An inquisitive dynamic epistemic logic

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Outline

Main result

A system that combines the main features of dynamic epistemic logic with those of inquisitive semantics

Roadmap

1. Brief review of DELQ (van Benthem, Miniča, . . .)
2. Brief review of INQB (Ciardelli, Groenendijk, Roelofsen, . . .)
3. An inquisitive epistemic logic, IEL
4. An inquisitive dynamic epistemic logic, IDEL

Dynamic epistemic logic with questions

Language (simplified)

$$p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \rightarrow \psi \mid K_a\varphi \mid [!\varphi]\psi \mid [?\varphi]\psi$$

- $[!\varphi]\psi$ = 'asserting φ leads to a state where ψ holds'
- $[?\varphi]\psi$ = 'asking whether φ leads to a state where ψ holds'

Dynamic epistemic logic with questions

Epistemic issue models

$$M = \langle W, \sim_{\mathcal{A}}, \approx_{\mathcal{A}}, V \rangle$$

- $\sim_{\mathcal{A}} = \{\sim_a \mid a \in \mathcal{A}\}$

  A set of equivalence relations on W encoding epistemic indistinguishability for each agent

- $\approx_{\mathcal{A}} = \{\approx_a \mid a \in \mathcal{A}\}$

  A set of equivalence relations on W encoding the issues that have been raised by each agent

Dynamic epistemic logic with questions

Interpretation

- The static fragment of the language is interpreted as usual
- Dynamic speech act operators change the model of evaluation
  - Assertions provide information; they change $\sim_{\mathcal{A}}$
  - Questions raise issues; they change $\approx_{\mathcal{A}}$
- Crucial clauses:
  - $M, w \models [!\varphi]\psi$ iff $M^{!\varphi}, w \models \psi$
  - $M, w \models [?\varphi]\psi$ iff $M^{?\varphi}, w \models \psi$

Dynamic epistemic logic with questions

Discussion

- The basic static fragment of the language, and its semantic interpretation, are completely classical
- Questions enter the picture at the level of speech acts
- The basic static language does not contain sentences that are interrogative in any systactic sense, or inquisitive in any semantic sense

Dynamic epistemic logic with questions

Alternative approach

- Change the semantics of the basic static fragment of the language in such a way that the meaning of a sentence embodies both its informative and its inquisitive content
- Add interrogative sentences, $?\varphi$, to the static language
- The dynamic part of the language can then be simplified. We just need a single general purpose speech act operator:

  $[\varphi]\psi$ = 'uttering φ leads to a state where ψ holds'

Dynamic epistemic logic with questions

Main advantage of the alternative approach

If inquisitiveness enters the picture at the syntactic/semantic level, it becomes possible to deal with embedded questions

(1)	John knows who will come to the party.	$K_a?x.Px$
(2)	John knows whether Mary will come to the party.	$K_a?p$
(3)	If it rains, will Mary still come to the party?	$p \rightarrow ?q$

Embedded questions cannot be dealt with straightforwardly in DELQ, because sentences like $K_a?q$ and $p \rightarrow ?q$ are not in $\mathcal{L}_{DELQ}$

Inquisitive semantics

Language

$$p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \rightarrow \psi \mid ?\varphi$$

- Interrogative sentences, but
- no knowledge operators, and
- no speech act operators

Inquisitive semantics

Models

- Sentences are evaluated relative to information states, i.e., sets of possible worlds
- The central notion is support, rather than truth
- In uttering a sentence φ, a speaker proposes to update the common ground in such a way that it comes to support φ

Inquisitive semantics

Support

1. $s \models p$ iff $\forall w \in s : w(p) = 1$
2. $s \models \neg\varphi$ iff $\forall w \in s : \{w\} \not\models \varphi$
3. $s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$
4. $s \models \varphi \vee \psi$ iff $s \models \varphi$ or $s \models \psi$
5. $s \models \varphi \rightarrow \psi$ iff $\forall t \subseteq s$: if $t \models \varphi$ then $t \models \psi$
6. $s \models ?\varphi$ iff $s \models \varphi$ or $s \models \neg\varphi$

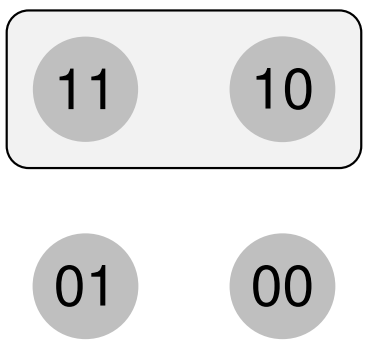
Propositions and possibilities

- $[\varphi] =$ the set of all states supporting φ
- A possibility for φ is a maximal state supporting φ

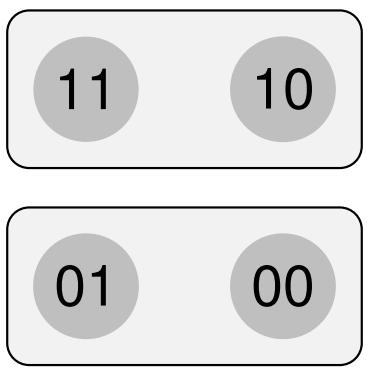
Inquisitive semantics

Illustration

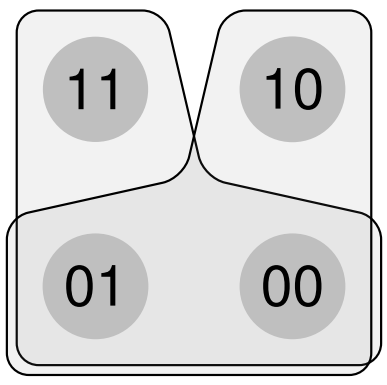
The semantics applies deals in a uniform way with declaratives, interrogatives, and embedded interrogatives:



p



$?p$



$p \rightarrow ?q$

Inquisitive semantics

Informative content

In uttering a sentence φ, a speaker proposes to eliminate all worlds that are not contained in any state supporting φ

- $\text{info}(\varphi) = \bigcup[\varphi]$

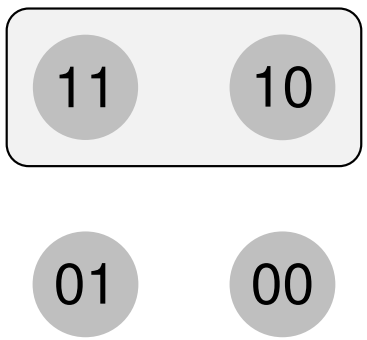
Informative and inquisitive sentences

- φ is informative iff $\text{info}(\varphi) \neq \mathcal{W}$
- φ is inquisitive iff $\text{info}(\varphi) \not\models \varphi$

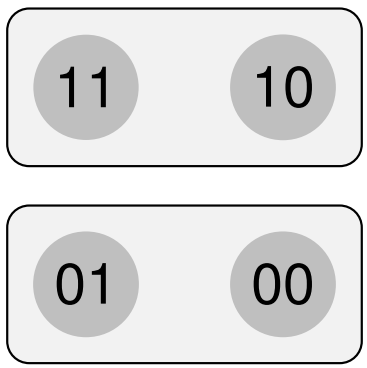
Inquisitive semantics

Questions and assertions

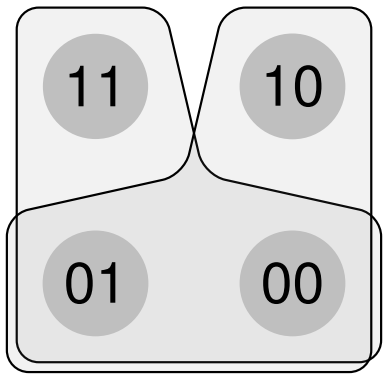
- φ is a question iff it is non-informative
- φ is an assertion iff it is non-inquisitive



assertion



question



question

Inquisitive semantics

Discussion

- Inquisitiveness enters the picture at the level of sentences and their semantic content
- The system deals straightforwardly with conditional questions
- It does not deal with knowledge-*wh* ascriptions yet, because $\mathcal{L}_{INQB}$ does not contain knowledge operators
- The system does not allow us to specify precisely what happens at the speech act level
- Thus, integrating inquisitive semantics with dynamic epistemic logic will help both traditions a step further

Inquisitive epistemic logic

Language

$$p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \rightarrow \psi \mid ?\varphi \mid K_a\varphi$$

- Interrogative sentences
- Knowledge operators
- No speech act operators

Inquisitive epistemic logic

States

- Sentences are still evaluated relative to states
- States are now sets of worlds in the canonical model for S5 (or some other epistemic logic)
- As before, the central notion is support, rather than truth
- In uttering a sentence φ, a speaker proposes to update the common ground in such a way that it comes to support φ

Inquisitive epistemic logic

Support

1. $s \models p$ iff $\forall w \in s : V^c(w, p) = 1$
2. $s \models \neg\varphi$ iff $\forall w \in s : \{w\} \not\models \varphi$
3. $s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$
4. $s \models \varphi \vee \psi$ iff $s \models \varphi$ or $s \models \psi$
5. $s \models \varphi \to \psi$ iff $\forall t \subseteq s$: if $t \models \varphi$ then $t \models \psi$
6. $s \models ?\varphi$ iff $s \models \varphi$ or $s \models \neg\varphi$
7. $s \models K_a\varphi$ iff $\forall w \in s : \sigma_{a,w} \models \varphi$

Propositions

- As before, $[\varphi] = \{s \mid s \models \varphi\}$

Inquisitive epistemic logic

Knowledge ascription

We now have a unified treatment of knowledge-*that* and knowledge-*wh* ascription:

(4) John knows that Peter will come. $K_a p$

(5) John knows whether Peter will come. $K_a ?p$

A state s supports $K_a ?p$ iff for every $w \in s$,
a's information state in w supports either p or $\neg p$.

Inquisitive epistemic logic

All the central notions from INQB carry over directly to IEL

- info(φ) $= \bigcup[\varphi]$
- φ is informative iff info(φ) $\neq \mathcal{W}$
- φ is inquisitive iff info(φ) $\not\models \varphi$

This brings us to the final step: adding a dynamic layer

Inquisitive dynamic epistemic logic

Language

$$p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \rightarrow \psi \mid ?\varphi \mid K_a\varphi \mid [\varphi]_a\psi$$

- Interrogative sentences
- Knowledge operators
- One speech act operator

  $[\varphi]_a\psi$ = 'an utterance of φ by a leads to a state supporting ψ'

Inquisitive dynamic epistemic logic

Discourse contexts

- Sentences will be evaluated relative to a discourse context
- A discourse context is a pair $\langle s, T \rangle$, where:
  - s is a state

    $\Rightarrow$ representing the information that has been provided so far
  - T is a stack of IEL-propositions

    $\Rightarrow$ representing the proposals that have been made so far

Inquisitive dynamic epistemic logic

Changing the discourse context

- Utterances change the discourse context
- $s^{\varphi_a} = \{w \in s \mid \sigma_{a,w} \subseteq \text{info}(\varphi)\}$
- $T^{\varphi_a} = T + [\varphi]$

Inquisitive dynamic epistemic logic

Support

1. $\langle s, T\rangle \models p$ iff $\forall w \in s : V^c(p, w) = 1$
2. $\langle s, T\rangle \models \neg\varphi$ iff $\forall w \in s : \langle \{w\}, T\rangle \not\models \varphi$
3. $\langle s, T\rangle \models \varphi \wedge \psi$ iff $\langle s, T\rangle \models \varphi$ and $\langle s, T\rangle \models \psi$
4. $\langle s, T\rangle \models \varphi \vee \psi$ iff $\langle s, T\rangle \models \varphi$ or $\langle s, T\rangle \models \psi$
5. $\langle s, T\rangle \models \varphi \to \psi$ iff $\forall s' \subseteq s$: if $\langle s', T\rangle \models \varphi$ then $\langle s', T\rangle \models \psi$
6. $\langle s, T\rangle \models ?\varphi$ iff $\langle s, T\rangle \models \varphi$ or $\langle s, T\rangle \models \neg\varphi$
7. $\langle s, T\rangle \models K_a\varphi$ iff $\forall w \in s : \langle \sigma_{a,w}, T\rangle \models \varphi$
8. $\langle s, T\rangle \models [\varphi]_a\psi$ iff $\langle s^{\varphi_a}, T^{\varphi_a}\rangle \models \psi$

Note that the first seven clauses are essentially the same as in IEL

Inquisitive dynamic epistemic logic

Discussion

- IDEL brings together the main features of DEL and INQ
- Main vantage points from the viewpoint of DEL:
  - Inquisitiveness at the level of semantic content
  - Allows for a straightforward account of embedded questions
- Main vantage points from the viewpoint of INQ:
  - Perspicuous representation of the conversational participants' epistemic states
  - Explicit account of how utterances affect the discourse context

Thank you for your attention

www.illc.uva.nl/inquisitive-semantics

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