Inquisitive epistemic logic

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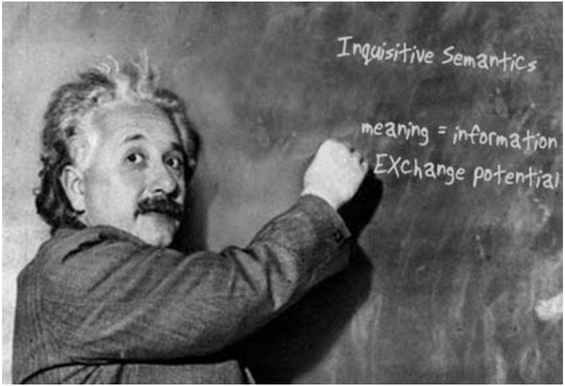

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Motivation

One of the primary applications of logic

- Modeling information exchange through communication between a number of agents

Epistemic logic

- Allows us to model what the facts are in a given situation and what all the agents know about these facts and about each other
- Allows us to capture the informative content of sentences

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- Allows us to model what the facts are in a given situation and what all the agents know about these facts and about each other
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What is missing from this picture?

Motivation

What is missing?

- It is not only important to model what the agents know, but also what they want to know, i.e., the issues that they entertain

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Moreover:

- Agents do not only use declarative sentences to provide information
- They also use interrogative sentences to request information
- To capture the meaning of both declaratives and interrogatives, we need to be able to capture both informative and inquisitive content

Motivation

Summing up

- Epistemic models need to be enriched with issues
- Our logical language needs to be enriched with interrogatives
- Our notion of meaning needs to be enriched with inquisitive content

Epistemic logic

Epistemic models

An epistemic model is a triple $M = \langle \mathcal{W}, V, \{\sigma_a \mid a \in A\} \rangle$, where:

- $\mathcal{W}$ is a set, whose elements are called possible worlds
- $V : \mathcal{W} \to \wp(\mathcal{P})$ is called the valuation function
- $\sigma_a : \mathcal{W} \to \wp(\mathcal{W})$ is called the epistemic map of agent a

  For any world w, $\sigma_a(w)$ is an information state satisfying:

  - Factivity: $w \in \sigma_a(w)$
  - Introspection: for all $v \in \sigma_a(w)$: $\sigma_a(v) = \sigma_a(w)$

Epistemic logic

- The language $\mathcal{L}_{EL}$ is a propositional language enriched with knowledge modalities K_a.
- The semantics is given by a recursive definition of truth w.r.t. a world.
- The proposition expressed by a sentence φ in a model M is the set of worlds where the sentence is true:

$$|\varphi|_M = \{w \in \mathcal{W} \mid \langle M, w\rangle \models \varphi\}$$

- An agent knows φ iff φ is true in all worlds in the agent's info state:

$$\langle M, w\rangle \models K_a\varphi \iff \text{for all } v \in \sigma_a(w),\ \langle M, v\rangle \models \varphi$$

- Notice that K_a expresses a relation between two sets of worlds: a's information state and the proposition expressed by φ

$$\langle M, w\rangle \models K_a\varphi \iff \sigma_a(w) \subseteq |\varphi|_M$$

Modeling issues

How to model issues?

- We want to equip agents not only with an information state, but also with an inquisitive state that describes the issues that they entertain.
- But how to model issues?

Key idea

- Model an issue as a set of information states
- Namely, those information states in which the issue is resolved

Example

- The issue 'who won the elections' is modeled as the set of information states in which it is known who won the elections

Two constraints

Does any set of information states properly represent an issue?

– No, there are two constraints.

Issues are downward closed

- If an issue I is resolved in an information state s, then it will also be resolved in any more informed information state $t \subset s$
- So issues are downward closed: for any $s \in I$, if $t \subset s$, then $t \in I$

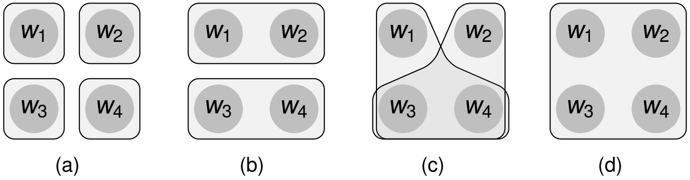
Issues always contain the inconsistent information state

- The inconsistent information state is the empty state, $\emptyset$
- It is standardly assumed that in $\emptyset$ everything is known
- Similarly, we assume that in $\emptyset$ every issue is resolved
- This means that an issue always contains $\emptyset$
- Equivalently, issues are always non-empty sets of info states

Definition of issues and examples

Issues

- An issue is a non-empty, downward closed set of information states.
- We say that an issue I is an issue over a state s in case $s = \bigcup I$.
- The set of all issues is denoted by Π.



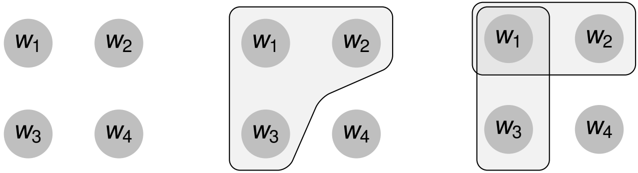
Four issues over the state $\{w1, w2, w3, w4\}$.

Inquisitive epistemic models

A model should now provide, for every possible world w:

1. a specification $V(w)$ of the basic facts;
2. a specification $\sigma_a(w)$ of the agents' information;
3. a specification $\Sigma_a(w)$ of the agents' issues.

where $\Sigma_a(w)$ should be an issue over $\sigma_a(w)$.



(a) $\mathcal{W}$ (b) $\sigma_a(w_1)$ (c) $\Sigma_a(w_1)$

Inquisitive epistemic models

Can we simplify?

- Do we need to specify all three components explicitly?
- Recall that $\Sigma_a(w)$ should be an issue over $\sigma_a(w)$.
- But this means that $\sigma_a(w) = \bigcup \Sigma_a(w)$.
- So if we know $\Sigma_a(w)$, we also know $\sigma_a(w)$.
- This means that $\sigma_a(w)$ does not need to be specified explicitly.
- $\Sigma_a(w)$ already encodes both information and issues.

Definition of inquisitive epistemic models

Inquisitive epistemic models

An inquisitive epistemic model is a triple $M = \langle \mathcal{W}, V, \{\Sigma_a \mid a \in A\} \rangle$, where:

- $\mathcal{W}$ is a set, whose elements are called possible worlds
- $V : \mathcal{W} \to \wp(\mathcal{P})$ is called the valuation function
- $\Sigma_a : \mathcal{W} \to \Pi$ called the inquisitive state map of agent a

  For any w, $\Sigma_a(w)$ is an issue satisfying:

  - Factivity: $w \in \sigma_a(w)$
  - Introspection: for all $v \in \sigma_a(w)$: $\Sigma_a(v) = \Sigma_a(w)$

where for any w, $\sigma_a(w) := \bigcup \Sigma_a(w)$ is the information state of a at w.

Enriching the logical language

- We have enriched epistemic models with issues
- The next step is to enrich our logical language with interrogatives

Syntax

- Declaratives: $\alpha ::= p \mid \bot \mid \alpha \wedge \alpha \mid \alpha \rightarrow \alpha \mid K_a\varphi \mid E_a\varphi$
- Interrogatives: $\mu ::= ?\{\alpha_1, \ldots, \alpha_n\} \mid \mu \wedge \mu \mid \alpha \rightarrow \mu$

Abbreviations

- $\neg\alpha := \alpha \rightarrow \bot$
- $\alpha \vee \beta := \neg(\neg\alpha \wedge \neg\beta)$
- $?\alpha := ?\{\alpha, \neg\alpha\}$

Examples

- $K_a?K_b?p$
- $p \rightarrow ?K_bp$
- $K_a?p \rightarrow ?K_bK_a?p$

From truth conditions to support conditions

- We have enriched our models and our logical language.
- Next, we need to enrich the semantics.
- In EL, the semantics specifies truth conditions wrt worlds.
- For interrogatives, this does not work.
- Rather, we should give resolution condition wrt information states.
- We could give a simultaneous definition of truth and resolution.
- But there is a better solution: we will lift the interpretation of declarative sentences from worlds to information states as well.
- We define a support relation, $s \models \varphi$, where intuitively:
  - $s \models \alpha$ amounts to: α is established, or true everywhere in s;
  - $s \models \mu$ amounts to: μ is resolved in s.

Support conditions

Definition (Support)

1. $\langle M, s\rangle \models p \iff p \in V(w)$ for all worlds $w \in s$
2. $\langle M, s\rangle \models \bot \iff s = \emptyset$
3. $\langle M, s\rangle \models ?\{\alpha_1, \ldots, \alpha_n\} \iff \langle M, s\rangle \models \alpha_i$ for some $i \in \{1, \ldots, n\}$
4. $\langle M, s\rangle \models \varphi \wedge \psi \iff \langle M, s\rangle \models \varphi$ and $\langle M, s\rangle \models \psi$
5. $\langle M, s\rangle \models \alpha \rightarrow \varphi \iff$ for any $t \subseteq s$, if $\langle M, t\rangle \models \alpha$ then $\langle M, t\rangle \models \varphi$
6. $\langle M, s\rangle \models K_a\varphi \iff$ for any $w \in s$, $\langle M, \sigma_a(w)\rangle \models \varphi$
7. $\langle M, s\rangle \models E_a\varphi \iff$ for any $w \in s$ and for any $t \in \Sigma_a(w)$, $\langle M, t\rangle \models \varphi$

Fact (Persistence, empty state)

- Persistence: if $\langle M, s\rangle \models \varphi$ and $t \subseteq s$ then $\langle M, t\rangle \models \varphi$.
- Empty state: $\langle M, \emptyset\rangle \models \varphi$ for any sentence φ.

Deriving truth conditions from support conditions

Definition (Truth)

φ is defined to be true at a world w, $\langle M, w\rangle \models \varphi$, just in case $\langle M, \{w\}\rangle \models \varphi$

Fact (Truth conditions)

1. $\langle M, w\rangle \models p \iff p \in V(w)$
2. $\langle M, w\rangle \not\models \bot$
3. $\langle M, w\rangle \models ?\{\alpha_1, \ldots, \alpha_n\} \iff \langle M, w\rangle \models \alpha_i$ *for some index* $1 \leq i \leq n$
4. $\langle M, w\rangle \models \varphi \wedge \psi \iff \langle M, w\rangle \models \varphi$ *and* $\langle M, w\rangle \models \psi$
5. $\langle M, w\rangle \models \alpha \rightarrow \varphi \iff \langle M, w\rangle \not\models \alpha$ *or* $\langle M, w\rangle \models \varphi$
6. $\langle M, w\rangle \models \neg\alpha \iff \langle M, w\rangle \not\models \alpha$
7. $\langle M, w\rangle \models \alpha \vee \beta \iff \langle M, w\rangle \models \alpha$ *or* $\langle M, w\rangle \models \beta$
8. $\langle M, w\rangle \models K_a\varphi \iff \langle M, \sigma_a(w)\rangle \models \varphi$
9. $\langle M, w\rangle \models E_a\varphi \iff$ *for any* $t \in \Sigma_a(w)$, $\langle M, t\rangle \models \varphi$

Inquisitive epistemic logic

Definition (Proposition)
The proposition expressed by φ in M is the set $[\varphi]_M = \{s \mid \langle M, s\rangle \models \varphi\}$.

Definition (Truth-set)
The truth-set of φ in M is the set of worlds $|\varphi|_M = \{w \mid \langle M, w\rangle \models \varphi\}$.

Fact (Truth-sets and propositions)
For any φ and any M:

$$|\varphi|_M = \bigcup [\varphi]_M$$

Truth for declaratives and interrogatives

Truth for declaratives

- The semantics of a declarative is determined by its truth conditions:

$$\langle M, s\rangle \models \alpha \iff \text{for all } w \in s,\ \langle M, w\rangle \models \alpha$$

- Thus, for any declarative α we have $[\alpha]_M = \wp(|\alpha|_M)$.

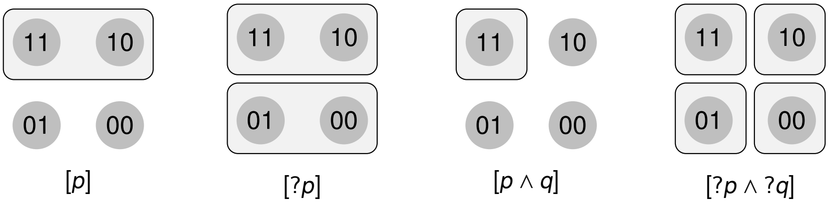
Truth for interrogatives

- $\langle M, w\rangle \models \mu$ just in case $w \in s$ for some state s that supports μ.
- That is, μ is true at a world just in case it can be truthfully resolved.

Back to the support conditions: basic cases

Definition

1. $\langle M, s \rangle \models p \iff p \in V(w)$ for all worlds $w \in s$
2. $\langle M, s \rangle \models \bot \iff s = \emptyset$
3. $\langle M, s \rangle \models ?\{\alpha_1, \ldots, \alpha_n\} \iff \langle M, s \rangle \models \alpha_i$ for some index $1 \leq i \leq n$
4. $\langle M, s \rangle \models \varphi \land \psi \iff \langle M, s \rangle \models \varphi$ and $\langle M, s \rangle \models \psi$



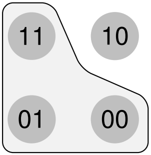
Support for implication

Definition

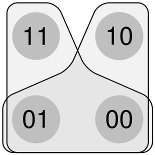
- $\langle M, s\rangle \models \alpha \to \varphi \iff$ for any $t \subseteq s$, if $\langle M, t\rangle \models \alpha$ then $\langle M, t\rangle \models \varphi$

Fact

- $\langle M, s\rangle \models \alpha \to \varphi \iff \langle M, s \cap |\alpha|_M\rangle \models \varphi$



$[p \to q]$



$[p \to ?q]$

The knowledge modality

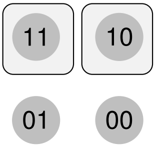
General definition

- $\langle M, w\rangle \models K_a\varphi \iff \langle M, \sigma_a(w)\rangle \models \varphi$

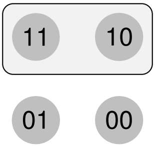
Knowing a declarative

For a declarative α, support amounts to truth at each world, so in this case we recover the standard clause:

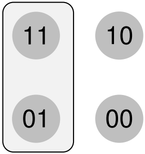
- $\langle M, w\rangle \models K_a\alpha \iff$ for all $v \in \sigma_a(w)$, $\langle M, v\rangle \models \alpha$



$\Sigma_a(w_{11})$



$[p]$



$[q]$

The knowledge modality

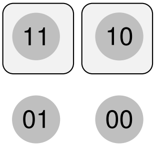
General definition

- $\langle M, w\rangle \models K_a\varphi \iff \langle M, \sigma_a(w)\rangle \models \varphi$

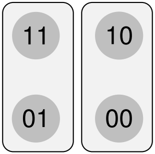
Knowing an interrogative

- $\langle M, w\rangle \models K_a\mu \iff \sigma_a(w)$ resolves μ.

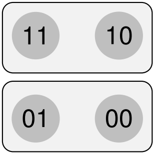
Example: $K_a?p \equiv K_a p \vee K_a\neg p$



$\Sigma_a(w_{11})$



$[?q]$



$[?p]$

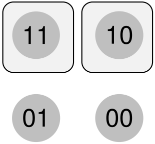
The entertain modality

General definition

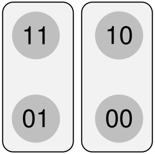
- $\langle M, w\rangle \models E_a\varphi \iff$ for any $t \in \Sigma_a(w)$, $\langle M, t\rangle \models \varphi$

Declaratives and interrogatives

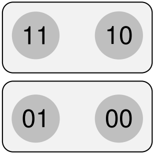
- For a declarative α, $E_a\alpha \equiv K_a\alpha$.
- For an interrogative μ, $E_a\mu$ is true just in case whenever the internal issues of a are resolved, μ is also resolved.
- Intuitively, $E_a\mu$ is true iff every state that a wants to reach is one that supports μ



$\Sigma_a(w_{11})$



$[?q]$



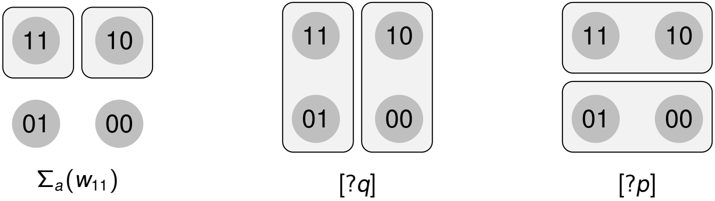
$[?p]$

Wondering = entertaining without knowing

- The truth conditions for $E_a\mu$ are close to those for *a* wonders about μ
- But one exception: if *a* already knows how to resolve μ, $E_a\mu$ is true but we would not say that *a* wonders about μ.
- So to wonder about μ is to entertain μ without knowing μ.

$$W_a\varphi := E_a\varphi \wedge \neg K_a\varphi$$

- With this definition, $W_a\varphi$ is a contradiction if φ is a declarative.



Comparison with basic EL: the modalities

- Our operators K_a and E_a are not Kripke modalities.
- However, like Kripke modalities in epistemic logic, they express a relation between two semantic objects of the same type:
  - a state associated with the world
  - the proposition expressed by the prejacent
- In EL, states $\sigma_a(w)$ and propositions $|\varphi|_M$ are simple sets of worlds:
  - $\langle M, w \rangle \models K_a\varphi \iff \sigma_a(w) \subseteq |\varphi|_M$
- In IEL, both states $\Sigma_a(w)$ and propositions $[\varphi]_M$ are more structured, namely, they are issues.
  - $\langle M, w \rangle \models K_a\varphi \iff \bigcup \Sigma_a(w) \in [\varphi]_M$
  - $\langle M, w \rangle \models E_a\varphi \iff \Sigma_a(w) \subseteq [\varphi]_M$

Comparison with basic EL: the logic

IEL is a conservative extension of EL

- Any IE-model M induces a standard epistemic model M^e, obtained simply by forgetting the issues for each agent.
- For any IE-model M and formula $\alpha \in \mathcal{L}_{EL}$,

$$M, w \models \alpha \iff M^e, w \models \alpha$$

Conclusion

- Our goal was to develop a logic to model information exchange, seen as a process of raising and resolving issues.
- We enriched epistemic models with a description of agents' issues.
- We enriched the logical language with interrogatives.
- We formulated a uniform, support-based semantics for declaratives and interrogatives.
- The result is a conservative extension of standard epistemic logic.
- K_a was generalized to describe which issues agents can resolve.
- New modalities E_a and W_a we introduced to describe which issues agents entertain and wonder about.


THANK
YOU
GRACIAS
ARIGATO
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