text{M}_{?!}) = \boxed{?} \circ \boxed{!}$

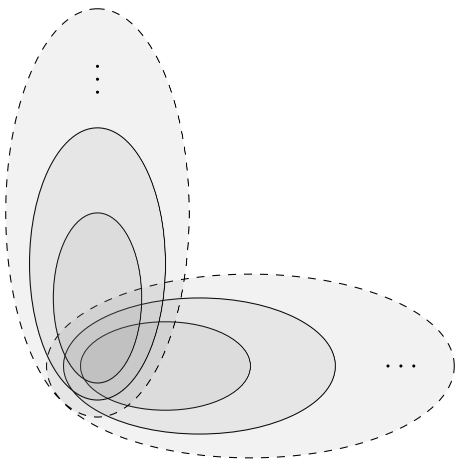


Figure 7: A state set with two distinct infinite inclusion hierarchies.

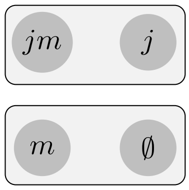


Figure 8: Visualisation of the polar question *Did John smile?*

The corresponding derivation is sketched in (30), and the resulting visualisation can be found in figure 8.

(30) Did John smile?

$$T$$
$$\boxed{?}(\boxed{!}(\lambda p_{\langle s,t\rangle}.\exists x \forall w(p(w) \to \mathit{smile}(w)(x))))$$

$$\textsc{m}_{?!} :: \langle T, T\rangle \qquad \boxed{?} \circ \boxed{!}$$

$$T \qquad \lambda p_{\langle s,t\rangle}.\exists x \forall w(p(w) \to \mathit{smile}(w)(x))$$

$$\text{John} :: \langle\langle e, T\rangle, T\rangle \qquad \lambda P_{\langle e,T\rangle}.P(\mathbf{j})$$

$$\text{smile} :: \langle e, T\rangle \qquad \lambda x_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \mathit{smile}(w)(x))$$

Finally, we need to mention that there are cases in which the declarative syntactic marker has to appear not at the top of the syntactic structure, but right above the object. At the moment, we can only motivate this semantically; quite possibly, though, there also is an independent syntactic motivation for this distribution of $\textsc{m}_!$. We leave this question for future work. To understand how the problem manifests itself, consider example (31), which can be resolved by replies like *John saw someone*, but also by the more informative *John saw Mary*. The latter response, however, is clearly over-informative; thus, we do not want it to correspond to a possibility in the denotation of (31). What the declarative projection operator does is to pool together the different states introduced by *someone*. Without this projection, these alternative states would persist in the derivation and we would derive over-informative answers like *John saw Mary*. With the projection, on the other hand, we arrive at the sentence denotation given below: it contains all states p such that there is a *specific* individual y (all worlds in p agree on which y this is, e. g. John) for whom there exists *some* individual x (whose identity varies across the worlds in p) such that y saw x.

(31) Who saw someone?

$$T$$
$$\boxed{?}(\lambda p_{\langle s,t\rangle}.\exists y(\forall w'(p(w') \rightarrow \exists p'(\exists x(\forall w(p(w) \rightarrow see(w)(x)(y))) \wedge p'(w')))))$$

$$\text{M}_? :: \langle T,T\rangle \qquad \boxed{?}$$

$$T$$
$$\lambda p_{\langle s,t\rangle}.\exists y(\forall w'(p(w') \rightarrow \exists p'(\exists x(\forall w(p(w) \rightarrow see(w)(x)(y))) \wedge p'(w'))))$$

$$\text{who} :: \langle\langle e,T\rangle,T\rangle \qquad \lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists x P(x)(p)$$

$$\lambda_1$$

$$T$$
$$\lambda p_{\langle s,t\rangle}.\forall w'(p(w') \rightarrow \exists p'(\exists x(\forall w(p(w) \rightarrow see(w)(x)(x_2))) \wedge p'(w')))$$

$$\text{M}_! :: \langle T,T\rangle \qquad \boxed{!} = \lambda P_T.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow \exists p'(P(p') \wedge p'(w)))$$

$$T$$
$$\lambda p_{\langle s,t\rangle}.\exists x(\forall w(p(w) \rightarrow see(w)(x)(x_2)))$$

t_1 saw someone

3.6 Variable binding

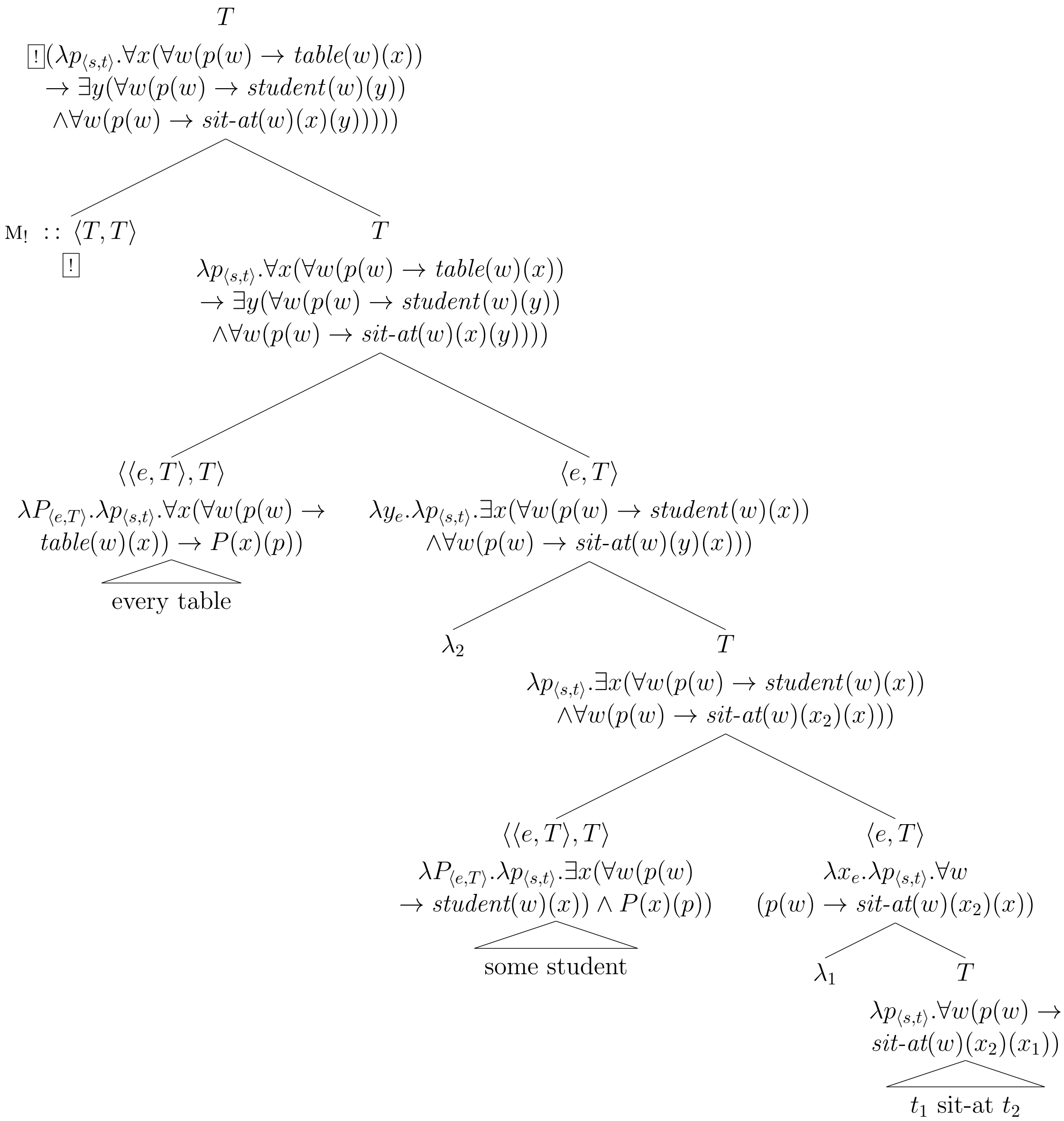
To conclude our exposition of the proposed type-theoretical inquisitive system, we will briefly demonstrate how this system is able to deal with bound variable pronouns, quantified DPs in object position and inverse quantifier scope. The scope taking mechanism employed for these phenomena is *quantifier raising*: we assume that a DP α_i can move out of its base position, subsequently c-commanding its co-indexed trace t_i. To ensure that the trace variable x_i is bound, predicate abstraction takes place, with the index of movement on α_i acting as the λ-binder.

Example (32) illustrates this mechanism for the case of reflexive pronouns in object position; in examples (33) and (34), readings with inverse quantifier scope are derived: (33) clearly does not express that *one* student was simultaneously sitting at every table; rather, it has to be a *different* student at every table. This scope configuration emerges if the universally quantified DP *every table* moves to a higher syntactic position than the existentially quantified DP *some student*. In contrast, (34) allows readings both corresponding to surface and to inverse quantifier scope. Which constraints govern the availability of the respective readings goes beyond the span of this account. The purpose of the below examples merely is to demonstrate that the different scopal configurations can be derived within the proposed grammar fragment in a completely standard way.

(32) Everybody defended himself.

- T
  $\boxed{!}(\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\forall x \forall w(p(w) \to \mathit{defend}(w)(x)(x)))$
  - $\textsc{m}_{!} :: \langle T, T\rangle$
    $\boxed{!}$
  - T
    $\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\forall x \forall w(p(w) \to \mathit{defend}(w)(x)(x))$
    - everybody $:: \langle\langle e,T\rangle,T\rangle$
      $\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\forall x P(x)(p)$
    - $\langle e,T\rangle$
      $\lambda x_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \mathit{defend}(w)(x)(x))$
      - λ_1
      - T
        $\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \mathit{defend}(w)(x_1)(x_1))$
        - t_1 defended himself$_1$

(33) Some student was sitting at every table.



(34) a. Who did everybody see? (surface scope: $\exists > \forall$)

T
$\boxed{?}(\lambda p_{\langle s,t\rangle}.\exists y \forall x \forall w(p(w) \rightarrow see(w)(y)(x)))$

- $\mathrm{M}_{?} :: \langle T,T\rangle$
  $\boxed{?}$
- T
  $\lambda p_{\langle s,t\rangle}.\exists y \forall x \forall w(p(w) \rightarrow see(w)(y)(x))$
  - $\langle\langle e,T\rangle,T\rangle$
    $\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists x P(x)(p)$
    who
  - $\langle e,T\rangle$
    $\lambda y_e.\lambda p_{\langle s,t\rangle}.\forall x \forall w(p(w) \rightarrow see(w)(y)(x))$
    - λ_1
    - T
      $\lambda p_{\langle s,t\rangle}.\forall x \forall w(p(w) \rightarrow see(w)(x_1)(x))$
      - $\langle\langle e,T\rangle,T\rangle$
        $\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\forall x P(x)(p)$
        everybody
      - $\langle e,T\rangle$
        $\lambda x_e.\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow see(w)(x_1)(x))$
        see t_1

b. Who did everybody see? (inverse scope: $\forall > \exists$)

- T
 $\boxed{?}(\lambda p_{\langle s,t\rangle}.\forall y \exists x \forall w(p(w) \rightarrow see(w)(y)(x)))$
  - $\textsc{M}_? :: \langle T, T\rangle$
 $\boxed{?}$
  - T
 $\lambda p_{\langle s,t\rangle}.\forall y \exists x \forall w(p(w) \rightarrow see(w)(y)(x))$
    - $\langle\langle e, T\rangle, T\rangle$
 $\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\forall x P(x)(p)$
 everybody
    - $\langle e, T\rangle$
 $\lambda y_e.\lambda p_{\langle s,t\rangle}.\exists x \forall w(p(w) \rightarrow see(w)(y)(x))$
      - λ_2
      - T
 $\lambda p_{\langle s,t\rangle}.\exists x \forall w(p(w) \rightarrow see(w)(x_2)(x))$
        - $\langle\langle e, T\rangle, T\rangle$
 $\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists x P(x)(p)$
 who
        - $\langle e, T\rangle$
 $\lambda x_e.\lambda p_{\langle s,t\rangle}.\forall w (p(w) \rightarrow see(w)(x_2)(x))$
          - λ_1
          - T
 $\lambda p_{\langle s,t\rangle}.\forall w(p(w) \rightarrow see(w)(x_2)(x_1))$
            - t_1 see t_2

4 Comparison with Hamblin semantics

At first glance, inquisitive semantics and Hamblin semantics seem akin both in their empirical target (questions) and their semantic machinery (sets of "alternatives"). Closer inspection, however, reveals rather principled differences between the conceptual foundations underlying either framework. In particular, the systems differ in their conception and implementation of alternatives—with the notion of downward-closedness playing a crucial role. We will address the conceptual differences and their practical consequences in section 4.1. In addition, however, there are purely technical differences as well: the mode of semantic composition in Hamblin semantics is not the same as that in the type-theoretical inquisitive system proposed in the previous section. This is why certain technical difficulties arising from the combination of Hamblin alternatives with variable binding are not an issue in our framework.

We will get to these matters in detail in section 4.2.

4.1 Notion of alternatives

Conceptual point of departure

Inquisitive semantics and Hamblin semantics approach the notion of answerhood from different points of departure. Inquisitive semantics starts with the pre-theoretic notion of what it takes to resolve a question. As we have already seen, these resolution conditions are formally captured as information states, which, taken together, make up the denotation of a question. They are those pieces of information that settle the issue raised by a question. This means that sentence denotations necessarily are downward-closed: because, if some state s settles a given issue, then any substate $t \subseteq s$ will settle that issue as well; after all, t specifies the location of the actual world within ω with higher precision than s. Under this resolution-centric view, answerhood becomes a derived notion—a desirable outcome, since it is an intuitively more vague concept than resolution, and the algebraic perspective taken in inquisitive semantics allows us to define different conceptions of answerhood in a natural and formally sound way. For example, *basic* answers could reasonably be defined as the alternatives for a question, that is, as those pieces of information *minimally* needed to settle the issue raised by the question. *Exhaustive* answers on the other hand can simply be defined in terms of intersection—which only becomes possible since sentence denotations are downward-closed. Why this is will fall into place shortly when we look at coordinated questions in the next section.

In conclusion, inquisitive semantics departs from an intuitively clear pre-theoretic notion and, based on this, is flexible enough to derive different conceptions of what constitutes an answer—something which is intuitively less clear.

In Hamblin semantics, in comparison, it is the notion of answerhood which is conceptually prior. The denotation of a question consists of all *basic* answers to that question, where basic answerhood is a pre-theoretic concept. On a theory-internal level, however, Hamblin semantics does not provide a precise characterisation of basic answerhood (as opposed to non-basic answerhood). In this respect, the conceptual point of departure taken in Hamblin frameworks appears less solid than that of inquisitive semantics.

Conjunction and explanatory adequacy

Chomsky (1965) suggests that, when evaluating a grammar, there are distinct, hierarchically ordered levels of adequacy to take into account: starting with the most elementary level, these are *observational*, *descriptive* and *explanatory* adequacy. While Chomsky's criteria for each level are mostly geared to theories of syntax, Groenendijk and Stokhof (1984 : 10ff) adapt them to the evaluation of semantic frameworks, in particular spelling out relevant requirements for explanatory adequacy. In their view, explanatory adequacy demands a certain systematicity in constructing the semantic space: the computation of meanings has to proceed *compositionally*, and the notions and principles employed by the semantic theory have to be *general*: they should be applicable outside the theory's specific domain as well. For example, consider a theory that aims at capturing the meaning of sentences coordinated by *and*. In order to make this precise, let $[\![\cdot]\!]$ be a function specified by the semantic theory which translates natural language expressions to semantic objects. Further, let α and β be natural language sentences. Then, the requirements for descriptive adequacy would amount to the following. In order for the computation to be compositional, α *and* β has to translate as $[\![\alpha \textit{ and } \beta]\!] = [\![\alpha]\!] \odot [\![\beta]\!]$ where, in order to satisfy the generality requirement, $\odot$ has to be a suitably domain-independent operation for sentence conjunction. What constitutes a suitable operation in a given framework, is usually determined by the framework itself. If our semantic account is based on set theory for example—that is, $[\![\alpha]\!]$ and $[\![\beta]\!]$ are sets of e. g. possible worlds—then, $\odot$ will be intersection: $[\![\alpha \textit{ and } \beta]\!] = [\![\alpha]\!] \cap [\![\beta]\!]$. However, we can view this from a yet more general perspective if we look at the semantic space as a partially ordered set: the elements in this set are propositions and a reasonable choice for the partial order is the entailment relation between propositions. Then, we desire of a suitable operation $\odot$ for sentence conjunction that (a) it is commutative, associative and idempotent, and that (b) $[\![\alpha]\!] \odot [\![\beta]\!]$ is the "weakest" proposition which entails both $[\![\alpha]\!]$ and $[\![\beta]\!]$. This is precisely what characterises a so-called *meet* operation in a partially ordered set (Roelofsen 2012). Hence, now speaking in full generality, what we demand of $\odot$ is that it is a meet operation. Under this view, set intersection becomes just one specific implementation of such an operation.

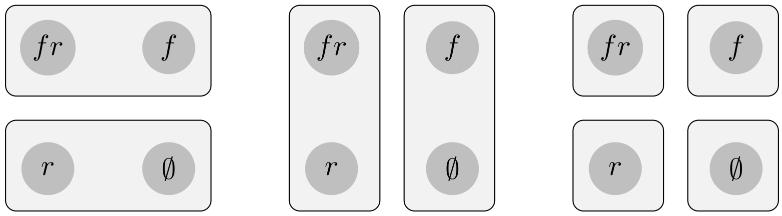
Now, applying these criteria to the two frameworks at hand, clearly, both inquisitive and Hamblin semantics are compositional; it is mostly their generality in the sense described above which needs further investigation. There would indeed be a lot to say about the algebraic foundations of inquisitive semantics and in particular about the treatment of disjunction they give rise to. Here, however, we will just point the reader to Roelofsen (2012) and instead continue investigating the more straightforward example of sentence conjunction.

Since in both inquisitive semantics and Hamblin semantics sentence denotations are sets of propositions[9], the conjunction of sentences would classically amount to set intersection. We will see that, while inquisitive semantics allows us to adhere to this classical picture[10], in Hamblin semantics it has to be given up. At least in this respect, inquisitive semantics thus achieves a higher degree of explanatory adequacy.

Consider example (36) from Ciardelli et al. (2012). We can capture its meaning using the lexical entry (35) for *and*, which applies uniformly to declaratives and interrogatives. Taken individually, each of the two polar questions has a denotation with exactly two possibilities—one corresponding to a positive and one correspond-

[9]Again, we use the term *proposition* in the classical sense. For us, a proposition is a set of possible worlds. Hence, the terms *state* and *proposition* refer to the same kind of semantic object.

[10]Or rather, the algebraic foundation underlying inquisitive semantics is even more general: the space of semantic meanings Σ, ordered by an informativeness ordering $\leq$, forms a Heyting algebra $\langle \Sigma, \leq \rangle$ with *meet*, *join* and (*relative*) *pseudo-complement* operators (Roelofsen 2012). In this setting, intersection becomes just a specific instantiation of the *meet* operator.



(a) Does John speak French? (b) Does John speak Russian? (c) Does John speak French, and does he speak Russian?

Figure 9: Conjunction of interrogatives

ing to a negative reply (figure 9(a) and (b)). When the questions are coordinated by *and* as in (36), though, the resulting denotation contains exactly four possibilities (figure 9(c)): the coordinated question can only be settled by exhaustively specifying which of the two languages John speaks. Crucially, however, the meaning of the entire question is obtained by intersecting the meanings of the two subquestions. This is exactly the classical treatment of conjunction we had been after—only now the objects that get intersected are more fine-grained: in the classical setting, semantic objects are world-sets; now they are downward-closed sets of world-sets. Observe that downward-closedness really is vital in order for the intersection to yield the desired result: since the denotations of the two conjuncts are downward-closed, they do not only contain the maximal states depicted in figure 9(a) and (b), but also all subsets of these maximal states, including—and this is what it comes down to—those states that are contained in the denotation of the entire conjunction.

(35) $Tr(\text{and}) = \lambda S_T.\lambda S'_T.\lambda p_{\langle s,t\rangle}.S(p) \wedge S'(p)$

(36) Does John speak French, and does he speak Russian?

$$\lambda p_{\langle s,t\rangle}.(\forall w(p(w) \to \textit{speak-French}(w)(\mathbf{j})) \vee \boxed{\neg}\forall w(p(w) \to \textit{speak-French}(w)(\mathbf{j})))$$
$$\wedge\, (\forall w(p(w) \to \textit{speak-Russian}(w)(\mathbf{j})) \vee \boxed{\neg}\forall w(p(w) \to \textit{speak-Russian}(w)(\mathbf{j})))$$

T

$\boxed{?}(\boxed{!}(\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \textit{speak-French}(w)(\mathbf{j}))))$

M?! does John$_1$ speak French

and :: $\langle T, \langle T, T\rangle\rangle$

$\lambda S_T.\lambda S'_T.\lambda p_{\langle s,t\rangle}.S(p) \wedge S'(p)$

T

$\boxed{?}(\boxed{!}(\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to \textit{speak-Russian}(w)(x_1))))$

M?! does he$_1$ speak Russian

In Hamblin semantics, on the other hand, the denotations of the subquestions are not downward-closed:

(37) $[\![\,\text{Does John speak French?}\,]\!]_{\text{Hamblin}} = \{\,\{fr, f\}, \{r, \emptyset\}\,\}$

(38) $[\![\,\text{Does John speak Russian?}\,]\!]_{\text{Hamblin}} = \{\,\{fr, r\}, \{f, \emptyset\}\,\}$

In order to combine these sets in a way that produces the desired meaning for sentence (36), clearly, we cannot just intersect them. What we need instead is an operation for pointwise intersection. While such a mechanism reliably yields the appropriate interpretation of coordinated questions, in a framework based on set-theory, it seems a less generally motivated choice than intersection. In particular, pointwise intersection is not idempotent and hence not a meet operation.

4.2 Mode of semantic composition

4.2.1 Pointwise functional application and projection operators

In the proposed inquisitive framework, semantic composition is driven by classical rules for functional application and predicate abstraction (see (9)). In Hamblin semantics, however, we need pointwise versions of both rules since all denotations are set-valued. While it is easy to find a suitable pointwise formulation of functional application, we will soon see that pointwise predicate abstraction poses a more serious problem.

(39) **Pointwise functional application**[11]

$$\lambda X_\tau.\exists A(\exists B((Tr(\alpha))(A) \wedge (Tr(\alpha))(B) \wedge X = A(B))) :: \langle \tau, t\rangle$$

$$Tr(\alpha) :: \langle\langle \sigma, \tau\rangle, t\rangle \qquad Tr(\beta) :: \langle \sigma, t\rangle$$

The pointwise fashion of composition has direct repercussions on the treatment of certain operators in Hamblin systems: In the inquisitive grammar fragment, we are able to specify lexical entries for the projection operators $\boxed{!}$ and $\boxed{?}$. In Hamblin semantics, this is not possible; instead, such operators require syncategorematic translation rules. To see why, consider e. g. the case of $\boxed{!}$, which turns a possibly multi-membered set of states into one with at most a single possibility. This single

[11]The remainder of this paper will make frequent references to work in the generative tradition where the pointwise functional application and predicate abstraction rules are usually not stated within the two-step approach (first translation, then interpretation) to compositional semantics adopted here. Instead, these rules are formulated with respect to a direct interpretation function $[\![\cdot]\!]^{M,g}$ that maps natural language expressions to model-theoretic interpretations relative to the model M and variable assignment g. In that framework, the pointwise functional application rule would take the following form:

$$\{f(x) \mid f \in [\![\alpha]\!]^{M,g} \wedge x \in [\![\beta]\!]^{M,g}\} :: \langle \tau, t\rangle$$

$$[\![\alpha]\!]^{M,g} :: \langle\langle \sigma, \tau\rangle, t\rangle \qquad [\![\beta]\!]^{M,g} :: \langle \sigma, t\rangle$$

possibility could be thought of as a "large classical disjunction" of the individual pieces of information from the old set. In order for an operator to produce such a disjunction, all states from that old set have to be "simultaneously" available to the operator. If the set is processed pointwise, however, only one state will be avaible at a time.

4.2.2 Combining alternatives and quantifier raising

While the notion of alternatives has found wide applications in formal semantics (e. g. Rooth 1985; Kratzer and Shimoyama 2002; Alonso-Ovalle 2006), one of its most prominent uses is the analysis of in-situ *wh*-questions (Hamblin 1973). As pointed out by Romero and Novel (2013), sets of alternatives in *wh*-questions essentially can be thought of as a scoping mechanism solely taking place in the *semantics*. In contrast, scope configurations deviating from surface scope can also be obtained by *syntactic* means: quantifier raising (QR) is an example. Romero and Novel (henceforth R&N) give a concise summary of semantic versus syntactic scoping mechanisms, including empirical differences between the phenomena traditionally handled by either one. Here, we will follow suit with a small body of previous work (R&N, Shan 2004) and focus on the interplay of semantic alternatives with syntactic movement, since the combination of these two mechanisms has been a noted source of technical difficulties for existing frameworks.

The difficulties with syntactic movement stem from the fact that it involves variable binding: the displaced element α_i c-commands its co-indexed trace t_i. To ensure that the trace variable x_i is bound, predicate abstraction takes place, with the index of movement on α_i acting as the λ-binder.

In existing Hamblin frameworks, however, predicate abstraction and alternative sets, do not work together seamlessly. N&R show how it can be done, but they need to design their system with great caution, making certain assumption about the status of assignment functions and the semantics of *wh*-phrases. Our discussion will be centred around two of the three problems identified by R&N and Shan (2004).[12] We will demonstrate that in the inquisitive framework these problems do not arise in the first place.

Problem 1: Type mismatch

R&N's problem 1 pertains to sentences like (40) in which both variable binding (through quantifier raising) and alternatives (through the *wh*-phrase) are present.

(40) a. Who saw nobody?
b. Nobody λ_1 [who saw t_1]

[12] We will omit their third problem, since—as R&N point out themselves—it does not primarily have to do with semantic *composition* in a Hamblin framework, but rather with the presuppositional treatment of *wh*-phrases.

Assume we are working in a Hamblin system. Further assume that, in the semantic derivation for (40), we use the point-wise functional application rule in (39), but naively apply the ordinary predicate abstraction rule from (9b). Then, the translation of "λ_1 *who saw* t_1" will be interpreted as a function into sets (it will have type $\langle e, \langle \tau, t \rangle \rangle$). The quantifier *nobody*, however, expects as its argument not a function into sets, but a set of functions (type $\langle \langle e, \tau \rangle, t \rangle$). To avoid a type-clash, the denotation *can* be transposed (and Kratzer and Shimoyama (2002) present a rule for alternative-friendly predicate abstraction which aready takes care of this). But as Shan (2004) points out, transposing is only possible at the sacrifice of losing information: a function into sets holds less information with respect to ordering than a set of functions. As a consequence, the resulting denotation can contain spurious alternatives, making incorrect empirical predictions.

Let us now switch back to the inquisitive system. Here, we can use the ordinary rules for functional application and predicate abstraction. As already shown in section 3.6, variable binding happens quite naturally. We also obtain the desired result for (41), the problematic example from R&N. No type-conflict ensues between the quantifier and its argument. Hence, no transposing is needed and no spurious alternatives are generated.

In fact, this does not come as a surprise, since we deliberately chose the denotations so as to guarantee they will fit together compositionally. In a Hamblin system, by contrast, the denotations do not fit; the derivation relies on special rules for functional application and predicate abstraction to fix this.

(41) Who saw nobody?

T

$\boxed{?}(\lambda p_{\langle s,t\rangle}.\exists y \neg \exists x \forall w$
$(p(w) \to \mathit{see}(w)(x)(y)))$

$\text{M}_? :: \langle T, T\rangle$
$\boxed{?}$

T
$\lambda p_{\langle s,t\rangle}.\exists y \neg \exists x \forall w$
$(p(w) \to \mathit{see}(w)(x)(y))$

who $:: \langle\langle e, T\rangle, T\rangle$
$\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\exists x P(x)(p)$

$\langle e, T\rangle$
$\lambda y_e.\lambda p_{\langle s,t\rangle}.\neg \exists x \forall w$
$(p(w) \to \mathit{see}(w)(x)(y))$

λ_2

T
$\lambda p_{\langle s,t\rangle}.\neg \exists x \forall w$
$(p(w) \to \mathit{see}(w)(x)(x_2))$

nobody $:: \langle\langle e, T\rangle, T\rangle$
$\lambda P_{\langle e,T\rangle}.\lambda p_{\langle s,t\rangle}.\neg \exists x P(x)(p)$

$\langle e, T\rangle$
$\lambda x_e \lambda p_{\langle s,t\rangle}.\forall w$
$(p(w) \to \mathit{see}(w)(x)(x_2))$

λ_1

T
$\lambda p_{\langle s,t\rangle}.\forall w(p(w) \to$
$\mathit{see}(w)(x_1)(x_2))$

t_2 saw t_1

Problem 2: Binding into the *wh*-phrase

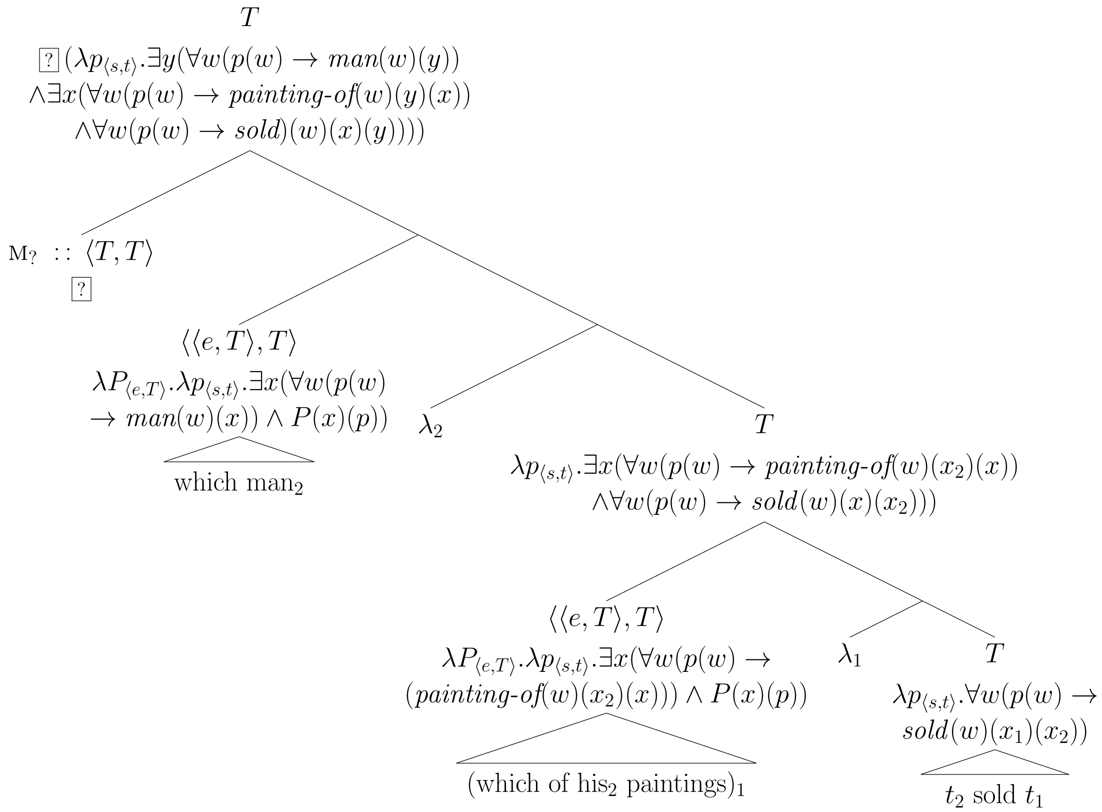
Shan (2004) also calls attention to another problem, which, just as problem 1, has to do with conflicting types. It arises from sentences like (42) (R&N's (38)), in which a pronoun inside the alternative-generating phrase is bound from the outside.

(42) Which man_1 sold which of his_1 paintings?

Intuitively, the denotation of the *wh*-phrase *which of* $\textit{his}_1$ *paintings* depends on which man it is combined with: we only want to match each man with his *own* paintings—not with those of a different painter. In Hamblin semantics, this requires a lot of careful combinatorics, since the denotation of the *wh*-phrase is spelled out as a set of individuals: this makes an explicit pair-forming operation necessary. In particular, in order to guarantee that each painter is only paired with his own paintings, it seems that a function from individuals (painters) to sets of individuals (paintings) is needed. Such a function, however, would be another instance of the problematic type $\langle e, \langle \tau, t \rangle \rangle$ already encountered above.

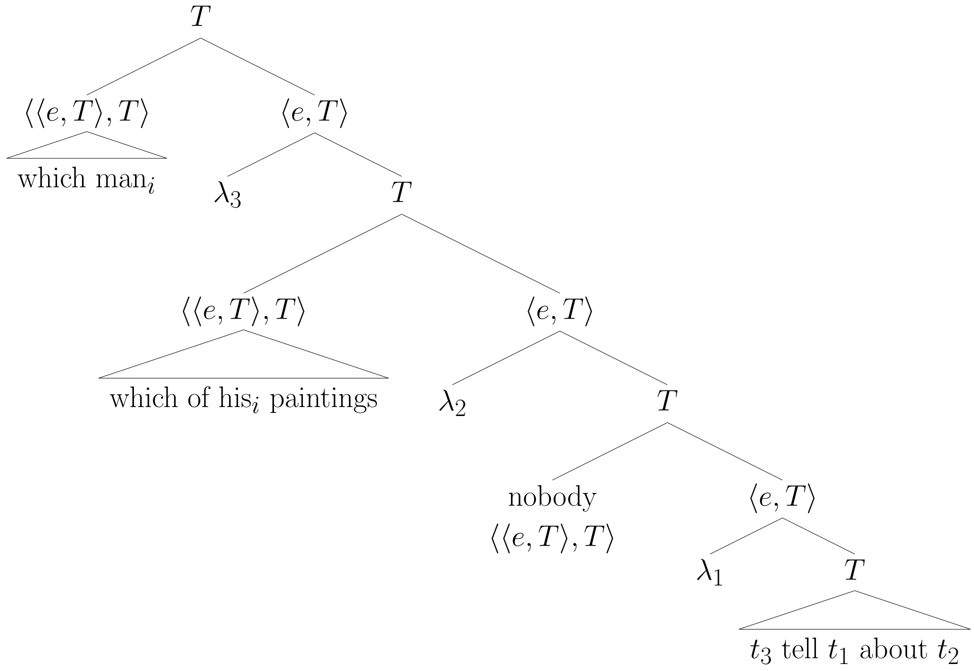
In our inquisitive framework, this problem does not arise, since we do not have to perform any of the explicit combinatorics required in Hamblin semantics. That painters and paintings are matched correctly is guaranteed simply by a standard variable binding mechanism.

(43) Which man_2 sold which of his_2 paintings?



Shan (2004) also identifies a variation of problem 2, in which quantifier raising and binding into the *wh*-phrase take place within the same sentence. Because the co-occurrence of these two mechanisms requires the types $\langle \langle e, \tau \rangle, t \rangle$ for quantifier raising and $\langle e, \langle \tau, t \rangle \rangle$ for binding into the *wh*-phrase (see example (43)) to "interleave" in a certain way, he calls this problem the *interleaving type problem.* We will not go into why exactly this example is problematic for existing accounts, but only sketch the derivation to show that similar difficulties do not ensue in our framework. As can be seen in (44), type-wise, nothing extraordinary is happening in the derivation. The desired scope configuration (which man / which paintings > nobody) comes about by quantifier raising each of the three DPs; and the binding works just as in example (43).

(44) Which man$_i$ told nobody about which of his$_i$ paintings?



5 Conclusion

We have seen how the inquisitive conception of sentence meaning can inspire the setup of a type-theoretical alternative semantics. A grammar fragment for such a semantics has been specified and shown capable of accounting for a standard range of phenomena in the realm of question semantics and variable binding. Although this type-theoretical inquisitive system bears resemblance to Hamblin semantics, there are fundamental conceptual and technical differences between both frameworks. Conceptually, it seems that the notion of resolution conditions allows us to formulate a flexible and theoretically solid definition of answerhood in the inquisitive system. The Hamblin notion of basic answerhood, on the other hand, appears to lack a precise formal definition. Technically, semantic composition in the inquisitive system—as opposed to semantic composition in Hamblin frameworks—does not rely on pointwise versions of functional application and predicate abstraction. As a consequence, in particular, the problems associated with pointwise predicate abstraction do not arise in the inquisitive system.

Appendix

A Intensional theory of types

A.1 Syntax

We start by defining the set of types $\mathbf{T}$. It is the smallest set such that:

(i) $e, s, t \in \mathbf{T}$

(ii) If $\sigma, \tau \in \mathbf{T}$, then $\langle \sigma, \tau \rangle \in \mathbf{T}$.

For each type σ, the vocabulary of intensional type theory contains the infinite set VAR_σ of variables of type σ and the (possibly empty) set $\text{CON}_\sigma^{\mathcal{L}}$ of constants of type σ.

Based on this, we can define the syntax of an intensional, type-theoretical language $\mathcal{L}$. By $\text{WE}_\sigma^{\mathcal{L}}$, we refer to the set of all well-formed expressions of type σ in $\mathcal{L}$. Under this terminology, formulas are the elements of $\text{WE}_t^{\mathcal{L}}$.

(i) If $\alpha \in \text{VAR}_\sigma$, or $\alpha \in \text{CON}_\sigma^{\mathcal{L}}$, then $\alpha \in \text{WE}_\sigma^{\mathcal{L}}$.

(ii) If $\alpha \in \text{WE}_{\langle \sigma,\tau \rangle}^{\mathcal{L}}$ and $\beta \in \text{WE}_\sigma^{\mathcal{L}}$, then $(\alpha(\beta)) \in \text{WE}_\tau^{\mathcal{L}}$.

(iii) If $\varphi, \psi \in \text{WE}_t^{\mathcal{L}}$, then $\neg\varphi, (\varphi \wedge \psi), (\varphi \vee \psi)$ and $(\varphi \to \psi) \in \text{WE}_t^{\mathcal{L}}$.

(iv) If $\varphi \in \text{WE}_t^{\mathcal{L}}$ and $v \in \text{VAR}_\sigma$, then $\forall v \varphi, \exists v \varphi \in \text{WE}_t^{\mathcal{L}}$.

(v) If $\alpha \in \text{WE}_\sigma^{\mathcal{L}}$ and $\beta \in \text{WE}_\sigma^{\mathcal{L}}$, then $(\alpha = \beta) \in \text{WE}_t^{\mathcal{L}}$.

(vi) If $\alpha \in \text{WE}_\sigma^{\mathcal{L}}$ and $v \in \text{VAR}_\tau$, then $\lambda v \alpha \in \text{WE}_{\langle \tau,\sigma \rangle}^{\mathcal{L}}$.

(vii) For any σ, all elements of $\text{WE}_\sigma^{\mathcal{L}}$ are constructed in a finite number of steps using (i)–(vi).

A.2 Semantics

Here, we start by specifying domains of interpretation for the different types. A domain $\boldsymbol{D}_{\sigma,D,W}$ for type σ is defined based on a set of possible worlds W and a domain of individuals D.

(i) $\boldsymbol{D}_{e,D,W} = D$

(ii) $\boldsymbol{D}_{s,D,W} = W$

(iii) $\boldsymbol{D}_{t,D,W} = \{1, 0\}$

(iv) $\boldsymbol{D}_{\langle \sigma,\tau \rangle,D,W} = \{f \mid f : \boldsymbol{D}_{\sigma,D,W} \to \boldsymbol{D}_{\tau,D,W}\} =: \boldsymbol{D}_{\sigma,D,W}^{\boldsymbol{D}_{\tau,D,W}}$

A model $\mathbf{M} = (D, W, I)$ for an intensional type-theoretical language $\mathcal{L}$ consists of a non-empty domain D, a non-empty set of possible worlds W and an interpretation function I. The *extension* $[\![\alpha]\!]_{\mathbf{M},w,g}$ of an expression α is defined relative to such a model, a possible world $w \in W$ and an *assignment function* g (a function mapping variables of type σ to objects in D_σ). In the below setup, all worlds share a common domain and the accessibility relation is universal.

(i) If $\alpha \in \mathrm{CON}^{\mathcal{L}}_\sigma$, then $[\![\alpha]\!]_{\mathbf{M},w,g} = I(\alpha)(w)$.
If $\alpha \in \mathrm{VAR}_\sigma$, then $[\![\alpha]\!]_{\mathbf{M},w,g} = g(\alpha)$.

(ii) If $\alpha \in \mathrm{WE}^{\mathcal{L}}_{\langle\sigma,\beta\rangle}$ and $\beta \in \mathrm{WE}^{\mathcal{L}}_\sigma$, then $[\![\alpha(\beta)]\!]_{\mathbf{M},w,g} = [\![\alpha]\!]_{\mathbf{M},w,g}([\![\beta]\!]_{\mathbf{M},w,g})$.

(iii) If $\varphi, \psi \in \mathrm{WE}^{\mathcal{L}}_t$, then:
$[\![\neg\varphi]\!]_{\mathbf{M},w,g} = 1$ iff $[\![\varphi]\!]_{\mathbf{M},w,g} = 0$.
$[\![\varphi \wedge \psi]\!]_{\mathbf{M},w,g} = 1$ iff $[\![\varphi]\!]_{\mathbf{M},w,g} = [\![\psi]\!]_{\mathbf{M},w,g} = 1$.
$[\![\varphi \vee \psi]\!]_{\mathbf{M},w,g} = 1$ iff $[\![\varphi]\!]_{\mathbf{M},w,g} = 1$ or $[\![\psi]\!]_{\mathbf{M},w,g} = 1$.
$[\![\varphi \to \psi]\!]_{\mathbf{M},w,g} = 0$ iff $[\![\varphi]\!]_{\mathbf{M},w,g} = 1$ and $[\![\psi]\!]_{\mathbf{M},w,g} = 0$.

(iv) If $\varphi \in \mathrm{WE}^{\mathcal{L}}_t$ and $v \in \mathrm{VAR}_\sigma$ where $\sigma \neq s$, then:
$[\![\forall v\varphi]\!]_{\mathbf{M},w,g} = 1$ iff for all $\mathbf{d} \in \mathbf{D}_\sigma$: $[\![\varphi]\!]_{\mathbf{M},w,g[v/\mathbf{d}]} = 1$.
$[\![\exists v\varphi]\!]_{\mathbf{M},w,g} = 1$ iff for some $\mathbf{d} \in \mathbf{D}_\sigma$: $[\![\varphi]\!]_{\mathbf{M},w,g[v/\mathbf{d}]} = 1$.

(v) If $\varphi \in \mathrm{WE}^{\mathcal{L}}_t$ and $v \in \mathrm{VAR}_s$, then:
$[\![\forall v\varphi]\!]_{\mathbf{M},w,g} = 1$ iff for all $w' \in W$: $[\![\varphi]\!]_{\mathbf{M},w',g} = 1$.
$[\![\exists v\varphi]\!]_{\mathbf{M},w,g} = 1$ iff for some $w' \in W$: $[\![\varphi]\!]_{\mathbf{M},w',g} = 1$.

(vi) If $\alpha \in \mathrm{WE}^{\mathcal{L}}_\sigma$ and $v \in \mathrm{VAR}_\tau$, then $[\![\lambda v\alpha]\!]_{\mathbf{M},w,g}$ is that function $h \in \boldsymbol{D}_\sigma^{\boldsymbol{D}_\tau}$ such that for all $\mathbf{d} \in \boldsymbol{D}_\tau : h(\mathbf{d}) = [\![\alpha]\!]_{\mathbf{M},w,g[v/\mathbf{d}]}$.

(vii) If $\alpha \in \mathrm{WE}^{\mathcal{L}}_\sigma$ and $v \in \mathrm{VAR}_s$, then $[\![\lambda v\alpha]\!]_{\mathbf{M},w,g}$ is that function $h \in \boldsymbol{D}_\sigma^W$ such that for all $w' \in W : h(w') = [\![\alpha]\!]_{\mathbf{M},w',g}$.

References

Luis Alonso-Ovalle (2006). *Disjunction in alternative semantics.* Ph.D. thesis, University of Massachusetts Amherst.

Noam Chomsky (1965). *Aspects of the Theory of Syntax.* Massachusetts Institute of Technology. M.I.T. Press.

Ivano Ciardelli (2009). Inquisitive semantics and intermediate logics. Master Thesis, University of Amsterdam.

Ivano Ciardelli (2010a). A first-order inquisitive semantics. In: *Logic, Language and Meaning*, pp. 234–243. Springer.

Ivano Ciardelli (2010b). A first-order inquisitive semantics. In: Maria Aloni, Harald Bastiaanse, Tikitu de Jager and Katrin Schulz (eds.), *Logic, Language, and Meaning: Selected Papers from the Seventeenth Amsterdam Colloquium*. Springer.

Ivano Ciardelli, Jeroen Groenendijk and Floris Roelofsen (2012). Inquisitive semantics. *NASSLLI 2012 lecture notes*.

Jeroen Groenendijk and Martin Stokhof (1984). *Studies on the Semantics of Questions and the Pragmatics of Answers*. Ph.D. thesis, University of Amsterdam.

Andreas Haida (2007). *The indefiniteness and focusing of wh-words*. Ph.D. thesis, Humboldt Universität Berlin.

Charles L. Hamblin (1973). Questions in Montague grammar. *Foundations of language*, **10**(1):41–53.

Irene Heim and Angelika Kratzer (1998). *Semantics in generative grammar*, vol. 13. Blackwell Oxford.

Lauri Karttunen (1977). Syntax and semantics of questions. *Linguistics and philosophy*, **1**(1):3–44.

Angelika Kratzer and Junko Shimoyama (2002). Indeterminate pronouns: The view from Japanese. In: *3rd Tokyo conference on psycholinguistics*.

Paula Menéndez-Benito (2005). *The grammar of choice*. Ph.D. thesis, University of Massachusetts Amherst.

Richard Montague (1973). The proper treatment of quantification in ordinary English. In: *Approaches to natural language*, pp. 221–242. Springer.

Floris Roelofsen (2012). Algebraic foundations for the semantic treatment of inquisitive content. *Synthese*, pp. 1–24.

Maribel Romero and Marc Novel (2013). Variable Binding and Sets of Alternatives. In: Anamaria Falaus (ed.), *Alternatives in Semantics*, chap. 7, pp. 174–208. Palgrave.

Mats Rooth (1985). *Association with Focus*. Ph.D. thesis, University of Massachusetts.

Hotze Rullmann and Sigrid Beck (1996). Degree Questions, Maximal Informativeness, and Exhaustivity. In: *Proceedings of the 10th Amsterdam Colloquium*, vol. 10, p. 73. ILLC, University of Amsterdam.

Chung-chieh Shan (2004). Binding alongside Hamblin alternatives calls for variable-free semantics. In: *Proceedings of SALT*, vol. 14, pp. 289–304.