An Inquisitive Semantics with Witnesses

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Frankfurt, March 9, 2012

Netherlands Organisation for Scientific Research

Mission statement

Main aims of inquisitive semantics

- Develop a notion of meaning that captures both informative and inquisitive content in order to provide a logical foundation for the analysis of discourse that is aimed at the exchange of information.
- A sentence is taken to express a proposal to update the common ground of a conversation in one or more ways.
- If a sentence proposes two or more alternative updates it is inquisitive, it requests a response that establishes one of the alternative updates.
- The semantics is intended to give rise to a logical pragmatical notion of compliant responsehood.

Mission statement

Main aims of this paper

- We address a foundational issue for an inquisitive semantics for a language of first-order logic.
- The notion of compliant responsehood that our inquisitive semantics for a language of propositional logic gives rise to does not fit its most straightforward lift to the first order case.
- This is illustrated by Ciardelli's infamous boundedness sentences, which are a symptom of the disease.
- We present an inquisitive witness semantics that takes care of the symptoms in that it can deal with compliant responses to the boundedness sentences.
- However, there still remain open questions as to whether the new semantics is also a cure for the disease as such.

Overview

1. Propositional inquisitive semantics
2. First order inquisitive semantics
3. Problems with the boundedness sentences
4. Inquisitive witness semantics
5. Conclusions and open question

1. Propositional inquisitive semantics

Propositional inquisitive semantics

Language of propositional logic

- Let $\mathcal{P}$ be a finite set of proposition letters. We denote by $\mathcal{L}_\mathcal{P}$ the set of formulas built up from letters in $\mathcal{P}$ and $\bot$ using the binary connectives $\wedge$, $\vee$ and $\rightarrow$. We will refer to $\mathcal{L}_\mathcal{P}$ as the propositional language based on $\mathcal{P}$.

Abbreviations

- $\neg\varphi$ for $\varphi \rightarrow \bot$
- $!\varphi$ for $\neg\neg\varphi$ (non-inquisitive projection)
- $?\varphi$ for $\varphi \vee \neg\varphi$ (non-informative projection)

Worlds and states

Definition (Worlds)

- A $\mathcal{P}$-world is a function from $\mathcal{P}$ to $\{0, 1\}$.
- We denote by $\mathcal{W}_{\mathcal{P}}$ the set of all $\mathcal{P}$-worlds.

Definition (States)

- A $\mathcal{P}$-state is a set of $\mathcal{P}$-worlds.
- We denote by $\mathcal{S}_{\mathcal{P}}$ the set of all $\mathcal{P}$-states.

- We will usually drop reference to $\mathcal{P}$ and simply talk about worlds and states.
- We also take the common ground to be a state.

Propositions as proposals

Meaning and support

- We look upon the meaning of a sentence φ, the proposition expressed by φ, as a proposal to update the common ground in such a way that the new common ground supports φ.
- In the semantics we recursively specify the notion of when a state $s \in \mathcal{S}_{\mathcal{P}}$ supports a sentence $\varphi \in \mathcal{L}_{\mathcal{P}}$, denoted by $s \models \varphi$.

Support

Definition (Support)

Let $s \in \mathcal{S}_{\mathcal{P}}$ be a $\mathcal{P}$-state and $\varphi \in \mathcal{L}_{\mathcal{P}}$.

1. $s \models p$ iff $\forall w \in s : w(p) = 1$ for $p \in \mathcal{P}$
2. $s \models \bot$ iff $s = \emptyset$
3. $s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$
4. $s \models \varphi \vee \psi$ iff $s \models \varphi$ or $s \models \psi$
5. $s \models \varphi \rightarrow \psi$ iff $\forall t \subseteq s$: if $t \models \varphi$ then $t \models \psi$

Note: the absurd state $\emptyset$ supports any formula φ.

Definition (Entailment, and equivalence)

1. $\varphi \models \psi$ iff for all s: if $s \models \varphi$, then $s \models \psi$
2. $\varphi \equiv \psi$ iff $\varphi \models \psi$ and $\psi \models \varphi$

Some facts

Fact (Persistence)

- *If* $s \models \varphi$ *then for every* $t \subseteq s$*:* $t \models \varphi$

Fact (Singleton states behave classically)

- $\{w\} \models \varphi \iff w \models \varphi$ *in classical propositional logic*

Fact (Support for negation and the projection operators)

1. $s \models \neg\varphi$ *iff* $\forall w \in s : \{w\} \not\models \varphi$
2. $s \models\, !\varphi$ *iff* $\forall w \in s : \{w\} \models \varphi$
3. $s \models\, ?\varphi$ *iff* $s \models \varphi$ *or* $s \models \neg\varphi$

Propositions, truth sets, and alternatives

Definition (Propositions and truth sets)

1. The proposition expressed by φ, $[\varphi] := \{s \in \mathcal{S} \mid s \models \varphi\}$.
2. The truth set of φ, $|\varphi| := \{w \in \mathcal{W} \mid \{w\} \models \varphi\}$.

Definition (Alternatives)

1. Every maximal element of $[\varphi]$ is called an alternative for φ.
2. The alternative set of φ, $[\![\varphi]\!]$, is the set of alternatives for φ.

Propositions and alternatives

The alternative set of a sentence completely determines the proposition that is expressed by that sentence, and vice versa.

Fact (Propositions and alternatives)

$$s \in [\varphi] \iff s \text{ is contained in some } \alpha \in [\![\varphi]\!]$$

Thus, an utterance of φ proposes to update the common ground in such a way that the new common ground is contained in one of the alternatives for φ.

Informative content

- Worlds that are not contained in any state supporting φ will not survive any of the updates proposed by φ.
- In other words, if any of the updates proposed by φ is executed, all worlds that are not contained in $\bigcup[\varphi]$ will be eliminated.

Definition (Informative content)

- $\text{info}(\varphi) := \bigcup[\varphi]$

Fact (Informative content is classical)

- *For any formula φ: info$(\varphi) = |\varphi|$*

Informativeness and inquisitiveness

Informativeness and inquisitiveness

- A sentence φ is informative in a state s iff it proposes to eliminate at least one world in s, i.e., iff $s \cap \text{info}(\varphi) \neq s$.
- A sentence φ is inquisitive in s iff in order to reach a state $s' \subseteq s$ that supports φ it is not enough to incorporate the informative content of φ itself into s, i.e., $s \cap \text{info}(\varphi) \not\models \varphi$.
- This means that φ requests a response from other participants that provides additional information.

Informativeness and inquisitiveness

Definition (Inquisitiveness and informativeness in a state)

- φ is informative in s iff $s \cap \text{info}(\varphi) \neq s$
- φ is inquisitive in s iff $s \cap \text{info}(\varphi) \not\models \varphi$

Definition (Absolute inquisitiveness and informativeness)

- φ is informative iff it is informative in $\mathcal{W}$, i.e., iff $\text{info}(\varphi) \neq \mathcal{W}$
- φ is inquisitive iff it is inquisitive in $\mathcal{W}$, i.e., iff $\text{info}(\varphi) \not\models \varphi$

Fact (Alternative characterization of inquisitiveness)

- *φ is inquisitive iff $[\![\varphi]\!]$ contains at least two alternatives.*

Three semantic categories

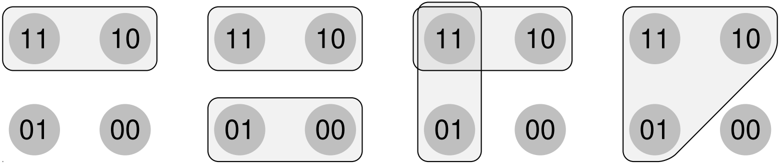
Definition (Questions, assertions, and hybrids)

- φ is a question iff it is not informative;
- φ is an assertion iff it is not inquisitive;
- φ is a hybrid iff it is both informative and inquisitive.

Fact (Characterizing questions and assertions)

- *φ is an assertion iff $\varphi \equiv\, !\varphi$.*
- *φ is an question iff $\varphi \equiv\, ?\varphi$.*

Two possibilities for disjunction



(a) p (b) $?p$ (c) $p \vee q$ (d) $!(p \vee q)$

(a) p is informative but not inquisitive ⇒ an assertion.

(b) $?p$ is inquisitive but not informative ⇒ a question.

(c) $p \vee q$ is both informative and inquisitive ⇒ a hybrid.

(d) $!(p \vee q)$ is informative but not inquisitive ⇒ an assertion.

Compliance

- Inquisitive semantics is intended to give rise to a logical notion of compliant responsehood.
- Here we only consider the most basic type of compliance.

Definition (Basic compliant responses)

- ψ is a basic compliant response to φ just in case

$$[\![\psi]\!] = \{\alpha\} \text{ for some } \alpha \in [\![\varphi]\!]$$

- Recall: an alternative α for φ is a $\subseteq$-maximal supporting state.
- A basic compliant response to an inquisitive sentence is a minimally informative issue resolving assertion.
- Propositional inquisitive semantics deals with (basic) compliance in a satisfactory way.

2. First order inquisitive semantics

Worlds based on a discourse model

Definition (Discourse models)

Let $\mathcal{L}$ be a first-order language with *n*-ary predicate symbols and *n*-ary function symbols (individual constants are 0-ary function symbols).

- A discourse-model $\mathbb{D}$ for $\mathcal{L}$ is a pair $\langle D, I \rangle$, where D is a domain and I an interpretation of all function symbols in $\mathcal{L}$.

Definition ($\mathbb{D}$-worlds)

Let $\mathbb{D} = \langle D, I \rangle$ be a discourse model.

- A world w based on $\mathbb{D}$, or simply a $\mathbb{D}$-world, is a model $\langle D_M, I_M \rangle$ such that $D_M = D$ and I_M coincides with I as far as the function symbols are concerned.
- The set of all $\mathbb{D}$-worlds is denoted by $\mathcal{W}_{\mathbb{D}}$.

States based on a discourse model

Definition ($\mathbb{D}$-states)

Let $\mathbb{D}$ be a discourse model.

- A state based on a discourse model $\mathbb{D}$ or simply a $\mathbb{D}$-state, is a set of $\mathbb{D}$-worlds.

Some properties of states

- A state is a set of first-order models for $\mathcal{L}$, called worlds.
- All worlds in a state share the same domain and the individual constants and function symbols are rigid designators in a state.
- Since the discourse model does not assign an interpretation to the predicate symbols in $\mathcal{L}$, the worlds in a state may assign different denotations to them.

Why discourse models are called like that

The shared discourse model assumption

- We assume that the information states of all participants in a conversation are based on the same discourse model, and (hence) the common ground of the conversation is too.
- The interpretation of all individual constants and function symbols in the language is part of the common ground.
- The exchange of information in a conversation thus only concerns the denotation of the predicate symbols.
- Of course this is an idealization that can (and must) eventually be relaxed.

Note on the interpretation of terms

- The interpretation function I of a discourse model extends in a natural way to an interpretation of all terms $t \in \mathcal{L}$.
- If the free variables of t are $x_1, \ldots, x_n$, then $I(t)$ will be the function $D^n \to D$ which maps a tuple $(d_1, \ldots, d_n) \in D^n$ to the element $d \in D$ denoted by the term t in the discourse model $\mathbb{D}$ when x_i is interpreted as d_i for all $i = 1, \ldots, n$.
- Roughly, think of $I(t)$ as $\lambda x_1 \ldots \lambda x_n.t$, where $x_1, \ldots, x_n$ are the free variables occurring in the term t, in that order.
- Note that if a term t is a variable x, then $I(t) \simeq \lambda x.x$ is the identity function on D.
- We will make use of this notion of the interpretation of terms in the witness semantics.

Basic first order support

Definition (First-order support)

Let s be a $\mathbb{D}$-state, g an assignment, and φ a formula in $\mathcal{L}$.

1. $s \models_g \varphi$ iff $\forall M \in s : M \models_g \varphi$ for atomic φ
2. $s \models_g \bot$ iff $s = \emptyset$
3. $s \models_g \varphi \wedge \psi$ iff $s \models_g \varphi$ and $s \models_g \psi$
4. $s \models_g \varphi \vee \psi$ iff $s \models_g \varphi$ or $s \models_g \psi$
5. $s \models_g \varphi \rightarrow \psi$ iff $\forall t \subseteq s$: if $t \models_g \varphi$ then $t \models_g \psi$
6. $s \models_g \forall x.\varphi$ iff $s \models_{g[x/d]} \varphi$ for all $d \in D$
7. $s \models_g \exists x.\varphi$ iff $s \models_{g[x/d]} \varphi$ for some $d \in D$

Examples quantifiers and inquisitiveness

- The existential quantifier inherits the inquisitive features of disjunction.
- $s \models_g \exists x.P(x)$ iff there is some object $d \in D$ such that d is in the denotation of P in all worlds $w \in s$. The state has to contain the information that some specific object d has the property P.
- $s \models_g ?!\exists x.P(x)$ iff either in all worlds $w \in s$ de denotation of P is non-empty, or in all worlds $w \in s$ de denotation of P is empty.
- $s \models_g \forall x.?P(x)$ iff in all $w \in s$ the denotation of P is the same.

Basic first order semantics

Except for relativization to assignment functions

- The basic facts of persistence and classical behavior of singleton states is preserved.
- The definitions of entailment and equivalence are preserved.
- The definitions of informative content, informativeness and inquisitiveness are preserved.
- The definitions of assertion, question and hybrids are preserved.
- The facts concerning the characterization of assertions and questions are preserved.

3. Problems with the boundedness sentences

The maximality problem

- One feature of the propositional semantics system is not preserved in the basic first order setting:
- The proposition expressed by a sentence is no longer fully determined by the alternative set of that sentence.
- It is not always the case that every state supporting φ is contained in a maximal state supporting φ.
- Ciardelli has shown there are first-order formulas that do not have any maximal supporting states in the basic first-order system.

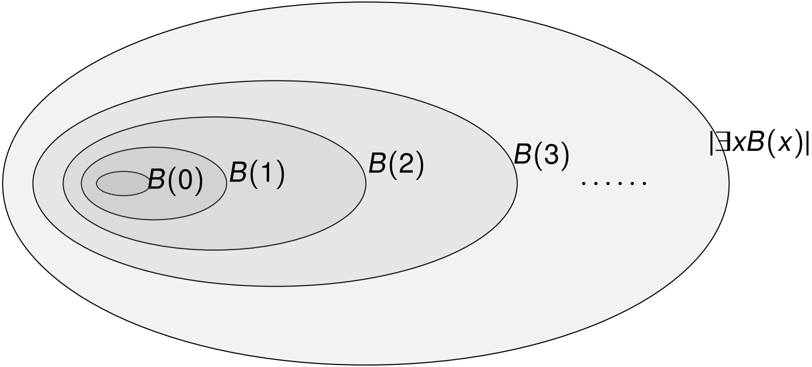
A specific discourse model for a specific language

- Consider a language which has a unary predicate symbol P, a binary function symbol $+$, and the set $\mathbb{N}$ of natural numbers as its individual constants.
- Consider the discourse-model $\mathbb{D} = \langle D, I \rangle$, where $D = \mathbb{N}$, I maps every $n \in \mathbb{N}$ to itself, and $+$ is interpreted as addition.
- Let $x \leq y$ abbreviate $\exists z(x + z = y)$.
- Let $B(x)$ abbreviate $\forall y(P(y) \rightarrow y \leq x)$.
- For every $n \in \mathbb{N}$, let $B(n)$ abbreviate $\forall y(P(y) \rightarrow y \leq n)$.
- Intuitively, $B(n)$ says that n is greater than or equal to any number in P.
- In other words, $B(n)$ says that n is an *upper bound* for P.

The boundedness formula

- A state s supports a formula $B(n)$, for some $n \in \mathbb{N}$, if and only if $B(n)$ is true in every world in s, that is, if and only if n is an upper bound for P in every w in s.
- Now consider the formula $\exists x.B(x)$, which intuitively says that there is an upper bound for P.
- This formula, which Ciardelli refers to as the boundedness formula, does not have any maximal supporting state.

The boundedness formula



The intended alternatives $|B(n)|$ for the boundedness formula and its truth set $|\exists x B(x)|$, which is not itself an alternative.

The maximality problem

- Let s be an arbitrary state supporting $\exists x.B(x)$.
- Then there must be a number $n \in \mathbb{N}$ such that s supports $B(n)$, i.e., $B(n)$ must be true in all worlds in s.
- Now let w^* be the world in which P denotes the singleton set $\{n + 1\}$.
- Then w^* cannot be in s, because it does not make $B(n)$ true.
- Thus, the state s^* which is obtained from s by adding w^* to it is a proper superset of s itself.
- However, s^* clearly supports $B(n + 1)$, and therefore also still supports $\exists x.B(x)$.
- This shows that any state supporting $\exists x.B(x)$ can be extended to a larger state which still supports $\exists x.B(x)$.
- Therefore no state supporting $\exists x.B(x)$ can be maximal.

The compliance issue

- This example shows that our notion of basic compliant responses, which makes crucial reference to maximal supporting states, does not always yield satisfactory results in the first-order setting.

- At first sight, it is tempting to conclude from this that there must be something wrong with the given notion of basic compliant responses.

- However, the problem is deeper than that.

- Ciardelli's next example shows that the very notion of meaning assumed in basic inquisitive semantics is not fine-grained enough to serve as a basis for a suitable notion of compliance in the first-order setting.

The boundedness problem

- Consider the following variant of the boundedness formula:

$$\exists x(x \neq 0 \wedge B(x))$$

- This formula says that there is a positive upper bound for P.
- Intuitively, it differs from the ordinary boundedness formula in that $B(0)$ is not a compliant response.
- However, in terms of support, $\exists x(x \neq 0 \wedge B(x))$ and $\exists x.B(x)$ are equivalent.
- The current notion of support is not fine-grained enough to capture the fact that these formulas intuitively do not have the same range of compliant responses.

Intermediate conclusions

Under the support semantics presented above:

$$\exists x(x \neq 0 \wedge B(x)) \equiv \exists x.B(x)$$

- This does not imply that the semantics does not give an appropriate account of the logical-semantical notions of informativeness and inquisitiveness in the first order setting.

  Purely in terms of these notions the equivalence should hold!

- It does imply that the semantics is not fine-grained enough to provide a general account of compliance.

  In terms of compliance the equivalence should not hold.

4. Inquisitive witness semantics

Introducing witnesses

- In the basic semantics for a state to support $\exists x.Px$, there should be a witness in the sense that: there is some object such that in every world in the state, that object belongs to the denotation of P.
- There can be very many such witnesses in a supporting state (Cf. the boundedness sentences, where if there is one witness in this sense, there are infinitely many).
- However, states as such only embody information, they are not rich enough to represent that some specific object has been introduced by the discourse as a witness.
- We will enrich states to allow for this.

Witnesses

- For a formula like $\exists x.Px$ an object $d \in D$ suffices as a witness.
- But when an existential is embedded under a universal, as in $\forall x \exists y.Rxy$, this no longer suffices.
- Intuitively, support of $\forall x.\exists y.Rxy$ does not require that a specific object $d \in D$ is known to stand in the relation R with all other objects in D.
- To avoid problems of this sort, we will take witnesses to be functions from D^n to D, where $n \geq 0$.
- Notice that some of these functions are 0-place functions into D, which can be simply identified with objects in D.
- So witnesses can still be objects in D. But they can be other things as well.

Witnesses and states

- We assume a fixed first-order language $\mathcal{L}$ and a fixed discourse-model $\mathbb{D} = \langle D, I \rangle$ for $\mathcal{L}$.

Definition (Witnesses)

- For any $n \in \mathbb{N}$, let $D^\star_n$ be the set of functions $\delta \colon D^n \to D$.
- Then $D^\star = \bigcup_{n \geq 0} D^\star_n$ is the set of all $\mathbb{D}$-witnesses.

Definition (States with witnesses)

- A $\mathbb{D}$-state is a pair $\langle V, \Delta \rangle$, where V is a set of $\mathbb{D}$-worlds and Δ is a finite set of $\mathbb{D}$-witnesses, which contains the identity function $id : D \to D$.
- The set of all $\mathbb{D}$-states is denoted by $\mathcal{S}_{\mathbb{D}}$.
- If $s = \langle V, \Delta \rangle$ is a $\mathbb{D}$-state, then: $\text{worlds}(s) = V$ & $\text{witn}(s) = \Delta$.

Extension

- In what follows, we will usually drop reference to $\mathbb{D}$, and simply refer to a $\mathbb{D}$-state as a state.
- The set of states is partially ordered by the extension relation.

Definition (Extension)

Let s and t be two states. Then we say that s is an extension of t, $s \geq t$, iff $\text{worlds}(s) \subseteq \text{worlds}(t)$ and $\text{witn}(t) \subseteq \text{witn}(s)$.

- Notice that there is a minimal state, namely $\text{top} = (\mathcal{W}, \{id\})$, of which any other state is an extension.

Witness feeds

Definition (Witness feeds)

- A witness feed ϵ is a finite set of elements of D.

The role of witness feeds

- The role of witness feeds will be similar to that of assignments: they will be used to store certain information in evaluating whether or not a certain state supports a certain sentence.

- In particular, they play a role in evaluating existentially quantified sentences in the scope of one or more universal quantifiers.

Definition (Witness support)

Let s be a $\mathbb{D}$-state, g an assignment, ϵ a witness feed, and $\varphi \in \mathcal{L}$.

1. $s \models_{g,\epsilon} R(t_1, \ldots, t_n)$ iff
   (i) $\text{worlds}(s) \subseteq |R(t_1, \ldots, t_n)|$ and
   (ii) $I(t_i) \in \text{witn}(s)$ for $i = 1, \ldots, n$
2. $s \models_{g,\epsilon} \bot$ iff $\text{worlds}(s) = \emptyset$
3. $s \models_{g,\epsilon} \varphi \wedge \psi$ iff $s \models_{g,\epsilon} \varphi$ and $s \models_{g,\epsilon} \psi$
4. $s \models_{g,\epsilon} \varphi \vee \psi$ iff $s \models_{g,\epsilon} \varphi$ or $s \models_{g,\epsilon} \psi$
5. $s \models_{g,\epsilon} \varphi \to \psi$ iff $\forall t \geq s$: if $t \models_{g,\epsilon} \varphi$ then $t \models_{g,\epsilon} \psi$
6. $s \models_{g,\epsilon} \forall x.\varphi$ iff $s \models_{g[x/d],\epsilon \cup \{d\}} \varphi$ for all $d \in D$
7. $s \models_{g,\epsilon} \exists x.\varphi$ iff $s \models_{g[x/\delta(e_1,\ldots,e_n)],\epsilon} \varphi$ for some
   (i) $\delta \in \text{witn}(s)$ and
   (ii) $e_i \in \epsilon$ for $i = 1, \ldots, n$

Atomic sentences

$$s \models_{g,\epsilon} R(t_1, \ldots, t_n) \quad \text{iff}$$

$$\text{(i) worlds}(s) \subseteq |R(t_1, \ldots, t_n)| \text{ and}$$

$$\text{(ii) } I(t_i) \in \text{witn}(s) \text{ for } i = 1, \ldots, n$$

- Only the last part is special: for each term t_i that forms an argument of the predicate symbol R, the function $I(t_i)$ it denotes must be available as a witness in witn(s).
- In case an argument t_i is a variable x, $I(t_i)$ is the identity function, which is always present in the witness set.
- In case an individual constant c is an argument of R, the object it denotes must be available as a witness.
- In case $f(x)$ is an argument of R, since $I(f(x)) = I(f)$, it is this 1-place function that must be available as a witness.

Atomic sentences introduce new witnesses

- In line with our general view of propositions as proposals to update the common ground:

  In uttering an atomic sentence $R(t_1, \dots, t_n)$, a speaker proposes to add $I(t_1) \dots I(t_n)$ to the witness set of the common ground.

  (next to proposing to eliminate worlds from the common ground where the atomic sentence does not hold.)

- Thus, we can think of atomic sentences like $R(t_1, \dots, t_n)$ as introducing new witnesses.

- The reason for this is that we assume the participants in a conversation to share the same discourse model, and hence to share the rigid interpretation of all function symbols.

Implication

$$s \models_{g,\epsilon} \varphi \rightarrow \psi \text{ iff } \forall t \geq s : \text{if } t \models_{g,\epsilon} \varphi \text{ then } t \models_{g,\epsilon} \psi$$

- It may be that all the extensions of s that support φ contain certain witnesses that are not contained in witn(s) itself.
- This means that if ψ requires certain witnesses, as long as we need to introduce them to support φ, it is not necessary for s as such to already contain them for the implication to be supported in s.
- Example: top $\models_{g,\epsilon} P(a) \rightarrow \exists x.P(x)$.

Universal quantification

$s \models_{g,\epsilon} \forall x.\varphi$ iff $s \models_{g[x/d],\epsilon\cup\{d\}} \varphi$ for all $d \in D$

- In determining whether a state s supports a formula $\forall x.\varphi$ we do not only set the current assignment g to $g[x/d]$, but we simultaneously augment the current witness feed ϵ with the same object d.
- Then we check whether φ is supported by s relative to the adapted assignment and the augmented witness feed.
- As we will see below, the augmented witness feed is put to use when φ contains an existential quantifier.

Existential quantification

$$s \models_{g,\epsilon} \exists x.\varphi \quad \text{iff} \quad s \models_{g[x/\delta(e_1,\ldots,e_n)],\epsilon} \varphi \quad \text{for some}$$

$$\text{(i) } \delta \in \text{witn}(s) \text{ and}$$

$$\text{(ii) } e_i \in \epsilon \text{ for } i = 1, \ldots, n$$

- In checking whether $s \models_{g,\epsilon} \exists x.\varphi$ holds, we have to check whether $s \models_{g[x/d],\epsilon} \varphi$ holds, where d is an object that we obtain by applying some witness $\delta \in \text{witn}(s)$ to objects $e_1, \ldots, e_n$ in the witness feed.
- Thus, as desired, support of $\exists x.Px$ now really requires the presence of a witness which is known to have the property P.
- This means that in uttering $\exists x.Px$, a speaker requests a response that introduces a suitable witness and then establishes that this witness has the property P.

Existentials in the scope of universals

- In order to determine whether $s \models_g \forall x.\exists y.Rxy$, we have to check whether $s \models_{g[x/d],\{d\}} \exists y.Rxy$ for all $d \in D$.
- We have to verify whether for every $d \in D$, there is a witness $f \in \text{witn}(s)$ such that $s \models_{g[x/d][y/f(d,\ldots,d)],\{d\}} Rxy$.
- The witness f used may be an element of the domain or a function of arity $n \geq 1$.
- It may also be the identity function, then objects d introduced by the universal can serve as witnesses for the existential.
- Universal quantifiers add objects to the witness feed which can serve as input for functional witnesses that may be needed for existentials in its scope.
- In this way, the witness that is required for the embedded existential in $\forall x.\exists y.Rxy$ may functionally depend on the value that the current assignment assigns to x.

Three (of the many) states that support $\forall x.\exists y.Rxy$

1. The witness feed is not used.
   $\text{witn}(s) = \{id, I(a)\}$
   $\langle d, I(a)\rangle \in w(R)$ for all $d \in D$ and all $w \in \text{worlds}(s)$.
   Basic compliant response $\forall x.R(x, a)$ supported.
2. Identity function is applied to the object in the witness feed.
   $\text{witn}(s) = \{id\}$
   $\langle d, d\rangle \in w(R)$ for all $d \in D$ and all $w \in \text{worlds}(s)$
   Basic compliant response $\forall x.R(x, x)$ supported.
3. Additional function in witness set applied to object in feed.
   $\text{witn}(s) = \{id, I(f)\}$
   $\langle d, I(f)(d)\rangle \in w(R)$ for all $d \in D$ and all $w \in \text{worlds}(s)$
   Basic compliant response $\forall x.R(x, f(x))$ supported.

Linguistic relevance

Natural language examples

(1) Everyone loves someone.

(2) Who does everyone love?

a. Everyone loves Mary.
b. Everyone loves himself.
c. Everyone loves his mother.
d. John loves Mary and everyone else loves Sue.

- When we translate (1) as $\forall x.\exists y.Rxy$, then (a)-(c) correspond to the three responses discussed above.
- And (d) can be accounted for along the same lines.
- This may take us to assume that at the logical level the interrogative (2) is also associated with $\forall x.\exists y.Rxy$

Some logical facts

Fact (Persistence)

- *If* $s \models_{g,\epsilon} \varphi$ *and* $t \geq s$, *then* $t \models_{g,\epsilon} \varphi$

Fact (Support for negation)

- $s \models_{g,\epsilon} \neg\varphi$ *iff* *for all* $w \in worlds(s)$: $w \not\models_g \varphi$ *classically*
- $s \models_{g,\epsilon} !\varphi$ *iff* *for all* $w \in worlds(s)$: $w \models_g \varphi$ *classically*

Propositions and alternatives

Definition (Propositions)

1. $[\varphi]_g := \{s \in \mathcal{S}_{\mathbb{D}} \mid s \models_g \varphi\}$

Definition (Alternatives)

Let φ be a formula and g an assignment.

1. Every $\geq$-minimal element of $[\varphi]_g$ is called an alternative for φ relative to g.
2. The alternative set of φ relative to g, $[\![\varphi]\!]_g$, is the set of alternatives for φ relative to g.

Two questions:

- Does $[\varphi]_g$ always have at least one $\geq$-minimal element?
- Does $[\![\varphi]\!]$ always give rise to an appropriate characterization of basic compliant responses?

Entailment and equivalence

Convention

We write $s \models_g \varphi$ for $s \models_{g,\epsilon} \varphi$, when $\epsilon = \emptyset$.

Definition (Entailment and equivalence)

- $\varphi \models \psi$ iff for all s and g: if $s \models_g \varphi$, then $s \models_g \psi$
- $\varphi \equiv \psi$ iff $\varphi \models \psi$ and $\psi \models \varphi$

- The entailment relation also concerns the availability of witnesses. That means for example that:

$$\forall x.Px \not\models P(a)$$

- We introduce a notion of factual entailment in terms of factual support under which, e.g., universal instantiation does hold.

Factive support and entailment

Definition (Factive (*) support, entailment, and equivalence)

- $V \models^\star_g \varphi$ iff $s \models_g \varphi$ for some state s with worlds$(s) = V$
- $\varphi \models^\star \psi$ iff for all V, g: if $V \models^\star_g \varphi$, then $V \models^\star_g \psi$
- $\varphi \equiv^\star \psi$ iff $\varphi \models^\star \psi$ and $\psi \models^\star \varphi$

The following proposition shows that as soon as we disregard witness issues, the witness semantics coincides with the basic semantics.

Fact (Factive support and basic support)

$$V \models^\star_g \varphi \quad \Leftrightarrow \quad V \models_g \varphi \textit{ in basic inquisitive semantics}$$

Witness insensitivity

Definition (Witness insensitivity)

- φ is witness insensitive iff for all s, g:

$$\text{if} \quad \text{worlds}(s) \models^{\star}_{g} \varphi \quad \text{then} \quad s \models_{g} \varphi$$

Fact (Partial characterization of witness insensitivity)

1. *For atomic φ, including $\bot$, φ is witness insensitive iff there is no occurrence of a constant or a function symbol in φ;*
2. *If φ and ψ are witness insensitive, then $\varphi \vee \psi$ and $\varphi \wedge \psi$ are.*
3. *If ψ is witness insensitive, then $\varphi \rightarrow \psi$ is witness insensitive;*
4. *$\exists x.\varphi$ is not witness insensitive for any φ;*
5. *$\forall x.\varphi$ is witness insensitive iff φ is witness insensitive.*

Informative content is classical

Definition (Informative content)

- $\text{info}_g(\varphi) = \bigcup\{\text{worlds}(s) \mid s \in [\varphi]_g\}$.

Fact (Informative content is classical)

- *For every φ and every g:* $\quad \textit{info}_g(\varphi) = |\varphi|_g$

Informativeness and inquisitiveness

Definition (Inquisitiveness and informativeness in a state)

- φ is informative in s w.r.t. g iff $\text{worlds}(s) \cap \text{info}_g(\varphi) \neq \text{worlds}(s)$
- φ is inquisitive in s w.r.t. g iff $\text{worlds}(s) \cap \text{info}_g(\varphi) \not\models^{\star}_{g} \varphi$

Definition (Absolute inquisitiveness and informativeness)

- φ is informative iff for some g: $\text{info}_g(\varphi) \neq \mathcal{W}$
- φ is inquisitive iff for some g: $\text{info}_g(\varphi) \not\models^{\star}_{g} \varphi$

Informativeness and inquisitiveness

- All notions in the basic system that are defined in terms of informativeness and inquisitiveness, such as assertion question and hybrid, remain precisely the same.
- But among assertions and questions there is a further distinction now between witness sensitive and witness insensitive ones.
- All formulas in (1)-(4) are factively equivalent, (1) is fully equivalent with (2), and (3) is fully equivalent with (4), but, e.g., (1) and (3) are not fully equivalent.

(3) $P(a)$

(4) $\exists x(x = a \wedge P(x))$

(5) $\forall x(x = a \rightarrow P(x))$

(6) $!P(a)$

The boundedness problem solved

Fact (The boundedness formulas)

The boundedness formula and the positive boundedness formula are not equivalent in the witness semantics

Proof

Consider a state s such that:

- $\text{worlds}(s) = \{w\}$, where $w(P) = \{0\}$
- $\text{witn}(s) = \{id, 0\}$

State s factively supports both $\exists x.B(x)$ and $\exists x.(x > 0 \wedge B(x))$. However, while the boundedness formula is supported in s *tout court*, $s \models \exists x.B(x)$, the positive boundedness formula is not, $s \not\models \exists x.(x > 0 \wedge B(x))$. So, the boundedness sentence and the positive boundedness sentence are not equivalent in the witness semantics (although they are factively equivalent). □

Compliance and the boundedness sentences

- If we copy the definition form the basic system, the basic compliant responses to a sentence φ are characterized as those responses that provide a minimal amount of information and witnesses to establish a state that supports φ.

Definition (Basic compliant responses)

- ψ is a basic compliant response to φ just in case:

$$[\![\psi]\!] = \{\alpha\} \text{ for some } \alpha \in [\![\varphi]\!]$$

Fact (Compliant responses to the boundedness formulas)

- *For any $n \geq 0$, $B(n)$ is a basic compliant response to $\exists x.Bx$*
- *For any $n > 0$, $B(n)$ is a basic compliant response to $\exists x.(x \neq 0 \wedge Bx)$, and $B(0)$ is not.*

The compliance problem is not solved in general

- Let $B_P(x) = \forall y(P(y) \rightarrow y \leq x)$
- Let $B_Q(x) = \forall y(Q(y) \rightarrow y \leq x)$
- So, $\exists x.B_P(x)$ and $\exists x.B_Q(x)$ are two distinct boundedness formulas.
- Now consider $\exists x.B_P(x) \wedge \exists x.B_Q(x)$.
- since $B_P(3)$ is a compliant response to $\exists x.B_P(x)$ and $B_Q(4)$ is a compliant response to $\exists x.B_Q(x)$, we would expect $B_P(3) \wedge B_Q(4)$ to come out as a compliant response to the conjunction.
- However, $B_P(3) \wedge B_Q(4)$ strictly entails $B_P(4) \wedge B_Q(4)$ which is issue-resolving, so that $B_P(3) \wedge B_Q(4)$ does not count as a compliant response.
- Intuitively, this is of course wrong: in fact $B_P(3) \wedge B_Q(4)$ is preferable as a response over $B_P(4) \wedge B_Q(4)$.

5. Conclusions and an open question

Conclusions

- Our goal was to provide a notion of meaning that embodies the informative and inquisitive content of a sentence, and determines the range of compliant responses to a sentence.
- Basic compliant responses are assertions that resolve a given issue without providing more information than necessary.
- Inquisitive semantics intends to provide a semantic framework in which this notion can be formalized in a satisfactory way.
- In the propositional case the basic compliant responses to a sentence correspond to the alternatives for that sentence.
- This simple picture breaks down in the basic first order semantics, its notion of meaning is in need of refinement.
- Such a refinement is offered by the witness semantics.
- It allows us to distinguish sentences with the same informative and inquisitive content but different compliant responses.

Open questions

- Does the witness semantics achieve the goal for which it was designed?
- Does it generally make satisfactory predictions concerning compliant responses to first-order formulas?
- We have seen that the notion of alternatives, defined as $\geq$-minimal supporting states, does not exhaustively characterize basic compliant responses in all cases.
- It did so for the boundedness sentences as such, not for conjunctions of them.
- Open questions: is there a different notion of compliant responses, richer than the notion in terms of $\geq$-minimal supporting states, that fits the witness semantics?
- Or, are we in need of a further refinement of the witness semantics as such?

Thank you for your attention

The full paper van be found at our website

www.illc.uva.nl/inquisitive-semantics

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