Erotetic Languages

The Inquisitive Hierarchy

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Erotetic languages

Definition (Erotetic languages)

A logical language $\mathcal{L}$ is an *erotetic language* iff

1. The *semantics of $\mathcal{L}$ is inquisitive:* it distinguishes both informative and inquisitive content of the sentences in $\mathcal{L}$.
2. For some $\varphi \in \mathcal{L}$: φ is informative

   for some $\varphi \in \mathcal{L}$: φ is inquisitive.
3. $\varphi \in \mathcal{L}$ is a *tautology* iff φ is neither informative nor inquisitive.
4. For some $\varphi \in \mathcal{L}$: φ is a tautology.

That there are tautologies is to guarantee that there is some logic

Assertions, Questions and Hybrids

Definition (Assertions and questions)

Let $\mathcal{L}$ be an erotetic language, $\varphi \in \mathcal{L}$.

1. φ is an *assertion* iff φ is not inquisitive.
2. φ is a *question* iff φ is not informative.
3. φ is a *hybrid* iff φ is informative and inquisitive.

- These are semantic categories

Classical erotetic languages

Definition (Classical erotetic languages)

A logical language $\mathcal{L}$ is a *classical erotetic language* iff

1. $\mathcal{L}$ is an erotetic language which has two syntactic sentential categories of indicatives $\mathcal{L}_!$ and interrogatives $\mathcal{L}_?$, where
2. $\mathcal{L} = \mathcal{L}_! \cup \mathcal{L}_?$ and $\mathcal{L}_! \subset \mathcal{L}$ and $\mathcal{L}_? \subset \mathcal{L}$ and $\mathcal{L}_! \cap \mathcal{L}_? = \emptyset$.
3. Every $\varphi \in \mathcal{L}_!$ is an assertion and

   Every $\varphi \in \mathcal{L}_?$ is a question.

Fact

There are no hybrids in a classical erotetic language.

Truth. Questions are intensional

Classical evaluation

- $v \models \varphi$, sentence φ is true in world (model) v

Classical meaning

- $\text{info}(\varphi) = \{v \in \omega \mid v \models \varphi\}$

Informativeness

- φ is informative iff $\text{info}(\varphi) \neq \omega$

Can't work for questions

- By definition: questions are not informative
- If φ is a question: $\text{info}(\varphi) = \omega$
- Every question is true in every single world.

Inquisitive hierarchy

Pairs of worlds? Risky

- What if $\{v, u\} \models \varphi$ and $\{v, w\} \models \varphi$ and $\{w, u\} \models \varphi$, whereas $\{u, v, w\} \not\models \psi$?
- If this could happen (classically it couldn't), just considering pairs and not bigger sets might give the wrong results.
- And this can repeat itself at every level

Remark

This observation can also be used against trying to find a many-valued solution for the evaluation of questions

Fact

For pairs of worlds 5 values suffice
(ESSLLI 2008 Lecture Notes)

But what if pairs do not suffice?

Information states

- We need arbitrary sets of worlds to evaluate sentences of an erotetic language, and we call them states

Definition (States)

Let $\mathcal{L}$ be an erotetic language, and ω the set of suitable worlds for $\mathcal{L}$. The set of *suitable states for* $\mathcal{L}$, $S_{\mathcal{L}}$ is the set of all subsets of ω.

Definition (Extension)

Let $s, t \in S_{\mathcal{L}}$. *s is an extension of t* iff $s \subseteq t$.

Support semantics

Assumption (Standard structure of support semantics)

Let $S_{\mathcal{L}}$ be the set of suitable states for $\mathcal{L}$.

- *A* support semantics *for $\mathcal{L}$ characterizes the notion of when a state $s \in S_{\mathcal{L}}$ supports a sentence $\varphi \in \mathcal{L}$, which we denote as $s \models \varphi$.*
- *The logical notions of* validity, entailment *and* equivalence *are defined as:*
  1. $\models \varphi$ *iff for all* $s \in S_{\mathcal{L}}$: $s \models \varphi$
  2. $\varphi \models \psi$ *iff for all* $s \in S_{\mathcal{L}}$: *if* $s \models \varphi$, *then* $s \models \psi$
  3. $\varphi \equiv \psi$ *iff* $\varphi \models \psi$ *and* $\psi \models \varphi$
- *The notion of meaning is defined as:* $[\varphi] = \{s \in S_{\mathcal{L}} \mid s \models \varphi\}$

From validity to support

Assumption (Tautologies and validity)

$\models \varphi$ iff φ is a tautology.

Fact (Validity)

$\models \varphi$ iff φ is neither informative nor inquisitive.

Definition (Absolute informativeness and inquisitiveness)

1. φ is *informative* iff for some state $s \in S_{\mathcal{L}}$: φ is informative in s.
2. φ is *inquisitive* iff for some state $s \in S_{\mathcal{L}}$: φ is inquisitive in s.

Definition (Support)

$s \models \varphi$ iff φ is not informative in s and φ is not inquisitive in s.

Informativeness in a state

Definition (Informative content)

$\text{info}(\varphi) = \{v \in \omega \mid \{v\} \models \varphi\}$

By definition questions are not informative:

Fact (Questions)

φ is a question iff info$(\varphi) = \omega$

Non-informativeness of a sentence φ in a state s:
the update of s with the informative content of φ has no effect:

Definition (Informativeness in a state)

φ is *informative* in s iff $s \cap \text{info}(\varphi) \neq s$.

Inquisitiveness in a state

- If $s \not\models \varphi$ this may be due to φ being informative in s or inquisitive in s or both
- We have decided what φ being informative in s means
- We can neutralize that aspect:
- Add info(φ) to the information that is already contained in s
- If then s still does not support φ, then it must be because φ is inquisitive in s

Definition (Inquisitiveness in a state)

φ is inquisitive in s iff $s \cap \text{info}(\varphi) \not\models \varphi$.

General inquisitive semantics

Language is a standard propositional langage

Definition (General propositional inquisitive semantics)

1. $s \models p$ iff $\forall w \in s : w(p) = 1$
2. $s \models \bot$ iff $s = \emptyset$
3. $s \models \varphi \to \psi$ iff $\forall t \subseteq s$: if $t \models \varphi$ then $t \models \psi$
4. $s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$
5. $s \models \varphi \vee \psi$ iff $s \models \varphi$ or $s \models \psi$

Definition (abbreviations)

1. $\neg\varphi := \varphi \to \bot$
2. $!\varphi := \neg\neg\varphi$ (*non-inquisitive closure*)
3. $?\varphi := \varphi \vee \neg\varphi$ (*non-informative closure*).

(Ciardelli, Groenendijk, Roelofsen)

Inquisitive disjunction

Fact (Hybrid disjunction)

- *$p \vee q$ is a hybrid sentence*
- *$p \vee q$ is informative: $\omega \cap info(p \vee q) \neq \omega$*
- *$p \vee q$ is inquisitive: $\omega \cap info(p \vee q) \not\models p \vee q$*

Fact (Inquisitive question)

- *$?p = p \vee \neg p$ is an inquisitive question*
- *$p \vee \neg p$ is not informative: $\omega \cap info(p \vee \neg p) = \omega$*
- *$p \vee q$ is inquisitive: $\omega \cap info(p \vee \neg p) = \omega \not\models p \vee \neg p$*

Inquisitive Hierarchy

Definition

Let S^n denote the set of states s such that $|s| \leq n$.
φ is *n-inquisitive* iff $\exists X \subseteq S^n : \forall s \in X : s \models \varphi$ and $\bigcup X \not\models \varphi$.

Fact

Let $\mathcal{L}$ be an arbitrary propositional language. For any sentence $\varphi \in \mathcal{L}$: φ is not 1-inquisitive.

Classical logic when you evaluate relative to state with single word

Theorem (General inquisitiveness)

Let $\mathcal{L}_{\mathcal{P}}$ be a general inquisitive propositional language with a countably infinite set of proposition letters $\mathcal{P}$.

- *For any number $n > 1$ there is a sentence $\varphi \in \mathcal{L}_{\mathcal{P}}$ such that φ is n-inquisitive and φ is not k-inquisitive for all $k < n$.*

(Ciardelli and Roelofsen, ‘Inquisitive Logic’, JPL 2011)

Failure of pair-semantics

Under a pair-semantics there are four possibilities for $p \vee q \vee r$

Chris Potts calculator for pair semantics:

$$\{\,\{TTT, TTF, TFT, TFF\}$$
$$\{TTT, TTF, TFT, FTT\}$$
$$\{TTT, TTF, FTT, FTF\}$$
$$\{TTT, TFT, FTT, FFT\}\,\}$$

- Pair semantics gives wrong results
- we need general inquisitive semantics to get things right

Remember

Definition (Classical erotetic languages)

A logical language $\mathcal{L}$ is a *classical erotetic language* iff

1. $\mathcal{L}$ is an erotetic language which has two syntactic sentential categories of indicatives $\mathcal{L}_!$ and interrogatives $\mathcal{L}_?$, where
2. $\mathcal{L} = \mathcal{L}_! \cup \mathcal{L}_?$ and $\mathcal{L}_! \subset \mathcal{L}$ and $\mathcal{L}_? \subset \mathcal{L}$ and $\mathcal{L}_! \cap \mathcal{L}_? = \emptyset$.
3. Every $\varphi \in \mathcal{L}_!$ is an assertion and

   Every $\varphi \in \mathcal{L}_?$ is a question.

Fact

There are no hybrids in a classical erotetic language.

Classical inquisitive semantics

Standard propositional language with an aditional operator

Definition (Classical propositional erotetic language)

1. $\varphi \in \mathcal{L}_!$, for all $\varphi \in \mathcal{P}$
2. $\bot \in \mathcal{L}_!$
3. If $\varphi \in \mathcal{L}_!$, then $?\varphi \in \mathcal{L}_?$
4. If $\varphi \in \mathcal{L}_!$ and $\psi \in \mathcal{L}_{c \in \{!,?\}}$, then $(\varphi \to \psi) \in \mathcal{L}_c$
5. If $\varphi, \psi \in \mathcal{L}_{c \in \{!,?\}}$, then $(\varphi \wedge \psi) \in \mathcal{L}_c$

Definition (Classical abbreviations)

1. $\neg\varphi := (\varphi \to \bot)$
2. $(\varphi \vee \psi) := \neg(\neg\varphi \wedge \neg\psi)$

Classical inquisitive semantics

Standard propositional language with an additional operator

Definition (Classical propositional inquisitive semantics)

1. $s \models p$ iff $\forall w \in s : w(p) = 1$
2. $s \models \bot$ iff $s = \emptyset$
3. $s \models ?\varphi$ iff $s \models \varphi$ or $\forall t \subseteq s$: if $t \models \varphi$, then $t = \emptyset$
4. $s \models \varphi \rightarrow \psi$ iff $\forall t \subseteq s$: if $t \models \varphi$ then $t \models \psi$
5. $s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$

Fact

If we apply classical propositional inquisitive semantics to a classical propositional erotetic language it meets the semantic and syntactic criteria for being what it is called.

Classical inquisitive semantics is 2-inquisitive

Fact (Pair distributivity)

Let $\mathcal{L}$ be a classical propositional erotetic language, $\varphi \in \mathcal{L}$.

- *For every state s: $s \models \varphi$ iff for all $v, w \in s$: $\{v, w\} \models \varphi$.*

Expressive limitations of the classical erotetic language

- In general inquisitive semantics we can express alternative questions by means of $?(\varphi_1 \vee \ldots \vee \varphi_n)$
- The classical language as it is is capable to express alternative questions with two alternatives by $?(\varphi \vee \psi) \wedge ((\varphi \vee \psi) \rightarrow ?\varphi)$
- Due to the fact that it is a pair semantics it lacks the expressive power to deal properly with three or more alternatives.

Adding classical alternative questions

Alternative questions

- If Φ is a finite subset of $\mathcal{L}_!$, then $?\Phi \in \mathcal{L}_?$
- $s \models ?\Phi$ iff $\exists\varphi \in \Phi : s \models \varphi$, or $\forall\varphi \in \Phi : s \models \neg\varphi$

No pair distributivity

With this addition of alternative questions pair-distributivity doesn't hold anymore.

- There are big differences between the general and the classical inquisitive language
- But every meaning that can be expressed by a single sentence in general inquisitive semantics can be expressed by a pair of sentences in classical inquisitive semantics

A hybrid set of two sentences

- $\{p \vee q, ?\{p, q\}\}$
- By using disjunctive normal form for general inquisitive semantics, we can use this pattern to translate every general inquisitive sentence into a pair of classical ones.
- There is also a simple recursive translation procedure in the other direction.

Conclusions

- Inquisitive semantics is a general erotetic semantic framework
- It is not inherently linked to a mono-categorial language or inquisitive disjunction
- It can just as well be used in combination with bi-categorial languages
- The semantic framework can be used as a tool to compare different erotetic systems
- By the way, both general and classical inquisitive semantic are conservative extensions of classical logic
- In the classical inquisitive case the logic of $\mathcal{L}! \subset \mathcal{L}$ is classical.
- In the general inquisitive case the logic of the disjunction-free fragment of $\mathcal{L}$ and the language $\{!\varphi \mid \varphi \in \mathcal{L}\}$ is classical