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Modals in Legal Language

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Chapter 1

Introduction

This dissertation proposes a novel treatment of deontic modals such as permission, obligation and prohibition with special attention to applicability to legal language. The first chapter serves as an overview of the structure and topics of the dissertation. We will go through the entire argument at a general level, with emphasis on the connections between the different topics we discuss and the motivation for the choices we make. Detail will have to be sacrificed, but hopefully the outline will serve to guide the reader through the remaining chapters.

1.1 On intended audiences

This dissertation is written as a continuous argument from start to finish, but there are two intended audiences with different areas of expertise and we have tried to keep the relevant sections distinct as much as possible.

In chapter 2 and chapter 7 section 2, this dissertation investigates interpretation problems of natural language connectives in courts of law from the perspective of lawyers and judges. The discussion of these puzzles of interpreting *or* and *conditional sentences* is intended for practitioners of law, more so than for linguists. We will argue that the accrued knowledge and formal tools from the semantics literature can be of assistance in courtrooms, and while doing so we attempt to keep the content easily accessible for people with little or no training in formal tools. This might make certain sections overly familiar for linguists. These sections include, for example, chapter 2, where we discuss the interpretation puzzles, and the second section of chapter 7, where we apply the formal framework to these puzzles. We still recommend that any linguists reading this dissertation take a look at those chapters for two reasons. First, we discuss the empirical observations that motivate some of the choices regarding formal methods in other chapters and, secondly, we illustrate how the formal tools semanticists use on a daily basis could be applied to problems outside of their field.

On the other hand, chapters 3 to 6 are primarily intended for semanticists as they concern the standard treatment of modal expressions in the literature and its puzzles. We will propose a new semantics for permission, prohibition and obligation modals in formal terms. As the intended audience is semanticists, the chapters are probably inaccessible to people without training in formal tools. We try to take this into account in the *summary for lawyers* section of chapter 7 which is once again intended to be read without formal training. We will make reference to the semantic framework developed in the preceding chapters but reintroduce it in minimal terms to make it more accessible.

1.2 Legal language

This dissertation aims to improve our understanding of the common but mostly neglected problem of language interpretation in law. Courts of law routinely interpret legal texts in accordance with rules of legal interpretation but, in this dissertation, we are interested in a particular kind of interpretation that occurs in courts of law - interpretation stemming from the meaning of expressions and their relations to each other.

The fact that legal cases can depend on the interpretation of natural language expressions in legal texts has been noted by several authors in legal theory. Mellinkoff wrote in his influential book "The language of the law" [76] that judges have struggled with ambiguous language for centuries. Ambiguity is a pervasive feature of natural language that comes in many forms, and in this dissertation we will focus on ambiguities in the interpretation of the connectives *or* and *if-then* in legal language.

1.2.1 *Or* in legal language

Or is infamous in legal language because of the question whether *or* is *inclusive* or *exclusive*.

To see how the *inclusive/exclusive or* puzzle creates problems in a courtroom, consider a simple everyday example in which the interpretation of a rule could lead to a legal dispute. Imagine a canteen with a lunch offer in which a starter or a dessert is included in the price of the meal. The canteen would probably have the following sign.

(1) You may take a starter or a dessert.

From (1), it is clear that when the client adds only a starter to their meal, it is free of charge. And if the client adds only a dessert, it is also free of charge. But what if the client adds both a starter and a dessert to their meal? Assume that the owner of the canteen demands a client who took both a starter and a dessert to pay for the dessert. The client then seeks legal arbitration to avoid paying. The judge would then likely need to decide on the interpretation of *or* in (1).

If the *or* in (1) is read *inclusively*, then adding both is also free of charge. However, if the *or* in (1) is read *exclusively*, then one would have to pay for the dessert. Most people say that intuitively (1) only gives permission to take one of the two, but providing arguments for and against this interpretation can be difficult.

Judges can of course use their discretion to provide an interpretation with little regard for the meaning of the terms. But such use of their authority has an important downside. When judges determine what a statute says without being limited by the words and expressions used therein, a subject to the rule will find it difficult to predict what is prohibited and what is permitted. After all, a judge could change this at will. So for the sake of legal certainty, it would be preferable if, whenever possible, laws were interpreted as closely to the meaning of the expressions in which they have been written.

When faced with the ambiguities of natural language, we can study the literature in semantics which has collected and stored a wealth of knowledge regarding natural language, and its ambiguities. In this dissertation, we intend to bring together the field of law and linguistics to see whether and to which extent linguists can help interpret the language of the law.

The primary topic of chapter 2 is the discussion of natural language interpretation in courts of law. As our starting point, we will critically investigate Solan's [94] ground-breaking book "The language of judges" and examine his examples of *inclusive/exclusive or* from American court cases. Solan was probably the first author to have applied formal tools from linguistics to the interpretation problems of lawyers.[1]

1.2.2 Examples from the World Trade Organisation

We extend the scope of the investigation by introducing a new source of examples: World Trade Organisation (WTO) legal texts. According to Matsushita et. al. [70, p. 1], the WTO is an international body whose purpose is "to develop and coordinate international trade." It achieves its objectives, partly, through member countries signing multilateral trade agreements the terms of which are binding to its members. Disagreements between member countries about the proper interpretation of the meaning of the text of the agreement are resolved by the WTO adjudicative body which hears the arguments of the two parties and prepares an interpretation of the dispute in the form of a *panel report*.

These *panel reports* are an invaluable source of examples for linguists interested in legal language for several reasons. First, these additional examples demonstrate that natural language interpretation puzzle arise across different legal domains.

Second, WTO cases are recorded and presented with meticulous attention to the statements of the parties' and the adjudicators' so that the context of the statements and reasoning behind the decisions is available for linguistic study. This careful preparation of the materials turns out to be of great value for investigative purposes.

Third, WTO adjudicators are more limited in their interpretation of legal texts as they "cannot add to or diminish the rights and obligations provided in the covered agreements" (*Dispute Settlement Understanding of the WTO*, Art. 3.2, available here: http://bit.ly/XHAvD4). This forces

[1] Leaving aside the literature on translating legal texts into logic, spearheaded by Pfeiffer [77], Tammelo[99] and Allen[7].

the adjudicators in the WTO to pay close attention to the natural language meaning of the terms of the agreements.

1.2.3 Conditional sentences

We further increase the scope of the investigation by introducing interpretation puzzles regarding the connective *if-then.* According to an authoritative legal drafting textbook by Haggard and Kuney [46, p. 211], such *conditional sentences* can leave *substantive gaps* or *ambiguity.* Consider example (2) by Haggard and Kuney.

(2) If Buyer requests it, Seller will ship by United Parcel.

The salient reading of (2) says that when the *antecedent* of the sentence is the case, i.e., in all cases where a Buyer requests shipping by United Parcel, the *consequent follows*, i.e., the Seller will ship by United Parcel. But what happens when the Buyer does not request shipping by United Parcel? There are several competing interpretations. According to one, the Seller is free to choose any shipping company, including United Parcel. However, a competing interpretation says that if the Buyer does not request shipping by United Parcel, the Seller is prohibited from shipping by United Parcel. This and similar puzzles will be considered in detail in chapter 2.

With due regard to Solan's trailblazing contribution to the literature, his approach is limited in certain crucial aspects. He utilized tools from mainly syntax and psycholinguistics, relying on classical propositional logic for the semantics. The problem with such an approach is that the linguistic analysis of the example in (1) is not a simple matter due to the inclusion of the modal auxiliary *may* which grants *permission.* Also, (2) concerns the *obligation* of the Seller to ship by United Parcel and the puzzle whether there is an additional *prohibition* to not ship by United parcel when the Buyer does not request it. This is why we need to utilize additional tools.

1.3 Modals and conditionals

Permission, *obligation* and *prohibition* are traditionally analysed in a richer semantic framework than those which Solan considered. A treatment of modal auxiliaries *may*, *must* as permission and obligation is called *deontic* after the ancient greek word $\delta\epsilon o\nu$ which means "that which is binding, needful, right".[2] Under a deontic reading (3-a) says that John has an obligation to pay his taxes and (3-b) says that John has permission to drive a car. Deontic modals provide an additional challenge to linguists which will concern us throughout this dissertation.

(3) a. John *must* pay his taxes.
 b. John *may* drive a car.

Alongside their deontic meaning, the same modal auxiliaries *may* and *must* can also denote what is known.

[2] Liddell-Scott-Jones dictionary.

(4) a. John *must* be paying his taxes.
b. John *may* be driving a car.

With the modal auxiliaries construed as *epistemic modals*, (4-a) says that as far as we know, John is paying his taxes. There is no evidence to the contrary. (4-b) says that as far as we know, it is possible that John is driving a car. But in this dissertation, we are mostly concerned with deontic modals.

Solan's omission of discussion of semantic frameworks was probably intentional as not all linguistic tools are equally suitable for solving the language interpretation problems that courts face, so we must make a choice as to which semantic framework to apply to the puzzles of interpretation of legal texts. We will evaluate existing semantic framework to analyze the puzzles of *or* and *conditional sentences* which we will discuss in chapter 2. In the end, we end up proposing our own semantics to apply to these puzzles.

A further reason to take a closer look at modal semantics comes from the fact that in linguistics the most widely accepted semantic framework for the analysis of *conditional sentences*, Kratzer semantics [60, 57, 58, 59], depends on the semantic treatment of modals.

Before we discuss Kratzer semantics, in chapter 3 we will first consider *conditional sentences* from a linguistic perspective. Traditionally in propositional logic, the connective for *conditional sentences* has been analyzed as *material implication*. Alas, this logical connective bears little resemblance to the use of *conditional sentences* in natural language. To expand on this we will discuss *material implication* and its many puzzles in the beginning of chapter 3. Some of the puzzles we touch on are the *false antecedent*, *true consequent*, *contraposition* and *strengthening the antecedent* puzzles that are well known in the literature. The reason for doing this is to clarify intuitions regarding the natural language interpretation of *conditional sentences* and to list certain well known puzzles that any semantic treatment ought to solve.

As we already said, Kratzer semantics for conditionals is dependent on the treatment of modals. This is because Kratzer takes the antecedent of a conditional to restrict the domain for a (generally covert) operator that quantifies over the consequent. She considers the most likely candidate for this hidden operator to be the epistemic modal *must* but if there is an overt deontic modal, e.g., *may* or *must*, then the antecedent restricts this permission, obligation or prohibition modal instead.

Clearly, such a treatment of the connective is dependent on the treatment of modals. Kratzer builds on what is known as standard modal logic (SML), which treats modal expressions as *quantifiers over accessible worlds*. The most frequently cited modal operators are necessity and possibility, represented by the universal and existential quantifier, respectively.

Standard modal logic takes the context into account as the accessibility relation that determines the truth value of a modal statement can change from context to context. To illustrate, the accessibility relation for deontic modals depends on a set of rules. So if you consider a modal statement you know to be true in another country, where the rules captured by the accessibility relation are different, the same modal statement can be false.

1.3.1 Puzzles of modal logic

As we said earlier, the semantics of modals has proven challenging for linguists and the standard account to deontic modals suffers from several well known puzzles. The puzzles are equally problematic for both SML and its refinement, Kratzer semantics. Unless specifically specified, we will refer to SML formulations for these puzzles.

The first puzzles concern *contrary to duty* situations where one violates rules. We will give a sketch of what goes wrong with the puzzles here, and elaborate on it in chapter 3.

As we said, an obligation sentence such as (5) in SML holds when all accessible worlds are p[3] worlds.

(5) A country must establish a research center. $\Box p$

The salient reading of (5) says that in all situations in which a country has not established a research center that country incurs a violation of the rule in (5). So SML captures the salient reading but problems arise when someone does not act according to the rules. For example, if North Korea does not establish a research center there can be a secondary rule which states that the violator of the rule in (5) has to pay a fine, i.e., (6) will hold.

(6) North Korea must pay a fine. $\Box q$

SML counter-intuitively predicts that (6) cannot hold. This is because for (5) to hold, in all accessible worlds countries establish a research center. But as all accessible worlds are already p worlds, North Korea could not have invoked (6) by bringing about $\neg p$.

Kratzer semantics avoids the above puzzle by adding a second layer of contextual dependence through an ordering of worlds. Her semantics allows one to differentiate between worlds that are closer to the ideal from those that are further. So, even if there is a violator of (5), the rule still holds if all those worlds that are closest to the ideal are p worlds. It is also better if the violator pays a fine and, thus, worlds where North Korea pays a fine should be closer to the ideal than those worlds where it does not, so (6) holds too.

We stated earlier that we need to choose an appropriate semantic framework to discuss disjunction, conditionals and modals in legal language, so in chapter 4 we discuss the puzzles of SML and Kratzer semantics to determine their suitability. Many of the puzzles we discuss have been known in the literature for a long time, but have recently been reasserted by authors such as Cariani [22] and Lassiter [64]. We will consider these puzzles in turn.

The first problem arises from the fact that according to SML and Kratzer semantics, deontic modals have the property of upward monotonicity, which is to say that any valid inference in the propositional case is a valid inference embedded under deontic modals. But there is a difference between the propositional case and the deontic case with, for example, *or* sentences. The propositional case is illustrated by (7).

[3] For ease of notation, worlds are represented as a sequence of the elements of the set of propositions that corresponds to it; for example, instead of $\{p, q\}$, we will write pq.

(7) a. You mailed the letter. p
b. You mailed the letter or burned it. $p \vee q$

Most people who accept that (7-a) is the case also accept that (7-b) holds. This is because when you mailed the letter, you also mailed or burned it. Yet, an *or* sentence does not behave the same way when embedded under an obligation modal. Consider (8) based on an example by Ross [86].

(8) a. You must mail the letter. $\Box p$
b. You must mail the letter or burn it. $\Box(p \vee q)$

The SML analysis of *must* as a universal quantifier over accessible worlds says that (8-a) holds when in all accessible worlds you mail the letter. This also satisfies the conditions for (8-b) to hold as in all accessible worlds you either mail the letter or burn it. But this prediction is counter-intuitive as most people who accept (8-a) do not accept (8-b).

The difference between the propositional and deontic case is not limited to Ross's puzzle with disjunction. In chapter 4 we discuss similar puzzles with conjunction, both in the original formulation by Prior [81] and Jackson's [50] version which is more widely known in the semantics literature. This means that solutions that focus on the connective *or* will be unable to provide a uniform solution to these puzzles.

Jackson also formulated a puzzle regarding the divergent behaviour of conditional sentences embedded under deontic modals, but we discuss this so called *conditional ought's* puzzle separately. Conditional sentences demonstrate that it is not only upward monotonicity which creates problems for Kratzer semantics. Based on some examples by Priest [80], we formulate the illustrative *all or nothing* puzzle.

In the most basic standard case, where conditionals are analyzed as material implication, it is not valid to infer from (9-a) being the case that (9-b) also holds[4].

(9) a. $(p \wedge q) \rightarrow r$
b. $p \rightarrow r$

But Kratzer's treatment of conditionals, where the antecedent restricts the permission modal analyzed as an existential quantifier over the best worlds, predicts that whenever (10-a) is the case, i.e., one of the best worlds is a *pqr* worlds, so is (10-b) as the existence of one *pqr* world is sufficient for (10-b) to hold.

(10) a. $(p \wedge q) \rightarrow \Diamond r$
b. $p \rightarrow \Diamond r$

This means that a conditional permission statement where several conditions have to be met for the permission to be granted can be arbitrarily weakened. This is counter-intuitive in natural language. Consider (11).

[4]The same holds if (9-a) and (9-b) are analyzed in Kratzer semantics with the antecedent restricting an epistemic necessity modal scoping over the consequent.

(11) a. If the car passed its technical inspection and you have your license, then you may drive.
b. If the car passed its technical inspection, then you may drive.

Most people do not accept that when (11-a) holds, then so does (11-b). This is because when a license is required to gain permission to drive, one cannot ignore this requirement.

As we saw, this counter-intuitive inference is not a relic of the propositional case brought into the deontic case by upward monotonicity. Instead, Kratzer's treatment of deontic modals and conditionals brings about the puzzle.

Another type of puzzles for SML concerns *deontic conflicts*. We stated earlier that Kratzer semantics avoids the problems of SML in cases where the rules are violated. But Kratzer's treatment of deontic modals fails in situations in which it is impossible to avoid all violations. Consider (12).

(12) a. Mother: You must leave your room. $\Box p$
b. Father: You must not leave your room. $\neg\Box p$

The situation in (12) is unfortunate but commonplace. Such *conflicts of obligations* exist, but Kratzer's treatment predicts that they do not. In Kratzer semantics (12-a) is the case when in the best worlds you leave your room and (12-b) is the case when in the best worlds you do not leave your room. As both of these cannot be the case, both (12-a) and (12-b) are predicted to be false in Kratzer semantics. But this is counter-intuitive, a deontic conflict arises exactly because of the fact that both (12-a) and (12-b) hold simultaneously.

One of the issues with working with deontic modals is that the topic has generated a larger number of puzzles than can be reasonably discussed in one dissertation. As references to these puzzles are common in the literature, it might be expected that we say something about them. One such topic is gradability; Lassiter [64], among others, argues that deontic modals ought to be analyzed with the same formal tools as gradable adjectives. Yet, as Lassiter admits himself [64, p. 144], the deontic modals we are concerned with here, *must* and *may*, are not gradable in natural language - either an obligation and permission hold, or they do not. Due to our focus on *must* and *may*, we will not discuss gradability in this dissertation.

Another issue we are setting aside for now is a class of puzzles regarding decision-making in deontic contexts such as Kolodny and MacFarlane's [55] *miner's puzzle*. The discussion gravitates around the way in which the semantics interacts with contextual information, and we leave such puzzles for future work.

The puzzles of upward monotonicity with *or* and *and*, together with the puzzles with *conditional sentences* and *deontic conflicts* leads us to abandon SML and Kratzer semantics for conditionals and to look for a new account of deontic modals which can be applied to puzzles in legal language.

There are a number of alternative treatments of deontic modals to choose from. For example, Cariani [22] and Lassiter [64] suggest that the problem lies with upward monotonicity and propose treatments of deontic modals that hew closely to SML with the exception of adhering to the principle of upward monotonicity. But we saw with regard to *conditional sentences* and *deontic conflicts* that we need a more radical approach. So we look instead to the literature on another well known puzzle for deontic modals: *free choice*.

The *free choice* puzzle has been one of the most studied puzzles the literature on deontic modals since it was investigated by Kamp [51]. Similarly to Ross's puzzle from earlier, the puzzle concerns the behaviour of *or* under permission. Unlike the propositional case in (13-a) which is the case when either disjunct is the case, (13-b) is the case when permission is granted to both disjuncts.

(13) a. A country established a research center or a laboratory.
b. A country may establish a research center or a laboratory.

The salient reading of (13-a) says that a country, say Estonia, established a research center or a laboratory but it is not known which one. Without additional information, a country might have also established both. The salient reading of (13-b), on the other hand, says that permission is granted to establish a research center and permission is granted to establish a laboratory. It is not necessarily the case that permission is granted to establish both simultaneously but establishing both is also not prohibited by (13-b).

An SML treatment of permission says that (13-b) is the case when in the accessible worlds permission is granted to establish a research center or a laboratory, but it is not known which one. This prediction is parallel to the propositional case where it is not known which disjunct is the case.

Crucially, a *free choice* example under negation once again behave similarly to the propositional case.

(14) a. A country did not establish a research center or a laboratory.
b. A country may not establish a research center or a laboratory.

The salient reading of (14-a) says that a country established neither a research center nor a laboratory and the salient reading of (14-b) similarly states that permission is granted to establish neither a research center nor to establish a laboratory. This fact confounds proposed solutions to the *free choice* puzzle which deviate from a standard treatment of disjunction[5] and suggests that the solution involves either pragmatics or an alternative account of deontic modals.

In chapter 4, we investigate the viability of a solution with the use of pragmatics and find that several natural language examples behave differently from what would be expected were the *free choice* effect pragmatic. We then turn to two different types of alternative treatments of deontic modals.

First, Kratzer and Shimoyama [61] popularized *alternative-based* semantics in which several connectives, including *or*, are analyzed in terms of sets of propositions. Several authors including Simons [92], [93] and Aloni

[5]See for example Zimmermann [102].

[9] have suggested that the puzzle arises because *or* introduces alternatives that correspond to the denotations of the disjuncts. In such *alternative-based* accounts, deontic modals are made sensitive to the alternatives. For example, Aloni [9] proposed that with permission, every alternative satisfies permission such that in (13-b) there is first universal quantification over alternatives and then standard existential quantification over accessible worlds within each alternative. This makes the correct prediction with regard to (13-b) as there is at least one best world in which a country establishes a research center and one best world in which a country establishes a laboratory. Our eventual proposal will incorporate Aloni's *alternative-based* account but to solve other puzzles alongside *free choice*, we need to also look to other proposals.

Several authors including Asher and Bonevac [17] and recently Barker [18] have suggested that deontic modals ought to be treated as an Andersonian [15] reduction, which is to say that the meaning of permission $\Diamond p$ is an implication from p to the absence of a violation occurring. The violation is a proposition similar to p itself and provides the information that rules have been violated. Such an approach to deontic modals corresponds well with the way in which WTO judges discuss rules in real life examples as they explicitly discuss violations of legal agreements. Previous Andersonian accounts differ both in the way that WTO judges reason about laws and the account purported in this dissertation in that they do not consider the possibility of several violations being at play simultaneously.

We will also accommodate an Andersonian approach to deontic modals but, as we discuss in chapter 4, the original Andersonian approach shares many of the puzzles of SML and suffers from some that are unique to a treatment of deontic modals as implication. Neither Asher's and Bonevac's nor Barker's proposal avoids all of the puzzles listed in chapter 4. In this dissertation we will modify the Andersonian approach in the way outlined in the next section.

1.4 A new treatment of deontic modals

As SML and Kratzer semantics suffers from several puzzles, in chapter 5 we propose a new treatment of deontic modals.[6] The building blocks for this new account are discussed below.

In accordance with WTO texts and the reductionist Andersonian treatment of deontic modals we incorporate *violations* into our semantics. We adopt the framework of *inquisitive semantics*[7] for a treatment of alternatives so that we can incorporate the idea developed in *alternative-based* semantics that deontic modals quantify over alternatives. We combine the two accounts by proposing a new semantics for deontic modals in which permission, defined similarly to implication, quantifies over alternatives such that in each of the alternatives no violation occurs.

[6] The approach builds on previous work [4, 6].

[7] See e.g., Groenendijk and Roelofsen [43], Ciardelli and Roelofsen [26], Ciardelli et al. [25] or the website https://sites.google.com/site/inquisitivesemantics/.

We will be focusing on the crucial feature of inquisitive semantics that its treatment of disjunction formalizes the intuition that *or* sentences serve to offer alternatives[8]. The more general aim of inquisitive semantics is to develop a notion of meaning that sheds light on information exchange, building on Stalnaker's [96] idea that the meaning of a sentence can be understood in terms of the context change. According to Stalnaker, the goal of interlocutors is to establish common ground - a set of propositions every participant of the conversation has accepted. Inquisitive semantics recasts Stalnaker's idea by taking a sentence to express a proposal to update the common ground. Each participant in a conversation has an information state that embodies what this participant believes to be the case, where this information state includes only the information that the participant is aware of knowing. An information state is represented (as is standard) by a set of possible worlds – in other words, ways in which the participant can imagine the world to be. If the set is empty, the information state is *inconsistent*. A sentence is *informative* in a state if updating with it eliminates worlds from the state and *inquisitive* if it represents at least two possibilities for updating the common ground.

Inquisitive semantics has the noteworthy property of characterizing *assertions* and *questions* in the semantics, rather than in the syntax as was the case in, for example, Groenendijk and Stokhof [41] or Hamblin [47]. In inquisitive semantics an *assertion* is a sentence that is *informative* but not *inquisitive* and a *question* is a sentence that is *inquisitive* but not *informative*. The properties can also overlap so when a sentence is both *informative* and *inquisitive* it is a *hybrid*. Note that whenever we discuss standard inquisitive semantics, we are referring to *basic inquisitive semantics* in Ciardelli et al. [25].

We use the radical version of inquisitive semantics (RIS) [44] as it has several advantages over basic inquisitive semantics when dealing with *or* and *conditional sentences* in deontic contexts. For example, RIS provides, among other things, a suppositional account of *conditional sentences* which replicates the desired predictions of the standard account of conditionals. Furthermore, we saw in conjunction to the *free choice* puzzle that *or* behaves similarly to the propositional case when permission is under negation. Radical inquisitive semantics allows one to capture this fact as alternatives are also maintained under negation and we can define prohibition, the contrary to permission, also as universal qualification over alternatives.

It follows that in our proposal permission $\Diamond\varphi$ takes all alternative ways in which φ could be the case and states that in each of them no violation occurs. So within the alternative, there is an implication form φ to $\neg v$. The negation of a permission statement $\neg\Diamond\varphi$ is a prohibition which states that in all alternative ways in which φ could be the case a violation does occur. According to which permissions and prohibitions are salient, φ can be permitted, prohibited or neutral.

Even though we accept, in the Andersonian tradition, that deontic modals are similar to implication, we do not believe they can be defined via implica-

[8] Alongside alternative semantics, see also Huddleston et al. [48, p. 1294].

tion as deontic modals and implication differ with regard to their behaviour under negation. To see this, consider the following sentences.

(15) a. It is not the case that if a country establishes a research center or a laboratory, then it will not violate rules.
b. A country may not establish a research center or a laboratory.

Granted that (15-a) is very difficult to parse, its salient reading says that a country will violate rules if it establishes a research center or a country will violate rules if it establishes a laboratory, but it is not known in which case it will violate rules. However, (15-b) says something stronger - a country will violate its permissions both when it establishes a research center and when it establishes a laboratory.

The intuitive reason for this difference is that permissions, obligations and prohibitions provide information regarding governing rules and, thus, leave no room for *ignorance readings* in which it is not known whether one or another rule holds. These are only possible in statements about rules, rather than in rules themselves.

We capture this difference between implication and modals by having existential quantification over alternatives in the negation of implication but universal quantification over alternatives in the negation of modals.

The resulting semantic framework is called MadRis which stands for Modified Andersonian Deontic Radical Inquisitive Semantics. It is an extension of radical inquisitive semantics, and it modifies Andersonian deontic modals as it introduces quantification over alternatives. Furthermore, deontic modals are similar to implication, but differ with respect to their behaviour under negation.

The resulting deontic modals are non-monotonic but they are not defeasible. In defeasible semantics, whether entailments go through or not depends on the context. MadRis makes intuitive predictions regarding entailments without making use of such context-based formal devices.

1.4.1 Solutions to the puzzles

We will demonstrate in chapter 5 that this treatment of modals accounts for the intuitive readings associated with the *free choice* puzzle. At the end of chapter 5, we also discuss the conceptualization of *deontic conflicts* and suggest that different rules ought to be generally analyzed with different violations. This leads to the following solution regarding puzzles of *deontic conflicts*.

When there are several rules that are in conflict, the fact that we incorporated propositions about violations allows us to differentiate between the rules by assigning to each rule a different violation. This mirrors the way WTO judges discuss violations of rules. We will see that MadRis gives an intuitive account of *deontic conflicts* as one cannot derive the counterintuitive predictions of the standard account and MadRis captures the fact that subjects to conflicting rules have a choice as to which rule to violate, they merely cannot avoid violating all rules.

Note that non-absurd readings in cases of *deontic conflicts* are not intuitively acceptable in all cases. When a single rule is in internal conflict, such as in (16) where it is both prohibited and permitted to do p, MadRis correctly predicts that the sentence is odd.

(16) $\Diamond p \wedge \neg\Diamond p$

MadRis predicts that the only way in which (16) could hold is when p is not the case. This is because all situations in which p is the case are absurd as a violation both occurs and does not occur. As we treat deontic modals as a kind of implication, p is supposed by both $\Diamond p$ and $\neg\Diamond p$ but (16) results in supposition failure. The version of MadRis put forward in this dissertation does not incorporate formal tools to capture suppositional content explicitly, but we discuss the in-development *suppositional inquisitive semantics* to show the potential for future work.

In chapter 6 we return to the puzzles outlined in chapters 3 and 4 in detail and demonstrate that these puzzles are solved in MadRis. Alongside demonstrating that MadRis avoids these puzzles, we also discuss the potential criticism of MadRis on the grounds that it is not monotonic. We discuss the problematic examples and show that the non-monotonic account in MadRis makes intuitive predictions regarding the infelicity of certain deontic statements that have been put forward in the literature as evidence for the upward monotonicity of deontic modals. This includes negative polarity items such as *any*.

We conclude the chapter by discussing the inference pattern *modus tollens* which says that if a conditional $\varphi \rightarrow \psi$ is the case, then it is also the case that the negated consequent implies the negation of the antecedent: $\neg\psi \rightarrow \neg\varphi$. This inference pattern is often considered a desirable feature of the semantics of conditional sentences, despite the fact that it has been shown to be counter-intuitive as early as in 1894 by Carroll's barber shop example [23]. As noted by Yalcin [103], the *modus tollens* inference pattern crucially misbehaves when implications or deontic modals are embedded in the consequent of a conditional sentence. As MadRis is built on the intuition that implication and deontic modals are similar, we investigate *modus tollens* and demonstrate that the inference pattern in not valid in MadRis. Because of this, MadRis avoids making the counter-intuitive predictions that Yalcin discusses.

1.5 Puzzles of legal language revisited

The fact that the SML and Kratzer semantics suffers from puzzles led to the development of MadRis. In chapter 7 we will apply MadRis to the puzzles of *or* and *conditional sentences* in legal language. We will demonstrate how MadRis brings clarity to the discussion of the problematic examples by representing their meaning with semantic tools.

The goal of this dissertation with regard to the interpretation problems lawyers and judges have with *or* and *conditional sentences* is admittedly modest, as we merely aim to propose a framework that provides semantic tools to capture standard intuitions in the literature. We will not discuss several connected topics as the syntax or pragmatics associated with these sentences. We outline these limitations in the concluding chapter and discuss the potential for further work. We conclude that MadRis is a suitable, but not all encompassing, tool to analyze legal language.

Chapter 2

Legal Language and its Puzzles

2.1 Introduction

Laws and legal reasoning require natural language which is ambiguous, so lawyers routinely encounter difficulties in interpreting legal texts. Mellinkoff [76] showed that judges have been aware of the need to interpret the language of the law for centuries. He claims that the earliest authenticated treatise on statutory interpretation (ca. 1571) already discussed the meaning of basic connectives in language. One of the problematic instances concerned a reading of *and* that was more similar to *or*.

(1) "... two such thinges so contraryant are coupled together that they can not drawe under one yocke."

The legal interpretation debate took a notable twist in the 1950's, when a number of scholars proposed that logic can replace natural language to avoid syntactic ambiguities. As they saw it, the problem with natural language was that it was not suitable for the precise nature of the law.

For example, Pfeiffer [77] wrote about the Prudential Life Insurance Company having its contract translated into symbolic logic. The goal was to find unnecessary prolixity, loop-holes or inconsistencies in the natural language of the contract. Pfeiffer himself believed that translations into logic would solve these ambiguity problems for lawyers.

Tammelo [99] and Allen [7] attempted to provide a logic that could be used to assist legal drafting. The core idea was to break complex expressions into constitutive parts, such as propositions and connectives, so that these could be translated into Allen's logical framework. It was assumed that the translation would reveal any drafting errors. Unfortunately, the debate ended after Summers [98] criticized Tammelo and Allen.

> "Professor Tammelo also claims that lawyers can profitably use symbolic logic to combat the vagueness or open texture of legal concepts. Professor Allen has likewise made this claim. Obviously, the claim cannot be assessed until efforts are made to show how symbolic logic can be used in this way." [98, p. 63]

There was a long pause in the literature in which no successful attempts were made to use symbolic logic to cast light on natural language ambiguities in legal language. Solan [94] reinvigorated the debate on the role of symbolic logic in legal interpretation by studying American Court cases with the aid of tools from theoretical linguistics. Among other phenomena, Solan investigated the connective *or* in legal texts and demonstrated that linguistic methods can be applied to puzzles of legal interpretation. This was surprising as lawyers and linguists rarely work together.

Shuy, another linguist that works with legal cases, observed that the reason why there is so little cooperation is partly because "[the] courts do not know what linguists do" and "may even labor under false impressions about our field" [95, p. 8]. In this case, the lack of information goes both ways. While lawyers and judges have little experience regarding what linguistic tools have to offer, linguists are equally unaware of the usefulness of linguistics tools for legal interpretation.

Another issue arises from the wide range of tools in theoretical linguistics, not all of which are equally suitable for the interpretation of legal texts. We will see later in this chapter that many puzzles for lawyers require one to determine the most plausible interpretation of a sentence or expression in the context of a particular text and situation. This suggests that it is more likely that tools from the linguistic study of semantics and pragmatics will be of assistance to lawyers and judges, rather than, for example, the study of morphology or syntax.

Solan's trailblazing study of American court cases was based on tools from the linguistic study of syntax and also from psycholinguistics, with the semantics drawn from classical propositional logic. As such, the choice of tools limited Solan's ability to analyze a wide range of problematic examples. For example, many of the cases Solan discussed involved expressions of obligation and permission, containing modal auxiliaries such as *must* and *may.* These modals pose well-known problems for a standard semantic analysis and require a richer semantic framework capable of representing the logic of modals. Solan had very little to say regarding the linguistically more puzzling aspects of legal language and, thus, also did not introduce the legal community to tools available to handle them.

This is not to take away from Solan's contribution to the study of legal puzzles with the tools from linguistics. The debate in the linguistics literature regarding the usefulness and applicability of different theoretical tools is still in its infancy so Solan's examples and his analysis remain a reasonable starting point. We will also begin this study by critically investigating Solan's analysis of the familiar puzzle of *or* in legal cases. This gravitates around the notorious question of whether *or* should be read inclusively or exclusively. To illustrate this puzzle, consider a simple example.

(2) I will invite John or Mary to the party.

When (2) is read inclusively, it is possible that I will invite both John and Mary to the party. On an exclusive reading of (2), I will invite either John or Mary but not both. There is broad acceptance in the linguistics literature (see, for example, Huddleston [48, p. 1294]) that *or* does offer alternatives without sacrificing the inclusive reading in examples such as (2).

This chapter will continue the study of the role of tools from theoretical linguistics in legal interpretation. We will investigate two different natural language examples that have caused interpretation difficulties for lawyers and judges: *or* and *conditional sentences*. We will begin with Solan's examples and add to them examples from the World Trade Organisation (WTO) adjudicative bodies.

2.1.1 Introduction to the WTO

The WTO proceedings provide an intuitive account of the legal context and we will return to it in chapter 5 when we present the semantic framework. In particular, the way that the WTO adjudicative bodies discuss violations of agreements will prove relevant to our semantic approach.

To see why this is the case, we will discuss the WTO procedures in detail. According to Matsushita et. al. [70, p. 1], the WTO is an international body whose purpose is "to develop and coordinate international trade." More specifically, it "exists to 'facilitate the implementation, administration, and operation and to further the objectives' of the WTO agreements" (*WTO Agreement* Article 3:1, available here: http://bit.ly/WyK9Vj). These agreements pertain to a wide range of trade-related matters and all members of the WTO must abide by their terms.

A disagreement between member countries about the proper interpretation of one of these agreements may give rise to a dispute in which a complainant country argues that a respondent country's interpretation of the agreement and the policies based on it are inconsistent with the agreement in question, thus violating its WTO obligations.[1] The respondent country then attempts to show that no such inconsistency exists and that its policy should be permitted under the text of the agreement.

The WTO requires the parties to a dispute to spend 60 days in consultations to attempt to find a mutually acceptable solution. However, if this consultation period does not result in a resolution of the dispute, the complainant may request that the WTO establish a three-member panel to resolve the dispute. This request involves "identify[ing] the specific measures at issue and provid[ing] a brief summary of the legal basis of the complaint" (*DSU*, Article 6.1, available here: http://bit.ly/XHAvD4). The *measures at issue* are the policies of the respondent that have given rise to the complaint; and the *legal basis* of the complaint is the aspect of a WTO agreement or agreements that the complainant takes these policies to violate.

For example, a lengthy dispute between the European Communities (EC) and Ecuador, Guatemala, Honduras, Mexico, and the United States, which concerned import restrictions that the EC imposed on bananas, revolved

[1]This is not the only reason why a dispute may arise. In particular, there are also "non-violation complaints," where a country's policy "does not conflict with any WTO agreement" (Matsushita et al. [70, 121]); and "situation complaints," which arise from "the existence of 'any situation' other than those covered by the violation and non-violation complaint procedures" [70, p. 123].

around the complainant's claim that the respondent countries' allocation of licences for the importation of bananas was inconsistent with Article III:4 of the General Agreement on Tariffs and Trade (GATT), which requires that one country treat the products of another country in a manner *no less favourable* than *like products* of any other country. This was because this licence allocation amounted to *a requirement or incentive* to purchase bananas from certain countries only –namely, those from 12 former European colonies.

Worth noting here is the centrality of the claim of *inconsistency* between a respondent country's policy and a provision in a WTO agreement, about which we shall have more to say in chapter 5.

The WTO panel hears the arguments of the two parties and prepares an interpretation of the dispute, in the form of a *Panel Report*, in seeking to resolve the dispute. Once the panel produces its report, it is circulated to WTO members and must be adopted by the WTO's Dispute Resolution Body within 60 days of its circulation. Before the report is adopted, though, the parties may appeal it before the Appellate Body, the WTO's version of a court of appeal, which then issues its own report. This report "may uphold, modify or reverse the legal findings and conclusions of the panel" (WTO 2012: http://bit.ly/YJyFzV).

This report is also circulated to WTO members and then adopted by the Dispute Resolution Body (unless it decides by consensus not to, something that has never happened according to Matsushita et al.[70, pp. 115-117]).

Also relevant here, as already noted, is the basis for interpreting WTO texts, which accords with "customary rules of interpretation of public international law" (*DSU* Article 3.2, available here: http://bit.ly/XHAvD4). According to Matsushita et al. [70, p. 27], this is understood "to mean, in large measure, the rules contained in the Vienna Convention on the Law of Treaties", Article 31(1) of which states that "[a] treaty shall be interpreted in good faith in accordance with the ordinary meaning to be given to the terms of the treaty in their context and in the light of its object and purpose". We will have more to say regarding this article when we discuss the role of natural language interpretation in law in the next section.

The reason for including material from the WTO is three-fold. First, we can use WTO examples to demonstrate that the puzzles that lawyers face in interpreting legal language are not a unique feature of American court cases but rather a pervasive aspect of the way that legal interpretation works.

Second, the WTO cases are recorded and presented with meticulous attention to the statements of the parties and the adjudicators so that the context of the statements and reasoning behind the decisions is available for linguistic study. This careful preparation of the materials turns out to be of great value for investigative purposes.

A final reason for this choice of materials is that - unlike the court context described by Solan, which may resort to sometimes very artificial rules of legal interpretation like the *and/or rule* discussed in the next section - WTO adjudicators are required to hew more closely to principles of ordinary language interpretation, as they "cannot add to or diminish the rights and obligations provided in the covered agreements" (*Dispute Settlement Un-*

derstanding of the WTO, Art. 3.2, available here: http://bit.ly/XHAvD4). This makes it possible to use analytical tools developed for ordinary language with fewer caveats and qualifications.

We will not yet commit ourselves to a particular set of tools for the analysis, focusing on clarifying intuitions regarding possible readings. Before we consider examples, we will discuss the role of language interpretation in law.

2.2 The role of language interpretation in law

Solan's [94] primary goal was not to demonstrate that linguistic analysis can assist lawyers and judges interpret legal texts but rather to argue that the way judges interpret connectives such as *or* does not match the interpretation in linguistics. While Solan's effort is groundbreaking, it is not obvious that applying linguistic methods to legal disputes is uncontroversial.

This section is intended to evaluate arguments for and against competing interpretations. Yet courts of law have at least two different types of arguments. They will apply what we will refer to as legal reasoning - which is the domain of lawyers and judges. They also do linguistic reasoning to ascertain what is the literal meaning of legal texts. To avoid the more controversial position of Solan, we will limit ourselves to the latter - investigating natural language examples. Whenever lawyers or judges make arguments based on legal principles, we will consider that outside of our domain.

2.2.1 Interpretation rules of law outside of the scope of linguistics

We will start with an example to demonstrate what we consider to lie outside the scope of linguists. Solan provided several examples in which natural language interpretation takes a secondary role. The role is secondary because the main goal of judges is to provide fair judgements, so instead of interpreting text, sometimes judges reinterpret texts using the following rule.

Solan [94, p. 45] observed that "[t]he difficulty in interpreting *and* and *or* is so well recognized in the law that a special hand-waving canon of construction [i.e., a legal rule of interpretation] exists in both federal law and the law of many states, neutralizing the difference between the two terms." The rule he discusses is provided in (3)

(3) Generally, the words *or* and *and* in a statute may be construed as interchangeable when necessary to effectuate legislative intent.[2]

Commentary to the above rule[3] states that drafters of legal texts sometimes use *or* when they mean *and* and *vice versa*, so judges can decide to correct such mistakes by changing one for the other. Note that this rule does not provide an interpretation of the meaning of *or* but provides judges with

[2] 425 U.S. at 410 n. 11

[3] ibid. 421 n.6

discretionary powers to decide whether the connective in a sentence should be interpreted as *or* or *and*.

This is not surprising considering that the role of a judge is to reach a just decision rather than to provide an interpretation of language. As Solan discusses [94, p. 53], the *and/or rule* serves to satisfy "our everyday sense of fairness" even if that means "pay[ing] little attention to the statutory language as written."

To illustrate this, Solan discusses several cases in which the *and/or rule* is used to correct apparent drafting errors. The first case concerns harassment in *People v. Caine*. A person is found guilty of harassment in New York if:

(4) a. "with intent to harass, annoy or alarm"
b. "he engages in a course of conduct or repeatedly commits acts which alarm or seriously annoy" (*NY Penal Code* §240.25 (5))

This provision states that in the eyes of the law, one commits harassment only if both condition (4-a) and condition (4-b) are satisfied. In *People v. Caine* the court recorded the following facts.

> "On February 20, 1972 the complaining police officer stopped the defendant for a traffic infraction and while writing the tickets the defendant approached and argued with the officer. He was advised by the officer to go back to his car but returned again. At this time the defendant stated that the officer should shove the summons up his F* a*. In response to the officer's questioning "what did you say?" the invective was repeated. At this point the officer alighted from his car and again directed the defendant to return to his vehicle. Again the defendant is alleged to have stated, "Go F* yourself" and in response to the officer's inquiry repeated the words."

The crucial fact is that the defendant repeated his acts, which aligns with the second disjunct "repeatedly commits acts which alarm or seriously annoy". The court, in response, did two things. First, it found that the defendant had demonstrated no "intent to harass, annoy or alarm" as required by New York, but instead merely expressed himself immaturely. This means that he did not fulfil the first condition (4-a) and was thus innocent.

Yet, as the second act, the court continued to state that the disjunction in the law is too lenient as it merely requires one to demonstrate a course of conduct or a repeated commiting of acts. They invoked the *and/or rule* which we introduced earlier (11) to mandate that the law be read differently (the changed word is underlined).

(5) "he engages in a course of conduct <u>and</u> repeatedly commits acts which alarm or seriously annoy."

After this strengthening, it is more difficult to be found guilty of harassment in New York.

As a linguist, there is very little to be said about this case. The judges apply a legal rule of interpretation, which is out of the scope of linguistic analysis and effectively replaced *or* with *and* for reasons of fairness. A linguist can analyse either *or* or *and* but exchanging one for a different one remains outside of the scope of linguistics.

Solan [94, p. 46] remarked that the application of the *and/or rule* also appears to be justified in *Beslity v. Manhattan Honda*. The case related to false advertising which is governed by the provision in (6).

(6) "Any person who has been injured by reason of any violation of [a prohibition on false advertising] [...] may bring an action in his own name to enjoin such unlawful act [...] *and* to recover his actual damages [...]" (*New York General Business* Law §350-d(3), *italics* by Solan)

The emphasized *and* in (6) is read so that a victim of false advertising may only bring an action to court if he has two goals: enjoining an act (i.e., forcing the false advertising to stop) and recovering his actual damages. In case the advertising has stopped, recovering damages via a court action is no longer possible.

The judge in *Beslity v. Manhattan Honda* found the text in (6) too limiting and used the *and/or rule* to reinterpret the *and* as *or* so that one could recover damages even after the advertising campaign had ceased.

One of the more probable reasons for the existence of such rules of interpretation is the possibility that due to the vast number of legal provisions, some rules are inconsistent or create absurd consequences. This was the case in *172-02 Liberty Avenue* which concerned forfeiture of property in cases where the owner knows about but does not consent to criminal activity. The governing rule is given in (7).

(7) "no property shall be forfeited [...] by reason of any act or omission established by that owner to have been committed or omitted without the knowledge or consent of that owner." (*U.S.C.* §881(a)(7))

The act or omission in (7) refers to criminal acts which might lead to forfeiture. According to Solan [94, p. 50], a standard reading of *without* scoping over *or* requires that the owner neither knows about nor consents to the criminal activity.[4]

In *172-02 Liberty Avenue* the owner of the property in question had been cooperating with the police in a drug trafficking matter and thus obviously knew that his property was being used in criminal activity. It would be absurd for the owner to lose his property through forfeiture for helping the police so the *and/or rule* was applied to reinterpret (7) such that forfeiture is prohibited in case the owner knew about the criminal activity but did not consent to it.

All of the three cases above were outside of the scope of linguistic analysis. To find common ground between the work of judges and linguists, we need to find language interpretation in courts of law that can be analyzed using linguistic methods.

[4] This is in line with an analysis of *or* according to *De Morgan's laws*, which, in propositional logic, refer to the equivalences $(p \wedge q)$ with $(\neg p) \vee (\neg q)$ and $\neg(p \vee q)$ with $(\neg p) \wedge (\neg q)$.

2.2.2 The potential role of linguists in assisting courts interpret legal texts

It is a discernible fact across legal systems that natural language interpretation does play a role in courts of law. For example, the interpretation of international law is governed by article 31.1 of the Vienna Convention on the Law of Treaties.

(8) "A treaty shall be interpreted in good faith *in accordance with the ordinary meaning* to be given to the terms of the treaty in their context and in the light of its object and purpose." (*italics* added)

The provision that, under international law, text interpretation should incorporate the investigation of the ordinary meaning of terms opens the door for the study of the meaning of language.

Matsushita et al. [70, p. 112] notes that "Article 31 is cited "[i]n almost every report of Panels and the Appellate Body", "and dictionary meanings of the relevant words in the provision in question are discussed".

The rule of interpretation in this article of the Vienna Convention does not, however, exhaust those relevant to the interpretation of WTO texts. Article 32, though rarely invoked, is also important. It states that "[r]ecourse may be had to supplementary means of interpretation, including the preparatory work of the treaty and the circumstances of its conclusion, in order to confirm the meaning resulting from the application of Article 31, or to determine the meaning [...]"

A linguist should not compete with a lawyer in terms of knowledge of laws, procedures and principles of law but their expertise is relevant in questions concerning the ordinary meaning of language.

There are also instances of the interpretation of the *ordinary meaning of terms* in other legal domains. An article by the Estonian judge Saarmets [87] discusses similar provisions within the Estonian legal system where the interpretation of acts is governed by (9).

(9) A provision of an Act shall be interpreted together with the other provisions of the Act pursuant to the *wording*, spirit and purpose of the Act. (*General Part of the Civil Code Act (Ts´US)* §3, *italics added*)

Saarmets [87, p. 1] comments that it would not be false to say that the principle of interpretation in (9) applies in other areas of law beside the Civil Code. Yet, it is helpful to concentrate on this specific instance of the principle as there are clarifying comments and examples that we can use to investigate the role of language interpretation in law.

For example commentary of the law by the legal theorist Kull [62, p. 9] explains that "The interpretation of law is always the interpretation of text."[5] He expands on the comment by referring to two cases in the highest

[5]Translated from Estonian.

court of Estonia. “Terms of a law must be first interpreted grammatically[6], but one cannot be limited to grammatical interpretation, if that leaves the purpose of the regulation unclear.”[7]

What Kull’s commentary says is that the interpretation of the natural language sentences in legal texts is a part of the interpretation of law. He qualifies this by saying that it can be insufficient and can be overruled by other methods of interpretation. It is interesting to note that the highest court’s comments[8] strengthen the reading of the Civil Code Act. In the Act, the interpretation of wording was merely one possible interpretation method but the highest court considers interpreting the words of a text the first method to use.

Leaving aside the exact role of language interpretation but accepting that it is a part of the interpretation done by judges, we can take a look at some examples where courts have used natural language interpretation as an argument in a case. For example, in a case about the interpretation of the constitution, the judges wrote: “To determine the content of the relevant prohibition one must explain what the words “to change” ... mean.”[9] They continue: “In textual interpretation, one’s primary source is the ordinary meaning of the words in natural language.”[10]

The judges in the above case determined the meaning of the words by looking at a dictionary. This sufficed in the above case, but one is in trouble if the words in question are connectives such as *and*, *or* or *if-then*. A dictionary provides the information that they are connectives and will provide some examples, but their meaning is different from a noun or a verb in that they connect different parts of a sentence, rather than referring to any state of affairs. A dictionary will be of little help as a dictionary does not explain the way in which a connective joins all possible clauses. But this is where linguistic methods become helpful.

2.3 Interpretation puzzles

We will take a look at some examples of expressions that have created interpretation problems for lawyers such that judges have had to provide interpretations of the meaning of connectives. We will first discuss examples of the disjunction and then conditional sentences. We will then demonstrate how linguistics would approach the same interpretation problems.

2.3.1 Disjunction

Solan [94] discussed the following *or* sentence at the heart of *Department of Welfare of City of New York v. Siebel*. In 1957[11], a stepmother of a boy com-

[6] RKHKo 27.05.2008 3-3-1-24-08, p. 12. This and other cases are available at http://www.nc.ee/?id=11&pre=T

[7] RKHKo 03.06.2010 3-1-1-42-10 p. 7

[8] RKHKo 27.05.2008 3-3-1-24-08, p. 12.

[9] RḰUKo 23.02.2009 3-41-18-08, §11, translated from Estonian.

[10] RḰUKo 23.02.2009 3-41-18-08, §12, translated from Estonian.

[11] 465 U.S. 605

mitted to a school for delinquent children was ordered to bear some of the expense of incarceration. She sued the city on grounds that the disjunction in the relevant law (reproduced in (10)) ought to have an exclusive interpretation. It followed her reading of this sentence that as the boy's father already contributed then there is no obligation for her to also contribute.

(10) "to compel such parent *or* other person legally chargeable to contribute" (*Domestic Relations Court Act of the City of New York*, §56-a, *italics* added)

According to an exclusive interpretation of *or*, the law compels one of the two, either a parent or another legally chargeable person to contribute, but not both. Were the interpretation of *or* inclusive, both a parent and legally chargeable person could be compelled to contribute at the same time.

The stepmother won her case at trial level on the basis of an exclusive interpretation of *or* but was overturned in the New York Court of Appeals, New York's highest court. The latter stated that *or* should be interpreted as *and* whenever possible and as *or* in other cases.[12] So the Court of Appeals referred to the *and/or rule*, repeated here as (11).

(11) Generally, the words *or* and *and* in a statute may be construed as interchangeable when necessary to effectuate legislative intent.[13]

As we remarked earlier, the *and/or rule* does not provide an interpretation of the meaning of *or*. To begin to analyze disjunction, we ought to provide a simple example. For this purpose, we will repeat the example from chapter 1.

Imagine a canteen with a lunch offer in which a starter or a dessert is included in the price of the meal. The canteen would probably have the following sign.

(12) You may take a starter or a dessert.

When a client reads (12), it is clear that taking one or the other of the two items is safe, there would be no additional costs. But what if the client wants to take both? Most people intuitively say that such a client deviates from expected behaviour, but providing arguments for and against can be difficult.

The example in (12) is not a standard example from a linguistic standpoint because it involves the modal auxiliary *may* which makes it a permission sentence. Such examples provide an additional challenge to linguists, and we will discuss the puzzles associated with such examples at length in chapter 4, but nothing in the nature of such sentences makes them less suitable as example sentences to discuss the *exclusive or* puzzle.

The consequences of the difficulty of interpreting *or* sentences is demonstrated by the following example from the WTO. In dispute number 345, India also argued that *or* ought to have an exclusive interpretation.

[12] Such interpretations of *or* and *and* are far from natural language which places the rule entirely outside of the scope of semantics.

[13] 425 U.S. at 410 n. 11

The situation was the following. The US customs office required a deposit on incoming goods from India, and also required that the deposit be paid half in bonds and half in cash. India considered this unfair and referred to an Ad Note to the WTO Anti-Dumping Agreement that states (13).

(13) "a contracting party may require reasonable security (bond or cash deposit)".

The example gives permission to ask for reasonable security, and the form of the security (but not the sum total) is guaranteed to be reasonable if it is a bond or a cash deposit. India argued that the *or* in between bond and cash deposit should be read exclusively such that it only allows for either a bond or a cash deposit, but not a combination of both.

Readers can test whether their intuitions align with the US or India with the aid of the following simplified example. Imagine that you have been grounded for trying to sneak into a bar while being under legal drinking age. You have spent a week at home as punishment but on the 8th day, you find the following note from your parents.

(14) You may visit reasonable entertainment (cinema or theatre).

The above example gives permission to visit reasonable entertainment and the items in the brackets are guaranteed to be reasonable. Yet, is visiting both a cinema and a theatre also reasonable?

The judges of the WTO disagreed with India and ruled that the language does not suggest that a combination of both bonds and a cash deposit is necessarily unreasonable. But they did not explain the point, but merely asserted this. The statement is provided below in (15).

(15) "we see nothing in the text ... to suggest that the combination of both (otherwise reasonable) forms of security necessarily results in a measure that is unreasonable. In particular, the text ... does not provide that the form of security will only be reasonable if either (i) cash deposits or (ii) bonds are required"

Before investigating in detail to see whether disjunction is exclusive or inclusive, we ought to introduce examples of a different connective to show that issues of language interpretation are not restricted to *or*.

2.3.2 Conditional sentences

The source of the following examples is different. Instead of looking at cases in courts of law, we can look into legal drafting textbooks. We will see which kind of examples have been identified as sufficiently problematic for the legal community to invest time into training young lawyers to be aware of their interpretation issues.

Haggard and Kuney [46] discuss conditionals or *if-then sentences* as having specific interpretations that can cause problems. Classifying conditionals is by no means uncontroversial in linguistics but it will not influence the discussion here. Instead we will limit ourselves to the basic case of a conditional shown in the following example.

(16) If I agree with you, then we will both be wrong.

The *if* clause provides the antecedent or the condition. When the sentence as a whole is the case, then a situation which satisfies the antecedent will also satisfy the consequent, or the *then* clause. In the case of (21), the sentence provides the information that any time I agree with you, we will both be wrong.

Haggard and Kuney discuss the following two conditionals. The first assumes that a collective bargaining agreement contains the following provision.

(17) If an employee requests an unpaid personal leave in writing at least 10 days in advance and has no unused vacation time, then the shift supervisor shall approve the request.[46, p. 279]

The rule posits three conditions that must be satisfied. The request for vacation time must be in writing, it must be 10 days in advance and there must not be unused vacation time. If those conditions are met, the shift supervisor must approve the request. But, what if the conditions are not met, for example when the request is handed in 7 days in advance and yet the shift supervisor approves the request? Haggard and Kuney state that a union would file a grievance because the rule has a negative inference that when the conditions are not met, then the the consequent must be false too.

A simplified example with the same structure is the following.

(18) If Odysseus' plan works, then he will be a hero.

The *if* clause states the condition to be satisfied - Odysseus' plan must be considered a success. If (18) is the case, and the condition is satisfied, then it is also the case that Odysseus is considered a hero.

We can introduce two atoms to represent the sentences in the antecedent and the consequence. Let the antecedent be p and the consequent q. So with the conditional (18), p represents "Odysseus' plan works" and q represents "[Odysseus] is a hero." The conditional as a whole can be represented by an arrow $p \rightarrow q$.

The issue introduced by Haggard and Kuney with example (17) is whether $p \rightarrow q$ licences a negative inference that when the antecedent is not the case, then the consequent is not either. We use an $\neg p$ to represent that sentence p is not the case and the $\models$ symbol to represent a semantic inference. So the issue we are investigating with regard to example (17) is whether the following inference holds.

(19) $p \rightarrow q \models \neg p \rightarrow \neg q.$

With regard to example (18), the question is whether from (18) it follows that the following sentence holds as well.

(20) If Odysseus' plan does not work, then he will not be a hero.

We will leave this issue for now, and introduce the second example from the legal drafting textbook.

(21) If delivery is before June 30, 2004, then purchaser will tender cash.

(21) uncontroversially places an obligation on a purchaser to pay cash - which is preferable compared to, for example, payments by cheque. Yet, the obligation comes about only on condition that delivery is timely. If the delivery is after the 30th of June, then the purchaser may choose a different means of payment.

The interpretation issue arises when the purchaser chooses to tender cash or, in other words, to satisfy the consequent. Does that place an obligation on the deliverer to arrive before the 30th of June? This would mean that whenever $p \rightarrow q$ is the case, $q \rightarrow p$ is the case as well. So the semantic inference we are investigating is the following.

(22) $p \rightarrow q \models q \rightarrow p.$

In terms of (21) this means that whenever (21) is the case, (23) is as well.

(23) If purchaser will tender cash, then delivery is before June 30, 2004.

We have now listed the puzzles to be solved. We will discuss the problematic examples in detail in this chapter but we will spend the following chapters motivating and describing a semantics to deal with these examples. We will present the final version of the solutions in chapter 7. But before we can elaborate on the examples, we need to discuss the role of language interpretation in law.

2.4 Intuitions

To understand how to analyze the meaning of connectives using linguistic methods, it might be useful to begin with the principle of compositionality.

Compositionality aims to answer the question how we can understand sentences despite not having ever heard them before. Stanford Encyclopaedia of Philosophy explains compositionality as follows.

> The meaning of a complex expression is determined by its structure and the meanings of its constituents. [88]

The principle of compositionality captures the idea that there is an inherent logic in the structure of language that allows us to unravel the meaning from the way the structure and constituent parts interact. In what follows, we want to get at the structure.

In both the WTO and in Estonian legal practice we saw instances of interpretation problems being solved by use of a dictionary. In light of the principle of compositionality, a dictionary suffices, for most purposes within a court of law, to provide the meaning of nouns and verbs, but it does little to explain the meaning that arises from the structure of a complex expression. The way a linguist would approach the structure is by trying to uncover the inherent logic of connectives.

Native speakers of a language grasp the internal logic of language intuitively, and recognize when a structure violates what's possible with a language. For example, we know from linguistic literature that *any* requires negation or another downward entailing context. We can demonstrate with the use of examples that our intuitions verify this fact.

(24) a. Charles does not have any potatoes.
b. #Charles does have any potatoes.

A native speaker will find (24-a) a perfectly ordinary sentence, while (24-b) will seem odd and difficult to utter. By collecting such judgements about language from native speakers, a linguist can unravel the structure of expressions such as *any* and also of connectives.

2.4.1 Disjunction

We will first clarify native speaker intuitions on the puzzle whether *or* is exclusive. It is helpful to start with the claims of the stepmother in the case against New York (see example (10)) and India in the case against the USA (see example (13)). Both state that disjunction is always exclusive. This is to say that both disjuncts cannot be the case for a disjunction to be the case. For example, when permission is granted to take either a starter or a dessert (as in example (12)), then a person that takes both a starter and a dessert is not covered by the permission.

The meaning of exclusive disjunction ($\veebar$) can be represented with the following truth table.[14]

	A	B	$A \veebar B$
i	T	T	F
ii	T	F	T
iii	F	T	T
iv	F	F	F

Table 2.1: Exclusive *or*

The left-hand side of the truth table describes the four possible situations (i-iv) with two disjuncts, A and B. They can both be true (i), one of them can be true and the other false (ii-iii) or they can both be false (iv). For example, one can take both a starter and a dessert, one of the two, or neither of them. The right-hand side of the truth table says whether in the described situation the operator, in this case exclusive *or* is true or false. As we can see, in a situation where both disjuncts A and B are true, exclusive *or* is false.

At this early stage, we will assume a simple syntax where permission scopes over everything that follows it.[15] In this case, permission for an

[14] Other semantic representations of exclusive or exist, but the truth table will suffice for our purposes.

[15] We will revise this later.

exclusive disjunction provides permission for all those situations in which the disjunction is true in the truth table, ie. A and not B (ii) and not A and B (iii). Yet, permission is not granted for the case where both A and B are the case (i).

With the background in place, we can investigate the claim that *or* might be exclusive. Solan [94, p. 54] offered the following remark about the meaning of the statute in (10): "If *or* in the statute [was] interpreted to mean either one or the other but not both, then the fact that the Department had already compelled the father to contribute would be sufficient to let Mrs. Siebel off the hook. If, on the other hand, *or* [was] construed to mean either one or the other or both, then Mrs. Siebel [would have to] pay [...] In essence, the court [...] held that *or* is to be construed as *and* whenever possible, and as *or* otherwise, an interpretation somewhat different from both the logical and everyday meanings of *or*."

These remarks reinforce Solan's interpretation [94, pp. 45-46] of *or* as earlier in the book he states that "[w]hile logicians use [*or*] to mean *and/or*", "when it comes to the interpretation of legal documents, *and* generally means *and* and *or* is construed disjunctively, as meaning *either/or*".

Solan thus rather supports an exclusive interpretation of *or*, which does not fit the widely accepted view in linguistics that *or* is inclusive. For example, the influential reference work by Huddleston et al. [48, p. 1294] discusses *or* as inclusive and states that "the 'only one' reading commonly associated with or" is an implicature - an inference regarding the intent of the speaker, rather than the meaning of the connective itself.

In law, however, the view that *or* can be construed exclusively is more widespread. For example, the authoritative contract-drafting manual by Adams [1, p. 124] says that "[*o*]*r* is typically used when one wishes to convey that only one of the propositions is correct–in effect, when one wants the *or* to be exclusive"; and thus "the normal interpretation" of a sentence like (10) "would be that the legislature intended to convey that only one of the propositions [...] was correct, and there is no basis for suggesting that this language conveys the meaning *or both*.[16]

The exclusive interpretation of *or* seems to follow some miscommunication between linguists and law theorists as Adams [1, p. 124] cites Huddleston and Pullum [49] (chapter 15 of which is Huddleston et. al [48]) in support of these claims, despite the fact that elsewhere Huddleston et. al. [48, p. 1294] clearly state that "*or* doesn't **mean** that only one of the alternatives is true" and that a disjunction "is perfectly consistent with both component propositions being true–and indeed I might say it knowing that both are true."

The evidence towards an inclusive reading of *or* comes from intuition tests such as the following. First, we can try to find an example where our intuition says that *or* is inclusive. This is what we will see in example (25). Later we can also test for contradiction.

Too see an example in which *or* receives an intuitively inclusive reading, imagine a rule in a school.

[16]See also the associated article by Adams and Kaye [2].

(25) If a pupil smokes or drinks alcohol, then the pupil will be expelled.

The example says that whenever the *if* clause is satisfied, the violator will be expelled. Yet, when the *or* in the antecedent is read exclusively, when a pupil both smokes and drinks alcohol, then the *if* clause is not satisfied and there are no grounds for expelling him. Our intuition says that either smoking or drinking is sufficient to be expelled and doing both of it makes it only worse. Thus, we can find examples where our intuition says that *or* is inclusive.

Further evidence against an analysis of *or* as exclusive comes from the behaviour of *or* when embedded under negation. In such cases, the salient reading of *or* is one in which both disjuncts are negated, so that (26-a) can be rephrased as (26-b) and (27-a) as (27-b), respectively.

(26) a. She drove the car without the knowledge or consent of the owner.[17]
b. She drove the car without the knowledge and without the consent of the owner.

(27) a. Do not break or soil airplane seats.
b. Neither break nor soil airplane seats.

Furthermore, we can test for contradiction. We saw with example (24-a) that native speakers can detect when the inherent logic of an expression is contradicted. We can see from the truth table of exclusive disjunction that if *A or B* is the case, then it's not the case that both *A and B* are the case. So, to test for contradiction, we must construct a sentence where we have both *A or B* and *A and B* as that is predicted to be contradictory.

We will follow up the *or* sentence with an expression that represents the conjunction of its disjuncts. This is illustrated in (28-a). We can compare this to (28-b) where the primary sentence expresses that *both* is out of the question and the follow-up with *both* clearly leads to intuitive infelicity.

(28) a. John is a liar or a fool. In fact, he is both.
b. John is neither a liar nor a fool. #In fact, he is both.

Unlike (28-b), native speakers judge (28-a) and other such sentences to be acceptable, which means that the disjunction in (28-a) cannot be exclusive.

Despite the wealth of evidence in favour of an inclusive analysis of *or*, there are cases in which the salient reading is exclusive and others where the only reading is exclusive. Huddleston et al. [48, p. 1295] provided the following examples of cases where only exclusive readings are available.

(29) a. He was born on Christmas Day 1950 or 1951.
b. The fugitive is wanted dead or alive.

Intuitively, both (29-a) and (29-b) only allow one disjunct to be the case but this does not change our interpretation of the meaning of *or* as the readings of the sentences in (29-a) and (29-b) result from "narrowing down the range

[17] Based on Solan [94, p. 52]

of possible contexts for the whole coordination", based respectively on "our knowledge that one cannot be born on successive Christmas Days" and that a fugitive cannot be both dead and alive.[48, p. 1296]

Examples where the exclusive reading is merely the salient reading poses a different type of challenge to the inclusive treatment of *or*. Consider (30).

(30) I will invite John or Mary to the party.

When someone utters (30), it would be strange if both John and Mary were invited. Thus, this intuition seems to contradict the previous result that disjunction is not exclusive. Yet, we can test this example in the same way we tested (28-a) and add a *both* option at the end.

(31) I will invite John or Mary, and possibly both, to the party.

Once again, no contradiction arises. Note that the sentence does sound a bit odd because adding *possibly both* feels superfluous. Without the word *possibly* it is even worse because one cannot invite one and both at the same time. Yet this oddity does not interfere with our test as an exclusive reading of disjunction would render it impossible that both are invited to the party, which would make (31) as bad as (28-b).

Furthermore, if we modify the sentence by adding *only* we coerce an *only one* reading and then one cannot add *possibly both*.

(32) #I will invite only John or only Mary, and possibly both, to the party.

This example feels like a contradiction. But as there is no contradiction in example (31), we need an alternative explanation for the intuition that when someone utters (30), that person does not intend to invite both of them. Following Huddleston et al. [48], the interpretation could be an inference. We guess from the speaker's actions what his intent is. As a rational person, the person who uttered (30) should have considered the possibility of uttering the stronger alternative.

(33) I will invite John and Mary to the party.

If the intent of the speaker were to invite both, this alternative would capture that idea much more precisely than (30). From the fact that the speaker chose to use (30), we can thus infer that both would not be invited. Such inferences can be excluded from consideration when investigating the semantics of disjunction, as they do not concern the meaning of *or* but rather the intent of the speaker. And while we can infer that both John and Mary will not be invited, the test in example (31) shows that nothing in the use of disjunction in that sentence excludes both of them being invited.

One could also argue that sometimes disjunction is inclusive and at other times it is exclusive, but such an *ad hoc* approach fails to explain how hearers are capable of distinguishing which disjunction a speaker intends to use considering that there are no distinguishing markers between them.

Furthermore, one might ask why, if disjunction is inclusive, one cannot merely replace an *or* with *and*? The answer is that *and* gives rise to its own ambiguities. Consider the following example.

(34) You may take a starter *and* a dessert.

(34) has a reading in which permission is granted only if you take both a starter and a dessert simultaneously. As this is the strongest reading of the utterance, in the sense that it is the most restrictive, it might be convincingly argued that the drafters of the text intended this rather than any weaker reading, as the weaker readings could be communicated using *or*.

In summary, a careful investigation of the intuitions native speakers have regarding disjunction do not support the hypothesis that disjunction is exclusive. There is evidence of an inference that the speaker who utters *A or B* does not think *A and B* is possible, but intuition tests show that this inference is not in the meaning of *or* and can be left out of the semantics to follow.

Recognizing the fact that the meaning of *or* is inclusive does not give us much insight into its actual use in legal or other discourse. Nor does it offer much help in bridging the divide that Solan[94, p .46] alludes to in speaking of how "logicians use the word [*or*]" and how "in natural language" the word "is frequently used to mean one but not both of two items".

The same divide is seen in the observation by Huddleston et al.[48, p. 1294] that a sentence like that in (29-a) or (29-b) involves "offering alternative explanations" rather than "envisaging the possibility that both might apply", notwithstanding the inclusive meaning of or. What is necessary to bridge this divide is a recognition of the role of pragmatic reasoning about what the speaker intended to convey, which may lead to the conclusion that the speaker has used *or* to signal that only one disjunct holds. Unfortunately, in this dissertation, pragmatic reasoning can only be discussed briefly in chapter 5.

2.4.2 Conditionals

Regarding conditionals, we need to clarify intuitions regarding the examples (17) and (21). We will discuss the simplified example (18), reproduced here.

(35) If Odysseus' plan works, then he will be a hero.

In the conditional (35), p represents "Odysseus' plan works" and q represents "[Odysseus] is a hero." The conditional as a whole can be represented by an arrow $p \rightarrow q$. The puzzle was whether the following semantic inference holds.

(36) $p \rightarrow q \models \neg p \rightarrow \neg q$

In other words, we want to know whether when (35) is the case, then (37) is the case as well.

(37) If Odysseus' plan does not work, then he will not be a hero.

The easiest way to falsify the semantic inference is by finding an example where our intuition tells us that $\neg p \rightarrow \neg q$ does not follow from $p \rightarrow q$. There are a large number of such examples in the literature but we can consider both of our examples with the help of the following story.

Consider a young chess player, John. He is currently practising the Sicilian opening. The interesting thing about the Sicilian opening is that it can be played both with the white and the black pieces. So whichever colour he gets, he tries to make the Sicilian work for himself. Let us assume that he has played a total of 20 games, 10 with white and 10 with black and in all of them he has played the Sicilian opening. In this case, the following is a reasonable sentence to utter.

(38) If John had white, then John played a Sicilian opening.

In (38) p represents "John had white" and q represents "John played a Sicilian opening." Chess also has a useful feature, as there are only white and black pieces, then not playing with white pieces means that the person was playing with black pieces. Thus, $\neg p$ represents playing with black pieces.

If the above semantic inference is the case, then it ought to be the case that when (38) is the case then (39) is the case as well.

(39) If John had black, then John did not play a Sicilian opening.

Yet, we know that John was practising the Sicilian opening in all his games, both when he had white and black pieces. So, based on this information, even though we agree with (38) we reject (39). Thus, we could find an example that contradicts the above inference.

We can use the same context to investigate the other example (repeated below) and its potential semantic inference.

(40) If a delivery is on or before June 30, 2004, then purchaser will render cash.

The puzzle was whether when $p \rightarrow q$ is the case, $q \rightarrow p$ is the case as well. So we are investigating the following semantic inference.

(41) $p \rightarrow q \models q \rightarrow p$

In terms of (38) this means that whenever (38) is the case, (42) is as well.

(42) If John played a Sicilian opening, then John played black.

But from the story we know that John played 10 Sicilian opening games with white. So we accept (38) but reject (42).

In summary, our intuitions do not support the claim that (17) and (21) have the discussed semantic inferences. But to investigate this further, we need a semantics for conditionals. Unfortunately, as we will discuss in the next sections, this is not a simple task for a number of reasons that directly concern legal discourse. But before we look into a semantics for legal discourse, we can ask how legal discourse can help semanticists in their own work.

2.5 Lessons for semanticists

Until recently, legal discourse has been of little interest to semanticists. Well known work on deontic contexts has taken examples from everyday use of permission and prohibition. In, for example, linguistic studies of modals by Kamp[51] and Simons [93], the examples concern situations where a mother gives permission for her children, rather than using language from a law or contract.

This section will do a case study of legal discourse to investigate whether legal discourse sets restrictions on semantic models. These restrictions will guide our work in the following sections. The case study develops our previous work [5] to draw general conclusions regarding models for legal discourse.

The background story to the example runs as follows. Recall the banana dispute from the introduction to the WTO in section 2.1.1. The complainant of WTO dispute number 27 can produce bananas cheaper than the respondent and so the respondent's bananas were no longer being sold. The respondent reacted by placing a tax on all bananas except his own. The respondent also provided a way to avoid the tax. If one buys the respondent's bananas and then sells them within the respondent's country for profit, one will be allocated licences that exempt them from the tax because they will be considered to be inside the *tariff quota*. The complainant finds this unfair as selling under the *tariff quota* still reduces his profits because he needs to first buy more expensive bananas instead of directly selling his own cheaper ones. This lead to the following exchange.

Note that a dispute is standardly begun by a complainant's claim that some policy is *inconsistent* with a specific WTO article. This follows the WTO regulation set down in the Dispute Settlement Understanding article 6.1.

(43) The request for the establishment of a panel [...] shall [...] identify the specific measures at issue and provide a brief summary of the legal basis of the complaint.

The specific measures at issue are the policies of the respondent that the complainant contests. The legal basis for a dispute should refer to a particular WTO law or laws that are being violated by the aforementioned policies. The example at hand adheres to these guidelines.

(44) Complainant:"[the respondent is] inconsistent with Article III:4 of GATT because this licence allocation amounts to a requirement or incentive to purchase [the respondent's] bananas."

The complainant claims that the respondent's licence allocation for bananas is the specific issue at hand. The legal basis for the case refers to Article III:4 of GATT. It is claimed that the article does not permit *requirements or incentives* to purchase goods. The wording of the article is reproduced in (45).

(45) "The products [...] of any Member imported into the territory of

> any other Member shall be accorded treatment no less favourable than that accorded to like products of national origin in respect of all laws, regulations and requirements[...]"

Note that the article specifically mentions *requirements*. It does not explicitly mention *incentives*. This can be shown to be the source of the alleged deviance in the behaviour of disjunction as it leads the respondent to reply to the complaint with the following utterance.

(46) Respondent:"[the respondent] does not force any trader to purchase any quantity of [the respondent's] bananas."

(44) includes a disjunction between *requirements* and *incentives*. Assuming that *forcing purchases* and *requiring a purchase* are synonymous in this case, then the respondent only negates the disjunct that mentions *requirements*. Standardly in propositional logic, the negation of a disjunct allows, through a process of elimination, to conclude that the remaining disjunct is true. This is the case because for a disjunction as a whole to be true, at least one of the disjuncts should be true. The panel deviates from this standard inference regarding disjunction, as it does not use this simple eliminative reasoning process.

The respondent probably realized that article III:4 of GATT does not refer to *incentives*, and hoped that the panel finds against the complainant because one cannot establish a link between incentives and the WTO article that only mentions *requirements*. Unfortunately for the respondent, the panel cited a prior case linking *incentives* and *requirements*.

(47) "this obligation [described in (45)] applies to any requirement imposed by a contracting party, including requirements 'which an enterprise voluntarily accepts to obtain an advantage from the government' "

The panel reasoned from the fact that the respondent provides *incentives* to buy its bananas (an uncontested fact), through the link between *incentives* and *requirements*, to the conclusion that the respondent has created a *requirement* that is not permitted under article III:4 of GATT (45).[18]

We can generalize two constraints from this example of legal discourse that will influence the choices we make in choosing our semantic framework.

2.5.1 First constraint: issue resolution

The first constraint arises from the fact that the judges always face an issue to solve. (43) specifies that a dispute is valid only when two criteria are satisfied. Firstly, the complainant must refer to some policy that is being implemented by the respondent and, secondly, the complainant must refer to a specific WTO article that prohibits that policy.

The complainant's role is to claim that the policy in question violates that particular article, while the respondent denies that claim. The judge

[18] For a more detailed explanation of the steps and utterances involved, see [5].

will thus need to resolve the issue whether the policy in question is *inconsistent* with the mentioned WTO article. An issue will be at the heart of every legal dispute and, thus, any model of legal discourse should be able to account for issue resolution.

What we mean by issue resolution is most often understood as accounting for the semantics of questions. A number of recent semantic accounts that deal with obligations and permission have included a variant of question semantics. We will later discuss the puzzles of deontic modals in chapter 4, with emphasis on the free choice puzzle. Recent proposals to solve the well-known puzzle of *free choice* inferences, such as Aloni's [9] and Simons's [93], have predicted the behaviour of disjunction in permission utterances with the help of introducing alternatives in question semantics. The focus of WTO cases on issue resolution provides additional motivation for considering such approaches to the semantics of modals.

Approaching the issue of deontic models not from a puzzle-oriented direction, but from a top-down perspective, one can argue that the central role of questions in dispute settlement should be recognized as the reason for question semantics to enter deontic models. Not every semantic problem and every solution to every problem will require question semantics, nor is question semantics the sole alternative for dealing with such phenomena, but a lack of attention to issue resolution should be a major problem for any semantic model that hopes to account for legal discourse. It will not be certain whether the model will make correct predictions once it is extended to other parts of legal discourse which will inevitably bring in issue resolution and, thus, such models would be open to criticism.

2.5.2 Second constraint: violations

Note that (43) consists of two parts. The complainant's case rests on the claim that the policy is *inconsistent* with article III.4. Note that this is not logical inconsistency. Instead, what is meant is that the policy violates the rules and regulations that govern the situation. This explicit claim is affirmed by the judges in their concluding statement.

(48) Judges: "we find the allocation ... inconsistent with the requirements of Article III:4 of GATT."

This repetition of the word *inconsistent* reveals that not only does each dispute gravitate around an issue, the issue is whether an inconsistency is the case or not.

Legal discourse introduces an intermediate step between determining the state of affairs and establishing the legal consequences. If the court finds that a specific policy is the case, they will then check whether that policy is *inconsistent* with a law. In other words, they will judge that the policy violates the law. If they find a violation, then they will also determine an appropriate punishment such as a fine. If, on the other hand, there is no violation, then there will be no need to consider punishments.

This intermediate step plays a crucial role in conjunction with issue resolution that is at the heart of dispute settlement. If it were removed, then one could only investigate whether the state of affairs is as described by the complainant - whether the respondent truly implements the policies that are ascribed to it. Or, also, one may investigate whether laws do in fact state what the complainant claims. Yet, this does not account for the intuitive case where both the policy and law exist, but they contradict each other. For the judges to be able to investigate this, a deontic model must add a new entity, inconsistency, into its framework. Only then could one ask the question whether such an inconsistency follows from the policy and laws in effect.

This idea is not new to semantics, Anderson [15] suggested a deontic logic that explicitly adds violations as a consequence of a prohibited act. Anderson eventually rejected his own logic because it allowed one to reason along the lines of the naturalistic fallacy. One could derive the prediction that everything that is the case is obligatory. But other authors such as Barker [18] have proposed new versions that make use of a similar idea. This case study demonstrates that the intuition behind Anderson's work was well grounded in legal discourse and violations play a role in legal discourse. We will return to violations once we introduce the semantic framework in chapter 5.

2.6 Conclusions

In this chapter we discussed the possibility of applying tools from theoretical linguistics to the interpretation issues of lawyers. We began by taking a critical look at Solan's work on the exclusive *or* puzzle and noted that his trailblazing study was limited in terms of the tools he applied. Solan did not consider recent developments in semantics, drawing the meaning of connectives instead from classical propositional logic. This opens the possibility of improving on Solan's initial attempts at solving the interpretation puzzles of lawyers by introducing new tools from theoretical linguistics.

Furthermore, Solan's view that *or* is exclusive matches the view in the legal drafting literature. But the consensus view in the linguistics literature which states that *or* is inclusive and any 'only one' readings are due to contextual restrictions or pragmatic inferences regarding the intent of the speaker. The semantics to follow will try to capture the consensus view in linguistics to see how the application of new theoretical tools to the puzzles of lawyers explains these examples.

We introduced several new additional puzzles from the WTO and a legal drafting textbook. This expanded the scope of the study to *if-then sentences* and also demonstrated that the interpretation difficulties are not limited to American court cases.

The investigation of example sentences from the WTO also revealed two constraints for the semantic study in the following chapters. The semantics has to be able to account for issue resolution as the procedures in the WTO revolve around questions regarding whether rules have been violated. We will approach these puzzles with the help of inquisitive semantics, a recent framework in the tradition of alternative semantics that is well suited to deal with issue resolution.

Furthermore, the mentioned violations, or the inconsistency of laws with international obligations, are at the center of WTO legal disputes. So in chapter 5, we will adopt an Andersonian treatment of obligation and permission sentences that allows us to explicitly talk about such violations.

The following chapter will consider the newly introduced puzzles with conditionals and expand on the standard view of conditionals in the linguistics literature.

Chapter 3

Indicative Conditionals and Modals

3.1 Introduction

The previous chapter discussed deontic contexts and we saw that lawyers have trouble with disjunction and conditionals. In this chapter we will focus on conditionals from a linguistic perspective. We will begin by considering puzzles of material implication as these help motivate what has become the standard account of modals and conditionals: Kratzer semantics [60, 57, 58, 59].

Kratzer builds on standard modal logic, so we will first introduce the standard account and then discuss Kratzer's modification to it. We will finish the chapter by foreshadowing the puzzles of deontic semantics that we will introduce in chapter 4.

3.2 What is an indicative conditional?

Indicative conditionals connect two sentences in the indicative mood. For example, we can take the following two sentences.

(1) a. I agree with you.
b. We will both be wrong.

Language needs a way to express the idea that (1-b) follows (1-a). That is where the indicative conditional comes in.

(2) If I agree with you, we will both be wrong.

If we represent (1-b) with C and (1-a) with A, then (2) can be represented by *If A, C* or *C, if A*. The *if* clause A provides the antecedent and C the consequent of an indicative conditional.

Classifying natural language conditionals is a controversial subject, so we shall restrict ourselves to the prototypical cases of the indicative conditional and put aside various versions such as biscuit conditionals [21] that require a non-standard approach.

The logical operator from propositional logic that best corresponds with the indicative conditional is material implication, which has the following truth conditions.[1]

	A	C	$A \to C$
i	T	T	T
ii	T	F	F
iii	F	T	T
iv	F	F	T

Table 3.1: Material implication

Intuitively, an indicative conditional says nothing about whether the two sentences in the antecedent and consequent are the case. It merely expresses that whenever A is the case, so is C. In this case, we are only looking at situations where the antecedent is the case. In situations (iii) or (iv) where the antecedent is false, the material conditional cannot be falsified and is trivially true. In situation (i) where the antecedent and consequent are both true, the conditional is true as when A is the case, so is C. And in situation (ii) where the antecedent is true but the consequent is false, we can say that the conditional is false.

An account based on material implication faces a large number of logical puzzles which has led the majority of linguists to consider its original formulation insufficient to account for natural language indicative conditionals.

3.3 Puzzles of material implication

We will only consider five puzzles of material implication although there are many more in the literature. For more puzzles, see, for example, Priest's *Introduction to Non-Classical Logic* [80] or Bennett's *Philosophical Guide to Conditionals* [19]. For an inquisitive semantics perspective on puzzles of material implication, see Lojko's master's thesis [67].

We will focus on those puzzles of material implication that help motivate the most prominent account of conditionals in the literature: Kratzer semantics.

3.3.1 False antecedent

At the root of several material implication puzzles is the fact that when the antecedent is false, the implication cannot be falsified. Thus, a false antecedent entails ($\models$) any conditional. We understand entailment in the classical sense: φ entails ψ iff whenever φ is the case, so is ψ. We will denote material implication with $\to$ and negation is represented by $\neg$. The false antecedent puzzle concerns the following inference, which is valid with material implication.

[1] Clarence Irving Lewis[65] proposed that strict implication ($\Box(p \to q)$) would be a better alternative, as it represents implication so that in all models when A is the case, so is C. But the strict implication account falls victim to many of the same puzzles a material implication so we will not discuss it separately in this dissertation.

(3) $\varphi \models \neg\varphi \to \psi$

We will omit proofs of these basic entailments and instead provide examples with atomic sentences to illustrate how the entailment comes about. Consider the simple example in (4).

(4) $p \models \neg p \to q$

The atomic sentences will be illustrated with pictorial representation. In the following figures, we will draw the truth table in the previous section such that each situation is represented by a possible world: a circle with a sequence of atomic sentences. p represents that p is the case in that world and $\neg p$ represents that p is not the case.

We will also draw a line around the worlds that make the sentence the case. For example, in figure 3.1 we have drawn an opaque rectangle around the two worlds that make p the case: pq and $p\overline{q}$. As one can see, all non-p worlds are outside of the rectangle. This allows easy distinction between worlds that make the sentence the case and those that do not.

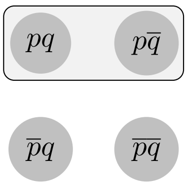


Figure 3.1: p

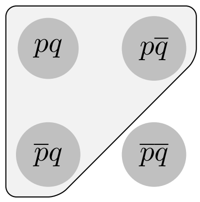


Figure 3.2: $\neg p \to q$

As one can see from the figures, both worlds which make p the case are also included in the worlds that make $\neg p \to q$ the case. So, whenever p holds, so does $\neg p \to q$. Yet, this leads to counter-intuitive predictions when applied to natural language.

(5) The butler did it; hence, if he didn't, the gardener did.

People often joke around that in all crime novels it's either the butler or the gardener that's guilty. But, jokes aside, intuitively the inference in (5) does not hold. There can be various other people that can be guilty when the butler is absolved. Yet, material implication analysis predicts that the inference in (5) is valid. So material implication makes counter-intuitive predictions when the antecedent is false.

3.3.2 True consequent

Material implication also makes any inference from a true consequent trivially true.

(6) $\psi \models \varphi \to \psi$

The inference in (6) follows from the fact that material implication can only be falsified in situations where the consequent is false. In other cases, either the antecedent is the case, which makes the implication follow, or the antecedent is false, in which case material implication is trivially the case. To illustrate this, consider (7)

(7) $q \models p \to q.$

The relevant sentences are illustrated by the following figures.

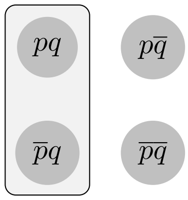


Figure 3.3: q

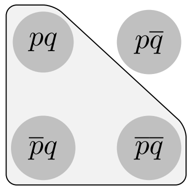


Figure 3.4: $p \to q$

As one can see, all worlds which make q the case also make $p \to q$ the case, so that it cannot be that q is true and $p \to q$ is false. Yet, this also predicts counter-intuitive inferences such as the following.

(8) John is in his office. Hence, if John was killed by a bomb this morning, then John is in his office.

We know that if John was killed, he will not be in his office. But if we are looking at John in his office, material implication predicts that (49) must be the case.

3.3.3 Strengthening the antecedent

Consider the interaction of material implication with conjunction. Lewis [66, p. 10] famously criticized the fact that material implication makes (9) a valid entailment.

(9) $\varphi \to \chi \models (\varphi \wedge \psi) \to \chi$

Consider its simple instance.

(10) $p \to r \models (p \wedge q) \to r$

The example in (11) shows that this inference is dubious.

(11) If I strike a match, it will light. Hence, if I strike a match and the match is wet, it will light.

Although most people accept that striking a match will make it light, few would also agree that a wet match will light. So the entailment in (10) is counter-intuitive.

To see how this inference follows from a material implication account, see the illustrative figures 3.5 and 3.6. In these figures, we must consider 8 possible worlds. We have drawn this figure by creating a mirror image to the right of the four world figure. We have added r worlds such that the four worlds to the left are $\neg r$ worlds and the four worlds to the right are r worlds. The top row is p worlds and outer columns are q worlds.

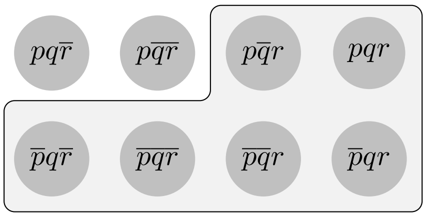


Figure 3.5: $p \to r$

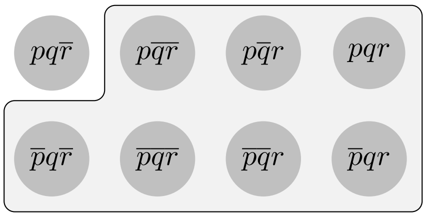


Figure 3.6: $(p \wedge q) \to r$

All worlds that make the premise of (10) the case also make the inferred conclusion the case. This means that it cannot be that $p \to r$ is the case, and $(p \wedge q) \to r$ is not. As one can see from this example, the interaction of conjunction with conditionals is also a source of difficulty for the material implication account of conditionals.

3.3.4 Contraposition

The following inference is known as contraposition.

(12) $\varphi \to \psi \models \neg\psi \to \neg\varphi$

Whenever a material implication is the case, it is also the case that if the consequent is false, the antecedent must be false as well.

At the conceptual level, contraposition is required for implication to be a valid form of inference. Intuitively, whenever an inference is valid, then knowing that the conclusion is false is sufficient to conclude that the premise is false as well. This is why classically when (13-a) holds, so does (13-b).

(13) a. $\varphi \models \psi$
b. $\neg\psi \models \neg\varphi$

To see why contraposition holds with material implication, consider the simple atomic case.

(14) $p \to q \models \neg q \to \neg p$

Material implication also makes (14) trivially true because both the premise $p \to q$ and the conclusion $\neg q \to \neg p$ are falsified by the same, $p\overline{q}$ world.

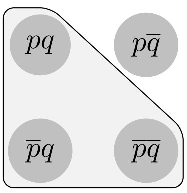


Figure 3.7: $p \to q$

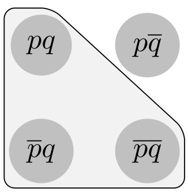


Figure 3.8: $\neg q \to \neg p$

From figures (3.7) and (3.8) we can see that any world that makes $p \to q$ the case also makes $\neg q \to \neg p$ the case. But Grice [39, pp. 78-79] provided the following story to question the validity of the inference.

Yog and Zog are playing chess with special rules. Yog gets white 9/10 times and there are no draws. They have already played around 100 games, and Yog emerged victorious in 80 out of 90 of the games in which Yog had white, but Zog won all the remaining games. Now, the following two sentences have different probabilities.

(15) a. If Yog had white, Yog won.
b. If Yog lost, Yog had black.

The probability that the sentence (15-a) holds is 8/9 but it is only 1/2 for sentence (15-b). The problem with this situation is that, as (16-a) illustrates, (15-a) and (15-b) are equivalent (represented as $\equiv$) if analyzed as material implication. This is because when you play chess, you use either the white or black pieces. So, playing with not white pieces is the same as playing with black pieces. And losing is the same as not winning when draws are taken out of the rules of chess. So if (15-a) is represented by $p \to q$ then its contraposition $\neg q \to \neg p$ is (15-b). But equivalent sentences should not have different probabilities. 8/9 and 1/2, respectively.

Also problematic, as illustrated by (16-b), (15-a) is made true in all cases when Yog had black as that makes the antecedent false. This seems insufficient to provide such an accurate probability assessment. The problematic cases are provided below.

(16) a. $p \to q \equiv \neg q \to \neg p$
b. $\neg p \models p \to q$

We already noted that the false antecedent inference (16-b) makes for an counter-intuitive prediction, but the differing probabilities also demonstrate that contraposition is counter-intuitive as an entailment because there exist counter-examples in natural language.

3.3.5 Negating conditionals

As we already briefly noted with respect to the true consequent puzzle, the negation of a conditional is also problematic for a material implication account. From the truth table, we can see that a material implication is false only when the antecedent is the case, and the consequent is not.

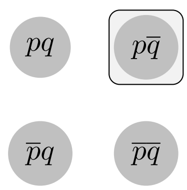


Figure 3.9: $\neg(p \to q)$

The problem with such a prediction of negation was discussed by Grice, although he credits Bromberger with the intuiton [39]. He gave the following example.

(17) It's not the case that if there's a god, then we are free to do whatever we like.

In fact, this should sound like a true statement to most people because, for example, the Bible contains a number of prescriptions on what humans ought to do. In a material implication account $\neg(p \to q)$ is the case when both p and $\neg q$ are the case. But, intuitively, it is not the case that someone who believes (17) is committed to the belief that god exists (p). One can be agnostic and yet know that religions have rules for people.

It is much more intuitive to think that the negation of a conditional is *If A, not C*. So, the following serves as a paraphrase of (17).

(18) If there's a god, then we are not free to do whatever we like.

This is illustrated in figure 3.10.

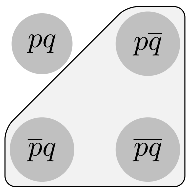


Figure 3.10: $p \to \neg q$

Grice [39] also discusses two other types of negations for conditionals. One is a game context. When people are playing Bridge, one can count cards and also use secret language. Assume that an arbitrary secret code represents (19). So the following sentence can be used to communicate the content of one's hand.

(19) If I have a red king, I have a black king.

Now, the partner can later say that you said something false. What he would mean by that is exactly the negation of a material implication - you had a red king but no black king. This corresponds with a *A and not B* treatment of the negation of implication. Yet, the intuition seems to be covered by the *If A, not C* treatment of the negation of implication equally well.

Another example is more problematic.

(20) It's not the case that if X is given penicillin, he will get better.

When a doctor says (20), he means that the drug will have very little effect. Such a sentence can be strengthened to mean *If A, not C* by adding "... as X is allergic to penicillin." Yet, what (20) itself means is that neither the consequent nor its negation follows from the antecedent. In this sense, the negation of a conditional is ambiguous between the readings *If A, not C* and *If A, C might not occur.*

The material implication negation of conditionals also fails to account for Frank Ramsey's intuition on conditionals. Ramsey motivated a suppositional approach to conditionals: "If two people are arguing *If p, will q?* and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; so that in a sense *If p, q* and *if p, ¬q* are contradictories." [82, p. 15a] With material implication, $p \to q$ and $p \wedge \neg q$ are contradictories. So, one cannot pose a question *If p will q?* and hope that the possible answers are as Ramsey suggested.

3.4 Standard account of modals and conditionals

Kratzer semantics [59] has become the prevalent theory on conditionals in the linguistics literature. It is known as the restrictor account, because instead of defining a two-place connective, Kratzer takes the antecedent to restrict the domain for a (generally covert) operator that quantifies over the consequent. She considers the most likely candidate for this hidden operator to be the epistemic modal *must*. An example such as (21) is interpreted such that, restricted to the cases when Pete called, it must be the case that Pete won.

(21) If Pete called, he won.

This interpretation of (21) makes Kratzer's a suppositional account. We suppose that the antecedent is the case to see whether the consequent is necessarily the case. As Kratzer assumes that the covert operator is an epistemic necessity modal, we must introduce modal logic before we can further expound upon her semantics for conditionals.

3.4.1 Standard Modal Logic (SML)

SML[2] attempts to capture the semantics of expressions that qualify statements about the world.[3] This paper will be limited to the semantics of modal auxiliaries *must* and *may*[4] such as illustrated in the following sentences.

(22) a. John *must* pay his taxes.
b. John *may* drive a car.

These modal auxiliaries can have several interpretations. For example they may receive an *epistemic* interpretation according to which (22-a) says that it is known that John pays his taxes.

In deontic contexts, such as concern us in this dissertation, these modals are standardly analysed as expressions of obligation and permission. In this

[2]SML as we approach it here is generally associated with von Wright, see for example his "Deontic Logic" [101].

[3]This is a rather narrow construal of modal logic. According to the Stanford Encyclopedia of Philosophy entry on modal logic [37], in the philosophy literature the main goal of modal logic is to study the deductive behaviour of modal expressions.

[4]There exist other modal auxiliaries such as *might, ought, should, could, etc.* and also modal adverbs, nouns, adjectives and more.

case, (22-a) is interpreted as expressing an obligation for John to pay his taxes.

Other interpretations are, for example, *bouletic* which concerns the extent to which things are desireable, and *circumstantial/dynamic* modals which express possibility and necessity with respect to situational circumstances.

Kratzer builds on SML, which makes it expedient to begin by introducing this simpler logic as a foundation. SML treats modal expressions as *quantifiers over worlds*. These quantifiers are restricted by an accessibility relation.

The most frequently cited modal operators are necessity and possibility, represented by the universal and existential quantifier, respectively. Necessity and possibility are defined with respect to a Kripke structure where a model is the triple $M = \langle \omega, R, V \rangle$ consisting of a non-empty set of worlds ω, accessibility relations between worlds R and valuations V of atomic sentences in worlds.

A possible world w in ω together with the valuation V says which atomic sentences of p, q, r hold in that world. Accessibility relations signify which worlds are possible with respect to each other. This allows necessity and possibility to be defined in the following way.

Definition 1. Necessity and possibility

1. $[\![\text{Necessarily } \varphi]\!]^{M,w} = 1$ iff $\forall w'$ such that wRw': $[\![\varphi]\!]^{M,w'} = 1$
2. $[\![\text{Possibly } \varphi]\!]^{M,w} = 1$ iff $\exists w'$ such that wRw': $[\![\varphi]\!]^{M,w'} = 1$

When φ necessarily holds, any world accessible from the world of evaluation is such that φ is the case in it. When φ possibly holds, there must be a world accessible from the world of evaluation wherein φ is the case.

The accessibility relation R can have various modal interpretations. For example, in the legal context discussed earlier *must* would have a deontic reading ie. the accessible worlds are those in which all rules hold. For example, *must* φ would signal an obligation to do something if the rules state that all deontically accessible worlds are φ worlds. For a person under obligation, there would not be a choice, the only way to fulfil an obligation is to do what is obligated.

The interpretation of the accessibility relation R can also be epistemic - all accessible worlds are compatible with what is known such that when something *must* be, one cannot imagine a situation where the opposite is the case. And likewise the accessibility relation can be modified for other interpretations of modality. As one can see, the different interpretations are captured by the same semantics for the modal operator, all that varies is the accessibility relation R.

The standard account thus treats modals as either universal or existential quantifiers over worlds, such that their meaning is relative to the accessibility relation: it can be interpreted epistemically, deontically, or according to other modal interpretations. As is well known in the literature, such an approach encounters several puzzles.

3.4.1.1 From SML to Kratzer semantics

There are several reasons why Kratzer semantics are preferred over SML but we will focus on the advantages of Kratzer semantics over SML with the help of deontic puzzles. For example, *contrary to duty* puzzles are a problem for SML but they receive a solution in Kratzer's account.

3.4.1.2 Contrary to duty puzzles

We will make use of von Fintel's version [30, pp. 3-4] of a standard *contrary to duty* example used to illustrate the advantages of adding an ordering source.[5] Following von Fintel's version of this story, imagine a city where double-parking is illegal so that (23) holds.

(23) You must not double park.

Furthermore, anyone that violates (23) and parks next to another car parked at the curb will have to pay a fine. So, when a man called Robin is found guilty of double-parking in this city, then, intuitively, (24) holds.

(24) Robin must pay a fine.

The relevant modal base contains all the rules that govern the city, including (23). But if we consider a SML treatment of obligation in (23) as universal quantification over accessible worlds then (23) only holds if in none of the worlds in the modal base double parking occurs. But if no double-parking occurs, then Robin could not have been found guilty of double parking and does not have to pay a fine. So SML counter-intuitively predicts that (24) is false.

3.4.2 Kratzer's Modal Logic

Kratzer builds on SML so she accepts the standard account that possibility modals are existential quantifiers over accessible worlds and necessity modals are universal quantifiers over accessible worlds.[6] Furthermore, the quantifier is restricted by the domain of quantification - the modal base f. So, for example, deontic modals are restricted by the relevant set of laws which determine what is obligatory, prohibited and permitted.

Kratzer adds a second layer of context by introducing an ordering source g. The ordering source states which possible worlds are closer to the ideal state of affairs. The more propositions in a world w differ from the propositions in the ideal world w', the further that world w is from w'. Kratzer never uses this terminology, but an intuitive way to think about the ordering source g in deontic contexts is that worlds are ordered based on which of them has the least *violations*. As the ideal world has no violations, each

[5] Probably the most famous version of a *contrary to duty* puzzle is due to Chisholm [24]

[6] Strictly speaking, Kratzer does not refer to an accessibility relation but rather a function from the world of evaluation to a set of propositions. She calls this function the *conversational background*.

distinguishing feature from the ideal world is a violation.[7] This terminology will be useful to keep in mind later in the dissertation.

For example, imagine a situation where there are two salient obligations.

(25) a. You must invite your spouse to your party.
b. You must invite your best friend to your party.

We will represent the obligations (25-a) and (25-b) as follows.

(26) a. $\Box p$
b. $\Box q$

Intuitively, we are fine if we invite both the spouse and the friend, but in trouble in all other cases. Yet, the situation where we do not invite either of them is distinctly worse than either of the situations where we invite only one of them. In the worse situation both your spouse and your best friend will be angry with you, in the other two cases only one of them will be angry.

The ordering that follows has an ideal situation - where neither obligation is violated, the situations where one of the two obligations is violated is worse than the ideal and the worst possible scenario is one in which both are violated. Such an ordering is demonstrated in figure 3.11.

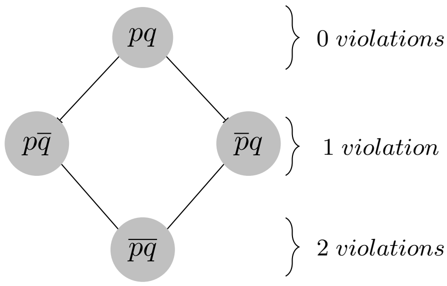


Figure 3.11: $\Box p \wedge \Box q$

The ordering source can also be applied to deontic conflicts[8] as the ordering source is well formed even when there is no ideal world. A deontic conflict is a situation in which there is no ideal world. Consider a simple deontic conflict situation where the spouse and best friend have had a terrible falling out and having them in the same room would be a bad idea. In this case, the following obligation is added to (25-a) and (25-b).

(27) You must not invite both your spouse and your best friend to your party.

We will represent this added obligation as (28).

(28) $\Box\neg(p \wedge q)$

[7]Different violations can naturally have different weight but we will restrict ourselves to outlining the simple case.

[8]There are numerous other applications for the ordering source that do not concern us here.

In such a situation there is no ideal world. If you invite only one of the spouse or the best friend, the other will be angry. And if you invite both of them, there will be an ugly fight between them. Yet, it is distinctly worse to invite neither of them than to invite both of them, as then both your spouse and your best friend will be angry.

According to the ordering, there are three non-ideal worlds $p\overline{q}$, $\overline{p}q$ and pq which violates one of the three obligations but not the others. There is also one clearly worse world in which two obligations are violated: $\overline{pq}$.

The second ordering is demonstrated on the following figure 3.12.

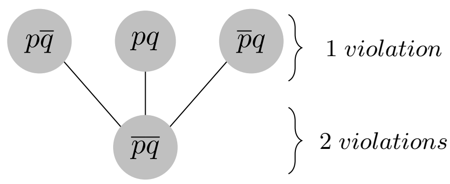


Figure 3.12: $\Box p \wedge \Box q \wedge \Box\neg(p \wedge q)$

The ordering source provides a comparison of deontically accessible worlds. The ordering source where w is at least as close to the ideal as w' is denoted as $w \leq w'$ and a set of propositions is denoted by σ.

Definition 2 (Ordering source)**.** For all worlds w and $w' \in W$: $w \leq_\sigma w'$ iff $\{p : p \in \sigma \wedge w' \in p\} \subseteq \{p : p \in \sigma \wedge w \in p\}$

The above definition states that the world w is at least as close to the ideal as w' iff all propositions of w' are also true in w. For example, every law that is satisfied in w' is also satisfied in w. Yet, as the ordering is defined through a subset relation, w can satisfy more laws than w'.

The modal base for a deontic modal is a function f, such that $f(w)$ represents the content of a body of laws in a world w. The ordering source (g), together with a modal base (f), allows Kratzer to define modals such as *must* in a way that avoids the puzzles of deontic conflicts that SML suffers from.

Definition 3 (Necessity)**.** $[\![\text{Must } \varphi]\!]^{M,w} = \forall w' \in \bigcap f(w) \exists w \in \bigcap f(w)$ $[w \leq_{g(w)} w' \wedge \forall w'' \in \bigcap f(w) : (w'' \leq_{g(w)} w) \rightarrow w'' \in \varphi]$

What the definition says is that φ is necessary when there's a world that is closest or equally close to the ideal and any world that is at least equal to the best world is a φ world.

Yet, unlike the SML formulation, the above definition provides sensible truth conditions in deontic conflict examples. Recall that SML predicted that when $\neg p$ is the case, $\Box p$ has to be false. This is because the ideal world is one in which the obligation is satisfied, and, thus, p is the case in it. Without an ideal world, one cannot satisfy $\Box p$. Kratzer's formulation returns a consistent result also when the ideal world is not available. It will create an ordering based on which worlds are closest to the ideal and $\Box p$ is the case when all of those worlds are p worlds.[9]

[9] We will have more to say on whether the solution Kratzer proposes is sufficient when we consider puzzles of deontic conflicts in chapters 4 and 6.

After this brief introduction to modality in Kratzer semantics, we can outline Kratzer's account of conditionals.

3.4.3 Kratzer Conditionals

Recall that Kratzer's restrictor theory on conditionals takes the antecedent to restrict the domain for an operator that quantifies over the consequent. The operator can either be an overt modal such as in example (29-a) or covert as in example (29-b).

(29) a. If my hen has laid eggs today, then the Cologne Cathedral must collapse tomorrow morning.
b. If my hen has laid eggs today, then the Cologne Cathedral will collapse tomorrow morning.

To analyze (29-b), Kratzer assumes that there is a covert epistemic necessity in the consequent for the antecedent to restrict. This definition for conditionals makes use of Kratzer's additions to standard modal logic that we already introduced. Assuming a modal base f and an ordering source g, a conditional is defined as follows.

Definition 4 (Conditional)**.** $[\![\varphi \rightarrow \psi]\!]^{M,w}_{f,g} = [\![\psi]\!]^{M,w}_{f',g}$ where $\forall w, f'(w) = f(w) \cup \{w' | [\![\varphi]\!]^{M,w'}_{f,g} = 1\}$[10]

The definition states that we take the set of propositions that make up the modal base $f(w)$ and suppose that the antecedent φ holds. This eliminates all worlds where the antecedent is false and provides a new base of evaluation for the consequent. The consequent is the case if in the restricted modal base and ordering, all accessible worlds satisfy ψ. Thus, we suppose that the antecedent is the case to see whether the consequent ψ holds in all remaining worlds.

Consider the indicative conditional in (29-a). According to this definition of conditionals, (29-a) holds if, after all worlds where the hen did not lay eggs are removed, all remaining accessible worlds are such that the Cologne Cathedral will collapse tomorrow morning. And (29-a) is not the case when there exists one world, after all worlds where the hen did not lay eggs are removed, such that the Cologne Cathedral will not collapse tomorrow. As such, Kratzer provides a suppositional treatment of conditionals where we suppose that the antecedent holds and see whether the consequent holds as well. But this treatment is dependent on an account of modals and inherits the puzzles of modal logic that we will discuss in the next chapter.

3.5 Conclusion

In this chapter we considered simple examples of indicative conditionals. One of the primary logical connectives associated with conditionals is material implication but there exist several puzzles that cast doubt on its viability

[10]Note that Kratzer in [59, p. 94] points at the inability of this definition to account for several consecutive modals, but this definition suffices for our purposes.

as the representation of natural language conditionals. We sketched five puzzles of material implication that have been mentioned in the literature as motivating factors toward a suppositional accounts of conditionals.

The currently prevalent theory of conditionals in the linguistics literature is due to Kratzer. According to Kratzer semantics, a conditional is analyzed by restricting the evaluation of the modal operator scoping over the consequent to only those worlds that support the antecedent. When no modal operator is present in the consequent, it is assumed that there exists a covert epistemic necessity modal.

As such a treatment of conditionals is dependent on an account of modals we briefly outlined standard modal logic and Kratzer semantics. We will not adopt the Kratzer account of conditionals in this dissertation, though, because both SML and Kratzer semantics suffer from well-known unsolved semantic puzzles. The next chapter will introduce these puzzles.

Chapter 4

Puzzles of Modal Logic

4.1 Modal logic

We are concerned with Standard Modal Logic (SML) in Kratzer's improved formulation. Kratzer's semantics for modals has three main components.

Quantifiers Modal operators are quantifiers over worlds.

Modal base The set of worlds is restricted to the modal base f.

Ordering source Worlds are ordered by the ordering source g according to their closeness to the ideal state of affairs.

There is a large body of literature on the problems for SML (see the relevant entry in the Stanford Encyclopedia of Philosophy [72]). Lassiter [64] and Cariani [22], among others, have recently raised similar issues with regard to Kratzer's theory on modals and conditionals. We will begin by discussing some of the puzzles of modal logic that they emphasize before moving on to consider puzzles with Kratzer conditionals in particular.

The last sections will discuss *free choice* phenomena because, as we will see in chapter 5, the extensive literature of proposals to solve the *free choice* provides a good starting point for a uniform solution to all of these puzzles.

4.2 Monotonicity

Standardly a sentential operator O is monotonic if it is either upward or downward monotonic [100].

Definition 5. Monotonicity:

Upward monotonicity (UM): O is UM iff $\varphi \models \psi$ implies $O\varphi \models O\psi$;

Downward monotonicity (DM): O is DM iff $\psi \models \varphi$ implies $O\varphi \models O\psi$;

Monotonicity: O is monotonic iff O is upward or downward monotonic.

Modal operators in SML and Kratzer semantics are upward monotonic. To illustrate this property, consider (1-a) and (1-c). According to SML, the obligation (1-a) holds if all accessible worlds are φ worlds. If (1-b) holds and all φ worlds are ψ worlds, then all accessible worlds are also ψ worlds. This means that if the entailment in (1-b) holds, then whenever (1-a) holds, so does (1-c).

(1) a. $\Box\varphi$
 b. $\varphi \models \psi$
 c. $\Box\psi$

Consider the same example with the following atomic sentences.

(2) a. $\Box(p \wedge q)$
 b. $p \wedge q \models p$
 c. $\Box p$

In SML, the obligation in (2-a) holds if all accessible worlds are pq worlds. (2-b) holds because all pq worlds are also p worlds. The obligation in (2-c) holds if all accessible worlds are p worlds. So, if (2-a) holds and, thus, all accessible worlds are pq worlds, then so does (2-c).

Kratzer semantics is upward monotonic in a parallel fashion, but we no longer consider only the accessible worlds, specified as the modal base, but rather the best worlds according to the ordering source. (1-a) holds if the best worlds are φ worlds and (1-c) holds if the best worlds are ψ worlds. If all φ worlds are also ψ worlds then when (1-a) is satisfied according to Kratzer semantics, so is (1-c).

Upward monotonicity naturally also applies to the possibility operator. According to Kratzer semantics, (3-a) holds if there is a φ world among the best accessible worlds. When (3-b) holds, all φ worlds are also ψ worlds, so whenever (3-a) is satisfied, so is (3-c).

(3) a. $\Diamond\varphi$
 b. $\varphi \models \psi$
 c. $\Diamond\psi$

The property of upward monotonicity is a source of puzzles for the standard treatment of modality, although we will eventually see that this property is not the only source of deontic puzzles for the standard account.

4.2.1 Ross's puzzle

When deontic modals are analyzed standardly as quantifiers over worlds, as is done in SML and Kratzer semantics, then the property of upward monotonicity becomes a feature of deontic modals.

Alf Ross [86, p. 41] noted that a quantifier based approach towards deontic logic yielded invalid inferences. He discusses imperatives in relation to disjunction.

(4) a. Slip the letter into the letter box!
 b. Slip the letter into the letter box or burn it!

Intuitively, it does not have to be that (4-b) is the case when (4-a) is the case. According to the standard analysis that Ross was concerned with, imperatives are analyzed in the same way as deontic necessity modals, i.e., as universal quantifiers. So whenever the imperative (4-a) is satisfied, so is (4-b). (4-a) is standardly translated as (5-a) and (4-b) as (5-b)[1].

(5) a. $\Box p$
b. $\Box(p \vee q)$

We also know that in the propositional case a disjunct entails its disjunction.

(6) $p \models p \vee q$

According to a standard account of imperatives, when (5-a) holds, all worlds need to be p worlds. For (5-b) to hold, all worlds need to be either p worlds or q worlds. But when all worlds are p worlds, according to (6), all worlds are p worlds or q worlds, so (5-b) holds as well. And this makes (7) a valid entailment.

(7) $\Box p \models \Box(p \vee q)$.

But this prediction does not fit our earlier intuition regarding (4-a) and (4-b).

While Ross intended to present a puzzle of imperatives, it is a symptom of a more general issue with standard deontic logics based on the quantifiers $\forall$ and $\exists$ over accessible worlds. The problem centers on these accounts being upward monotonic. This means that problematic inferences also exist for obligation sentences such as (8-a) and permission sentences such as (8-b).

(8) a. A country must establish a research center, hence, a country must establish a research center or invade its neighbour.
b. A country may establish a research center, hence, a country may establish a research center or invade its neighbour.

Neither of the above inferences is licensed in natural language and should not be valid in deontic logic. As we discussed earlier, upward monotonic SML and Kratzer semantics predict that such inferences are valid. Consider, for example, (8-b) in Kratzer semantics. The premise of (8-b) is standardly translated as (9-a) and the conclusion as (9-b).

(9) a. $\Diamond p$
b. $\Diamond(p \vee q)$

(9-a) holds in Kratzer semantics when, according to the modal base and ordering source, there is at least one p world among the best worlds. (9-b) holds when, according to the modal base and ordering source, there is at least one p world or at least one q world among the best worlds. This means that whenever (9-a) holds, so does (9-b) and the inference in (8-b) is predicted to be valid. The obligation in (8-a) is parallel to (5-a) and (5-b).

[1]According to the standard account, $\Box(p \vee q)$ and $\Box p \vee \Box q$ have the same meaning.

Both Lassiter [64] and Cariani [22] used the above inference patterns to criticize Kratzer's modal logic. In response, Kai von Fintel in [31] noted that Ross's example and the more famous *free choice* phenomenon are both monotonicity puzzles of the same kind. We will accept this suggestion as a starting point and investigate it in more detail later.

If upward monotonicity puzzles are connected to *free choice* then any solution for one ought to also be a solution for the other. Below we will introduce the *free choice* puzzle and add several other monotonicity puzzles.

4.2.2 The *Free Choice* Puzzle

The *free choice* puzzle has become the best documented deontic logic puzzle in the linguistics literature since it was investigated by Hans Kamp [51]. The *free choice* puzzle rests on the fact that when permission $\Diamond\varphi$ is analyzed standardly as the existential quantifier, it fails to correctly predict the interaction of permission with disjunction. A sentence such as (10) where the permission auxiliary *may* scopes over disjunction intuitively gives permission to establish a research center and a laboratory, and the choice is left to the discretion of the country.

(10) A country may establish a research center or a laboratory.

The salient reading of (10) is one in which permission is granted to establish a research center and permission is granted to establish a laboratory. It is not necessarily permitted, though, for a country to establish both a research center and a laboratory, rather than choosing one. There could exist a separate prohibition on establishing both. In this sense, (10) does not guarantee that bringing about both disjuncts simultaneously is also permitted.

When (10) is a law, then a country knows that it can choose to establish one of the two without incurring any trouble. But a quantifier-based analysis predicts something weaker - at least one of establishing a research center or establishing a laboratory is permitted, and it is unknown which one it is.

This means that according to the standard analysis of modals, a country has no guarantee that if it chooses one, for example to establish a laboratory, that no trouble ensues. It might have been that permission had been granted to bring about the second disjunct, i.e., to establish a research center instead. This prediction is weaker than our intuitions require and has become known as the *ignorance reading.*

It is common in the literature to claim that the intuition that (10) grants *free choice* is captured by an account in which (10) entails (11-a) and (11-b).

(11) a. A country may establish a research center.
b. A country may establish a laboratory.

The sentences are standardly translated as follows.

(12) a. $\Diamond(p \vee q)$
b. $\Diamond p$

If there exists an entailment from (12-a) to (12-b), then whenever (10) is the case, so is (11-a). While we agreed that (10) provides permission for both disjuncts, leaving the choice of disjunct at the discretion of the country, an entailment proposes something stronger. A treatment of *free choice* as entailment predicts that whenever (10) is the case, one could felicitously utter (11-a). But this leads to counter-intuitive predictions. According to classical entailment, when an inference is valid, and we learn that the conclusion is false, we know that the premise does not hold. This means that classically (13) holds.

(13) If $\varphi \models \psi$ then $\neg\psi \models \neg\varphi$

If *free choice* examples were classical entailment so (12-a) entails (12-b), then (14) would hold as well.

(14) $\neg\Diamond p \models \neg\Diamond(p \vee q)$

But if we accept (14), the we also have to accept that (15-a) entails (15-b).

(15) a. You may not burn a letter.
b. You may not burn a letter or mail it.

Intuitively, (15-a) does not entail (15-b) because the prohibition to burn a letter does not extend to a prohibition to mail the same letter. In fact, this is a variant of Ross's puzzle that we discussed earlier. Because we do not accept that (15-a) entails (15-b), the *free choice* effect cannot be explained via a classical entailment relation.

This observation allows us to refine the intuitions regarding what we need to capture with respect to the *free choice* puzzle. While the *free choice* inference grants permission to establish a research center or a laboratory, it does not grant permission to establishing one independent of the other.

Recall that we discussed the *inclusive/exclusive or* puzzle in chapter 2. The intuition that permission is not necessarily granted for both disjuncts has been developed by authors including Fox [33] and Aloni & Ciardelli [10] who have claimed that disjunction under permission is *exclusive*. For example, Fox argues that there's a scalar implicature such that from a *free choice* sentence such as (10), translated as (16-a), one can derive (16-b).

(16) a. $\Diamond(p \vee q)$
b. $\neg\Diamond(p \wedge q)$

An exclusive reading of disjunction is considered a special case rather than a salient reading of *free choice* permission. Fox credits Simons [92] with presenting the accepted inclusive reading in the literature and Simons says that there is a reading in which (10) is compatible with permission for both disjuncts at the same time. So the *both* reading is a contingent fact of *free choice*.

There are a number of examples in the literature that demonstrate that enacting both disjuncts is not prohibited. The following example is modified from Barker [18, p.17].

(17) A country may establish any number of scientific institutions, so a

country may establish a research center or a laboratory and even both of them at the same time.

This example demonstrates that there is no necessary contradiction in granting permission to establish both a research center and a laboratory after a *free choice* utterance. Such a prohibition would need to be explicitly added.

Due to these facts, the *free choice* phenomenon is more complicated than merely finding the mechanism for the entailment from (10) to (11-a). We must capture the intuition that permission is granted to each disjunct but not independently of the other. And, also, even though a *free choice* sentence such as (10) does not guarantee that bringing about both disjuncts is necessarily permitted, *free choice* examples do have *inclusive* readings.

Over time, three crucial empirical observations have arisen that make the *free choice* puzzle difficult to solve. Besides the intuition that permission is granted for both disjuncts, the weaker reading of disjunction predicted by standard quantifier-based modal logic can also be attained. When one adds "... but I do not know which" to (10), one generates what has become known as the *ignorance reading*.

(18) A country may establish a research center or a laboratory, but I do not know which.

The salient reading of (18) is that the speaker is certain that there exists permission to establish something, but it is not known whether it is a research center or a laboratory. (18) is a felicitous utterance in situations where it is not clear what is permitted. For example, when an international law student tries to recall what she learnt but cannot remember it precisely, she might utter (18).

Furthermore, disjunction under permission also realigns itself with predictions made by SML and Kratzer's modal logic in cases where permission scopes under negation.

(19) A country may not establish a research center or a laboratory.

The salient reading of (19) says that neither establishing a research center nor a laboratory is permitted. This fact is well known in the literature since it was noticed by Alonso-Ovalle [11].

In conclusion, there are three main observations.

Free choice *Free choice* examples like (10) grant permission to each disjunct.

Ignorance The *ignorance reading* in (18) reverts to a disjunctive reading.

Negation Permission sentences under negation such as in (19) extend the prohibition to each disjunct.

The combination of the three observations (10), (18) and (19) regarding the behaviour of disjunction under permission have left the *free choice* puzzle without a widely accepted solution. The standard approach in the large body of literature on solving the *free choice* puzzle is to assume that the quantifier-based account of modals is correct and that disjunction causes the problems. The authors then proceed to modify the semantics of disjunction or add a pragmatic framework to capture the salient reading of (10). Yet, as we will see later, such modification allows one to capture the *free choice* effect in (10) but not the salient reading of disjunctive permission under negation such as in example (19).

4.2.2.1 Is *free choice* an upward monotonicity puzzle?

Recall that von Fintel [31] claimed that the property of upward monotonicity is at the root of both Ross's puzzle regarding imperatives and the *free choice* puzzle. Now that we have introduced the *free choice* puzzle, we ought to also investigate the basis for von Fintel's claim that it is a monotonicity puzzle. This is important because if *free choice* is an upward monotonicity puzzle then any solution to upward monotonicity puzzles should also be able to explain all three primary intuitions of the *free choice* puzzle.

The following examples formed the basis for von Fintel [31, p. 6] to claim that anyone that attempts to account for monotonicity puzzles should have an account for the *free choice* puzzle. According to him, we infer permission sentences such as (20-b) from obligation sentences such as (20-a).

(20) a. You ought to mail the letter or burn it.
b. You may mail the letter.
c. You may burn the letter.

The sentences are translated as follows.

(21) a. $\Box(p \vee q)$
b. $\Diamond p$
c. $\Diamond q$

The inference from (20-a) to (20-b) is not entirely straightforward. To make the inference from the obligation sentence (20-a) to a permission sentence (20-b), we must invoke what is known in the literature as Kant's law [72]. This is an umbrella term for inferences such as the one below (in following examples, epistemic modals will be marked by $\Diamond_\epsilon$).

(22) $\Box p \models \Diamond p$

It is called Kant's law because for Kant, for something to be a moral obligation, it ought to be possible/permissible to enact it. The original use and the role of agency is beyond the scope of our current investigation but we are interested to see whether (22) is a fact about the meaning of obligation. McNamara [73] provides a counterexample.

(23) a. I'm obligated to pay you back \$10 tonight.
b. I can't pay you back \$10 tonight (e.g., I just gambled away my last dime).

The sentence in (23-a) is a deontic obligation and (23-b) is a circumstantial fact that says fulfilling the obligation is impossible. Intuitively, there is nothing odd about such a situation as people often take financial obligations they cannot meet. The sentences can be translated as follows.

(24) a. $\Box p$
b. $\neg\Diamond_{\epsilon} p$

As one can see, the application of Kant's law in (22) to the above formulas requires that (23-b) cannot follow from (23-a) but that is not intuitively the case. The two sentences seem consistent.

One could argue against such a claim by saying that Kant's law only applies to inferences from deontic obligation to deontic permission. This is quite far from the original meaning of Kant's law, but it is easy to test. Imagine a protagonist under investigation for money laundering and all his bank accounts have been frozen so that he has no money. This means he will also not have cash for payments. Under such a situation, one might utter the following.

(25) a. I'm obligated to pay you back \$10 tonight.
b. I do not have permission to make financial transactions tonight.

Intuitively, such a situation is no different from the inferences with (23-a) and (23-b), in that (25-a) and (25-b) are intuitively consistent sentences. Yet, they contradict deontic Kant's law. So, for the project of this dissertation, let us set aside Kant's law and simplify von Fintel's example to only touch upon *free choice*, such as in (10), reproduced below as (26-a).

(26) a. A country may establish a research center or a laboratory.
b. A country may establish a research center.
c. A country may establish a laboratory.

The sentences might be translated as follows.

(27) a. $\Diamond(p \vee q)$
b. $\Diamond p$
c. $\Diamond q$

Von Fintel claims that from (26-a), one can infer (26-b), which is to say that there exists an entailment from (27-a) to (27-b). While we agreed that (26-a) provides permission for both disjuncts, we demonstrated in earlier that this intuition should not be captured with an entailment relation between (26-a) and (26-b).

Another way to investigate whether *free choice* is an upward monotonicity puzzle is to check the entailment from the disjuncts to the disjunction. In propositional logic, the following entailment holds.

(28) $p \models p \vee q$

Modals are upward monotonic in Kratzer's semantics, as we demonstrated with Ross's puzzle but the following entailment also holds.

(29) $\Diamond p \models \Diamond(p \vee q)$

But intuitively when (26-b) is the case, (26-a) might not be. This is because permission to establish a research center does not extend permission to establish a laboratory. Thus, to the extent that Kratzer semantics makes an unintuitive prediction regarding entailment relations regarding disjunctive permission utterances, *free choice* is and should be analyzed as a upward monotonicity puzzle. Granted, this is only part of the puzzle. But we expect to find a uniform solution to Ross's puzzle and at least to the part of the *free choice* puzzle which is also an upward monotonicity puzzle.

The overall situation is made even more difficult by the fact that there are a number of other puzzles for Kratzer's account. We will introduce the notable ones below.

4.2.3 Monotonicity and conjunction

Arthur Prior [81] introduced the Good Samaritan paradox for upward monotonic deonic modals.

(30) a. It ought to be the case that Jones helps Smith who has been robbed.
b. It ought to be the case that Smith has been robbed.

Intuitively, most people accept (30-a) and reject (30-b) as characterizations of a Good Samaritan. According to Prior, (30-a) and (30-b) are generally translated into logic as follows.[2]

(31) a. $\Box(p \wedge q)$
b. $\Box q$

The sentence in (30-a) is analyzed as a conjunction under the scope of obligation. It has also been argued that Prior uses a non-restrictive relative clause that does not necessarily receive a conjunctive reading. This counterargument is not convincing, though, as we will see that the UM puzzle also works with explicit conjunction. We will first discuss the original puzzle but will focus on the similar dr. Procrastinate case that follows as that has explicit conjunction embedded under conjunction.

The puzzle arises because in the propositional case, a conjunction entails its conjuncts so that the following entailment holds.

(32) $p \wedge q \models q$

When obligation is analyzed in an upward monotonic account of deontic modals, the following entailment also holds.

(33) $\Box(p \wedge q) \models \Box q$

This means that in SML and Kratzer's modal logic, whenever (30-a) is the case, so is (30-b). But we saw that intuitively people tend to reject (30-b) even when they accept (30-a).

[2] Another translation for (30-a) which suggests itself is $q \to \Box p$.

It could be argued that the translation of (30-a) as conjunction is suspect, and thus the criticism is unwarranted. But Jackson [50] provided a more intuitive version of a monotonicity puzzle that gravitates around a conjunction entailing its conjuncts. This is called the dr. Procrastinate puzzle.

4.2.3.1 Dr. Procrastinate

According to the background story, dr. Procrastinate is an expert in her field but she never finishes her assignments. When dr. Procrastinate is asked to write a book review, as an expert in her field, she ought to accept to write the book review and actually write it. This means that intuitively the obligation (34-a) holds. But as dr. Procrastinate will not write the book review, she ought not to accept to avoid wasting the publisher's time. This means that the obligation (34-b) holds as well.

(34) a. Dr. Procrastinate ought to accept and write the book review.
b. Dr. Procrastinate ought not to accept.

Intuitively, it is not absurd that both of these two statements hold at the same time. So the first prediction a semantics has to make is that (34-a) and (34-b) can coexist. There is an intuitive conflict between the two obligations so that one cannot satisfy both (34-a) and (34-b) at the same time.

Furthermore, we accept that dr. Procrastinate will violate the obligation in (34-a) because she does not write reviews, but it would be worse if, on top of that, she accepts to write another review despite the fact that she does not write reviews. In the latter case, she violates not one but both obligations.

A standard treatment of deontic modals predicts the coexistence of (34-a) and (34-b) to be impossible. Jackson translated (34-a) into logic as (35-a) and (34-b) as (35-b). From the story we know that (36) holds as well.

(35) a. $\Box(p \wedge q)$
b. $\Box\neg p$

(36) $\neg q$

As we saw with the Good Samaritan puzzle, a conjunction entails its conjuncts.

(37) $p \wedge q \models p$

This means that in Kratzer's UM treatment of deontic modals, (35-a) entails (38).

(38) $\Box p$

The problem lies in the fact that (38) contradicts (35-b). For (38) to be the case, the best worlds have to be p worlds, and for (35-b) to hold, the best worlds have to be $\neg p$ worlds. But both of these cannot be the case.

The issue centers around the fact that a UM semantics counter-intuitively predicts that whenever (34-a) is the case, so is (39).

(39) Dr. Procrastinate ought to accept.

This is intuitively untenable. Dr. Procrastinate should only accept if she will also complete her task. If she does not write the review (which everyone in the field knows she will fail to do), then she ought not to accept.

The intuitions that need to be captured are the following. It should not be absurd that (34-a) and (34-b), ie. (35-a) and (35-b), are both the case. This requires that (35-a) does not entail (39).[3]

Furthermore, the semantics must be able to capture the intuitive deontic conflict. As we know that dr. Procrastinate will not write, we know that she will violate the obligation in (34-a). This is the starting point of the puzzle. But it should still be possible for the semantics to capture the fact that she has a choice whether to violate another obligation or not. It is more reproachable for dr. Procrastinate to violate both (34-a) and (34-b) than to only violate (34-a).

As SML and Kratzer semantics are unable to capture these intuitions in a straightforward manner, we will look for alternative accounts of deontic modals. But one might argue that merely adopting a non-monotonic account of deontic modals would suffice to solve these upward monotonicity puzzles. We already discussed that the *free choice* puzzle has intuitive aspects that go beyond upward monotonicity. The following section discusses other puzzles that do not rely entirely on upward monotonicity but come about through the interaction of the standard account of deontic modals with the standard account of conditionals.

4.3 Beyond monotonicity

The standard account of deontic modals suffers from more than just upward monotonicity puzzles. We will take a look at two puzzles that pose additional challenges to a standard modal account when it is coupled with the standard account of conditionals.

4.3.1 All or nothing

Recall that Kratzer semantics treats a conditional as a restricted modal statement. The antecedent of the conditional restricts the modal operator in the consequent. If no modal is found, it is assumed that there exists a covert universal epistemic modal. This account generates new puzzles where one can counter-intuitively weaken conditional permission statements.

(40) a. If the car passed its technical inspection and you have your license, then you may drive.
b. $(p \wedge q) \rightarrow \Diamond r$

The salient reading of (40-a) says that permission to drive the car is contingent on two facts: it must be the case that the car passed its technical

[3] Another translation for (35-a) which suggests itself is $p \rightarrow \Box q$ and such a treatment would make capturing this intuition easier.

inspection and you must also have your license. If either of those conditions is not satisfied, there is no guarantee that driving the car is permitted. In fact, it is likely that there exists a prohibition against driving a car that did not pass the inspection and another prohibition against driving without a license. The puzzle is called *all or nothing* because either all conditions are satisfied or no permission is granted.

Kratzer semantics analyses the sentence in (40-b) by restricting the modal base and ordering source for the permission modal in the consequent to only worlds in which both p and q are the case.[4] (40-b) is the case if, restricted to pq worlds, there is at least one r world among the best worlds. A world where both p and q is the case is a world where p is the case. So when (40-b) holds, there exists a world in which both p and r are the case, which means that (41-b) holds as well.

(41) a. If the car passed its technical inspection, then you may drive.
b. $p \rightarrow \Diamond r$

Whenever (40-b) holds, (41-b) holds, so the entailment in (41) holds as well.

(42) $(p \wedge q) \rightarrow \Diamond r \models p \rightarrow \Diamond r$

According to (42), one can weaken the antecedent of a conditional permission statement by removing conjuncts, but this leads to counter-intuitive predictions.[5] If (42) holds then whenever (40-a) is the case, so is (41-a).

Intuitively, someone who accepts (40-a) does not necessarily accept (41-a) because the latter grants more permission; irrespective of whether you have a license or not, as long as the car has passed its technical inspection, permission is granted to drive. But this is counter-intuitive. If permission is granted when both of the conditions are fulfilled, one cannot just dismiss one of the conditions.

4.3.2 Conditional oughts

The standard account of modal logic for *ought* also leads to problems with certain conditionals. Jackson noticed in [50, p. 191] that when the antecedent and consequent are the same, a standard account of deontic modals makes unintuitive predictions when there are modals in the consequent. These predictions were also discussed by Frank [36] and Zvolenszky [104].

(43) If Britney Spears drinks Coke in public, she must drink Coke in public.

Native speakers report that (43) is somehow odd and difficult to understand. But the above sentence has a possible contingent reading that whenever Britney Spears drinks Coke, there exists an obligation for her to do so. This reading seems to rely on the fact that Britney Spears dislikes Coke so much

[4] In the next section, we also discuss an alternative prediction in Kratzer semantics where a covert epistemic modal is added.

[5] While this is a puzzle concerning the weaking of the antecedent, we cannot call it that because *weakening the antecedent* standardly refers to $p \rightarrow r \models (p \vee q) \rightarrow r$.

that without being obligated to drink it, she would not touch it. As the reading is contingent someone can disagree with it. But Kratzer's account of modals and conditionals predicts that (43) cannot be false.

The conditional ought in (43) receives a counter-intuitive interpretation because of the interaction of the semantics of conditionals with upward monotonic modals. The following is a valid entailment in propositional logic.

(44) $p \models p$

Another way to say this is that whenever p holds, no world is a $\neg p$ world. In light of this, consider the following conditionals.

(45) a. $p \rightarrow \Diamond p$
b. $p \rightarrow \Box p$

According to restriction semantics for conditionals, we evaluate the modals in the consequents of (45-a) or (45-b) restricted to only p worlds. For (45-a) to be false, the best worlds need to be $\neg p$ worlds and for (45-b) to be false, at least one of the best worlds must be a $\neg p$ world. But as p is the case, then $\Diamond p$ and $\Box p$ cannot fail to hold as p in the antecedent has eliminated all worlds in which the modals in the consequent could be false.

Kratzer explores a possible solution to this puzzle [59, pp. 106-107]. One could assume that (43) is doubly modalized and takes the following form.

(46) $p \rightarrow \Box(\Box p)$.

The first modal in the consequent is a covert epistemic necessity modal and the second is the overt deontic modal. In this case, (43) receives an intuitive contingent reading because the antecedent restricts the epistemic modal and not the deontic obligation.

Kratzer correctly notes that for this solution to be tenable, (43) ought to be ambiguous between two readings - one in which a covert modal provides a contingent interpretation as in (46) and another where it does not (45-b). Otherwise all conditionals with modals in the consequent would receive the reading in (46). But the contingent interpretation is the only salient reading so the sentence does not seem to be ambiguous. Kratzer speculates that we might not perceive trivial interpretations when reasonable interpretations are available. She tests this with the help of the following example [59, p. 107].

(47) I could not possibly work more than I do.

Yet, (47) has two perceivable interpretations, one that is trivial and the other contingent. So the problem cannot be avoided by assuming that trivial interpretations are not perceived. The interpretation predicted by Kratzer's semantics for (43) is unintuitive.

Furthermore, the puzzle is not limited to universal quantification such as in (43). As Jackson [50] noted, deontic possibility sentences suffer from the same problem.

(48) If soldiers confiscate property, then soldiers may confiscate property.

Native speakers also say that this sentence is odd but similarly to (43), the sentence in (48) has a possible contingent reading. It is appropriate in situations where we have independent reasons for thinking that soldiers are trustworthy, so we can reassure someone that doubts the legitimacy of their actions by stating (48). And, crucially, it is also possible to disagree with (48) when we know that there are dishonest soldiers that confiscate property for selfish gain also when they have not been granted permission to do so.

But Kratzer semantics also predicts (48) to be a tautology . This reinforces the idea that the issue is not with the reading of the example in (43) but a problem with the restrictor account that relies on the standard account of modals. Note, that the example (48) is easier to interpret than (43), which means that eventually an account is needed to explain why (43) is more odd.

4.3.3 Preliminary conclusions regarding the standard account of modals

We have shown that upward monotonic modals in SML and Kratzer's account of modals lead to a number of problems. The problem arises both for universal quantification for necessity modals and existential quantification for possibility modals.

Furthermore, despite the fact that the initial proposed solutions to puzzles such as *Ross's puzzle* and *free choice* by, for example, Zimmermann [102] attempted to explain the problem by changing the definitions of disjunction, this approach fails as the standard approach to deontic modals also incurs puzzles with other connectives such as conjunction and conditionals, as shown by the *Good Samaritan/dr. Procrastinate* puzzle, *all or nothing* and *conditional oughts* puzzles.

Thus, a uniform solution to the entire range of the above mentioned puzzles requires a different approach. But before we consider solutions, we should consider a number of other puzzles for the standard account that concern conflicts of multiple deontic modals.

4.4 Deontic conflicts

We saw that monotonicity creates problems both for SML and Kratzer's modal semantics. But as noted by Lassiter [64, p. 151] we should also take a closer look at the puzzles that Kratzer's semantics is supposed to handle better than SML.

Recall that the difference between SML and Kratzer's account of modals is the use of the ordering source g. The ordering source can deal with deontic conflict examples as the ordering source is well formed even when there is no ideal world.

Consider again the simple conflict of obligations situation, repeated from chapter 3, section 3.4.2, where the spouse and best friend have had a terrible falling out and having them in the same room would be a bad idea. In this case, the following obligations hold.

(49) a. You must invite your spouse to your birthday party.
b. You must invite your best friend to your birthday party.
c. You must not invite both your spouse and your best friend to your birthday party.

We will represent the obligations as follows.

(50) a. $\Box p$
b. $\Box q$
c. $\Box(\neg(p \wedge q))$

In such a situation there is no ideal world. If you invite only one of the spouse or the best friend, the other will be angry. And if you invite both of them, there will be an ugly fight between them. Yet, it is distinctly worse to invite neither of them than to invite both of them, as then both your spouse and your best friend will be angry.

The situation in (49) is intuitively described by an ordering where there are three equally good non-ideal worlds $p\overline{q}$, $\overline{p}q$ and pq each of which violates one of the three obligations but not the others. There is also one clearly worse world in which two obligations are violated: $\overline{pq}$. This ordering is demonstrated on the following figure 4.1.

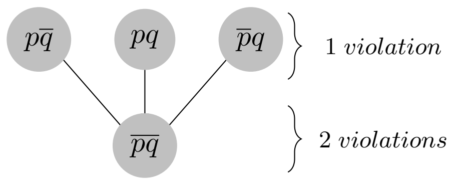


Figure 4.1: $\Box p \wedge \Box q \wedge \Box(\neg(p \wedge q))$

As we saw, unlike SML, Kratzer's semantics does not predict the above situation to be inconsistent. But it also does not provide a fully intuitive prediction in deontic conflict examples. The problem arises because all three obligations (49-a), (49-b) and (49-c) hold simultaneously, so that satisfying all of them becomes impossible. Yet, we will see that Kratzer semantics predicts that in such a situation, the obligations do not hold.

Recall that Kratzer defined obligation as universal quantification over a modal base. φ is obligated when φ holds in all worlds that are closest to the ideal state of affairs. As we can see in figure 4.1, all three worlds with a single violation are equally close to the ideal state of affairs. But because there's a violation in each world, none of the obligations in (49-a), (49-b) or (49-c) hold in this situation. Thus, Kratzer predicts that the following is the case.

(51) a. It is not the case that you must invite your spouse to your birthday party.
b. It is not the case that you must invite your best friend to your birthday party.
c. It is not the case that you must not invite both your spouse and your best friend to your birthday party.

But uttering (51-a), (51-b) and (51-c) sounds very odd in the described situation where each of the obligations in (49) hold. The whole issue arises because the three obligations hold simultaneously. And even though such a conflict of obligations requires that one of those obligations is violated, that does not invalidate any of the obligations. In fact, the obligations must hold for it to be possible to violate them. So, while Kratzer's account is a step forward from SML, it remains insufficient to deal with deontic puzzles.

4.5 Prior proposals revisited

The above puzzles are well known in the literature and there are a number of proposed solutions to them. This is especially true of the *free choice* puzzle. Recently the *free choice* puzzle has been linked to Ross's puzzle and *conditional oughts* [31]. If the puzzles are connected then one would expect them to receive a uniform solution.

We will begin our review of prior proposals by looking at alternative semantics for modals that abandon monotonicity. The discussion of these is necessarily brief, with the modest aim of pointing towards the way in which such approaches fail to provide a uniform solution the puzzles associated with deontic modals. We will then evaluate suggestions to solve the *free choice* puzzle. Even though a uniform solution to all the puzzles would be preferable, most of the *free choice* literature does not consider the other puzzles.

With respect to the *free choice* puzzle, we will first consider implicature-based solutions. Looking ahead, we know that the community has not come to accept any of these solutions to the *free choice* puzzle so we will also consider alternative-based semantics for *or* and modifications of the semantics of deontic modals.

4.5.1 Alternative accounts

4.5.1.1 Cariani's account

Cariani [22] suggests an alternative semantics for *ought* to solve upward monotonicity puzzles. In his system, the interpretation of an *ought* sentence depends on a set of contextually relevant options for an agent. These alternatives are determined by salient goals, values and desires. To illustrate Cariani's semantics, we will consider the following two sets of alternatives for an agent, Jenny.

(52) a. ALT{walking, driving}
b. ALT{walking in a blue dress, walking in a red dress, driving}

According to Cariani, if Jenny's goals are the only thing that matters, and Jenny's goal is simply to get to school without using gasoline, then (52-a) is a more appropriate set of options than (52-b). This is because it considers alternatives which get Jenny to school such that one can exclude those which use gasoline (driving). If, in addition, there exists an arbitrary rule that she'll get fined for wearing a blue outfit and she wants to avoid a fine, then (52-b) presents a better choice of options because it allows one to differentiate between non-gasoline options that will get her fined and those that do not.

Besides introducing a set of alternatives, Cariani also needs to add a benchmark. Alternatives above the benchmark are permissible options. For the first case with Jenny, Cariani considers the following ordering and benchmark.

(53) walking> **benchmark** > driving

Cariani's account of *ought* predicts that (54) is true if three criteria hold.

1. The set of alternatives must include an alternative that exceeds the benchmark;

2. every mentioned alternative meets or exceeds the benchmark;

3. the mentioned alternative is deontically ideal according to the ordering.

These criteria together with the alternative set (52-a) and the ordering (53) licence the following *ought* sentence.

(54) You ought to walk.

(54) is licensed because *walking* exceeds the benchmark, which is required to satisfy the first two criteria. There is no other alternative that is preferred to *walking*, so it is ideal according to the ordering and satisfies the third criterion as well.

The same criteria do not licence, for example, the following.

(55) a. You ought to walk or drive.

The example (55-a) fails because not every mentioned alternative meets the benchmark as *driving* is below the benchmark.

Unfortunately, Cariani distances himself from accounting for other puzzles with deontic modals. According to Cariani [22, p. 15], (56) is a valid principle.

(56) If $\varphi \models \psi$ then $\Diamond\varphi \models \Diamond\psi$.

But such an inference opens up the following reasoning.

(57) If $p \models p \vee q$ then $\Diamond p \models \Diamond(p \vee q)$.

Applying (57) allows us to construct unintuitive inferences akin to Ross's puzzle.

(58) a. A country may establish a research center.
b. Hence, a country may establish a research center or bomb their neighbour.

When one accepts (58-a), one is not intuitively compelled to accept (58-b). In fact, most people would be very reluctant to accept (58-b) as a permission utterance. As Cariani does not suggest general solutions for puzzles of SML or Kratzer semantics, his account cannot be taken as an alternative.

4.5.1.2 Lassiter's account

Lassiter proposes that the meaning of deontic modals is similar to gradable adjectives. Like for Cariani, a Lassiter modal establishes a threshold value, so $\Box p$ is the case only if the proposition is mapped to a point on the ordering which exceeds the threshold.

Lassiter proposes that the ordering is an *interval scale* rather than an *ordinal scale* as was suggested by Kratzer. An interval scale has the structure $\langle X, Y, \succcurlyeq_P \rangle$, where X is the domain of property P, Y is a set of pairs of objects in X, and $\succcurlyeq_P$ is a weak order on Y.[64, p. 156] Lassiter offers the following illustration. We can think of comparing time intervals: $(a, b) \succcurlyeq_{time} (c, d)$ iff a exceeds b with respect to property *time* by more than c exceeds d. So the length of time between point a and point b exceeds the length of time between c and d. The ordering is formulated as a *probability-weighted preference*, most famously knows as *expected utility*.

Applying Lassiter's terminology, there are three thresholds for different modals.

High scalar D-modals Examples: must, require, need and have to.

Mid-scalar D-modals Examples: ought, want, supposed to, should, and good.

Weak-scalar D-modals Examples: permitted, may and allowed.

Mid-scalars are sensitive to contextual alternatives. In what follows, the difference between *must* and *ought* will play an important role. We will denote *must* with $\Box$ but differentiate between *must* and *ought* by spelling out *ought*. Denoting expected utility as $\mathbb{E}$, Lassiter defines *ought* as follows.

(59) Ought(φ) is true iff $\mathbb{E}(\varphi) \geq \theta_{ought}$, where θ_{ought} is a value significantly greater than $\mathbb{E}(\bigcup ALT(\varphi))$

We will use the following example from Lassiter to illustrate.

(60) a. Mother: You ought to go to the grocery store.
b. Son: I don't want to go to the grocery store. I want to go to a movie.
c. Mother: Well, you ought to go to a *clean* movie, then.

A set of alternatives for (60-a) is presented below.

(61) (62) ALT={Son goes to the grocery store, son does not go to the grocery store.}

The definition together with the set of alternatives predicts that the son ought to go to the grocery store only if going to the grocery store is above the threshold compared to the union of the alternative set.

Furthermore, when (60-c) is the case, the expectation of going to a *clean* movie has to be significantly higher than the expectation for going to any movie as such a movie could be violent and thus not clean. So, if (60-c) is the case, (63) does not have to be.

(63) You ought to go to a movie.

This means that, contrary to the standard analysis, there is no entailment from (60-c) to (63), despite going to a movie entailing going to a *clean* movie in the propositional case, and, thus, *ought* is not UM under Lassiter's treatment.

Recall that Lassiter is concerned with providing a solution to contrary to duty puzzles reproduced here as (64).

(64) a. You ought to invite your spouse to your birthday party.
b. You ought to invite your best friend to your birthday party.
c. You ought not to invite your spouse and your best friend to your birthday party.

Kratzer semantics predicted that conflicts of obligation cannot occur because in such situations each obligation is predicted to be false. As Lassiter made *ought* sensitive to alternatives, ought (φ) and ought(ψ) can be simultaneously true for incompatible $\varphi \wedge \psi$ in cases where the alternatives, against which the two statements are evaluated, are different. The problems arise for Lassiter when we consider other modals.

While there are clearly lexical differences between *ought* and *must* the deontic conflict is equally plausible for both ought and must sentences.

(65) a. You must invite your spouse to your birthday party.
b. You must invite your best friend to your birthday party.
c. You must not invite your spouse and your best friend to your birthday party.

In fact, regarding laws, it seems much more likely that a person is faced with a conflict between laws worded with *must*. This is a problem for Lassiter's account. Lassiter defines *must* as high-scalar in the following way.

(66) $\Box\varphi$ is true iff

1. $\mathbb{E}(\varphi) \geq \theta_{must}$ where θ_{must} is a high threshold;
2. $prob(\varphi)$ is significantly greater than 0;
3. For all $\psi \subseteq \neg\varphi$: if $prob(\psi)$ is significantly greater than 0, then $\mathbb{E}(\psi) < \mathbb{E}(\top)$

The definition says that φ must be not only an extremely good option but the only option with significant probability which is better than indifference.

$\Box\varphi$ and $\Box\psi$ cannot both be true when $\varphi \wedge \psi$ is not possible. This means that thanks to *ought* being sensitive to alternatives, ought (φ) and ought(ψ) can be simultaneously true for incompatible $\varphi \wedge \psi$ in some cases, but $\Box\varphi$ and $\Box\psi$ cannot.

Lassiter provides the following examples to distinguish between *ought* and *must*.

(67) a. I ought to go to my parents'.
b. I ought to go to my grandparents'.

Lassiter claims that these two obligations can be simultaneously the case but the following cannot.

(68) a. I must go to my parents'.
b. I must go to my grandparents'.

There is reason to doubt Lassiter's data. If we add some background information, Lassiter's claim becomes less plausible. Imagine that on the night before Christmas, your parents and grandparents have a falling out. In such a situation you could expect to receive the following phone calls.

(69) a. Parents: You must come to spend Christmas with us.
b. Grandparents: You must come to spend Christmas with us.

There is nothing in the meaning of *must* that releases you from the obligations to visit both your parents and your grandparents. And, as is the essence of conflicts of obligation, one must disappoint either the parents or the grandparents by violating the respective obligation. So in our opinion, there is nothing in the semantics of *must* that allows one to avoid contrary to duty puzzles. In fact, as laws are quite likely to be worded with *must* and *shall* rather than *ought*, any conflicting laws would generate counterexamples to Lassiter's data.

In summary, Lassiter does not provide a uniform solution to all puzzles as with conflicts of obligation, as one requires different solutions depending on whether the puzzle is worded with *ought* or *must*. Furthermore, Lassiter and Cariani cannot capture the main feature of the *free choice* puzzle - that permission is granted to both disjunts such that choice between them falls to the hearer. In this sense their solution to upward monotonicity puzzles leaves closely connected phenomena unexplained.

4.5.2 *Free choice* literature

We saw that *free choice* is connected to a number of upward monotonicity puzzles. For example, Ross's puzzle is an upward monotonicity puzzle and, as such, can be solved with the help of Lassiter's and Cariani's non-monotonic semantics. In response, von Fintel linked Lassiter's and Cariani's puzzles to the *free choice* puzzle on grounds that what needs to be explained is not merely which inferences hold in deontic logic but also why disjunction, among other operators, behaves differently under deontic modals. Von Fintel [31, p. 8] believes the solution will be implicature-based.

4.5.2.1 Implicature-based accounts

An implicature-based solution is close to a standard account of *free choice* in the literature since Kamp [51] identified the problem and in 1979 [52] proposed that implicatures can solve the problem. Many following accounts accepted that the *free choice* phenomenon is essentially a pragmatic effect and suggested implicature-based accounts. This approach is supported by the salient reading of (18), reproduced here as (70).

(70) A country may establish a research center or a laboratory, but I do not know which.

The salient reading of (70) says that it is safe to establish only one of the institutions, either a research center or a laboratory, but it is unclear which of them is safe. So, by adding “but I do not know which”, there appears to be a way to cancel the *free choice* effect in disjunctive permission utterances.

Implicature-based accounts include Schultz [91], [27], Fox [34] and the game theoretic implicature account by Franke [35]. As approaches to *free choice* have been extensively discussed in the literature, for example by Schultz [90] or more recently by Barker [18], we will concentrate on examining Eckardt's solution [27] to expatiate on general issues with implicature-based solutions.

Implicature-based solution Illustrating implicature-based approaches to *free choice*, Eckardt derives the *free choice* effect utilizing an implicature through the maxims of manner and quality. A simplified version of her account says that if an informed speaker uses disjunction then either disjunct would be have been more economical. So, even though (71-b) is the stronger statement, the speaker chose to use 5.5.

(71) a. $\Diamond(\varphi \vee \psi)$
b. $\Diamond\varphi$

From this we infer that the governing permissions are best described by disjunction because either disjunct alone would be false.

This pragmatic inference provides the *free choice* effect: there must be some worlds where $\Diamond\varphi \wedge \neg\Diamond\psi$ holds and others where $\neg\Diamond\varphi \wedge \Diamond\psi$ holds.

Eckardt derives this effect through the following pragmatic steps. First, she notes that $\Diamond\varphi$ refers to a subset W of all deontic alternatives W_{deont} for a subject in world w^*. Secondly, she assumes a pluralistic predication over worlds. A property P is a property of a plurality W of worlds, denoted P(W), iff for all $w \in W : P(w)$.

Note that A and B denote predicates. The assumptions allow us to take the following steps of reasoning.

1. Speaker utters (A or B)(W) and has sufficient knowledge.

2. Speaker violated the maxim of manner: be brief, as the speaker uttered a disjunction, rather than either disjunct: A(W) or B(W).

3. Inference from the maxim of quality: speaker believes A(W) is not true and B(W) is not true.

4. *free choice* effect: (A or B)(W) in clause 1 is the case, despite clause 3, only if there are some worlds in W in which A is true and B is not, and other worlds in which B is true but A is not, so that there are some $w_1, w_2 \in W : (A \land \neg B)(w_1), (\neg A \land B)(w_2)$.

The weakness of this particular account lies in its conclusion. The intuition behind the deontic *free choice* effect is that the speaker believes that permission is granted to bring about both disjuncts. Contrary to this, this solution results in the speaker believing that in some worlds either A or B is not permitted.

It is worth highlighting that Eckardt's step 3 corresponds to the exclusive *or* reading. We discussed the intuition that disjunction embedded under deontic permission receives an exclusive reading in section 5.5.1 and showed that there is no necessary contradiction between the disjuncts embedded under a permission modal. The prohibition to bring about both disjuncts would needs to be explicitly added.

Motivating an implicature-based account Eckardt partly motivates proposing an implicature-based solution, as opposed to a purely semantic solution, by referring to examples such as (72) where disjunction receives a salient *free choice* reading[6] despite there being no (deontic) modal at play.[27, p. 2] As there is no modal, the semantics of modals cannot be the source of the *free choice* reading. Consider a waitress who has taken an order from everyone in the room.

(72) Waitress: Everybody ordered beer or pizza.

The salient reading of (72) says that at least one person in the room ordered beer and at least one person ordered pizza, and there is no person that ordered neither. This reading is similar to the *free choice* reading with deontic modals as there has to be an instance of ordering beer and an instance of ordering pizza. Eckardt also discusses examples with *some*.

(73) Teacher: Some pupils had chips or ice cream.

The salient reading of (73) says that some pupils had chips and some pupils had ice cream, there had to be instances of each. She takes these examples as evidence that disjunction receives *free choice* readings without deontic or epistemic modals being the case. Such cases are of interest in the *free choice* literature and deserve attention to move towards a uniform solution to existing puzzles in the literature. On the other hand, these examples fail to convince that the *free choice* phenomenon is an implicature for the following two reasons.

[6] In this section, we will refer to any reading where at least one instance of each disjunct has to be the case as a *free choice* reading. In fact, there might be significant differences between such examples and the traditional *free choice* phenomenon with permission which we will set aside due to space constraints.

First, Eckardt herself discusses instances where a disjunction is licensed only when both disjuncts are the case which are not standardly analyzed as implicatures. Consider (74-a) and (74-b).

(74) a. If you get an A or a B in the exam, I will take you out for dinner.
b. There isn't a cup or a glass on the table.

The salient reading of (74-a) says that receiving an A grade will result in being taken out for dinner *and* receiving a B grade will result in being taken out for dinner. The salient reading of (74-b) says that there isn't a cup on the table *and* there isn't a glass on the table. So both of these examples have salient *free choice* readings but they are standardly analyzed as receiving those readings straightforwardly through the semantics. For example, (74-b) is predicted to have the *free choice* reading through the semantics of negation and disjunction as standardly $\neg(p \vee q) \equiv \neg p \wedge \neg q$. If some of these examples receive a *free choice* reading without reference to implicatures, it could be that all of the examples have a straightforward semantic explanation as long as we modify the standard account. We will not be able to cover the full range of examples that Eckardt discusses in this dissertation so we will focus on deontic modals.

Secondly, an implicature-based account of *free choice* relies on a plausible implicature-based account, but we will see in the next section that there are a number of general problems with implicature-based accounts which suggest the need to focus on semantic explanations for the *free choice* phenomenon.

General problems with implicature-based accounts Simons [93, p. 14] argues generally against implicature based accounts on the grounds that there does not seem to be a distinction between what is said and what is implicated in *free choice* examples such as (10) repeated here as (75).

(75) A country may establish a research center or a laboratory.

Compare this to a classic example of generalized implicature from Grice [40, p. 32].

(76) X is meeting a woman this evening.

Grice states that such a statement generally implicates that the woman being met is not X's wife, mother, sister, etc. Thus, there exists a clear distinction between that which is said (X will meet a woman) and that which is implicated (X will meet a potential romantic acquaintance). The lack of such distinctions in *free choice* sentences such as (75) poses a challenge to any implicature based account.

Barker [18, p. 16] casts doubt on the existence of another marker of implicatures, namely cancellability. Observe the following example.

(77) You may eat an apple or a pear, although in fact you may not eat an apple.

When an implicature in cancelled, the utterance only has the meaning of what is said. If (76) were cancelled by "... but it's only her mother." then the utterance would lose the implicature that the woman is a romantic acquaintance. Yet, instead of reverting the phrase to that which is said, the added phrase in (77) appears to make the statement contradictory or offers a correction of the preceding information.

There appears to be other content that can be cancelled, which is illustrated by the following continuations.

(78) You may eat an apple or a pear, although in fact you may not eat both.

The consequence of uttering (78) does not cancel the free choice effect. Permission is given to eat an apple and permission is given to eat a pear. The continuation provides the additional information that eating both an apple and a pear is prohibited, giving (78) an exclusive reading. The additional information does not conflict with *free choice* readings.

But contrary to these facts, the ignorance reading in (18) does affect the *free choice* effect. Adding "...but I do not know which." intuitively suggests that the speaker does not know the governing permissions and thus such utterances do not give permission for both disjuncts. Any solution to the *free choice* effect must account for why such a cancellation is possible. Jumping ahead to content in chapter 5, we will show that the ignorance reading can be accounted for as a scope effect, similar to one that is in effect in the following example.

(79) There isn't an apple or a pear on the table, but I do not know which.

Assuming that it was expected that there would be an apple and a pear on the table, the utterance of (79) says that one of them is missing, but it is not necessary that both of them are missing as would be the case if the continuation "...but I do not know which." were omitted as in (80).

(80) There isn't an apple or a pear on the table.

(80) is standardly translated as (81-a), which is equivalent with (81-b).

(81) a. $\neg(p \vee q)$
b. $\neg p \wedge \neg q$

(79), on the other hand, is standardly translated as (82).

(82) $\neg p \vee \neg q$

(81-a) entails (82). So, the addition of "...but I do not know which" weakens the interpretation of (80) as the speaker needs to know less to utter (79). We will show in chapter 5 that the *free choice* ignorance reading can be accounted for in similar fashion.

Furthermore, an implicature based account relies on the assumption that a knowledgeable speaker utters the *free choice* sentence. Yet, the *free choice* effect does not disappear in contexts where such an assumption is manifestly false. For example, Kamp [52, p. 279] noticed that the *free choice* effect does not disappear in the antecedent of a conditional.

(83) If a country may establish a research center or a laboratory, then a country will choose to establish a research center.

One cannot talk about the speaker knowing that such disjunctive permission exists, as the antecedent of a conditional does not need to hold true for the conditional to be felicitous. But the *free choice* effect does not disappear. This suggests that the phenomenon is semantic rather than pragmatic.

Barker [18, p. 16] discusses other contexts in which the *free choice* phenomenon arises but standard assumptions regarding a knowledgeable speaker are not satisfied. For example, (84) demonstrates that the speaker can be manifestly ignorant and the *free choice* effect will still remain salient.

(84) I do not know whether it's true that a country may establish a research center or a laboratory.

The speaker of (84) has no knowledge about permissions that govern the establishment of research centers or laboratories. But (84) says that if permission is granted, then establishing either a research center or a laboratory is safe. So the *free choice* effect does not disappear as it should under an implicature-based account.

The well known fact that *free choice* effects also occur with indefinites such as *any* raises another challenge to an implicature-based account.

(85) You may take any card.

The salient reading of (85) says that it is safe to choose any card in the deck, as opposed to permission being granted to take a specific card. While this *free choice* reading could be accounted for with the help of implicature, van Rooij [85, pp. 308-309] notes that even though implicatures ought to be cancellable, the *free choice* reading of (85) cannot be cancelled. Either authors must weaken their proposals such that they cannot account for *free choice* with regard to indefinites or they need to explain why cancellation is not possible in examples such as (85).

4.5.2.2 Alternative-based semantics

Due to the numerous challenges to an implicature-based account of the *free choice* effect, several authors have proposed semantic solutions to the *free choice* puzzle.

Alternative-based semantics is an umbrella term to refer to approaches which suggest that certain natural language phenomena generate sets of propositional alternatives. Zimmermann [102] made the paradigmatic connection between alternatives and *free choice* effects and posited a pragmatic mechanism that reinterprets disjunction as a conjunctive list of epistemic possibilities.

(86) $\Diamond p \wedge \Diamond q$

Embedded under negation, (86) behaves like standard conjunction and distributes negation between disjuncts.

(87) $\neg\Diamond p \vee \neg\Diamond q$

Unfortunately, the standard reading of conjunction under negation does not account for the salient reading of disjunctive permission embedded under negation. The example, reproduced here as (88-a) receives the following translation.

(88) a. A country may not establish a research center or a laboratory.
b. $\neg\Diamond p \vee \neg\Diamond q$

Yet, the salient reading of (88-a) is one in which permission is not granted to establish a research center nor a laboratory. Zimmermann predicts with (88-b) that only one of the two disjuncts is negated - a country is prohibited to establish either a research center or a laboratory, but not necessarily both.

Following in Zimmermann's footsteps, Aloni [9] also suggested that the solution to the *free choice* puzzle is to be found in alternative semantics. In her account, both disjunction and the indefinite *any* insert alternatives into the semantics. Alternatives are generated by the function $\textsc{alt}\varphi$, which takes the formula φ and turns it into a set of propositional alternatives. For example, in case of disjunction, the alternatives are the denotations of the disjuncts.

Other empirical phenomena that are believed to introduce alternatives are interrogatives, indefinites and indeterminate pronouns. Alternative-based accounts have proposed by Kratzer and Shimoyama [61], Alonso-Ovalle [12, 13, 14], Geurts [38], Simons [92, 93], Menéndez-Benito [74, 75], Aloni [8, 9], and others.

The main goal of these alternative semantics is to facilitate composition of truth-conditional accounts when sets of alternatives are introduced. The resulting proposals to solve *free choice* also rely on additional rules of composition.

Groenendijk and Roelofsen have remarked in 2010 [42] and Roelofsen again in 2012 [83] that inquisitive semantics also makes use of the formal machinery of alternatives. The alternative semantics that is closest to the inquisitive semantics account presented in this dissertation is Aloni's [9] proposal to also modify the standard semantics of modals. We illustrate alternative-based semantics for deontic modals with the help of Aloni's proposal below.

Under the standard account, with respect to a sequence of alternatives, a possibility modal states that *one* proposition in the sequence is possible, and likewise for necessity. Aloni proposes that modals become operators over propositional alternatives such that *all* propositions in the sequence are possible or necessary.

Aloni's account also maintains the existential and universal quantifier account of modals, as her possibility operator $\Diamond\varphi$ says that *every* alternative proposition for φ (α) satisfies existential quantification over accessible worlds ($\forall\alpha\exists w$). The necessity operator $\Box\varphi$ says that *at least one* alternative satisfies universal quantification over accessible worlds ($\exists\alpha\forall w$). So for $\Diamond\varphi$, the standard account of possibility modals must be the case for every alternative.

The effect of this modification is that the universal quantification over alternatives in deontic possibility modals collapses the alternatives, distributing the standard deontic possibility modal into each disjunct, which provides a *free choice* reading. The universal quantification over alternatives is something that we will also utilize in chapter 5 of this dissertation.

Due to the fact that Aloni's modification of deontic modals is quite conservative, her modals remain monotonic and thus only solves the *free choice* puzzle but not the other puzzles mentioned in this chapter. Furthermore, the main problem that such approaches encounter is capturing the intuitive interpretation of disjunctive permission under negation as illustrated by (19), repeated here as (89).

(89) A country may not establish a research center or a laboratory.

If we are interested in an uniform solution, then any approach that limits itself to the *free choice* puzzle will fall short of the overall goal.

Chapter 5 will introduce an inquisitive semantics that makes use of alternatives but modifies modals to a larger degree than has so far been attempted in the literature. Those modals will be more similar to the reduction-based deontic modals discussed in the next subsection.

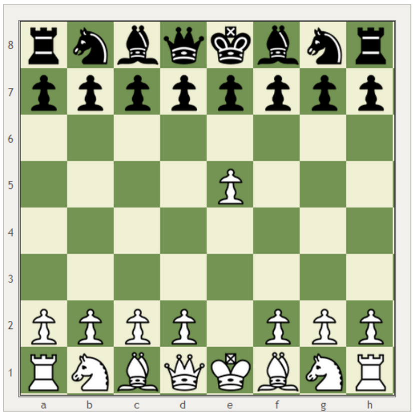
4.5.2.3 Reduction-based deontic modals

Other authors have chosen a broader approach and have changed the semantics of modal operators. Barker [18] proposes a semantic solution to the *free choice* puzzle that follows Kanger [53] and Anderson [15]. The proposals of Kanger and Anderson were developed independently but are nearly equivalent. We will explore Anderson's alternative to SML.

Intuitively, when some φ is obligatory, when you do not do φ then you have violated the obligation. Similarly, if some φ is permitted then it would be odd to find out that by doing φ you have incurred a violation. Simplifying slightly, these intuitions can be captured via the following formulas. To understand violations we introduce a distinguished proposition v to stand for sentences of the kind "Law X has been violated."

(90) a. $\Box\varphi \models \neg\varphi \rightarrow v$
b. $\Diamond\varphi \models \varphi \rightarrow \neg v$

A violation is not exactly a state of affairs or an unfortunate consequence but rather the observation that some rules have not been followed. Recall that Kratzer also measured the distance from the ideal world by counting the number of violations in a world. Anderson [15, p.347] provides a useful analogy with chess to explain violations. According to the rules of chess, a pawn may move at most two squares at a time. So, e5 which moves the pawn three squares violates that rule.



Naturally, nothing stops a player from lifting the pawn from e2 to e5, nor will a punishment necessarily follow. Yet, anyone that opens with e5 is not playing chess according to its rules. And v records the fact some rule is violated.

Anderson in his 1967 article [15] was not concerned with puzzles of upward monotonicity or *free choice*. Instead, he was trying to propose a reductionist modal logic that captures the idea that modality can be expressed with some form of implication. To solve puzzles of material implication, he adopted relevant implication (represented here by $\rightsquigarrow$). Relevant implication has several versions (See the relevant Standford Encyclopaedia article for an overview [69]) including C.I Lewis' strict implication: $\Box(p \rightarrow q)$ [68]. Anderson understood relevant implication as "p is a relevant and sufficient [but not necessarily a logically sufficient] condition for q."[15, 351] Regarding monotonicity puzzles, Anderson ensured that his logic satisfied upward monotonicity.

(91) If $\vdash p \rightsquigarrow q$ then $\vdash \Box(p \rightsquigarrow q)$

So, without further changes Anderson's violation-based modal logic does not solve upward monotonicity puzzles. Anderson's logic did, on the other hand, successfully avoid various other puzzles that concern material implication such as strengthening the antecedent (see chapter 3).

Barker introduces a Kangerian version of Anderson's reduction by positing a normative ideality δ such that if φ is permitted, then if φ then δ ($\Diamond\varphi =_{df} \varphi \rightarrow \delta$). What the distinguished proposition δ says is that the world is in an ideal state. This idea is similar to Kratzer's idea of a world being as close to the ideal as possible.

To avoid material implication puzzles, Barker's semantics uses linear logic that is resource sensitive. Inferences such as conjunct elimination are no longer straightforwardly valid. This means that he can avoid puzzles such as strengthening the antecedent that led Anderson to posit relevant implication.

In Barker's semantics, the solution to the *free choice* puzzle follows the standard interpretation of a disjunction in the antecedent of a conditional.

(92) $(\varphi \vee \psi) \rightarrow \delta$

Barker takes disjunction to be represented by additive disjunction $\oplus$ and implication as linear implication $\multimap$. As additive disjunction in the antecedent of a linear implication behaves standardly, both φ and ψ lead to the ideal set of affairs δ.

Unfortunately, the negation of (92) is also standard, as illustrated by (93).

(93) $\neg(\varphi \to \delta) \vee \neg(\psi \to \delta)$

And (93) is equivalent with (94)

(94) $\neg\Diamond\varphi \vee \neg\Diamond\psi$

(93) and (94) are read as φ or ψ does not lead to the ideal state δ, but it is not known which. This means that Barker's semantics does not capture the salient reading of disjunctive permission negated (19), repeated below as (95), which would require both φ and ψ to be prohibited.

(95) A country may not establish a research center or a laboratory.

Barker must rely on a pragmatic story to capture the salient reading of (95) which encounters all the challenges that he himself listed against a pragmatic account to the *free choice* phenomenon.

Furthermore, there is a conceptual difference between Kangerian and Andersonian reductions of modal logic. It makes a difference whether one reasons towards an ideal state as Kanger and Barker do or away from violations as Anderson does, and as we will in chapter 5. The prior analysis of World Trade Organization (WTO) examples in chapter 2 suggests that legal reasoning does not concern idealities but rather violations. While this might be contingent on the deontic context, in terms of legal language, the violation-based solution remains preferable.

At the level of semantic puzzles, the ideality-based approach struggles to account for conflicts of obligation. Let us again consider the scenario with three obligations.

(96) a. You must invite your spouse to your birthday party.
b. You must invite your best friend to your birthday party.
c. You must not invite both your spouse and your best friend to your birthday party.

In such a situation, no matter whether you invite your spouse, your best friend or both of them, you will not reach the ideal state δ. Asher and Bonevac [17] call this problem with Andersonian proposals the eternal damnation effect. Once one violates a single rule, nothing that follows can be ideal. This is shown in a modified example from [17, p. 307].

(97) a. The violation occurs, and you pay compensation.
b. So, you may not pay compensation.

(98) a. $v \wedge p$
b. $\neg\Diamond p$

(97-a) is translated as (98-a) and (97-b) as (98-b). As (98-a) entails (98-b), a standard Andersonian approach to permission counter-intuitively predicts that whenever (97-a) is the case, so is (97-b).

Asher and Bonevac propose an Andersonian solution to the *free choice* puzzle themselves but their solution to eternal damnation examples is to make their semantics defeasible. Reasoning is defeasible when the corresponding argument is rationally compelling but not deductively valid (see Koons [56]). Such an inference is considered defeasible because it can be defeated by additional information.

Asher and Bonevac [17] propose an Andersonian reduction where implication is analyzed defeasibly. Instead of evaluating a conditional based on a set of worlds, they restrict the set of worlds with a selection function $*$. This selection function excludes certain worlds from the evaluation of conditionals because these worlds are deemed abnormal. Worlds are abnormal if in those worlds conclusions do not follow from the premises but in the majority of worlds the conclusion do follow.

Asher and Bonevac claim that such an account is well suited to model permission.

> If you may have soup, for example, then your having soup normally will not result in any sanction being imposed. It might, however, if it is an abnormal soup-eating. ... Furthermore, soup-eating does not grant immunity from sanctions. Permission to have soup does not grant permission to have soup and pour it over the waiter's head. [17, p. 310]

What Asher and Bonevac claim is that pouring soup over a waiter's head will remove the permission to have soup. But this is intuitively a misrepresentation of the situation. One is still permitted to eat soup; what is prohibited is pouring soup over a waiter's head. So, one does not defeat the permission to eat soup, one merely breaks another rule.

The reason why Asher and Bonevac think that defeasible reasoning is required for permission is that they imagine only one sanction. Each permission, prohibition and obligation - whether it is walking your dog in a park, murder or paying your taxes - is governed by the same sanction. The sanction is the idea of doing anything wrong. But one can imagine a wealth of sanctions of different type and strength for these different situations. So, even though one does not incur a soup-sanction when eating soup is permitted, nothing stops one from getting a concurrent sanction for pouring the soup over a waiter's head. What is required is a semantics that allows reasoning with different sanctions.

To illustrate this, consider the following. We will modify the example of Asher and Bonevac to clarify intuitions.

(99) a. It is permitted to eat soup.
b. It is prohibited to eat soup by slurping it.

Intuitively, it does not follow from (99-b) that (99-a) cannot hold. Instead, (99-a) still holds, (99-b) merely provides the additional information that you incur a sanction if you slurp. So, if you assume that a person eats soup by slurping, then it would be correct to say that they do not incur the sanction for eating soup but they incur another sanction for slurping. So, Asher and Bonevac do not provide an account of what actually occurs in such a situation.

Furthermore, this line of thinking leads to perilous ground. If modals are defeasible then any law would be incomprehensible to the reader. Even though the law states that for example it is permitted to withhold paying taxes on charitable donations, the reader would not have any guarantee that not paying those taxes would not lead to a judgement that one has violated the law. One could only hope that they are not among the abnormal that would be penalized while others are not.

We discussed with the soup example that not being in violation of one law does not guarantee that one does not violate another. But reasoning with several sanctions is not defeasible reasoning. One can read every law to know what is permitted, obligated and prohibited. But one cannot know all the additional information that Asher and Bonevac suggest could defeat a law. In legal theory, the principle that a subject of laws ought to be able to regulate their conduct with certainty and be protected from arbitrary use of the power is known as *legal certainty*. Defeasible accounts of modals in legal contexts erode this principle.

4.6 Conclusions

We have thus seen that there is no uniform solution in the literature to all the puzzles discussed in the beginning of the chapter. Upward monotonicity puzzles, *free choice* and conflicts of obligation have been identified as interconnected, yet no solution is available. What we need is a solution that has 3 critical elements.

1. The proposal ought to solve upward monotonicity puzzles and *free choice* puzzles while accounting for conflicts of obligation.

2. The proposal ought not to be implicature-based due to numerous counterexamples.

3. The proposal ought not to incur any of the problems of Andersonian approaches such as strengthening of the antecedent or eternal damnation.

Chapter 5

Deontic modals in MadRis

5.1 Introduction

The sections below present and illustrate a semantics for deontic modals in the framework of inquisitive semantics. The semantics will build on radical inquisitive semantics (RIS) [44] which extends the conversational perspective on meaning by specifying the rejection conditions of sentences. Unlike basic inquisitive semantics, this allows inquisitiveness also in negated sentences.

We add to radical inquisitive semantics the basic operator $\Diamond_{v}\varphi$; abbreviated $\Diamond\varphi$ and read as 'φ is permitted'. The operator is a modified version of Andersonian [15] permission which reduces deontic modals to implications of the kind $\varphi \rightarrow \neg v$, read as 'if you do φ, the rule has not been violated.' In response to an intuitive difference between the way some implications and deontic modals are rejected, we do not reduce modals to implication but introduce them as basic operators similar to implication. This treatment of modals gives the account its name: modified Andersonian deontic radical inquisitive semantics, or MadRis.

To better motivate and explain the semantic clauses, we will present the semantics in steps. First, we will present the original support clauses for RIS as they were developed by Sano [89]. It is easier to explain a modified version of the semantics that uses abbreviated notation, so we will shortly afterwards introduce the modified version that we call MadRis to which we also add the clauses for deontic modals.The clauses are explained and illustrated with natural language examples at this stage.

MadRis provides straightforward semantic solutions to the well known deontic puzzles that we introduced in chapter 4 such as *free choice, Ross's puzzle, all or nothing* and more. After we explain the clauses for MadRis, we will discuss the *free choice* puzzle in detail to better illustrate the way the semantics is applied to examples. The solutions to the other puzzles will be left for the next chapter.

As discussed in chapter 4, the literature on modals also presents a type of puzzle which we will call *deontic conflicts.* These include several of the puzzles we discussed earlier such as *dr. Procrastinate, conflicts of obligations, contrary to duty* puzzles and more. We will demonstrate that MadRis allows the introduction of several different violations. These are conceptualized such that for each rule, standardly there exists one violation that corresponds to that rule. We will leave explaining how multiple violations straightforwardly account for this second set of puzzles for chapter 6.

5.2 Radical inquisitive semantics

We will consider a language of propositional logic with a fixed set of atomic sentences p, q, r and the connectives $\neg$, $\wedge$ and $\rightarrow$. Disjunction is defined as is standard $\varphi \vee \psi := \neg(\neg\varphi \wedge \neg\psi)$.

The basic semantic notion is that of an information state, denoted σ or τ, which is a set of worlds; a world w is a binary valuation of the atomic sentences in the language.

Let A be the set of all atomic sentences. A world w is a set which for each $\alpha \in A$ contains either α - meaning that α holds in w - or $\overline{\alpha}$ - meaning that α does not hold in w.

For ease of notation, a world w is represented as a sequence of the elements of the set that corresponds to it; for example, instead of $\{p, \overline{q}, v\}$, we will write $p\overline{q}v$. We also define ω as the set of all worlds w; ω corresponds to the state of ignorance.

One distinctive feature of radical inquisitive semantics is that its clauses recursively specify rejection conditions as well as support conditions for the sentences of the language. In the recursive semantics, it is defined when a state σ supports a sentence φ, $\sigma \models^{+} \varphi$, and when σ rejects φ, $\sigma \models^{-} \varphi$.

Unlike in basic inquisitive semantics (See Ciardelli et al. [25]), rejection conditions also pertain to both informative and inquisitive aspects. The rejection of a sentence may also embody a *rejection issue*, where the rejection of the sentence by a state requires that this issue already be resolved in the state. Conjunction, for example, is treated as rejection-inquisitive; the semantics dictates that for a state to reject a conjunction, it should reject one of its conjuncts.

Disjunction is defined via the negation of conjunction and, thus, disjunction receives a treatment familiar from alternative semantics, such as those discussed in chapter 4, where certain connectives introduce sets of alternatives.

Another distinctive feature of radical inquisitive semantics is its treatment of implication: all the alternatives for the antecedent need to be such that they support the consequent. These support conditions are similar to those in ordinary inquisitive semantics, where a state supports a conditional sentence if *all* of its substates supporting the antecedent also support the consequent.

To this, radical inquisitive semantics adds the rejection condition which asks for alternatives in which the antecedent is supported and the consequent rejected. A state σ rejects an implication if there exist *some* substates (consistent with the antecedent) of states that are always such that if they support the antecedent and are restricted to the state σ, then they reject the consequent.

The recursive semantics to which the notions defined above apply is stated below.

Definition 6. RIS

1. $\sigma \models^+ p$ iff $\forall w \in \sigma : p \in w$
   $\sigma \models^- p$ iff $\forall w \in \sigma : \overline{p} \in w$

2. $\sigma \models^+ \neg\varphi$ iff $\sigma \models^- \varphi$
   $\sigma \models^- \neg\varphi$ iff $\sigma \models^+ \varphi$

3. $\sigma \models^+ \varphi \wedge \psi$ iff $\sigma \models^+ \varphi$ and $\sigma \models^+ \psi$
   $\sigma \models^- \varphi \wedge \psi$ iff $\sigma \models^- \varphi$ or $\sigma \models^- \psi$

4. $\sigma \models^+ \varphi \to \psi$ iff $\forall \tau \subseteq \sigma.(\tau \models^+ \varphi$ implies $\tau \models^+ \psi)$
   $\sigma \models^- \varphi \to \psi$ iff $\exists \tau.(\tau \models^+ \varphi$ and $\forall \tau' \supseteq \tau.(\tau' \models^+ \varphi$ implies $\sigma \cap \tau' \models^- \psi))$

We will omit discussion of the clauses of radical inquisitive semantics, as we will introduce alternative notation in the next section that makes explaining the clauses easier and more insightful. The first three clauses are identical to those we adopt in the next section, so we will explain and illustrate them there. We will only briefly discuss the intuition behind the formulation of clause 4 to serve as background information for the simplified version proposed by Roelofsen [84] that we will adopt in the next section.

Consider clause 4 of RIS. For a state σ to support $\varphi \to \psi$, we look at substates of σ which support φ and if each of those supports ψ, then σ supports $\varphi \to \psi$.

To reject $\varphi \to \psi$, we need to find a state τ which supports φ such that every weakened state τ' which also supports φ is such that τ', restricted to σ, rejects ψ.

Finding τ is necessary but insufficient to reject $\varphi \to \psi$ as we might be supposing more than just φ so that φ is not the reason why ψ is rejected. So we try to weaken τ and take a look at each τ', superset of τ, and if any τ', restricted to σ, does not reject ψ then σ does not reject $\varphi \to \psi$. If such a weakening of τ cannot be done, then $\varphi \to \psi$ is rejected in σ.

We could also formulate the support clause for implication as the dual of the rejection clause so that it reads as in (1). As is standard with duals, the difference between the rejection and support clause is that it is sufficient to reject an implication if there *exists* a state that contradicts the implication but for support it is required that *every* state supports the implication.

(1) $\sigma \models^+ \varphi \to \psi$ iff $\forall \tau.(\tau \models^+ \varphi$ and $\exists \tau' \supseteq \tau.(\tau' \models^+ \varphi$ implies $\sigma \cap \tau' \models^+ \psi))$

We have not adopted the formulation in (1) as the support clause can be abbreviated. But if we accept that the support and rejection clauses can be parallel duals, we can use the following fact to simplify both of them.

In a finite setting, a state that supports the formula φ is always contained in a maximal state that supports φ, so we can simplify the support clause for implication such that every maximal state τ that supports φ, restricted to σ, supports ψ and the rejection clause states that some maximal state τ

that supports φ, restricted to σ, rejects ψ.[1] We will introduce the necessary notation for this abbreviation in the next section.

In the following section we will also add the clause for deontic permission which gives MadRis its name. The clauses of MadRis preserve the results of Ris such that the following illustration of MadRis also illustrates Ris.

5.3 MadRis

The set of all states that support a sentence φ is denoted $[\varphi]^+ = \{\sigma \subseteq \omega \mid \sigma \models^+ \varphi\}$ and the set of all states that reject a sentence φ is denoted $[\varphi]^- = \{\sigma \subseteq \omega \mid \sigma \models^- \varphi\}$. The recursive semantics guarantees that $[\varphi]^+$ and $[\varphi]^-$ are downward closed sets of states: if $\sigma \in [\varphi]^+$ and $\tau \subseteq \sigma$, then $\tau \in [\varphi]^+$, and likewise for $[\varphi]^-$. As the recursive semantics specifies both support and rejection conditions, the meaning of a sentence is determined as the pair $[\varphi] = \langle [\varphi]^+, [\varphi]^- \rangle$.

The following clauses of MadRis make use of the auxiliary notion of maximal supporting states. For the propositional case under consideration there is always at least one maximal supporting and one maximal rejecting state for a sentence as the absurd state supports and rejects everything. The set of all maximal supporting states for φ is denoted by $\textsc{max}[\varphi]^+ := \{\sigma \in [\varphi]^+ | \neg\exists\tau \in [\varphi]^+ : \sigma \subset \tau\}$ and the set of all maximal rejecting states for φ is denoted by $\textsc{max}[\varphi]^- := \{\sigma \in [\varphi]^- | \neg\exists\tau \in [\varphi]^- : \sigma \subset \tau\}$.

Maximal states represent alternative ways in which a sentence can be supported or rejected, and they play a crucial role in the explanation of *free choice* phenomena. As was discussed in chapter 4, this observation is not new (see for example Aloni [9]). Maximal supporting state notation also allows us to abbreviate the clauses for implication and permission such that they become easier to explain.

The key notions of *inquisitiveness* and *informativeness* are defined here along the lines of standard inquisitive semantics (see, e.g., Ciardelli et al. [25, p. 9)]). But unlike in basic inquisitive semantics, a sentence φ can be inquisitive or informative both on the support-side and rejection-side, which is mirrored in the definition.

Definition 7. Inquisitiveness and informativeness

φ is **support-inquisitive** iff at least two maximal states support φ.

φ is **rejection-inquisitive** iff at least two maximal states reject φ.

φ is **inquisitive** iff φ is support-inquisitive or rejection-inquisitive.

φ is **support-informative** iff $\bigcup[\varphi]^+ \neq \omega$.

φ is **rejection-informative** iff $\bigcup[\varphi]^- \neq \omega$.

φ is **informative** iff φ is support-informative or rejection-informative.

[1] Unlike the abbreviated version below, Ris does not require a finite setting, but for our purposes it is handier to work in the finite setting.

According to the clause for support-informativeness, a sentence φ is informative if the union of all its supporting states does not include all worlds, and likewise for rejection-informativeness.

The fact that meanings are determined by the pair of supporting and rejecting states is reflected in the notion of entailment as defined below, which combines support-entailment and rejection-entailment:

Definition 8. Entailment

Support-entailment: $\varphi \models_+ \psi$ iff $[\varphi]^+ \subseteq [\psi]^+$

Rejection-entailment: $\varphi \models_- \psi$ iff $[\psi]^- \subseteq [\varphi]^-$

Entailment: $\varphi \models \psi$ iff φ support-entails ψ and φ rejection-entails ψ.

According to definition 8, a sentence φ support-entails the sentence ψ if every state that supports φ also supports ψ. Likewise φ rejection-entails ψ if, in reverse order compared to the support case, every state that rejects ψ also rejects φ. This mirrors the classical picture where the negation of the conclusion entails the negation of the premise.

Classically the support and reject perspectives on entailment coincide.[2] In MadRis, considering both support-entailment and rejection-entailment is not redundant and the dual nature of entailment plays an important role in explaining various deontic puzzles. Crucially for the following discussion, it is sufficient for demonstrating entailment failure to show the failure of either support-entailment or rejection-entailment.

As usual, equivalence is defined as mutual entailment, where we again distinguish between support-equivalence and rejection-equivalence.

Definition 9. Equivalence

Support-equivalence:
$\varphi \equiv^+ \psi$ iff φ support-entails ψ and ψ support-entails φ.

Rejection-equivalence:
$\varphi \equiv^- \psi$ iff φ rejection-entails ψ and ψ rejection-entails φ.

Equivalence:
$\varphi \equiv \psi$ iff φ is support-equivalent and rejection-equivalent with ψ.

Mutual entailment of two sentences guarantees identity of the meanings of the propositions they express.

Fact 1. Equivalence guarantees identity: if $\varphi \equiv \psi$ then $[\varphi] = [\psi]$.

With these additional tools available to us, we can return to the recursive semantics. Below we will define the recursive semantics for MadRis. Before we provide the definitions, we will briefly outline how the maximal supporting state notation allows us to define implication and modals. More detailed explanations and their natural language illustrations follow after the definition.

[2]Classically, $\varphi \models \psi$ iff $\neg\psi \models \neg\varphi$.

The support clause for implication is defined through universal quantification over maximal states that support the antecedent. A state σ supports an implication if *all* of its maximal substates that support the antecedent, restricted to the state σ, also support the consequent.

The rejection clause for implication is defined through existential quantification over maximal states that support the antecedent. A state σ rejects an implication if *some* maximal state that supports the antecedent, restricted to the state σ, rejects the consequent.

To introduce deontic modals, a class of designated atoms $v_1, v_2, \ldots$ is added to the language to represent when a specific rule is violated. We use the designated atoms to define the sentential operator $\langle v \rangle \varphi$ read as 'φ is permitted'. In cases where there exists only a single relevant violation, we abbreviate $\langle v \rangle \varphi$ as $\Diamond\varphi$.[3] *Obligation* is also introduced in standard fashion: $\Box\varphi := \neg\Diamond\neg\varphi$.

In MadRis the definition of deontic permission is in the spirit of Anderson but it receives a *radical treatment*. As is standard, we will refer to the sentence embedded under a deontic modal as the prejacent. Similarly to implication, the semantics requires that for a state to support a permission statement, *all* maximal substates of it that support the prejacent support the deontic fact that *no violation results*. The v inside the diamond brackets refers to the particular violation that will not be incurred when φ is the case.

Unlike implication, the rejection clause for deontic permission statements is not defined through existential quantification over maximal states that support the prejacent; instead, the rejection clause mirrors the support clause in universally quantifying over the alternatives for the prejacent. Unlike the support clause where no violation occurs, in case a permission statement is rejected, bringing about the prejacent incurs a violation. So the rejection of a permission statement is a prohibition.

The semantics requires for a state to reject a permission statement that *all* maximal substates of it that support the prejacent support the deontic fact that *a violation results*.

The recursive semantics for MadRis is given below. The first three clauses are identical with RIS and the fourth clause is substantially the same as the RIS clause for implication, although it is provided using maximal supporting state notation which makes it easier to understand.

Definition 10. MadRis

1. $\sigma \models^+ p$ iff $\forall w \in \sigma : p \in w$
   $\sigma \models^- p$ iff $\forall w \in \sigma : \overline{p} \in w$
2. $\sigma \models^+ \neg\varphi$ iff $\sigma \models^- \varphi$
   $\sigma \models^- \neg\varphi$ iff $\sigma \models^+ \varphi$
3. $\sigma \models^+ \varphi \wedge \psi$ iff $\sigma \models^+ \varphi$ and $\sigma \models^+ \psi$
   $\sigma \models^- \varphi \wedge \psi$ iff $\sigma \models^- \varphi$ or $\sigma \models^- \psi$
4. $\sigma \models^+ \varphi \to \psi$ iff $\forall \tau \in \text{MAX}[\varphi]^+ : \tau \cap \sigma \models^+ \psi$
   $\sigma \models^- \varphi \to \psi$ iff $\exists \tau \in \text{MAX}[\varphi]^+ : \tau \cap \sigma \models^- \psi$
5. $\sigma \models^+ \langle v \rangle \varphi$ iff $\forall \tau \in \text{MAX}[\varphi]^+ : \tau \cap \sigma \models^- v$
   $\sigma \models^- \langle v \rangle \varphi$ iff $\forall \tau \in \text{MAX}[\varphi]^+ : \tau \cap \sigma \models^+ v$

[3] We discuss multiple violations at the end of this chapter and in chapter 6.

5.4 Illustration of the semantics

Below, we explain the clauses of MadRis one by one, illustrating them with examples and pictorial representations.

Atomic sentences An atomic sentence is illustrated by the natural language example in (2).[4]

(2) Sue sings.

a. *Positive response*: Yes, Sue sings.

b. *Negative response*: No, Sue does not sing.

According to clause 1 of definition 10, an atomic sentence p is supported by a state σ if p holds in every world w in σ; and p is rejected in σ if p holds in no world w in σ.

This means that there is a unique maximal state σ that supports p, a unique element of $\textsc{max}[p]^+$, which consists of all worlds where p holds; and a unique maximal state σ that rejects p, a unique element of $\textsc{max}[p]^-$, which consists of all worlds where p does not hold. The fact that there is a single maximal state means that atoms are neither support-inquisitive nor rejection-inquisitive.

As the maximal supporting state does not include worlds where $\neg p$ holds, and the maximal rejecting state does not include worlds where p holds, p is both support informative and rejection informative. We will generally omit discussion of informativeness below, unless a sentence is not informative.

The meaning of the atomic sentences p and q is depicted in figures 5.1, and 5.2, respectively, where the circles correspond to worlds that concern only the value of these two atomic sentences. Maximal states that support a sentence are indicated by solid lines; maximal states that reject a sentence are indicated by dashed lines.

Negation Negation is illustrated by the reversal of the atomic sentence in (2), given in (3).

(3) Sue does not sing.

a. *Positive response*: Yes, Sue does not sing.

b. *Negative response*: No, Sue sings.

According to clause 2 of definition 10, negation flips between support and rejection, so that a sentence $\neg\varphi$ is supported by a state σ if σ rejects φ and conversely for the rejection of $\neg\varphi$. This means that $\neg\varphi$ is support-inquisitive when φ is rejection-inquisitive, and *vice versa*. Consider the simple example $\neg p$, whose meaning is depicted in figure 5.3.

[4]The natural language examples are for illustration. The actual picture of positive and negative responses is naturally much more complicated. See for example Brasoveanu et al. [20]

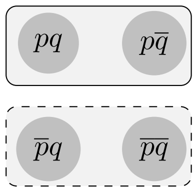


Figure 5.1: p

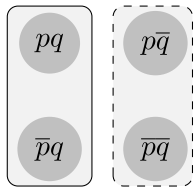


Figure 5.2: q

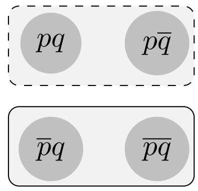


Figure 5.3: $\neg p$

Conjunction Consider the illustrating natural language example in (4).

(4) Sue sings and Mary dances.
  a. *Primary positive response*:
  Yes, Sue sings and Mary dances.
  b. *Primary negative responses*:
  No, Sue does not sing.
  No, Mary does not dance.

According to clause 3 of definition 10, a state σ supports a conjunction $\varphi \wedge \psi$ if σ supports both φ and ψ; and σ rejects this conjunction if σ rejects φ or σ rejects ψ.

Consider the simple example $p \wedge q$. A state σ supports $p \wedge q$ if σ supports both p and q. This means that $\textsc{max}[p \wedge q]^+$ consists of a single element, the state that consists of all worlds where both p and q hold, and is thus not support-inquisitive.

A state σ rejects $p \wedge q$ if it rejects either p or q. This means that $\textsc{max}[p \wedge q]^-$ consists of two elements, a state consisting of all worlds where p does not hold and a state consisting of all worlds where q does not hold. Hence, $p \wedge q$ is rejection-inquisitive. The meaning of $p \wedge q$ is depicted in figure 5.4.

Disjunction Disjunction $\varphi \vee \psi$ is defined as $\neg(\neg\varphi \wedge \neg\psi)$. Consider the simple example $p \vee q$ illustrated by the example in (5).

(5) Sue will sing or Mary will dance.
  a. *Primary positive responses*:
  Yes, Sue will sing.
  Yes, Mary will dance.
  b. *Negative response*:
  No, Sue won't sing and Mary won't dance.

According to this definition, the disjunction $p \vee q$ is supported by a state σ when σ rejects $\neg p \wedge \neg q$, so σ supports $p \vee q$ if σ supports either p or q, and rejects it if σ rejects both.

The meaning of $p \vee q$ is depicted in figure 5.5. As the diagram shows, there are two elements in $\textsc{max}[p \vee q]^+$: the set of worlds where p is the case and the set of worlds where q is the case; by contrast, there is only a single element in $\textsc{max}[p \vee q]^-$. This means that $p \vee q$ is support-inquisitive but not rejection-inquisitive.

Implication According to clause 4 of definition 10, a state σ supports $\varphi \to \psi$ if every maximal supporting state for the antecedent φ, restricted to the information contained in σ, supports the consequent ψ. A state σ rejects $\varphi \to \psi$ as soon as a maximal supporting state for φ, restricted to the information contained in σ, rejects ψ.

Consider the simple example $p \to q$, illustrated by the natural language example in (6).

(6) If Sue sings, then Pete plays the piano.

a. *Positive response*:
Yes, if Sue sings, then Pete will play the piano.

b. *Negative response*:
No, if Sue sings, then Pete won't play the piano.

As explained above, there is only one maximal supporting state for an atomic sentence p, consisting of all worlds where p is the case. The universal quantification in the support clause and the existential quantification in the reject clause both concern only this state.

A state σ supports $p \to q$ if the maximal substate of σ where p is the case supports q. So, in all worlds in σ where p is the case, q should be the case as well. A state σ rejects $p \to q$ if the maximal substate of σ where p is the case rejects q. So, in all the worlds in σ where p is the case, q should not be the case. Figure 5.6 shows the meaning of $p \to q$.

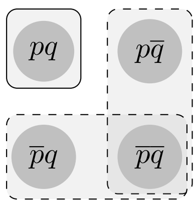


Figure 5.4: $p \wedge q$

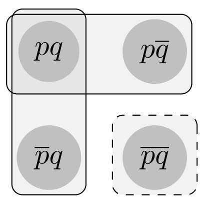


Figure 5.5: $p \vee q$

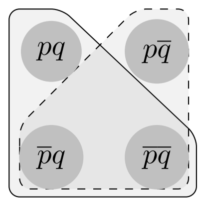


Figure 5.6: $p \to q$

More generally, as in basic inquisitive semantics, 2 and 3 hold.

Fact 2. : If ψ is not support-inquisitive, then $\varphi \to \psi$ is not support-inquisitive.

Fact 3. : If ψ is not rejection-inquisitive, then $\varphi \to \psi$ is not rejection-inquisitive.

In the example with $p \to q$, quantification over the maximal supporting states for the antecedent played no significant role due to the antecedent only having one maximal supporting state. This, however, is not the case for $(p \vee q) \to r$, where the antecedent is a support-inquisitive disjunction for which there are two maximal supporting states: the set of all worlds where p is the case and the set of all worlds where q is the case (see figure 5.5).

The natural language example in (7) illustrates $(p \vee q) \to r$.

(7) If Sue sings or Mary dances, then Pete will play the piano.

a. *Primary positive response*:
Yes, if Sue sings, Pete will play, and if Mary dances, he'll play too.

b. *Primary negative responses*:
No, if Sue sings Pete will not play.
No, if Mary dances, Pete will not play.

For a state σ to support $(p \vee q) \to r$, what should hold is that for each of the two maximal supporting states for $p \vee q$, when σ is restricted to it, the resulting substate of σ supports r. So, in each world in σ where p is the case, r should also be the case; and in each world in σ where q is the case, r should also be the case.

For a state σ to reject $(p \vee q) \to r$, what should hold is that for one (or both) of the two maximal supporting states for $p \vee q$: the maximal supporting state for p and the maximal supporting state for q, when σ is restricted to it, the resulting substate of σ rejects r.

Consider $(p \to r) \wedge (q \to r)$. The first conjunct $p \to r$ is supported in σ if the maximal state where p is supported, restricted to σ, also supports r. Likewise for $q \to r$. According to the clause for conjunction, the state σ supports $(p \to r) \wedge (q \to r)$ if both conjuncts are supported. So both the maximal supporting states for p and for q, restricted to σ, also support r.

According to the rejection clause for conjunction, a state σ rejects $(p \to r) \wedge (q \to r)$ if it rejects either conjunct: $p \to r$ or $q \to r$. A state σ rejects $p \to r$ if all maximal supporting states for p, restricted to σ, reject r. Likewise for $q \to r$.

This means that $(p \vee q) \to r$ is supported and rejected in the same states as $(p \to r) \wedge (q \to r)$ and hence that the two sentences are equivalent.

Fact 4. $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$

Classically this equivalence also holds and neither of the sentences is support-inquisitive, but MadRis also produces the result that both sentences are rejection-inquisitive. The maximal supporting state for $(p \vee q) \to r$ is shown in figure 5.7 and maximal rejecting states in figure 5.8.

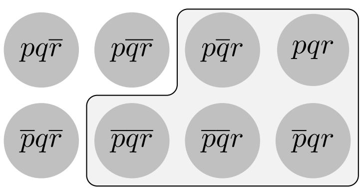


Figure 5.7: $[(p \vee q) \to r]^+$

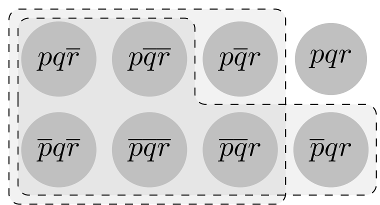


Figure 5.8: $[(p \vee q) \to r]^-$

Also consider the example $p \to (q \vee r)$ illustrated by the natural language example in (8).

(8) If Pete plays the piano, then Sue will sing or Mary will dance.

a. *Primary positive responses*:
Yes, if Pete plays the piano, Sue will sing.

Yes, if Pete plays the piano, Mary will dance.

b. *Primary negative response*:
No, if Pete plays the piano, Sue won't sing and Mary won't dance.

For a state σ to support $p \to (q \vee r)$, we consider the single maximal supporting state for p, when restricted to σ, the resulting state supports either q or r. So, either in each world in σ where p is the case, q should also be the case; or in each world in σ where p is the case, r should also be the case. As shown in figure 5.9, there exist two maximal supporting states for $p \to (q \vee r)$ which makes it support-inquisitive.

For a state σ to reject $p \to (q \vee r)$, what should hold is that the maximal supporting state for p, when restricted to σ, the state rejects both q and r. As there is only one maximal rejecting state for $p \to (q \vee r)$ (shown in figure 5.10), it is not rejection-inquisitive.

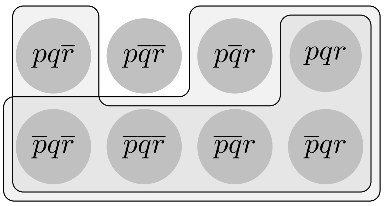


Figure 5.9: $[p \to (q \vee r)]^+$

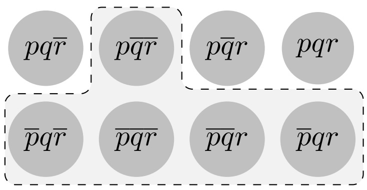


Figure 5.10: $[p \to (q \vee r)]^-$

Questions The question $?\varphi$ is defined via disjunction as $(\varphi \vee \neg\varphi)$. A state σ supports the simple polarity question $?p$ (illustrated by (9)) when σ supports p or when σ rejects p. Only an inconsistent state rejects $?p$.

(9) Will Sue sing?

a. *Possible answers*:
Yes, Sue will sing.
No, Sue will not sing.

The meaning of $?p$ is depicted in figure 5.11. As one can see, MAX$[?p]^+$ has two maximal supporting states corresponding to p and $\neg p$, respectively, so it is support-inquisitive. As $?p$ is rejected only by an empty state, it is not rejection-inquisitive. $?p$ is not support-informative as the union of all supporting states for $?p$ includes all worlds. $?p$ is rejection-informative but only because it is rejected by the inconsistent-state.

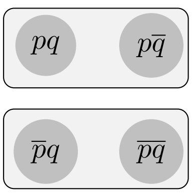


Figure 5.11: $?p$

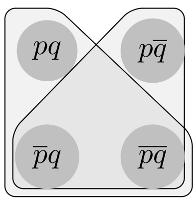


Figure 5.12: $p \to ?q$

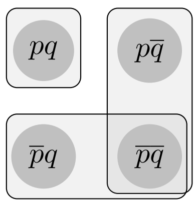


Figure 5.13: $?(p \wedge q)$

Together with the clauses of implication, the treatment of questions allows MADRIS to account for a number of intuitions, including Ramsey's [82] intuition that $p \to q?$ has the contrary answers: $p \to q$ and $p \to \neg q$. As figure 5.12 illustrates, a straightforward result in MADRIS is that Ramsey's question, if p, $q?$, represented in the framework by $p \to ?q$, has the possible answers $p \to q$ and $p \to \neg q$.

Importantly, a radical treatment can capture the intuition that a conjunctive question such as $?(p \wedge q)$ has three primary possible answers.

(10) Will both Sue and Mary sing?
   a. Yes, both Sue and Mary will sing.
   b. No, Sue won't sing.
   c. No, Mary won't sing.

As with $?p$, $?(p \wedge q)$ is only rejected by the empty state, so it is not rejection-inquisitive. On the support side, a state σ supports $?(p \wedge q)$ when the state supports both p and q or when it supports either $\neg p$ or $\neg q$, so $\textsc{max}?(p \wedge q)$ has three maximal supporting states illustrated in figure 5.13 and is thus support-inquisitive.

5.5 Violation semantics for deontic modals

We will now describe the properties of the deontic modal operator that figures in MADRIS. The central idea is that deontic statements offer ways to address the question of whether some state of affairs violates a specific rule.

According to clause 5 of definition 10, a state σ supports a permission statement $\Diamond\varphi$ if every maximal supporting state for the prejacent φ, restricted to the information contained in σ, rejects the violation v.

A state σ rejects $\Diamond\varphi$ if every maximal supporting state for φ, restricted to the information contained in σ, supports v. So, a state that rejects permission for φ supports the statement that φ is prohibited.

(11) A country may establish a laboratory.
   a. *Positive response*:
      Yes, a country may establish a laboratory.
   b. *Negative response*:
      No, a country may not establish a laboratory.

Consider the simple example $\Diamond p$ illustrated by (11).[5] As we have seen above, there is only one maximal supporting state for an atomic sentence p, consisting of all worlds where p is the case. The universal quantification in the support and rejection clause concerns only this state. A state σ supports $\Diamond p$ if the maximal substate of σ where p is the case supports $\neg v$. So, in all worlds in σ where p is the case, the violation v should not be the case.

[5] We continue to use the same example from chapter 4 that builds on an example from the WTO.

A state σ rejects $\Diamond p$ if the maximal substate of σ where p is the case supports v. So, in all worlds in σ where p is the case, the violation v should be the case as well. Of course, the simple example is structurally similar to implication, and the equivalence facts 5 and 6 can indeed be shown to hold. A comparison of clause 4 and 5 of definition 10, shows that the clause for implication and permission coincide on the support-side but the rejection clauses for implication and permission differ with respect to support-inquisitive antecedents.

Fact 5. : $\Diamond\varphi \equiv^{+} \varphi \rightarrow \neg v$

Fact 6. : If φ is not support-inquisitive, then $\Diamond\varphi \equiv \varphi \rightarrow \neg v$

These facts capture Anderson's [15] insight that the meaning of deontic operators is somehow related to implication. Fact 6 tells us that $\Diamond p$ is equivalent to $p \rightarrow \neg v$, illustrated in (12).[6]

(12) If a country establishes a laboratory, no violation occurs.

a. *Positive response*:
Yes, if a country establishes a laboratory, it incurs no violation.

b. *Negative response*:
No, if a country establishes a laboratory, it incurs a violation.

However, as this fact reflects, it is the non-inquisitiveness of p that guarantees the equivalence of $\Diamond p$ and $p \rightarrow \neg v$. Below, we shall consider examples with support-inquisitive prejacents. For now, though, we shall limit the discussion to illustrating the clauses with simple examples.

Figure 5.14 shows the meaning of $\Diamond p$ and $p \rightarrow \neg v$. For convenience, non-violation worlds ($\overline{v}$) are indicated in green and violation worlds (v) in red[7]. This figure also illustrates other key aspects of deontic sentences. The illustrative picture allows one to determine the deontic status of a state of affairs by seeing whether worlds that support a state of affairs p are within, outside or both with respect to the maximal state that supports the deontic statement in the figure.

The following picture illustrates permission, prohibition and neutrality:
1. The state where p *is permitted* has no pv world in the maximal supporting state, so looking at p worlds, $\neg v$ is also the case.
2. The state where p *is prohibited*, drawn dashed, has no $p\overline{v}$ world.
3. Both of these states are *deontically neutral towards* $\neg p$ as the maximal supporting states include both a $\overline{p}v$ and a $\overline{pv}$ world.

[6] Recall from chapter 3, section 3.4.1 that modal auxiliaries *must* and *may* can receive several interpretations alongside the deontic one. For example, the interpretation can be epistemic. One of the advantages of SML and Kratzer semantics, especially, is that they account for the intuition that different interpretations of modals share a structural similarity. We will not discuss epistemic modals here but will return to this point in chapter 7, section 7.1.1.

[7] If you are reading this in grayscale, violation worlds are darker and non-violation worlds are lighter.

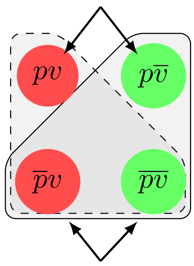


Figure 5.14: $\Diamond p/p \to \neg v$

A permission statement does not predetermine whether p is in fact the case. The set of worlds that supports $\Diamond p$ includes, for example, the world $\overline{p}v$ in which p is not the case. The inclusion of this world in a state where $\Diamond p$ is the case accounts for the intuition that permission statements do not require performance of the permitted act.

5.5.1 Inquisitiveness in *free choice* examples

Anderson's intuition that deontic modals can be reduced to implication leads to problems when permission statements with support-inquisitive prejacents are negated. Recall that $(p \vee q) \to r$ is rejection-inquisitive, so the same would hold for $(p \vee q) \to \neg v$ and, hence, if permission statements were reduced to implications, the free choice example $\Diamond(p \vee q)$, repeated here as (13-a), would also be rejection-inquisitive.

(13) a. A country may establish a research center or a laboratory.
b. A country may not establish a research center or a laboratory.

The salient reading of (13-a) is that a country incurs no violation by establishing a research center and a country incurs no violation when a country establishes a laboratory. Recall from 5.5.1 that a separate legal provision could exist that prohibits both the establishment of a research center and a laboratory even though one or the other action is legal. In this sense, (13-a) as such does not guarantee that performance of both the action described in one disjunct and the action described in the other is necessarily still legal. The provision in (13-a) merely guarantees that when a person chooses to perform the action described in either disjunct, the rules are not broken.

Similarly, the salient reading of (13-b) is that performing an action corresponding to either disjunct is prohibited. The negation of the permission statement $\Diamond\varphi$ is the prohibition $\neg\Diamond\varphi$ or, equivalently, the obligation $\Box\neg\varphi$.

Intuitively, prohibitions and obligations are not support-inquisitive. As (13-a) and (13-b) illustrated, the standard examples of free choice require that $\Diamond(p \vee q)$ and $\neg\Diamond(p \vee q)$ not be support-inquisitive: the authors of a rule establish what is permitted, what is prohibited, and what is obligatory, and this leaves no room for inquisitiveness.

It is, by contrast, only in everyday discussions of obligation or permission that ignorance of these could play a role. In other words, if a person does not know precisely what is permitted and what is required, he or she could, for example, utter a disjunction believing that some prohibition holds but not having enough information to specify which one. We will illustrate the ignorance case later with example (15). But to capture the non-inquisitive nature of modal statements, modal operators are defined as basic operators such that for no $\varphi : \Diamond\varphi$ is support-inquisitive or rejection-inquisitive.

To obtain an analysis of deontic modals whereby deontic statements are never inquisitive–unlike implication with universal quantification in the support clause and existential quantification in the rejection clause–permission has universal quantification scoping over the maximal supporting states for the prejacent in both support and rejection clauses.

Consider the standard free choice case, $\Diamond(p \vee q)$. Crucially, quantification occurs over multiple maximal supporting states for $p \vee q$, which are part of an inquisitive semantics analysis of $p \vee q$[8]. With $\Diamond(p \vee q)$, universal quantification in the support clause scopes over $\textsc{max}[p \vee q]^+$, which consists of two elements: the set of all worlds where p is the case and the set of all worlds where q is the case (see figure 5.5).[9] For a state σ to support $\Diamond(p \vee q)$, it should hold for each of the two maximal supporting states for $p \vee q$ that when σ is restricted to it, the resulting substate of σ rejects v.

The reading of (13-b) accords with the idea behind the rejection clause for permission modals that permission and obligation are not inquisitive. Thus, when the rule in (13-b) holds, any country that establishes a research center or a laboratory will be in violation of this rule. As in the support case, for a state σ to reject $\Diamond(p \vee q)$, it should hold for each of the two maximal supporting states for $p \vee q$ that when σ is restricted to it, the resulting substate of σ supports v. This means that a MadRis analysis correctly predicts that performing an action corresponding to either disjunct is prohibited.

The maximal state supporting (13-a) is shown in figure 5.15 and that supporting (13-b) is shown in figure 5.16.

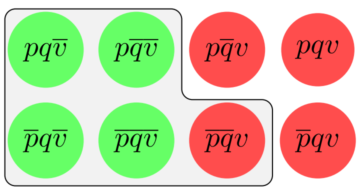


Figure 5.15: $[\Diamond(p \vee q)]^+$

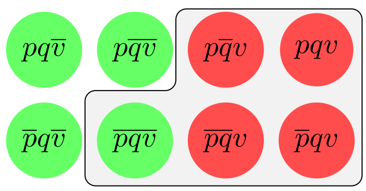


Figure 5.16: $[\neg\Diamond(p \vee q)]^+$

Worth noting is that the *free choice* effect does not correspond to an entailment relation. Even though the performance of p or q produces no violation, the following facts hold.

[8] And alternative semantics more generally.

[9] We will discuss alternative scope possibilities when we consider the ignorance reading.

Fact 7. $\Diamond(p \vee q) \not\models \Diamond p$

Fact 8. $\Diamond(p \vee q) \not\models \Diamond q$

We will briefly discuss why the facts hold. In MadRis the free choice example in (13-a) only support-entails its disjuncts.

Fact 9. $\Diamond(p \vee q) \models_+ \Diamond p$

Fact 10. $\Diamond(p \vee q) \models_+ \Diamond q$

As we discussed earlier, if a state σ supports the free choice example $\Diamond(p \vee q)$, it must also support $\Diamond p$ because one of the two maximal supporting states for $p \vee q$ is the maximal state that supports p. According to clause 5 of definition 10, when both maximal states that support $p \vee q$ are restricted to σ, $\neg v$ must hold. Hence fact 9 holds and likewise for 10.

However, consistent with classical entailment, MadRis entailment looks both at support-entailment and rejection-entailment; and the free choice example does not rejection-entail its disjuncts.

Fact 11. $\Diamond(p \vee q) \not\models_- \Diamond p$

Fact 12. $\Diamond(p \vee q) \not\models_- \Diamond q$

Fact 11 holds because there are states rejecting $\Diamond p$ that do not reject $\Diamond(p \vee q)$. A characteristic example of this is the state $\{pqv, \overline{p}q\overline{v}\}$ where q is the case and adding p would result in a violation. Such a state rejects $\Diamond p$ but not $\Diamond(p \vee q)$. This is exactly what is required to avoid this version of Ross's [86] paradox, illustrated in (14): $\neg\Diamond p$ illustrated in (14-a) should not entail $\neg\Diamond(p \vee q)$ in (14-b). Were we to accept that *free choice* constitutes classical entailment, we would have to accept that (14-a) entails (14-b).

(14) a. You may not burn a letter.
b. You may not burn a letter or mail it.

The entailment fails because we maintain the classical perspective on entailment such that $\varphi \models \psi$ iff $\neg\psi \models \neg\varphi$. In fact, we will see that this approach solves several puzzles of deontic logic.

We noted earlier that (13-a), represented as $\Diamond(p \vee q)$ does not necessarily guarantee that performing the actions in both disjuncts, ie. bringing about $p \wedge q$, is also without violation. So in addition to the regular *free choice* reading, the inclusion of additional information can bring about an *exclusive* reading in which bringing about both disjuncts is prohibited. In this case, we have to refer to a pragmatic strengthening of disjunction. MadRis does not yet include a pragmatic account, however, it is standard to assume that the prejacent $p \vee q$ can be pragmatically strengthened to $(p \wedge \neg q) \vee (\neg p \wedge q)$, the maximal supporting states of which are illustrated in figure 5.17.

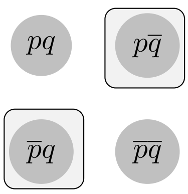


Figure 5.17: $[(p \wedge \neg q) \vee (\neg p \wedge q)]^+$

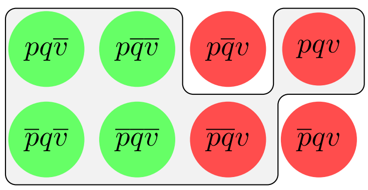


Figure 5.18: $[\Diamond((p \wedge \neg q) \vee (\neg p \wedge q))]^+$

Assuming this strengthening, consider $\Diamond((p \wedge \neg q) \vee (\neg p \wedge q))$. According to clause 5 of definition 10, a state σ supports $\Diamond((p \wedge \neg q) \vee (\neg p \wedge q))$ if both maximal supporting states in $\textsc{max}[(p \wedge \neg q) \vee (\neg p \wedge q)]^+$, the state that supports p but rejects q, and also the state that supports q but rejects p, when restricted to σ, are such that they support that no violation is incurred, i.e. in either state $\neg v$ holds.

In other words, MadRis predicts that choosing only p or only q will still not lead to a violation. Yet, as illustrated in figure 5.18, in contrast to the standard free choice example, the case in which both p and q hold is predicted to be neutral.

Furthermore, a separate rule could specify that $\neg\Diamond(p \wedge q)$ holds. A state σ supports $\neg\Diamond(p \wedge q)$ when the maximal supporting state for $p \wedge q$, ie. a state which supports both p and q, when restricted to σ, supports v. The maximal supporting state for $\neg\Diamond(p \wedge q)$ is shown in figure 5.19.

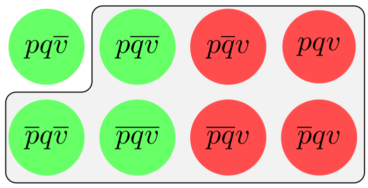


Figure 5.19: $[\neg\Diamond(p \wedge q)]^+$

In this case where both $\Diamond((p \wedge \neg q) \vee (\neg p \wedge q))$ and $\neg\Diamond(p \wedge q)$ are the case, performing both p and q is no longer neutral, instead the resulting state supports v, ie. a violation occurs. This accounts for the intuition that in *free choice* examples performing the action associated with each disjunct is not necessarily permitted.

An analysis of *free choice* permission statements also needs to consider the fact that when one continues (13-a), for example, with “but I do not know which” as shown in (15), the sentence no longer receives a free choice reading.

(15) A country may establish a research center or a laboratory, but I do not know which.

This example sentence would commonly be used by a layman who does not know which permissions and prohibitions govern the relevant situation. The speaker is certain that one of the two disjuncts is permitted but is unable to specify which of them is permitted in the current situation.

MadRis accounts for the ignorance reading of sentences like (15) by assuming that modals generally take the *strongest* scope, resulting in permission taking *wide scope* over the disjunction. Something additional is required for modals to take *weaker* or *narrow* scope. Scope *strength* can be determined by an inspection of entailment relations as is done in facts 13 and 14.

Fact 13. $\Diamond(p \vee q) \models \Diamond p \vee \Diamond q$ **Fact 14.** $\Diamond p \vee \Diamond q \not\models \Diamond(p \vee q)$

According to these facts, $\Diamond(p \vee q)$ is stronger than $\Diamond p \vee \Diamond q$. Thus, permission will first attempt to take *wide* scope which results in the *free choice* reading. In case a modal is blocked from taking *wide* scope, such as by the manifest ignorance in (15), then the modal will take *narrow* scope. The maximal supporting states for $\Diamond p \vee \Diamond q$ are shown in figure 5.20 and the maximal rejecting state in figure 5.21.

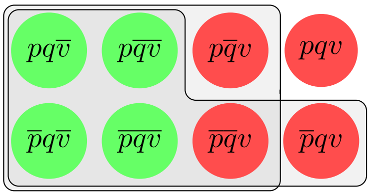


Figure 5.20: $[\Diamond p \vee \Diamond q]^{+}$

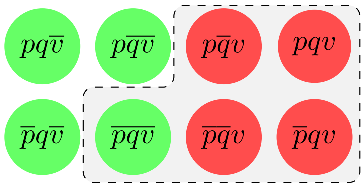


Figure 5.21: $[\Diamond p \vee \Diamond q]^{-}$

We showed that a MADRIS analysis can account for the basic facts of the free choice puzzle. However, the following problem calls for some further elaboration of such an analysis. Recall from the previous chapter the observation by Kamp [51] that a sentence like that in (16), can also receive a free choice reading, even if this is less salient than the ignorance reading.

(16) A country may establish a research center or a country may establish a laboratory.

(16) has the surface structure of *narrow* scope and MADRIS correctly predicts that the salient reading of (16) is the ignorance reading. The less salient free choice reading would also be accounted for by scope movement, where some additional information forces both modals to take *wide* scope over the disjunction. As they would have exactly the same effect, they would be represented again by $\Diamond(p \vee q)$.

Admittedly, however, this solution is not as neat as those that MADRIS provides for the other pieces of the free choice puzzle and may need to be revisited in future research. An avenue for doing so is offered by Simons [92], who accounts for this free choice reading in terms of across-the-board LF movement. However, such interaction between syntactic scope and semantics still remains unexplored in the MADRIS framework.

5.5.2 Deontic conflicts with multiple violations

So far in the explanation of MADRIS we have not utilized the fact that deontic statements can refer to multiple violations. This fact is reflected in the notation for permission that makes the particular violation explicit: $\Diamond_{v}\, \varphi$.

It could be the case that two seemingly contradictory permission statements hold. Imagine that a mother and father are both cross at a teenager. The mother thinks he spends too much time in the room and the father thinks he has stayed out too late. Unbeknownst to the other, mother and father utter (17-a) and (17-b).

(17) a. Mother: You must leave your room. $\boxed{v}p$
b. Father: You may not leave your room. $\neg\langle v\rangle p$

$\boxed{v}p$ is defined as $\neg\langle v\rangle\neg p$ and is supported by state σ if the maximal supporting state for $\neg p$, restricted to σ, also supports $\neg v$. $\neg\langle v\rangle p$ is supported by state σ if the maximal supporting state for p, restricted to σ, also supports $\neg v$. So, whether the teenager chooses to leave the room (p) or stay in the room ($\neg p$), he will incur a violation.

We will refer to such situations as deontic conflicts: situations where the relevant set of rules no longer allows for a situation in which all violations are avoided. As we saw in the previous chapter, standard accounts of modal logic struggle to provide an analysis of *deontic conflicts* but MadRis provides the tools to state that each alternative for the teenager results in a violation. Furthermore, MadRis allows for a more fine-grained analysis of such a *deontic conflict* through the introduction of multiple violations.

5.5.2.1 Multiple violations: rules versus authorities

One way to conceptualize multiple violations is to differentiate deontic authorities. We will not use this conceptualization but it is useful to consider it briefly to see its shortcomings.

In the above example, mother and father can be taken to represent different deontic authorities: each provides rules they enforce largely independently of the other. We could then say that there exists a violation for mother: v_1 and a violation for father: v_2. In this case, it is more accurate to introduce (17-a) and (17-b) as referring to different violations, as shown in (18-a) and (18-b).

(18) a. $\boxed{v1}p$
b. $\neg\langle v2\rangle p$

What the analysis gains from such a treatment is that we can now differentiate between different consequences of the inevitable breaking of the rules. The teenager can reason from the fact that mother's violation results in a stern look ($v_1 \to q$) and father's violation results in a more severe punishment ($v_2 \to r$) that, wishing to avoid r, it is advantageous to stay in the room $\neg p$.

But such a conceptualization is problematic as rules set by one authority, for example by mother, can be inconsistent and deontic conflicts can still occur. For reasons of forgetfulness, malice, etc. people create situations of *deontic conflicts*. So, it could easily be the case that mother uttered both (17-a) and (17-b) in which case the conceptualization does not allow us to reason about the consequences of choosing p and $\neg p$ in the same manner as before.

It is possible to reason that some rules are more important to follow than others, even when they come from the same authority. A single law can specify that the violation of one article is followed by a harsher punishment than another. Consider, for example, different degrees of murder: manslaughter receives fewer years in prison than murder even though the violations are considered from the perspective of one authority - the state.

So, as is generally accepted in law, it is more plausible to assume that each rule has its own violation associated with it, such that (17-a) being distinct from (17-b) would be the basis for associating (18-a) with v_1 and (18-b) with v_2.

A standard example of this in legal discourse, illustrated in (19), is a case when a court deems someone guilty of violating one article of a law, but judges that the defendant did not violate other articles of the same law.

(19) a. The jury finds the defendant in violation of article 1.
b. The jury find the defendant not in violation of article 2.

This example is based on a WTO panel report in DSU 344. Recall that the WTO uses the term *inconsistent* when it means a violation.

> "8.1 On the basis of the above findings, we conclude that:
>
> (a) Model zeroing[10] in investigations "as such" *is inconsistent* with Article 2.4.2 of the Anti-Dumping Agreement,
>
> (b) The USDOC *acted inconsistently* with Article 2.4.2 of the Anti-Dumping Agreement in the investigation on Stainless Steel Sheet and Strip in Coils from Mexico by using model zeroing,
>
> (c) Simple zeroing in periodic reviews is "as such" *not inconsistent* with Articles VI:1 and VI:2 of the GATT 1994 and Articles 2.1, 9.3 and 2.4 of the Anti-Dumping Agreement,
>
> (d) The USDOC *did not act inconsistently* with Articles VI:1 and VI:2 of the GATT 1994 and Articles 2.1, 9.3 and 2.4 of the Anti-Dumping Agreement by using simple zeroing in the five periodic reviews on Stainless Steel Sheet and Strip in Coils from Mexico." [Emphasis added.]

Were the conceptualization of multiple violations authority-based, the judgment would be inconsistent: the defendant both incurs and does not incur the same violation. But this is not plausible.

5.5.2.2 Further work on suppositions

A prevalent intuition regarding deontic statements says that sentences such as $\Diamond p$ should not provide information regarding whether p or $\neg p$ is the case.

[10] *Zeroing* is the "calculation of dumping margins... The "zeroing" methodology, generally speaking, involves treating specific price comparisons which do not show dumping as zero values in the calculation of a weighted average dumping margin." Source WTO DSU: http://bit.ly/13fj9RE *Model* and *simple* zeroing are different methodologies: generally, model zeroing compares average prices in one country with an average price in another, while simple zeroing compares the average price in one country to transaction prices. See http://1.usa.gov/YussFE for more discussion.

This intuition is straighforwardly accounted for in MadRis but it reappears with regard to certain deontic conflicts. Consider the conjunction in (20) on the assumption that both permission statements refer to the same violation.

(20) $\Diamond_v p \wedge \neg \Diamond_v p$

a. $\Diamond_v p$

b. $\neg \Diamond_v p$

The conjunction in (20) is supported by a state σ if both conjuncts, (20-a) and (20-b) are supported in the state. The first conjunct is supported by a state σ if the maximal supporting state for p, restricted to σ, supports $\neg v$. The second conjunct, (20-b), is supported by a state σ if the maximal supporting state for p, restricted to σ, supports v. The conjunction is supported by σ only if there exist no worlds that support p, ie. the prejacent is not the case.

It is problematic that the conjunction of two permission statements, neither of which alone provides information regarding whether p or $\neg p$ is the case, provides the information that $\neg p$ is the case. This is because both $\Diamond_v p$ and $\neg \Diamond_v p$ share the same prejacent p but the conjuncts provide contrary deontic information. The first states that no violation is incurred, and the other than a violation is incurred, which makes the two statements intuitively inconsistent.

MadRis does not yet have the tools to account for this type of an inconsistency, as it allows the prejacent to be vacuously supported by the empty state. The maximal supporting state for (20) where the prejacent p is rejected is illustrated in figure 5.22. Intuitively, this is a case of supposition failure as the supposition that the prejacent p is the case fails in all cases.

Not all deontic conflicts result in supposition failure. Most deontic conflicts can be intuitively avoided in case the permission and violation statements refer to different violations. But where such interpretations are infelicitous, and we have to assume that both deontic statements refer to the same violation, a deontic conflict results in supposition failure.

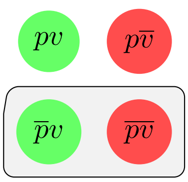


Figure 5.22: $[\Diamond_v p \wedge \neg \Diamond_v p]^+$

Groenendijk and Roelofsen have recently developed an extension of radical inquisitive semantics called suppositional inquisitive semantics [45] which adds suppositional content as a third component of meaning next to informative and inquisitive. In the extension, the rejection of the antecedent of a conditional or the rejection of the prejacent of a modal no longer vacuously supports the implication or modal statement as a whole. To more accurately account for examples such as (20), ongoing work attempts to add modified Andersonian deontic modals to suppositional inquisitive semantics.

Chapter 6

Puzzles Solved

6.1 Introduction

In the previous chapter we introduced the semantics MadRis that provides an alternative account of deontic modals. In this chapter we revisit the puzzles of standard modal logic that motivated its development.

The end of the previous chapter focused on multiple violations and deontic conflicts from the conceptual perspective. We will continue the theme in the beginning of this chapter by applying MadRis to the puzzles of deontic conflicts introduced in chapter 4.

Before we discuss these puzzles, recall from chapter 5 that in MadRis bringing about p can be *permitted, prohibited or neutral.*

1. The state where *p is permitted* has no pv world in the maximal supporting state, so looking at p worlds, $\neg v$ is also the case.

2. The state where *p is prohibited* has no $p\overline{v}$ world. So whenever p is the case, so is v.

3. The state where p is *neutral* includes both a pv and a $p\overline{v}$ world. So, the state does not say whether a violation occurs or does not occur.

Recall also that a permission or obligation statement does not predetermine whether p is in fact the case. For example, the maximal supporting state for $\Diamond p$ includes the world $\overline{p}v$ in which p is not the case. The inclusion of this world in a state where $\Diamond p$ is the case accounts for the intuition that permission statements do not require the permitted state of affairs to actually come about.

In line with standard examples in the literature, we will mostly consider conflicts of obligation. In MadRis obligation is the dual of permission such that fact 15 holds.

Fact 15. $\Box p \equiv \neg\Diamond\neg p$

Fact 15 means that the contrary of an obligation statement is a permission statement.

Fact 16. $\neg\Box p \equiv \Diamond\neg p$

And the contrary of a permission statement is a prohibition, which can be equivalently phrased as an obligation statement.

Fact 17. $\neg\Diamond p \equiv \Box\neg p$

With these facts in hand, we can proceed to discuss deontic conflicts.

6.2 Deontic conflicts

Recall from the previous chapter that deontic conflicts are situations where rules do not allow one to avoid all violations. Generally, this is not the case with deontic statements. Consider again the case of the teenager where there holds the simple obligation that is repeated here as (1).

(1) You must leave your room.

The obligation in (1) is standardly translated as $\Box p$. In MadRis it specifies that in those situations in which you leave your room (p), no violation (v) has to occur but, when you do not leave the room ($\neg p$), one incurs the violation for not fulfilling the obligation in (1). So, to avoid the violation, one would at least have to not bring about $\neg p$.

If we add more rules, the minimal case no longer guarantees that no violation is incurred. In chapter 5 we created a deontic conflict by adding a second rule that prohibits doing what is needed to avoid the violation in (1). The rule is repeated here as (2).

(2) You may not leave your room.

The prohibition in (2) is standardly translated as $\neg\Diamond p$. In MadRis it specifies that any situation in which you leave your room (p) incurs a violation (v). Notwithstanding the content of the obligation in (1), the prohibition in (2) says that one can avoid the violation v by not leaving the room $\neg p$.

The deontic conflict arises because the combination of (1) and (2) results in a situation where the two rules cancel out ways in which to avoid the violation of the other rule. Hence, whether one leaves the room or not, one incurs a violation.

6.2.1 Conflicts of obligation

The same deontic conflict can be discussed as a conflict of obligations puzzle. Both intuitively and accordance with fact 17 in MadRis the prohibition $\neg\Diamond p$ is equivalent with $\Box\neg p$. This means that MadRis correctly predicts that instead of (2) one could say (3).

(3) You must not leave your room.

To give a more precise picture of what is happening in this situation, we will assign different violations to (1) and (3). The former is represented by (4-a) which refers to violation 1 and (3) is represented by (4-b) which refers to violation 2.

(4) a. $\boxed{v1}\,p$
b. $\boxed{v2}\,\neg p$

Assigning multiple violations allows a formulation in which one or the other violation can be avoided. It is not possible to avoid both of them. When both (4-a) and (4-b) hold, if one brings about p, v_1 follows; and if one brings about $\neg p$, v_2 follows. The two resulting maximal supporting states for (4-a) and (4-b) are illustrated in figures 6.1 and 6.2, respectively.

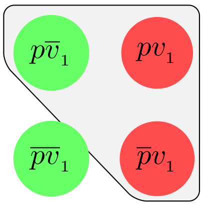


Figure 6.1: $[\boxed{v1}\,p]^+$

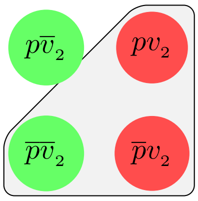


Figure 6.2: $[\boxed{v2}\,\neg p]^+$

These two figures illustrate the obligation statements in (1) and (2) independently of the other. Figure 6.3 illustrates the conjunction of the two. As we have several violations, we no longer end up with a simple picture with non-violation worlds (drawn green) and violation worlds (drawn red). Instead, we also get a third type of world in which one violation is the case and the other is not. We draw these orange.[1]

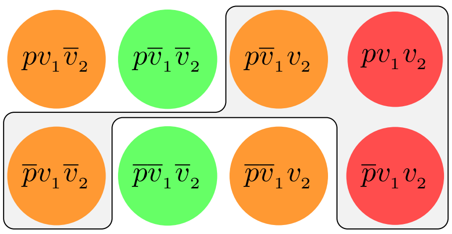


Figure 6.3: $[\boxed{v1}\,p \wedge \boxed{v2}\,\neg p]^+$

The maximal supporting state illustrated in figure 6.3 does not include green worlds; when both (1) and (3) are the case, at least one violation will be incurred.

Recall that Kratzer's solution to contrary to duty puzzles made the counter-intuitive prediction that in cases where one cannot satisfy all obligations, the obligations are false. This would be the case in MadRis if the conjunction of (4-a) and (4-b) rejected either obligation.

(4-a) is rejected by state σ if all maximal supporting states for $\neg p$, restricted to σ, support $\neg v_1$ and (4-b) is rejected by state σ if all maximal supporting states for p, restricted to σ, support $\neg v_2$.

Take a look at the maximal supporting state for the conjunction of the two violations in figure 6.3. The conjunction $\boxed{v1}\,p \wedge \boxed{v2}\,\neg p$ does not reject neither (4-a) nor (4-b). This means that MadRis gives the intuitively correct

[1]If you are reading this paper in grayscale, the non-violation worlds are lightest, worlds with one violation are darker and worlds with two violations are darkest.

prediction that the obligations hold, and the intuitive conflict arises because one cannot avoid incurring one of the violations.

This demonstrates that, unlike in SML or Kratzer semantics, such a conflict of obligations is not predicted to be absurd or false, rather it specifies a situation where there is a choice between different violations.

There are a number of ways in which the choice might be rationalized. Either one of p or $\neg p$ could be more desirable, or either one of v_1 or v_2 could be more repulsive. The choice could also be made off the basis of a combination of both factors. The full implications of any decision making mechanism by which the choice is made is left for future investigation, but incorporating multiple violations provide the means to outline a richer picture of such decision making.

6.2.2 Completely *free choice*

Introducing different violations for separate laws also allows one to account for a number of problems introduced by Asher and Bonevac [17, pp. 4-5]. We mentioned the puzzle of eternal damnation in chapter 4, but we will use another puzzle that Asher and Bonevac call *completely free choice* to demonstrate how to arrive at these types of solutions as the process is the same.

(5) a. You may have soup or not.
 b. Hence, you may rob the cash register.

In Andersonian semantics, (5-a) is problematic because it eliminates all violation worlds. So, as shown by the counter-intuitive continuation in (5-b), one would be able to do anything without incurring a violation. But representing the two permissions with different violations straightforwardly solves the puzzle.

(6) a. $\Diamond_{v1} (p \vee \neg p)$
 b. $\Diamond_{v2}\, q$

Both intuitively and in terms of the clauses of MadRis, (6-a) eliminates all v_1 worlds. Yet, (6-a) has no effect where v_1 does not hold. So, one can still differentiate between states that also support (6-b) and those that do not. So one can still prohibit robbing the cash register.

(7) $\neg \Diamond_{v2}\, q$

As one can see in figure 6.4, the intersection of (6-a) and (7) is not contradictory. Instead, $(\Diamond_{v1} (p \vee \neg p)) \wedge (\neg \Diamond_{v2}\, q)$ provides two types of deontic information. On the one hand, in this situation one cannot incur a soup violation v_1. On the other hand, when one robs the cash register, ie. when one brings about q, violation v_2 occurs. Hence, despite there existing no soup violations, robbing the cash register can still be prohibited.

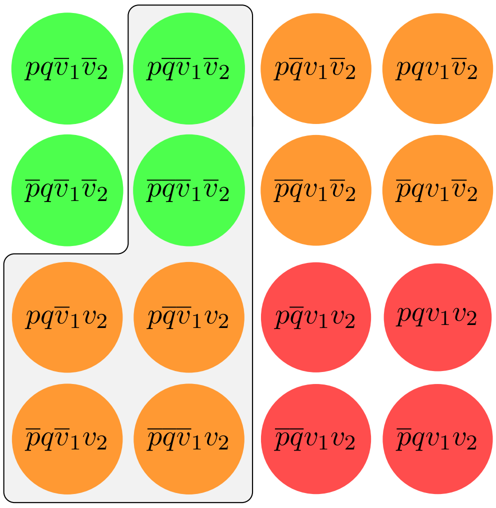


Figure 6.4: $[(\Diamond_{v1}(p \vee \neg p)) \wedge (\neg \Diamond_{v2}\, q)]^+$

Deontic conflicts and multiple violations will also play a role in some of the other puzzles that we will be looking at but we will first discuss the role of monotonicity in deontic semantics.

6.3 Monotonicity of deontic modals

In the previous chapter we discussed the *free choice* puzzle, which is considered at least partly an upward monotonicity puzzle. Monotonicity with respect to deontic modals is the property that any entailment relation that holds between two sentences in the propositional case also holds when both of those sentences are embedded under modals. To illustrate this property, we will designate a modal operator O which can refer to either $\Diamond$ or $\Box$.

Definition 11. Monotonicity:

Upward monotonicity (UM): O is UM iff $\varphi \models \psi$ implies $O\varphi \models O\psi$;

Downward monotonicity (DM): O is DM iff $\psi \models \varphi$ implies $O\varphi \models O\psi$;

Monotonicity: O is monotonic iff O is upward or downward monotonic.

As we outlined in chapter 4, there exist a number of monotonicity puzzles for deontic modals. To solve these, Lassiter [64] and Cariani [22], among others, have proposed non-monotonic accounts of deontic modals. Kai von Fintel [31], on the other hand, recently argued that monotonicity is a required property of modals.

The following subsections will demonstrate that MadRis modals are neither upward nor downward monotonic, so, alongside solving the free choice puzzle, MadRis provides a uniform solution to the many upward monotonicity puzzles in the literature. Yet, it is an interesting historical fact that MadRis modals were not designed with the intent of being non-monotonic, instead the definitions were motivated by WTO interpretations and salient readings from the *free choice* puzzle literature. We can thus expect that besides solving non-monotonicity puzzles, the modals have explanatory power with regard to other puzzles. In fact, as we will show in later sections in this chapter, not all of the pertinent puzzles of SML and Kratzer semantics are monotonicity puzzles. And the treatment of deontic modals in MadRis provides solutions to those as well.

6.3.1 Upward monotonicity puzzles

The alternative treatment of deontic modals in MadRis addresses all of the following puzzles. Such an account is preferential to the current pragmatic approaches to these puzzles because one does not need a separate story for each puzzle.

6.3.1.1 Ross's puzzle

We touched upon Ross's puzzle briefly in chapter 5, but it deserves a closer look. The standard example in the literature is formulated with obligation so we will begin with obligation sentences.

(8) a. A country must establish a research center.
b. A country must establish a research center or invade its neighbour.

The salient reading of (8-a) says that when a country does not establish a research center, the obligation is not satisfied. The salient reading of (8-b) says something intuitively weaker: a country gets a choice whether to establish a research center or invade its neighbour. The only situation that is against the rule in (8-b) is one in which a country does neither. As a situation where a country neither establishes a research center nor invades its neighbour is a situation in which a country does not establish a research center, (8-a) seems to entail (8-b).

Despite the fact that the latter is a weaker obligation than the former, it is highly counter-intuitive that someone who accepts (8-a) would also accept (8-b). Yet, a standard modal account predicts that the latter follows the former. These sentences are standardly represented as follows.

(9) a. $\Box p$
b. $\Box(p \vee q)$

As discussed in chapter 4, Ross's puzzle confounds semantic accounts where modals are upward monotonic as then both of the following entailments are valid.

(10) a. $p \models p \vee q$
b. $\Box p \models \Box(p \vee q)$

Most people are likely to accept (10-a) as a valid entailment, but the examples in (8-a) and (8-b) demonstrate that the entailment becomes suspect with modals.

In MadRis, a state σ supports $\Box p$ if it supports $\neg\Diamond\neg p$. This is the case if the maximal state that supports $\neg p$, restricted to σ, supports v. So, whenever one does not do p, a violation is incurred. The maximal supporting state that supports (9-a) is illustrated in figure 6.5.

A state σ supports $\Box(p \vee q)$ if it supports $\neg\Diamond\neg(p \vee q)$. Once again we look if the maximal rejecting state for $p \vee q$, restricted to σ, supports v. There is a single maximal rejecting state for $p \vee q$, the state which rejects both p and q. So, whenever one does neither p nor q, a violation is incurred. The maximal supporting state for (9-b) is illustrated in figure 6.6.

Fact 18. $\Box p$ support-entails $\Box(p \vee q)$

The figures illustrate that all states that support (9-a) support (9-b), so the former support-entails the latter. This accounts of the intuitive weakness of (8-b) compared to (8-a).

Recall that MadRis entailment maintains the classical view on entailment that when an inference is valid then if one can reject the conclusion, one can also also reject at least one of the premises. This is done by looking at both supporting and rejecting states.

For Ross's puzzle, for a valid entailment to hold, we must establish that all states that reject $\Box(p \vee q)$ also reject $\Box p$. This means that whenever we know that (8-b) is not the case, (8-a) ought not to be the case either.

In MadRis, a state σ rejects $\Box(p \vee q)$ if it supports $\Diamond\neg(p \vee q)$. This is the case if the maximal state that rejects $p \vee q$, restricted to σ, also rejects v. In other words, it would have to be permitted to do neither p nor q, as when one brings about $\neg(p \vee q)$, no violation is incurred. The maximal rejecting state for (9-b) is illustrated in figure 6.8.

A state σ rejects $\Box p$ if it supports $\Diamond\neg p$. For this to be the case, the maximal state that rejects p, restricted to σ, needs to reject v. In other words, it would have to be permitted not to do p, as when one brings about $\neg p$, no violation is incurred. The maximal rejecting state for (9-a) is illustrated in figure 6.7.

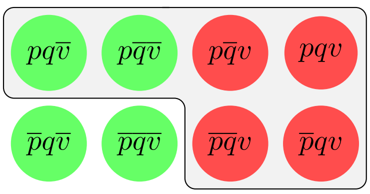


Figure 6.5: $[\Box p]^+$

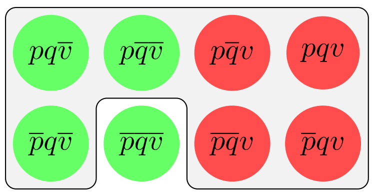


Figure 6.6: $[\Box(p \vee q)]^+$

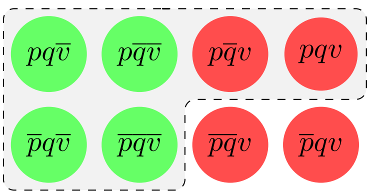


Figure 6.7: $[\Box p]^-$

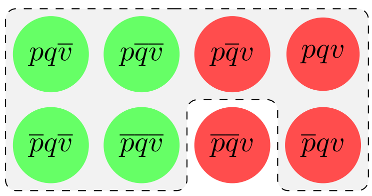


Figure 6.8: $[\Box(p \vee q)]^-$

As illustrated in figures 6.7 and 6.8, the maximal rejecting state for (9-b) is not a rejecting state for (9-a). This is because the world $\overline{p}qv$ is not in the maximal rejecting state for (9-a) but it is among the worlds that reject (9-b). Innocuous on its own, this inclusion of this world makes the rejection of (9-b) too weak to reject (9-a) in the following way.

The world $\overline{p}qv$ is one in which a country does not establish a research center, invades its neighbour and a violation is incurred. Compare this to its factual double. The world $\overline{p}q\overline{v}$ is one in which a country also does not establish a research center and invades its neighbour but no violation is incurred. The world $\overline{p}qv$ remains in both maximal states that reject both (9-a) and (9-b).

Because both of these worlds remain in the maximal rejecting state for (9-b), we say that invading a neighbour without establishing a research center is deontically neutral with respect to rejecting (8-b). This is because a state is deontically neutral with respect to a sentence p if it includes both pv and $p\overline{v}$ worlds.

To reject (8-a), i.e., $\Box p$, bringing about $\neg p$ would need to be permitted, ie. all $\neg p$ worlds need to be $\neg v$ worlds. A state that is deontically neutral towards $\overline{p}q$ does not give cause to reject the obligation $\Box p$ because $\overline{p}v$ worlds remain in the state.

So, with regard to $\Box p$, $\overline{p}qv$ is not included in its maximal rejecting state, but $\overline{p}q\overline{v}$ is. The situation is no longer deontically neutral with respect to (9-a), instead permission has been granted to bring about both $\neg p$ and q. This means that the situation in which a country does not establish a research center avoids incurring violations. This goes against the salient reading of (8-a) as that says that a country must establish a research center and is sufficient to reject $\Box p$.

Thus, the maximal rejecting state for (9-b) does not provide enough information to reject (9-a) because it is only concerned with the case when a country neither establishes a research center nor invades its neighbour. Other situations are neutral. This means that with respect to (9-b), cases where one does not establish a research center are deontically neutral, which does not provide enough information to reject (9-a) as this would require not establishing a research center to be permitted.

Fact 19. (9-a) does not rejection-entail (9-b).

As entailment requires both support-entailment and rejection-entailment, we conclude from fact 19 that fact 20 also holds.

Fact 20. (9-a) does not entail (9-b).

Thus, the obligation version of Ross's puzzle receives a straightforward semantic solution in MadRis as $\Box p$ does not entail $\Box(p \vee q)$.

We also need to consider the permission version of Ross's puzzle illustrated by (11-a) and (11-b).

(11) a. A country may establish a research center.
b. A country may establish a research center or invade its neighbour.

The salient reading of (11-a) says that permission is granted to establish a research center, so no violation is incurred when a country establishes a research center. (11-b) is a *free choice* example. It grants permission to establish a research center and to invade its neighbour. We will omit repeating the more fine grained intuitions regarding such examples that we discussed in chapter 5.

Intuitively, the free choice example is stronger because it grants permission to perform two different actions, while (11-a) only grants permission to establish a research center. The two sentences are standardly represented as (12-a) and (12-b).

(12) a. $\Diamond p$
b. $\Diamond(p \vee q)$

The fact that (11-b) is intuitively stronger than (11-a) is reflected in MadRis by the support-entailment fact, repeated here as fact 21.

Fact 21. $\Diamond(p \vee q)$ support-entails $\Diamond p$.

The puzzle lies in the fact that a standard account of modals not only fails to predict fact 21 but any semantics with UM deontic modals predicts that (12-a) entails (12-b).

(13) a. $p \models p \vee q$
b. $\Diamond p \models \Diamond(p \vee q)$

Intuitively the entailment in (13-b) is implausible. Someone who accepts (11-a) does not need to accept (11-b).

A state σ supports (12-a) when the maximal supporting state for p, restricted to σ, rejects v, ie. when establishing a research center does not incur a violation. The maximal supporting state is illustrated in figure 6.9.

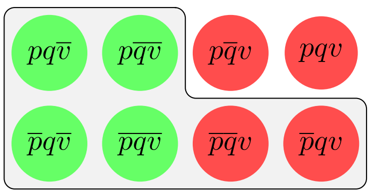


Figure 6.9: $[\Diamond p]^+$

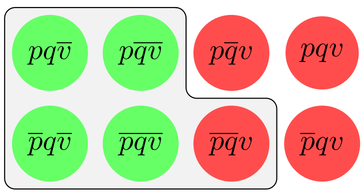


Figure 6.10: $[\Diamond(p \vee q)]^+$

On the other hand, the *free choice* example in (12-b) is supported by σ when the maximal supporting state for $p \vee q$, restricted to σ, rejects v, ie. when establishing either a research center or invading one's neighbour does not incur a violation. The maximal supporting state for (12-b) is illustrated in figure 6.10.

The maximal supporting state for $\Diamond p$ includes the world $\overline{p}qv$, which is a world where a research center is not established. Because of this, its factual information is not relevant when the only rule grants permission to establish a research center. Both the worlds $\overline{p}qv$ and $\overline{p}q\overline{v}$ are included in the maximal supporting state for (12-a), so we say that a situation in which one does not establish a research center and invades one's neighbour is deontically neutral.

There can exist another additional prohibition against invading one's neighbour.

(14) $\neg\Diamond q$

When (14) holds the world $\overline{p}q\overline{v}$ is eliminated as the maximal supporting state for q, restricted to σ, must support v. Consider the resulting intersection of (12-a) and (14).

(15) $(\Diamond p) \wedge (\neg\Diamond q)$

The maximal supporting state for this conjunction includes the world $\overline{p}qv$ but it no longer includes $\overline{p}q\overline{v}$. Thus the situation where one does not establish a research center but invades one's neighbour results in a violation.

The maximal supporting state for the *free choice* example in (12-b) does not include the world $\overline{p}qv$, but it does include the world $\overline{p}q\overline{v}$, which makes the situation under consideration permitted. Adding the prohibition in (14) would result in a deontic conflict as permission has been granted for something that is simultaneously prohibited.

Recall that permission comes about through the elimination of violation worlds. Thus, (12-a) is weaker than (12-b) because the former grants less permission than the latter. The maximal supporting state for (12-a) includes the world $\overline{p}qv$ but (12-b) does not. Because of this MadRis predicts that (11-a) does not support-entail (11-b).

Fact 22. $\Diamond p$ does not support-entail $\Diamond(p \vee q)$.

As entailment requires both support-entailment and rejection-entailment, we conclude from fact 22 that 23 also holds.

Fact 23. $\Diamond p$ does not entail $\Diamond(p \vee q)$.

For completeness, recall that we already discussed the prohibition version of Ross's puzzle in chapter 5. In conclusion, MadRis provides a straightforward semantic solution to Ross's puzzle. As the meaning of obligation, prohibition and permission sentences differs, the respective entailments also fail for different reasons.

6.3.1.2 Dr. Procrastinate

A number of the solutions to the *free choice* and Ross's puzzle have focused on the behaviour of disjunction embedded under deontic modals. But similar puzzles arise with other connectives.

The story goes that dr. Procrastinate is an expert in her field but she has a bad habit of never finishing assignments. As it's a fact in the story that dr. Procrastinate will not write the review, when she is asked to write a review, intuitively the following two obligations hold.

(16) a. Dr. Procrastinate ought to accept and write the review.
b. Dr. Procrastinate ought not to accept.

The story is set up in such a way that dr. Procrastinate will violate the writing conjunct in (16-a). According to the literature, there are two predictions to make. Firstly, the conjunction of (16-a) and (16-b) is not intuitively absurd as both can be the case simultaneously. Although due to a deontic conflict, it is not possible to avoid violating both (16-a) and (16-b). Secondly, we know that dr. Procrastinate will violate the obligation in (16-a) but could avoid violating the second obligation in (16-b). Building on the latter point, if dr. Procrastinate accepts despite the fact that she will not finish writing the review, her behaviour is more reproachable than when she does not accept.

(16-a) is generally represented by the embedded conjunction in (17-a). In SML obligation is upward monotonic, so the embedded conjunction (17-a) entails the embedded conjunct (18) because a conjunction entails its conjuncts.

Fact 24. $p \wedge q$ entails p.

In a semantics with UM deontic modals, one can conclude from fact 24 that (17-a) entails (18). And thus it is absurd that both (17-a) and (17-b) are the case simultaneously, as (17-b) is in conflict with (18).

(17) a. $\Box(p \wedge q)$
     b. $\Box\neg p$

(18) $\Box p$

In Kratzer semantics, when (17-a) is the case, the best worlds are pq worlds and when (17-b) is the case, the best worlds are $\overline{p}$ worlds. But both of them cannot be the case simultaneously.

MadRis captures the intuition that the conjunction of the obligations in (17-a) and (17-b) is not absurd. We will first assume the obligations refer to the same violation. When (17-a) holds, bringing about either $\neg p$ or $\neg q$ incurs a violation; when (17-b) holds, bringing about p incurs a violation and when (18) holds, bringing about $\neg p$ incurs a violation.

In MadRis, the rejection of (18) includes the state $\{p\overline{q}v, \overline{pqv}\}$ where *not writing* ($\neg q$) can lead to a violation. Because of this, the state does not reject (17-a), so fact 25 holds. As fact 25 holds, fact 26 holds as well.

Fact 25. (17-a) does not rejection-entail (18).

Fact 26. (17-a) does not entail (18).

In conclusion, this means that MadRis correctly predicts that (17-a) and (17-b) are not contradictory in the way those sentences are when obligation is UM. So, the upward monotonicity aspect of the dr. Procrastinate puzzle is straightforwardly solved in MadRis. We can move on to the second intuition.

The second intuition that needs to be covered concerns the possibility that dr. Procrastinate can avoid making the situation worse by fulfilling (16-b), despite violating (16-a).

The reason why dr. Procrastinate will violate (16-a) is that she will not write the review, ie. it is known that (19) holds.

(19) $\neg q$

For deontic conflict examples, we separate the obligations so they refer to different violations.[2] Introducing multiple violations allows one to quantitatively determine states with less violations.

(20) a. $\boxed{v1}\,(p \wedge q)$
b. $\boxed{v2}\,\neg p$

According to the story, dr. Procrastinate can salvage some of the situation by fulfilling (20-b), despite violating (20-a).

We will intersect (20-a), (20-b) and (19) and the maximal supporting state is shown in figure 6.11. In the figure the worlds factively eliminated by $\neg q$ are left grey. Green worlds contain no violations, orange worlds contain only one violation and red worlds contain two violations.

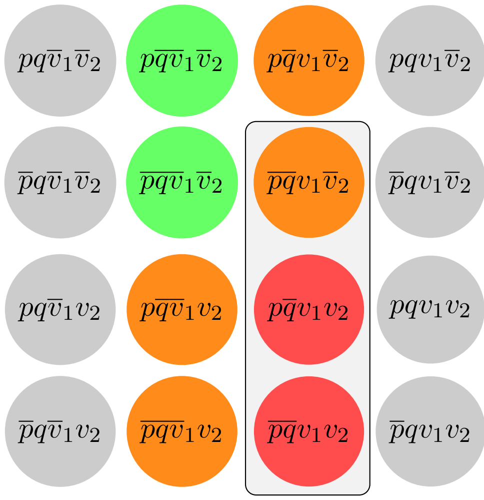


Figure 6.11: $[(\boxed{v1}\,(p \wedge q)) \wedge (\boxed{v2}\,\neg p) \wedge (\neg q)]^{+}$

As one can see, the maximal supporting state for the story contains three worlds. Each of them is a v_1 world, which correctly captures the intuition that as long as dr. Procrastinate does not write the review, she is doing something wrong.

Furthermore, there remains only one p world and in that world v_2 occurs. This means that MadRis predicts that in case dr. Procrastinate does accept to write a review, despite not writing it, then she will incur a second violation on top of v_1. Yet, the two remaining $\neg p$ worlds differ in that one is a v_2 world and the other is a $\neg v_2$ world (coloured orange because it contains only one violation). When both a violation and a non-violation follows $\neg p$, we say that this state is deontically neutral with respect to $\neg p$. Thus, dr. Procrastinate - barring additional information - can avoid the second violation by not accepting to write the review. And this is the second intuition that MadRis had to cover.

[2] Multiple violations also guarantee that (20-a) and (20-b) are not absurd but in a less interesting way.

6.3.2 Downward monotonicity: strengthening the antecedent puzzle

Recall that an operator is monotonic if it is upward or downward monotonic. We have demonstrated that deontic modals are not intuitively upward monotonic and that MadRis modals avoid the problems of UM modals because the modals are not upward monotonic.

Andersonian modals which reduce deontic modals to implication inherit the properties of implication; for example, material implication is downward monotonic. For convenience, the property of DM is repeated here as (21).

(21) An operator is DM iff $\psi \models \varphi$ implies $O\varphi \models O\psi$.

Downward monotonicity is generally regarded an unwanted property of deontic modals due to the *strengthening the antecedent* puzzle that we will discuss presently.

Strengthening the antecedent is a puzzle for material implication. The problem lies in the fact that in a material implication account an implication entails the implication where the antecedent has been strengthened with a conjunct: (23).

In the following, we will distinguish between the clauses in MadRis and material implication by representing the latter with $\rightarrow_m$. In propositional logic, a conjunction entails its conjuncts:

(22) $p \wedge q \models p$

We will make use of this entailment as in (23), the antecedent of the premise is p and the antecedent of the conclusion is $p \wedge q$.

(23) $p \rightarrow_m r \models p \wedge q \rightarrow_m r$

As discussed by Lewis [66, p. 80] and others, the entailment in (23) leads to counter-intuitive examples such as (24).

(24) a. If I strike a match, it will light.
b. Hence, if I strike a match and the match is wet, it will light.

Intuitively, we can accept (24-a) without accepting (24-b), yet material implication predicts that when (24-a) is the case, (24-b) cannot be false.[3] This is not to say that there do not exist natural language examples in which the inference is more plausible. Consider (25).

(25) a. If I walk the dog, I will get some fresh air.
b. If I walk the dog and whistle, I will get some fresh air.

Intuitively, we accept both (25-a) and (25-b). In fact, we can add any arbitrary conjunct in (25-b), such as whistling, because it does not change the outcome. But the existence of examples such as (24) demonstrates that

[3] Asher [16] suggested that *strengthening the antecedent* requires a default semantics. As we discussed in chapter 4 in conjunction with Asher's proposal, such accounts run into their own problems.

the plausibility of the inference in (25) cannot be a general inference rule for implication.

Strengthening the antecedent is also relevant for deontic modals. Recall that Anderson defined a permission utterance as relevant implication from the prejacent to the negation of a sanction s. If the modal were defined using material implication, then whenever (26-a) holds, (26-b) would hold as well.

(26) a. $p \rightarrow_m \neg s$
b. $(p \wedge q) \rightarrow_m \neg s$

This leads to examples such as the following.

(27) a. You may walk the dog.
b. You may walk the dog and kill the president.

Intuitively, no-one would accept that when permission is granted to walk the dog, this also grants permission to kill the president. So, *strengthening the antecedent* should not to be valid for neither implication nor modals in MadRis. Because (22) holds, if MadRis modals were DM, whenever (27-a) is the case, so would be (27-b).

In MadRis, *strengthening the antecedent* is not valid for implication or modals, which means deontic modals are not DM in MadRis.

Fact 27. $p \rightarrow r \not\models (p \wedge q) \rightarrow r$

Fact 28. $\Diamond p \not\models \Diamond(p \wedge q)$

To explain these facts, we will demonstrate how *strengthening the antecedent* fails in MadRis. The modal and implication case are parallel.

Consider the maximal supporting and rejecting states for the premise and conclusion in (27-a). A state σ supports $p \rightarrow r$ if the maximal supporting state for p, restricted to σ, supports r. This eliminates both $pq\overline{r}$ worlds and $p\overline{qr}$ worlds. On the other hand, a state σ supports $(p \wedge q) \rightarrow r$ if the maximal supporting state for $p \wedge q$, restricted to σ, supports r. This eliminates only $pq\overline{r}$ worlds. We thus conclude that every state that supports $p \rightarrow r$ also supports $(p \wedge q) \rightarrow r$ and fact 29 holds.

Fact 29. $p \rightarrow r$ support-entails $(p \wedge q) \rightarrow r$

This fact is illustrated in figures 6.12 and 6.13.

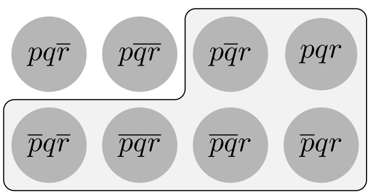


Figure 6.12: $[p \rightarrow r]^+$

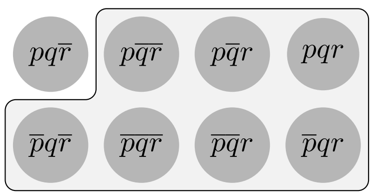


Figure 6.13: $[(p \wedge q) \rightarrow r]^+$

The support-entailment in fact 29 explains the intuitiveness of the inference in (25). When the additional information in the second conjunct does not have an effect on the implication, we do end up at only worlds in which $(p \wedge q) \rightarrow r$ also holds.

It is only when we begin to consider how one might reject the two sentences that the we see a difference. Recall that entailment looks at both supporting and rejecting states, such that when φ entails ψ, every state that supports φ must also support ψ and every rejecting state for ψ must be a rejecting state for φ.

Consider the maximal rejecting state for $(p \wedge q) \rightarrow r$ compared to the maximal rejecting state for $p \rightarrow r$. A state σ rejects $(p \wedge q) \rightarrow r$ if the maximal supporting state for $p \wedge q$, restricted to σ, rejects r. As we are interested only in worlds where both conjuncts holds, this eliminates only pqr worlds. So it is possible to reject $(p \wedge q) \rightarrow r$ with relatively little information.

Conversely, a state rejects $p \rightarrow r$ if the maximal supporting state for p, restricted to σ, rejects r. This eliminates both pqr worlds and $p\overline{q}r$ worlds. As we can see in figures 6.14 and 6.15, the maximal rejecting state for $(p \wedge q) \rightarrow r$ is not a rejecting state for $p \rightarrow r$ as it includes the world $p\overline{q}r$.

As $(p \wedge q) \rightarrow r$ only concerns the situation in which both p and q are the case, it does not provide as much information regarding when r follows as $p \rightarrow r$ which also concerns itself with $p\overline{q}$ worlds. This means that fact 30 holds.

Fact 30. $p \rightarrow r$ does not rejection-entail $(p \wedge q) \rightarrow r$.

From fact 30 we conclude that fact 27 also holds. Due to the weaker rejection-conditions, *strengthening the antecedent* is not a valid inference pattern, which explains the counter-intuitive examples in the literature.

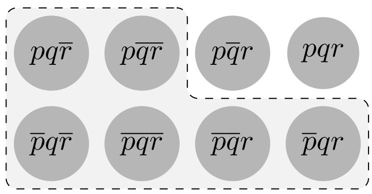


Figure 6.14: $[p \rightarrow r]^-$

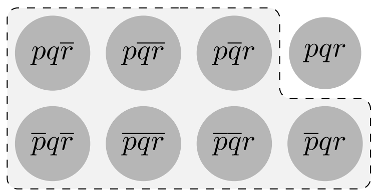


Figure 6.15: $[(p \wedge q) \rightarrow r]^-$

Also consider the deontic case. As with implication, the maximal supporting state for $\Diamond p$ supports $\Diamond(p \wedge q)$ and fact 31 holds.

Fact 31. $\Diamond p$ support-entails $\Diamond(p \wedge q)$

This can be determined by looking at figures 6.16 and 6.17. A state σ supports $\Diamond(p \wedge q)$ if the maximal supporting state for $p \wedge q$, restricted to σ, rejects v. As the maximal supporting state in figure 6.17 illustrates, the only world eliminated by $\Diamond(p \wedge q)$ is pqv. This world is also eliminated by $\Diamond p$ because a state σ supports $\Diamond p$ if the maximal supporting state for p, restricted to σ, rejects v. So, for $\Diamond p$, all p worlds where v is the case are eliminated. So both pqv and $p\overline{q}v$ worlds are eliminated.

From this we can conclude that $\Diamond(p \wedge q)$ grants less permission than $\Diamond p$. It only grants permission for those situations in which both p and q are the case, and does not say whether in $p\overline{q}$ worlds a violation is incurred or not. So it does not grant permission for cases where someone brings about p without bringing about q. In this sense, $\Diamond(p \wedge q)$ is a weaker permission statement than $\Diamond p$ that does grant permission to bring about p without bringing about q.[4]

On the other hand, as we can see in figures 6.18 and 6.19, the maximal rejecting state for $\Diamond(p \wedge q)$ is not a rejecting state for $\Diamond p$ as it includes the world $p\overline{qv}$.

$\Diamond(p \wedge q)$ is weaker than $\Diamond p$ as it only concerns the situation in which both p and q are the case. As such, for a state to reject $\Diamond(p \wedge q)$, the state cannot include $pq\overline{v}$, ie. it cannot be that both p and q are the case and no violation is incurred.

In the world $p\overline{qv}$, q is not the case, so it does not concern the conjunction example. Yet, the inclusion of this world does not satisfy the requirements for a state to reject $\Diamond p$. For a state to reject $\Diamond p$, when p is the case, a violation must occur. In the world $p\overline{q}v$, p is the case, but a violation does not occur, so a state that includes this world is not a rejecting state for $\Diamond p$. This means that fact 32 holds.

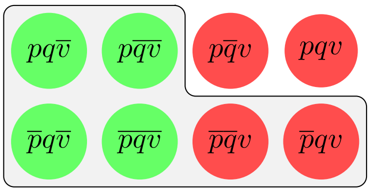


Figure 6.16: $[\Diamond p]^+$

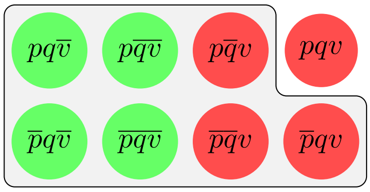


Figure 6.17: $[\Diamond(p \wedge q)]^+$

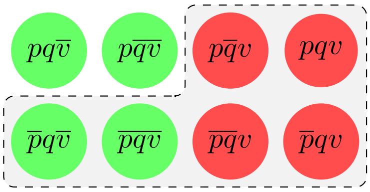


Figure 6.18: $[\Diamond p]^-$

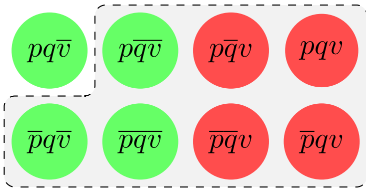


Figure 6.19: $[\Diamond(p \wedge q)]^-$

Fact 32. $\Diamond p$ does not rejection-entail $\Diamond(p \wedge q)$.

From fact 32, we conclude that fact 28 also holds. The implication fact 27 runs parallel to the deontic case outlined here. So MadRis provides a semantic solution to the puzzle of *strengthening the antecedent* and thus correctly predicts that deontic modals are not DM.

6.3.3 Do we still want monotonicity?

As we demonstrated in the previous sections, MadRis modals are neither upward nor downward monotonic. This aligns with the view of advocates of non-monotonicity for deontic modals such as Cariani [22] and Lassiter [64]

[4]For more permission, $\Diamond p \wedge \Diamond q$ grants more permission than $\Diamond p$.

who featured in chapter 4. Others, such as von Fintel[31], argue in favour of (upward) monotonicity. One of the more prominent examples cited in favour of monotonicity is the behaviour of deontic modals embedded under negation.

During the discussion of the free choice puzzle, we showed that MadRis provides an intuitive account of modals embedded under negation. But it might not be immediately clear what is at play in the examples von Fintel discusses so we will examine them in detail.

6.3.3.1 Contradictory modal statements

Consider von Fintel's example [31, p. 14], which we have modified slightly to demonstrate its similarity to the dr. Procrastinate puzzle.[5]

(28) a. #You don't have to bring wine to the party, but you do have to bring wine and beer.
b. $\neg\Box p \wedge \Box(p \wedge q)$[6]

The example is odd although it is not immediately clear why this is the case. (28-a) has several readings, including the following correcting reading. This correcting reading could be the reason why the sentence is intuitively odd but it is not the reading von Fintel discussed. So we will ultimately set the correcting reading aside after we expand on it briefly for clarity. To see this reading, consider the conjuncts separately.

(29) a. You have to bring wine to the party.
b. You have to bring wine and beer.

Intuitively, (28-a) is a way of specifying which obligations hold. If one has to bring wine and beer, it would be in some sense wrong to describe the situation such that one has to bring only wine as it does not give an adequately precise description of governing rules. So someone could respond by rejecting (29-a) and giving a more accurate description by uttering (29-b). Here we will have very little to say about this correcting reading.

The reading von Fintel intended is the one predicted by UM accounts of deontic modals, which says that (28-a) is an oxymoron because it says to bring and not bring wine. Consider that in propositional logic, a conjunction entails its conjuncts.

(30) $p \wedge q \models p$

In a semantics with UM deontic modals, because (30) holds, (31) must hold as well.

(31) $\Box(p \wedge q) \models \Box p$

According to standard entailment the negation of the conclusion entails the negation of the premise: so (32) holds too.

[5] The original example read: "#You don't have to bring alcohol to the party, but you do have to bring wine.

[6] The *but* is taken simply as conjunction to simplify the logical form.

(32) $\neg\Box p \models \neg\Box(p \wedge q)$

Thus, whenever (33-a) is the case, SML and Kratzer predict that so is (33-c).

(33) a. You don't have to bring wine to the party.
b. $\neg\Box p$
c. You don't have to bring wine and beer to the party.
d. $\neg\Box(p \wedge q)$

The problem is that in the original example (28-a), (33-a) was followed by (34-a).

(34) a. You have to bring wine and beer to the party.
b. $\Box(p \wedge q)$

But as a semantics with UM deontic modals predicts, (33-d) and (34-b) are contraries. Which is a way to explain why the sentence is odd.

In MadRis, one cannot use the same reasoning to come to this conclusion, yet MadRis also predicts that the sentence is odd for a similar reason. As this example is very similar to the dr. Procrastinate puzzle one would expect that an account that can solve that puzzle also explains what is happening in (28-a).

According to (28-a), it is obligatory to bring wine and it is not obligatory to bring wine. We can rephrase this as stating that not bringing wine is not permitted and, at the same time, not bringing wine is permitted. Intuitively, these sentences are in conflict.

In MadRis, deontic modals only concern the relation between the prejacent and violations. They say nothing about non-prejacent situations. (33-b) is supported by a state σ if the maximal supporting state for $\neg p$ rejects v and (34-b) is supported by σ if the maximal supporting state for $\neg p$ supports v. Thus, the conjunction in (28-b) eliminates all states that support $\neg p$ and it does not concern p states. This brings about the conflict that we discussed in section 5.2.2 in chapter 5. After conjoining the two sentences, when one brings about $\neg p$, a violation occurs and does not occur. There is a conflict although the situation is not absurd. As illustrated in figure 6.20, the conjunction can only be supported by a state in which p is the case.

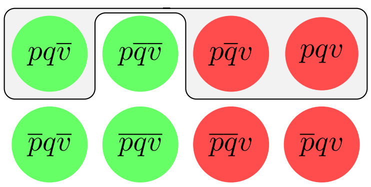


Figure 6.20: $[\neg\Box p \wedge \Box(p \wedge q)]^+$

We have demonstrated that the non-monotonic semantics for deontic modals MadRis also correctly predicts that (28-a) is odd because when one brings about p, v and $\neg v$ hold simultaneously. As (28-b) can only be supported by a state in which the prejacent is false, this appears to be a case of supposition failure. It is impossible to suppose the prejacent and that is why (28-b) is intuitively odd. As a case of supposition failure, the example lends itself to further study in suppositional inquisitive semantics discussed at the end of chapter 4.

This means that similarly to semantics with UM modals MadRis makes intuitive predictions about why (28-a) is odd but MadRis can also explain the following associated intuitions that a standard UM account of modals cannot.

The example in (28-a) is similar to the dr. Procrastinate puzzle and can thus be rephrased as a puzzle for the monotonic account. As in the dr. Procrastinate case, there exists a conflict between the modal sentences in (33-a) and (34-a). There also exists a way to make sense of this example by separating the sentences so that they refer to different violations. We can do this explicitly by having two different people utter them without being aware of each other's statements.

Imagine that a brother and sister are hosting a party. They agree to serve beer and wine. The brother leaves the meeting thinking that they will themselves buy the wine and have guests bring beer. Whenever he invites people, he writes that (35-a) is the case. The sister understood that they will have the guests bring both the beer and the wine. So when she invites people she writes that (35-c) is the case.

(35) a. Brother: You don't have to bring wine to the party.
b. $\neg\boxed{v1}\,p$
c. Sister: You have to bring wine and beer to the party.
d. $\boxed{v2}\,p \wedge q$

For any person that receives an invitation from both the brother and the sister, both (35-a) and (35-c) hold. Even though there is a conflict, it would not be wise to come to the party without wine, as the sister could justifiably say that she said to bring wine.

A guest cannot predict whether bringing wine will be obligatory or not. So, until it is known how the conflict is resolved, to avoid all violations of obligations, one would need to bring wine to avoid incurring the wrath of the sister. In conclusion, intuitively, (35-a) and (35-c) are in deontic conflict, and yet they can both hold simultaneously.

We saw that von Fintel claimed that the oddity in the example can be explained by a monotonic account but a full explanation of the situation requires an account with several violations similar to the previous conflicts of obligation that we discussed in this chapter.

The effect of (35-b) and (35-d) is illustrated in figure 6.21. We are mostly interested in the only two remaining $\neg p$ worlds, so see the bottom row. Both of those worlds are $\neg v_1$ and v_2 worlds. This means that when one does not bring wine, the brother thinks it is permitted and the sister thinks that you are in violation of her instructions. The only way to avoid both violations is to do $p \wedge q$, which is deontically neutral according to both rules.

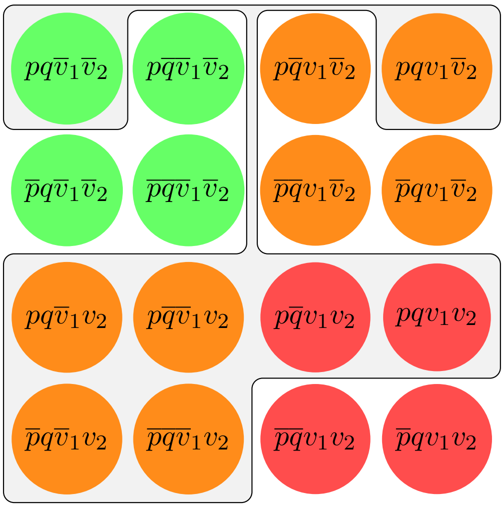


Figure 6.21: $[\neg \boxed{v1}\, p \wedge \boxed{v2}\, (p \wedge q)]^+$

In conclusion, von Fintel was correct to point out that (28-a) was odd but his claim that this is an argument in favour of UM for deontic modals fails as a semantics does not need UM deontic modals to predict that (28-a) is odd. MadRis also obtains that (28-a) is odd, although the sentence is not deemed inconsistent but rather an example of supposition failure.

As UM deontic modals predict that the sentence is inconsistent, they cannot account for the intuitive situation in which the conjuncts of (28-a) are uttered separately by different agents as in (35-a) and (35-c). Even though intuitively the brother and sister have created a deontic conflict, such a situation is not absurd. As MadRis can straightforwardly capture that (35-a) and (35-c) refer to different violations, we can provide an intuitive account of the governing rules when the inconsistency concerns two separate rules. So, both UM modals and MadRis can explain why (28-a) is odd, but only MadRis can capture the associated deontic conflict.

6.3.3.2 Negative polarity items (NPIs)

Non-monotonic accounts of deontic modals were also criticized by von Fintel for not accounting for the fact that negated modals can license NPIs. An NPI, such as *any*, is an expression that is generally not felicitous outside of the immediate scope of negation.

(36) a. Carl does not have any potatoes.
b. #Carl does have any potatoes.

The prevalent theory on NPI licensing is the Fauconnier-Ladusaw generalization [63] which states that NPIs are licensed in downward entailing positions. A UM deontic modal becomes DM under negation and so accounts for the fact that the NPI *any* is licensed in (37).

(37) You do not have to bring any alcohol to the party.

As von Fintel points out [31, p. 14], the Fauconnier-Ladusaw generalization is incompatible with non-monotonic accounts of modals as a non-monotonic modal does not become DM under negation. Of course, negation still scopes over the expression with *any*.

The interaction of modals with NPIs raises an interesting challenge for MadRis. There is no immediately available theory of NPI licensing but there are reasons to think that this phenomenon opens a promising new avenue into the study of NPIs.

There are a number of other recalcitrant examples that do not satisfy the Fauconnier-Ladusaw generalization. For example, interrogatives, *only*, superlatives, adversative attitude predicates and, most importantly for our purposes, the antecedents of conditionals also license NPIs. Consider (38).

(38) If you eat any vegetables, you'll be fine.

Standard accounts of conditionals, and also MadRis, do not predict that the antecedent of a conditional is downward entailing. As we mentioned earlier, if the antecedent were downward entailing, we would face the puzzle of strengthening the antecedent. Yet, NPIs like *any* can be licensed in the antecedent of a conditional.

As we analyze deontic modals as inquisitiveness-resistant implication, it would have been unexpected to find that their licensing of NPIs does not pattern at all with conditionals. As there does not appear to be a widely accepted proposal to explain why antecedents of conditionals license NPIs either, it shouldn't be expected from MadRis, taking into account the similarity of its modals with implication, that it provides an explanation of NPI licensing.

6.4 Beyond monotonicity

Not all of the problems of Kratzer semantics are upward monotonicity puzzles. Consider the *all or nothing* puzzle from chapter 4.

6.4.1 All or nothing

Recall that Kratzer semantics treats a conditional as a restricted modal statement. The antecedent of the conditional restricts the modal operator in the consequent. If no modal is found, it is assumed that there exists a covert universal epistemic modal. This account generates new puzzles where one can counter-intuitively weaken conditional permission statements. Consider (39-a).

(39) a. If the car passed its technical inspection and you have your license, then you may drive.
b. $(p \wedge q) \rightarrow \Diamond r$

The salient reading of (39-a) says that permission to drive the car is contingent on two facts: it must be the case that the car passed its technical inspection and you must also have your license. If either of those conditions is not satisfied, there is no guarantee that driving the car is permitted. In fact, it is likely that there exists a prohibition against driving a car that did not pass the inspection and another prohibition against driving without a license. We call this puzzle *all or nothing* because either all conditions are satisfied or no permission is granted.

Kratzer semantics analyses the sentence in (39-a) by restricting the modal base for the permission modal in the consequent to only worlds in which both p and q are the case. (39-b) is the case if this modal base contains at least one r world. A p and q world is also a p world. So when (39-b) holds, there exists a world in which both p and r are the case, which is sufficient to support (40-b).

(40) a. If the car passed its technical inspection, then you may drive.
b. $p \rightarrow \Diamond r$

Kratzer semantics analyses (40-b) by restricting the modal base for the permission modal in the consequent to only worlds in which p is the case. (40-b) is the case if this modal base contains at least one r world, which is the case when (39-b) holds. As whenever (39-b) is the case, so is (40-b), the entailment in (41) holds.

(41) $(p \wedge q) \rightarrow \Diamond r \models p \rightarrow \Diamond r$

According to (41), one can weaken the antecedent of a conditional permission statement by removing conjuncts, but this leads to counter-intuitive predictions.[7] Intuitively, someone who accepts (39-a) does not necessarily accept (40-a) because the latter grants more permission; irrespective of whether you have a license or not, as long as the car has passed its technical inspection, permission is granted to drive. But this is counter-intuitive. If permission is granted when both of the conditions are fulfilled, one cannot just dismiss one of the conditions.

MadRis provides a straightforward semantic solution to this puzzle. According to the clauses for implication and permission, a state σ supports (39-b) when the maximal supporting state for $p \wedge q$, restricted to σ, supports $\Diamond r$. In turn, a state σ supports $\Diamond r$ when the maximal supporting state for r, restricted to σ, rejects v. So a state that supports p, q and r rejects v. As illustrated in figure 6.22, the state σ includes the world $p\bar{q}rv$ because (39-b) only eliminates v worlds when all three of p, q and r are the case. So the situation where your car passed its technical inspection (p) but you do not have a driver's license($\neg q$) is neutral, i.e., permission has not been granted.

On the other hand, a state supports (40-b) when the maximal supporting state for p, restricted to σ, supports $\Diamond r$, which is the case when r, restricted to σ, rejects v. The state σ does not include the world $p\bar{q}rv$ because p and r are the case in this world, and thus v cannot be the case. This means that the maximal supporting state for (40-b), illustrated in figure 6.23, only includes $p\bar{q}r$ worlds which reject v, so that (40-b) gives permission to drive if the car passed its technical inspection (p), but you do not have a license($\neg q$).

[7] While this is a puzzle concerning the weaking of the antecedent, we cannot call it that because *weakening the antecedent* standardly refers to $p \rightarrow r \models (p \vee q) \rightarrow r$.

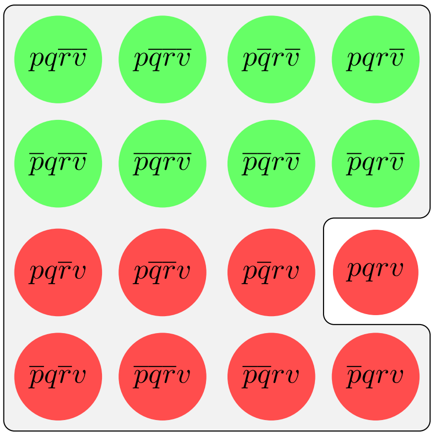


Figure 6.22: $[(p \wedge q) \rightarrow \Diamond r]^+$

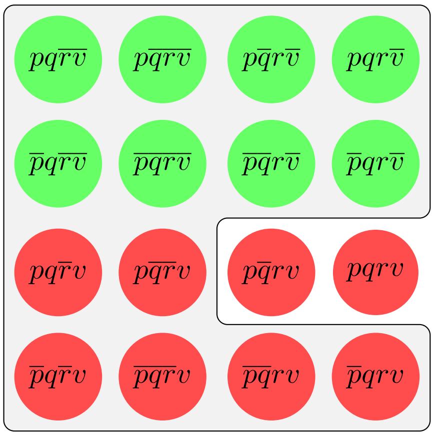


Figure 6.23: $[p \rightarrow \Diamond r]^+$

These figures illustrate that the maximal supporting state for $(p \wedge q) \rightarrow \Diamond r$ does not support $p \rightarrow \Diamond r$ because the former includes the world $p\bar{q}rv$, allowing us to conclude that fact 33 holds.

Fact 33. $(p \wedge q) \rightarrow \Diamond r$ does not support-entail $p \rightarrow \Diamond r$.

Fact 34. $(p \wedge q) \rightarrow \Diamond r$ does not entail $p \rightarrow \Diamond r$

As entailment requires both support-entailment and rejection-entailment to hold, we also conclude that fact 34 holds. So in MadRis, (39-a) grants less permission than (40-a), which accords with out intuitions about these sentences. This means that unlike Kratzer semantics, MadRis does not allow for counter-intuitive weakening of conditional permission sentences in the way that generate the *all or nothing* puzzle.

6.4.2 Conditional oughts

Recall from chapter 4 that conditionals of the following form provided problems for UM accounts of deontic modals.

(42) a. $p \rightarrow \Diamond p$
b. $p \rightarrow \Box p$

The reason why Kratzer semantics struggled with examples of the aforementioned kind is that the antecedent restricts the modal in the consequent. So, if one restricts the modal base to p worlds, every world will be a p world which satisfies both $\Diamond p$ and $\Box p$. Even if all worlds in the modal base are $\neg p$ worlds, restriction to p will leave the modal base empty, and both (42-a) and (42-b) will still be trivially true. This makes counter-intuitive predictions.

(43) a. If soldiers confiscate property, then soldiers may confiscate property.
b. If Britney Spears drinks Coke, then Britney Spears must drink Coke.

The sentence in (43-a) seems to say something about the trustworthiness of soldiers. These soldiers behave so well that they would never confiscate property unless they had been granted permission to do so. (43-a) says that all the times when these soldiers confiscate property, they have permission to do so, which has the effect of reporting that the soldiers have permission to confiscate property. Furthermore, the sentence can be, and probably often is, false. Unless soldiers are very trustworthy, they do confiscate property when they do not have permission for it. So, such a sentence ought not to be trivially true. On the other hand some native speakers reported that something is wrong with (43-a).

Example (43-b) is more difficult to understand. What it seems to say, if anything, is that Britney Spears would drink anything over Coke, and would only drink Coke when obligated to. Such a reading is not straightforward, though, and, intuitively, (43-b) requires correction.

In MadRis, sentences like (42-a) receive a straightforward contingent interpretation. Figure 6.24 illustrates that the maximal supporting state for (42-a) is the same as that for $\Diamond p$. This is the case because a state σ supports $p \rightarrow \Diamond p$ if the maximal supporting state for p, restricted to σ, supports $\Diamond p$. $\Diamond p$ is supported by σ if the maximal supporting state for p, restricted to σ, supports $\neg v$. As such, we restrict the maximal supporting state for p to σ twice, and the repetition adds nothing to the meaning of $p \rightarrow \Diamond p$ on top of the meaning of $\Diamond p$. But as $\Diamond p$ is contingent, MadRis does not make the counter-intuitive prediction that sentences like (42-a) are tautologies. Yet, as the meaning could be conveyed by the shorter $\Diamond p$, which explains why some native speakers find (43-a) odd.

The effect of (42-b) is shown in figure 6.25. As with UM deontic modals, MadRis also predicts that this conditional obligation sentence is trivially supported, albeit for a different reason. A state σ supports (42-b) if the maximal supporting state for p, restricted to σ, supports $\Box p$. But $\Box p$ is supported by σ if the maximal supporting state for $\neg p$, restricted to σ, supports v. This means that intersect both p and $\neg p$ with σ and then see if v holds. But as the restriction leaves only the empty state, and the empty state supports everything, all states support $p \rightarrow \Box p$.

In other words, the MadRis treatment of the conditional obligation in (42-b) requires you to restrict to a set of affairs p and then to answer whether that which has now become impossible ($\neg p$) leads to a violation or not. Yet, in such an inconsistent state, we can no longer distinguish between violation and non-violation worlds. As is standard in semantics, when a state is inconsistent in this way, MadRis predicts that every sentence is supported.

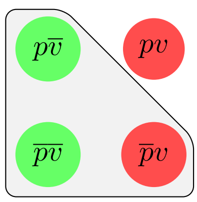


Figure 6.24: $[p \rightarrow \Diamond p]^+$

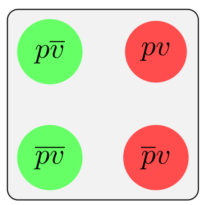


Figure 6.25: $[p \rightarrow \Box p]^+$

As things stand, MadRis does not offer a solution to the obligation variant of this puzzle but it does point at the probable solution. (42-b) is supported by every state due to the empty state. Recall that at the end of chapter 5 we discussed extending MadRis to the newly developed suppositional inquisitive semantics [45], which does not allow the empty state to support a sentence φ. In simple terms, it conceptualizes implications in such a way that a false antecedent leads to supposition failure.

When suppositional inquisitive semantics is extended with a MadRis treatment of deontic modals, a false prejacent will also result in supposition failure. Thus, the intersection of both p and $\neg p$ that occurs in (42-b) will no longer result in the state supporting v, as was the case here. Instead, a sentence of the kind in (42-b) will be predicted to suffer from supposition failure and there are no states that support it.

While the details will be left for future work, this approach promises an intuitive avenue to account for the counter-intuitive nature of (42-b).

6.5 Material implication puzzles

We will also consider conditionals to introduce the puzzle of *modus tollens* at the end of this chapter and to prepare for chapter 7 where we discuss the conditional puzzles from legal texts introduced in chapter 2. Alongside providing a radical treatment of modals, MadRis also inherits from radical inquisitive semantics (Ris) a radical treatment of implication. In this section we will discuss the puzzles for material implication from chapter 3 to show that similarly to Ris (see Lojko [67]), MadRis avoids these problems.

Chapter 3 listed puzzles that played a role in motivating suppositional accounts of conditionals. These were the false antecedent, true consequent and contraposition puzzles. Each of them is solved in MadRis.

6.5.1 False antecedent

This material implication puzzle rests on the fact that when the antecedent is false, the implication cannot be falsified. The false antecedent is represented by (44-a) and the implication by (44-b). We will represent material implication as $\rightarrow_m$.

(44) a. p
b. $\neg p \rightarrow_m q$

As one can see by comparing figures 6.26 and 6.27, whenever the antecedent is false, the material implication holds.

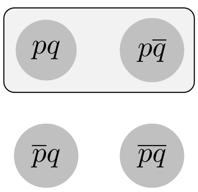


Figure 6.26: p

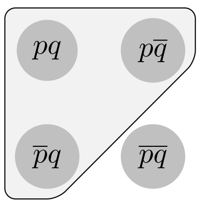


Figure 6.27: $\neg p \rightarrow_m q$

Thus, the false antecedent entails the material implication: (45).

(45) $p \models \neg p \rightarrow_m q$

The entailment allows for a number of counter-intuitive predictions such as the following.

(46) The butler did it; hence, if he didn't, the gardener did.

Intuitively, if the butler did not do it, a number of other candidates could have done it. So the entailment ought not to hold.

As this puzzle rests on the false antecedent, it receives an intuitive solution in suppositional inquisitive semantics discussed at the end of chapter 5. As the precursor to the suppositional account, a radical treatment of implication avoids the puzzle but does not entirely account for all the intuitions.

MadRis not only provides conditions under which a sentence is supported by a state but MadRis also specifies the conditions under which a state rejects (46). Consider the maximal rejecting state for (47) illustrated in figure 6.29.

(47) $\neg p \rightarrow q$

A state σ rejects (47) if the maximal supporting state for $\neg p$, restricted to σ, rejects q. As illustrated in 6.29, σ contains the world pq which supports p as the implication says nothing about whether the butler did it or not. This reflects the fact that to reject (47), one has to say something much weaker than to reject (44-a).

For example, take someone who knows that either the butler and gardener worked together or neither of them did it, ie. their information state contains pq and $\overline{pq}$ worlds. This person would reject the implication due to the $\overline{pq}$ world, but would not be in a position to reject (44-a) because in the world pq the butler might have done it. This means that the maximal supporting state for (47) does not reject (44-a) and fact 35 holds.

Fact 35. p does not rejection-entail $\neg p \rightarrow q$.

Fact 36. p does not entail $\neg p \rightarrow q$.

As entailment in MadRis requires both support-entailment and rejection-entailement, we can conclude from fact 35 that 36 also holds so, in MadRis, the counter-intuitive entailment is blocked because of rejection-entailment. This means that the two sentences are not sufficiently related for the negation of the conclusion to serve to reject the premise.

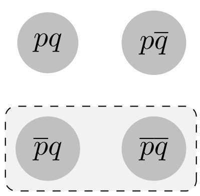


Figure 6.28: $[p]^-$

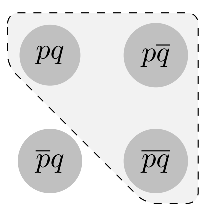


Figure 6.29: $[\neg p \rightarrow q]^-$

6.5.2 True consequent

The consequent (48-a) entails the material implication (48-b).

(48) a. q
b. $p \to_m q$
c. $q \models p \to_m q$.

The inference in (48-c) follows from the fact that material implication can only be falsified in situations where the consequent is false. As illustrated by figures 6.32 and 6.33, all states that make q the case also make $p \to_m q$ the case, so that it cannot be that (48-a) is the case without (48-b) being the case as well.

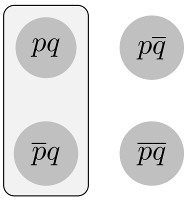


Figure 6.30: q

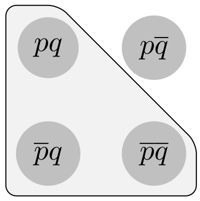


Figure 6.31: $p \to_m q$

This entailment leads to counter-intuitive inferences.

(49) John is in his office. Hence, if John was killed by a bomb this morning, then John is in his office.

We know that if John was killed, he will not be in his office. But if we are looking at John in his office, material implication predicts that (49) must be the case.

Material implication makes the problematic prediction because of the way in which implications are negated. Requiring that the antecedent is the case, and the consequent false makes rejecting an implication counter-intuitively difficult. In MadRis it is easier to reject the implication (50).

(50) $p \to q$

As is illustrated by figures 6.33 and 6.32, the maximal rejecting state for (50) is too weak to reject (48-a). This means that 6.33 does not rejection-entail (48-a).

Fact 37. *q does not rejection-entail $p \to q$.*

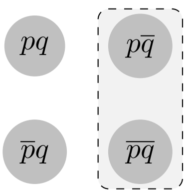


Figure 6.32: $[q]^-$

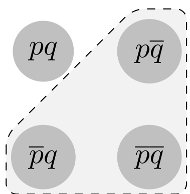


Figure 6.33: $[p \to q]^-$

For (48-a) to entail (50) in MadRis, we must establish both support-entailment and rejection-entailment. According to fact 37, (48-a) also does not entail (50).

Fact 38. *q does not entail $p \to q$.*

This accounts for the intuition that despite (48-a) including states in which (50) would be the case, there is no logical connection between (48-a) and the implication in (50).

6.5.3 Contraposition

For material implication, if the consequent is false, the antecedent must be false as well.

(51) $p \to_m q \models \neg q \to_m \neg p$

A comparison of figures 6.34 and 6.35 shows that the entailment in (51) has to hold. This captures the idea that a material implication is intuitively a valid inference, because when a inference is valid then the rejection of the conclusion would lead to the rejection of a premise. But a natural language conditional does not seem to be as strict.

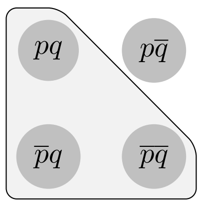


Figure 6.34: $p \to_m q$

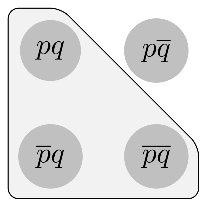


Figure 6.35: $\neg q \to_m \neg p$

Grice [39, pp. 78-79] told the following story to invalidate contraposition. Yog and Zog are playing chess with special rules. Yog gets white 9/10 times and there are no draws. They have already played around 100 games, and Yog emerged victorious in 80 out of 90 of the games in which Yog had white, but Zog won all the remaining games. In this case, the following two sentences have different probabilities.

(52) a. If Yog had white, Yog won. $p \to_m q$
b. If Yog lost, Yog had black. $\neg q \to_m \neg p$

The probability that the sentence (52-a) holds is 8/9 and the probability that the sentence (52-b) is the case is 1/2. The problem with this situation is that (52-a) and (52-b) are equivalent if analyzed as material implication. When you play chess, you use either the white or black pieces. So, playing with not white pieces is the same as playing with black pieces. And losing is the same as not winning when draws are taken out of the rules of chess. So if (52-a) is represented by $p \to_m q$ then its contraposition $\neg q \to_m \neg p$ is (52-b). But equivalent sentences should not have the different probabilities 8/9 and 1/2, respectively.

In MadRis, contraposition is not valid because (53-a) is not equivalent with (53-b). Consider the two sentences.

(53) a. $p \to q$
b. $\neg q \to \neg p$

As figures 6.36 and 6.37 illustrate, (53-a) is supported by the same states as (53-b). This corresponds to our intuitions regarding (52-a) and (52-b).

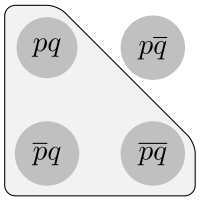


Figure 6.36: $[p \to q]^+$

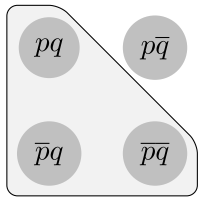


Figure 6.37: $[\neg q \to \neg p]^+$

To see where material implication and MadRis differ with respect to contraposition, consider (53-b). A state σ supports $\neg q \to \neg p$ if the maximal supporting state for $\neg q$, restricted to σ, rejects p. This eliminates all $p\overline{q}$ worlds. On the flip side, a state σ rejects $\neg q \to \neg p$ if the maximal supporting state for $\neg q$, restricted to σ, supports p. This eliminates $\overline{pq}$ worlds. The resulting maximal rejecting states for (53-a) and (53-b) are illustrated in figures 6.38 and 6.39, respectively.

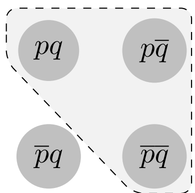


Figure 6.38: $[p \to q]^-$

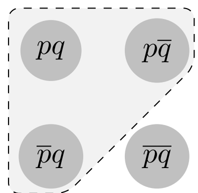


Figure 6.39: $[\neg q \to \neg p]^-$

Due to the different antecedent, the maximal rejecting state for (53-b) includes the world pq such that it is not a rejecting state for (53-a). This means that facts 39 and 40 hold.

Fact 39. $p \to q$ does not rejection-entail $\neg q \to \neg p$.

Fact 40. $p \to q$ does not entail $\neg q \to \neg p$.

As is standard, equivalence requires that two sentences mutually entail each other. Fact 40 is thus sufficient to demonstrate that fact 41 holds.

Fact 41. $p \to q$ is not equivalent with $\neg q \to \neg p$.

The different rejection conditions for the two sentences thus explain how two sentences with the same support conditions could differ in the way specified by Grice. If one is more likely to be rejected, it ought to be less likely that it holds.

6.6 Modus tollens

Modus tollens is a material implication inference pattern of the following form.

(54) $(\varphi \rightarrow_m \psi) \wedge \neg\psi \models \neg\varphi$

This inference patterns is connected to contraposition because it requires that when the consequent is false, the antecedent is false as well. There are a number of examples in which (54) appears to be intuitive. Here's an example by Yalcin [103].

(55) a. If a marble is big, then it is red.
b. This marble is not red.
c. Hence, this marble is not big.

(56) a. $p \rightarrow_m q$
b. $\neg q$
c. $\neg p$

Intuitively, someone who agrees with (55-a) and (55-b) would agree to (55-c). The reason for this is shown in figures 6.40 and 6.41. Every stateh that supports the conjunction also supports $\overline{p}$.

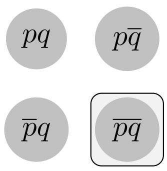


Figure 6.40: $(p \rightarrow_m q) \wedge \neg q$

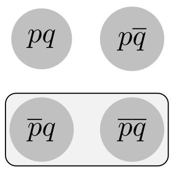


Figure 6.41: $\neg p$

Despite the support-side validity of the inference, there are a number of counterexamples. The oldest is Carroll's barbershop example [23]. While the original is a pleasant read, we will use Yalcin's formulation of the same puzzle. Imagine three barbers that work in such a way that one of them is always in the shop. This makes (57-a) true. Furthermore, one of the barbers, Allen, never leaves without Brown. This gives us the second premise (57-b).

(57) a. If Carr is out, then if Allen is out, Brown is in.
b. It's not the case that if Allen is out, Brown is in.
c. Hence, Carr is in.

A person that believes both (57-a) and (57-b), might still not believe (57-c) as it might be the case that Allen is in. This is problematic for modus tollens as the sentences are standardly represented with implication.

(58) a. $\neg p \rightarrow_m (\neg q \rightarrow_m r)$
b. $\neg(\neg q \rightarrow_m r)$
c. p

Carroll considered this puzzling because embedding $\neg q \rightarrow_m r$ in the consequent appears to fit the argument form of modus tollens, but we saw that intuitively (58-a) and (58-b) do not entail (58-c). But this is the case for material implication: (59)

(59) $(\neg p \rightarrow_m (\neg q \rightarrow_m r)) \wedge (\neg(\neg q \rightarrow_m r)) \models p$

The reason for the counter-intuitive result is the negation of material implication (58-b) which is rejected when both $\neg q$ and $\neg r$ are the case. As illustrated by figure 6.42, conjoining (58-a) with (58-b) eliminates most worlds, with the exception of one p world.

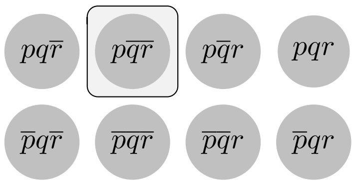


Figure 6.42: $(\neg p \rightarrow_m (\neg q \rightarrow_m r)) \wedge (\neg(\neg q \rightarrow_m r))$

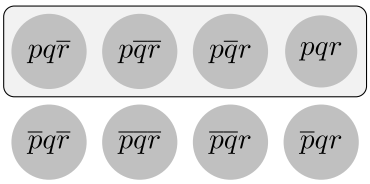


Figure 6.43: p

The rejection conditions for implication in MadRis are weaker than for material implication.

(60) a. $\neg p \rightarrow (\neg q \rightarrow r)$
b. $\neg(\neg q \rightarrow r)$

Consider the resulting maximal supporting state for the conjunction of (60-a) and (60-b) in figure 6.44.

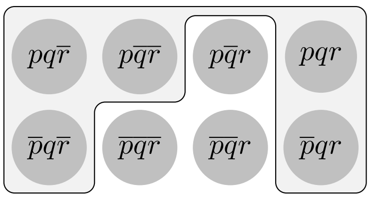


Figure 6.44: $[(\neg p \rightarrow (\neg q \rightarrow r)) \wedge (\neg(\neg q \rightarrow r))]^+$

As one can see in figure 6.44 the maximal supporting state for the conjunction of (58-a) and (58-b) in MadRis contains several more worlds than the material implication version. The state includes the worlds $\overline{p}q\overline{r}$ and $\overline{p}qr$ which reject p, thus, with the embedded implication (60-b), modus tollens fails both intuitively and in MadRis.[8]

There are natural language cases in which modus tollens appears to be intuitively valid. Yalcin's examples (57-a) and (57-b) do provide the information that (57-c) holds, but the fact is due to circumstances. As the inference in (58) is counter-intuitive, these examples cannot showcase a valid inference pattern.

[8]Lojko [67] showed that *modus ponens* remains valid in Ris and thus also in MadRis.

The validity of Yalcin's earlier example (55) rests on the fact that it includes only atomic sentences. A person that believes $p \to q$ and $\neg q$ intuitively believes $\neg p$. And, as illustrated by figures 6.45 and 6.46, MadRis also predicts that $(p \to q) \land \neg q$ support-entails $\neg p$.

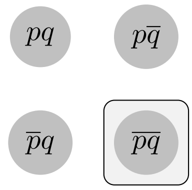


Figure 6.45: $[(p \to q) \land \neg q]^+$

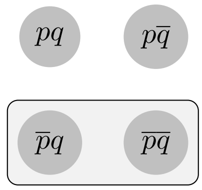


Figure 6.46: $[\neg p]^+$

Yalcin was mostly concerned with an example of modus tollens failure that concerned probability in the consequent but he also noticed that there is a macabre version of Carroll's puzzle with deontic obligation. Modifying Forrester's original example [32], one can provide the following counterexample to modus tollens.

(61) a. If you kill him, then you ought to kill him gently.
b. You ought not to kill him gently.
c. It's not the case that you will kill him.

Most people would agree to (61-a) and (61-b) but reject the conclusion that (61-c) holds. In fact, many people do the opposite of what they ought to do. Yet, this example once again has the argument form of modus tollens.

(62) a. $p \to \Box q$
b. $\neg \Box q$
c. $\neg p$

MadRis provides a straightforward explanation to the effect of deontic obligation in the consequent as it resembles the conditional case above. Consider figures 6.47 and 6.48.

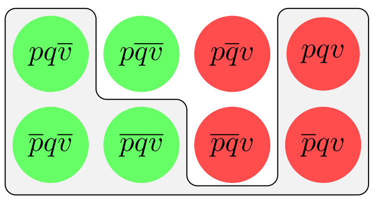


Figure 6.47: $[(p \to (\Box q)) \land (\neg \Box q)]^+$

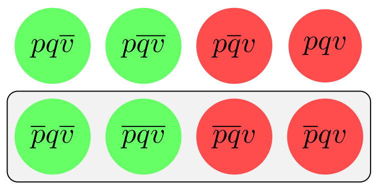


Figure 6.48: $[\neg p]^+$

As MadRis analyzes deontic obligation as a specific kind of implication, we see a similar result to Carroll's barbershop example. If modus tollens were valid, we would predict that all remaining worlds are $\neg p$ worlds, but figure 6.47 shows that the maximal supporting state for the conjunction of (62-a) and (62-b) includes two p worlds. This means that MadRis correctly predicts that in these cases, we cannot infer from (61-a) and (61-b) that (61-c) holds.[9]

[9]For more examples and insightful discussion of modus tollens, see Yalcin [103].

Chapter 7

Conclusions

This chapter concludes the dissertation but it does so in two very different sections. The first section of this chapter provides a summary of the entire dissertation. This summary is intended for linguists and we will make reference to the semantic puzzles that we addressed and provide a brief summary of the proposals. The section will include discussion of the formal tools, which might make the summary inaccessible to people without formal training. We will also collect examples from various chapters of the dissertation which open avenues for future work.

The second section is meant for lawyers and it is not a summary, *per se*, but rather an application of the proposed semantics to legal language. Recall that chapter 2 introduced a number of cases where lawyers faced interpretation problems with certain sentences. In chapter 2, we discussed the puzzling examples and associated examples from the linguistics literature in detail but without formal tools. The formal tools will be applied in section two of this chapter.

The reason why we did not apply formal rules in chapter 2 is because there was no uncontroversial candidate for a theoretic framework that would provide the required tools. Despite the fact that the simple versions of the examples themselves can be accounted for in basic propositional logic, chapters 3 to 4 discussed available frameworks and found that the standard accounts run into trouble with examples embedded under the deontic modals permission, prohibition and obligation. This makes the accounts unsuitable for application to puzzles of legal language as deontic modals are frequent in them. For example, the standard account for deontic modals in the literature does not account for the salient reading of *or* under permission, which limits our ability to discuss examples with *or* from legal texts. To solve these puzzles, in chapter 5 we introduced a new semantics called MadRis and demonstrated in chapter 6 that this new semantics avoids the puzzles for the standard account.

The summary for lawyers takes the puzzles from chapter 2 and demonstrates how they are solved with the help of MadRis. As the section is aimed to be readable by people with no formal training in logic or semantics, we will explain the solution, but omit the derivation itself. This means that we will present the puzzles again to pose the question and answer it by referring to the solution that MadRis gives us.

Note that we are not trying to change the consensus view but rather to provide a tool that can be applied to wider range of examples than those we had before. In section 2 of chapter 7, we will demonstrate how a MadRis analysis makes the meaning of puzzling examples explicit and illustrates the solutions graphically for easy presentation. This is the type of formal analysis that a lawyer can expect from a linguist, alongside the discussion of examples in chapter 2. We will show that MadRis captures the consensus view on the interpretation of *or* and *if-then* in the puzzling sentences from chapter 2.

The fact that this section is aimed at lawyers does not mean that semanticists should not read it at all. The final part of the section recounts some personal experience from working with lawyers. While this might not be of theoretical interest, it might be helpful in future interaction between people working in either field.

7.1 Summary of the dissertation

Despite the fact that there exists a lengthy history of documented cases where lawyers, judges and legal theorists complain about problems with interpreting natural language expressions in courts of law, there has been little cooperation between linguists and legal professionals. This dissertation follows in the footsteps of Solan's [94] trailblazing study of the application of formal methods to the interpretation problems of lawyers. Among other things, Solan discussed the *exclusive/inclusive or* puzzle in courtrooms. We investigated his examples and reinstated the consensus view in the literature that *or* is *inclusive*. We then proceeded to extend Solan's investigation of the applicability of formal tools to the interpretation problems of lawyers in three ways.

First, we added examples from an additional source: the World Trade Organisation (WTO) legal disputes. This demonstrated that similar problems with the interpretation of natural language occur also in other legal domains beside the US court system. Furthermore, WTO example cases are even more suited for linguistic analysis as WTO adjudicators have fewer discretionary powers to change the rights and obligations of WTO member states than, for example, judges in the American court system. This means that the interpretation of WTO agreements must remain close to the meaning of the natural language expressions in the legal text.

Secondly, we included a different type of problematic natural language expressions, *conditional sentences*, which have also been extensively studied in the literature.

Thirdly, Solan applied formal tools mainly from the syntax and psycholinguistics literature, while relying on propositional logic for the semantics. As the standard treatment of *conditional sentences* in the literature is based on a semantics for modal expressions, we need to move beyond propositional logic and incorporate formal tools from modal logic. This is a natural way to extend the study of puzzles for lawyers as legal texts commonly involve prohibitions, obligations and permissions, all of which are analyzed as deontic modalities.

Before we could apply formal tools to the problematic examples, we had to choose an appropriate formal framework. We began by surveying the literature on *conditional sentences*. Despite the fact that propositional logic has a connective for *conditional sentences*, *material implication*, its behaviour differs in many ways from natural language *conditionals*. We discussed some of the more prominent puzzles for material implication in chapter 3. These puzzles are both a motivating factor for the wide acceptance of the Kratzer treatment of *conditional sentences* in the literature and a challenge for any new treatment. We returned to these puzzles in chapter 6, after we had proposed a new semantics for deontic modals, and saw that the puzzles for material implication do not reappear in MadRis.

Standard modal logic suffers from a number of well-known puzzles. We reviewed the account, its refinement by Kratzer and the puzzles for both accounts in chapters 3 and 4. Due to the sheer number of puzzles for the standard account, we limited ourselves to four types of puzzles: *deontic conflicts*, *monotonicity puzzles*, *puzzles with conditionals and deontic modals* and the *free choice* puzzle.

We did not discuss gradability of deontic modals nor various puzzles regarding reasoning with deontic modals such as the *miner's puzzle*. As the treatment of deontic modals in MadRis differs from the standard treatment, it is unlikely that it faces the same puzzles with gradability of deontic modalities as the standard treatment but MadRis in its minimal formulation does not incorporate any formal tools to discuss gradability. The development of such tools is left for future work. MadRis also does not currently incorporate pragmatics, which means that a discussion of contextual factors is limited. For this reason, we have not discussed puzzles similar to the *miner's puzzle*. It is worth nothing, though, that the brief discussion of *exclusive or* in chapter 5 suggests that if pragmatic strengthening were included in the framework, the proposed deontic modals would remain well behaved.

A review of the literature on deontic puzzles and especially on *free choice* demonstrated that, broadly speaking, there were three types of solutions: pragmatic mechanisms, alternative-based semantics for *or* and non-standard definitions for deontic modals. There was also overlap between the proposals so these distinctions are only illustrative of general trends.

Contrary to several proposals that follow Kamp's [51] original suggestion, the literature includes a large number of observations which suggest that the *free choice* phenomenon is not pragmatic in nature. The standard pragmatic explanation of *free choice* states that when someone grants permission, they are not ignorant as to which permissions hold and, thus, the *ignorance reading*[1] is pragmatically replaced by a *free choice* reading. Yet, the *free choice* reading also arises in contexts where the speaker is manifestly ignorant. Furthermore, most prominently, *free choice* examples resist can-

[1]See for example chapter 5, section 5.5.1

cellation despite cancellation being a standard characteristic of pragmatic reasoning. For these and other reasons, we considered semantic solutions instead.

One of the core characteristics of natural language *or* is its ability to generate alternatives. This and related phenomena such as natural language *any* has led to the development of alternative-based semantics. Following Kratzer and Shimoyama [61], several authors have utilized an analysis where *or* generates sets of propositions, where the alternatives correspond to the denotations of the disjuncts. In this tradition, Aloni [9] suggested that the standard treatment of deontic modals ought to be modified such that every alternative satisfies permission. This is accomplished by quantifying over alternatives such that permission is applied to each alternative by universally quantifying over alternatives, which provides the *free choice* reading as permission is granted to bring about each alternative.

We adopt inquisitive semantics where we can also discuss alternatives. However, Aloni's definition for obligation differs from our proposal as it is applied only to at least one alternative by existentially quantifying over alternatives. We suggest that obligation, similarly to permission, ought to also apply to every alternative. Such a treatment of deontic modals is motivated by the idea that deontic modality rejects *ignorance readings* where it is not known which of the alternatives holds. Intuitively, the authors of a rule establish what is permitted, but also what is prohibited, and what is obligatory, and this leaves no room for ignorance. *Ignorance readings* are a feature of everyday discussions as a subject of rules might not know precisely what is permitted and what is required, so he or she could, for example, utter a disjunction believing that some prohibition holds but not having enough information to specify which one. So, we propose that deontic modals always quantify universally over alternatives.

Another approach to *free choice* examples provides an alternative semantics for permission and obligation. The main rival to the standard account is an Andersonian [15] reduction which treats deontic modals as implications to violations. For example *permission to drive* ($\Diamond p$) is analyzed as an implication - if you do drive, no violation occurs ($p \rightarrow \neg v$). And, if walking a dog is obligatory ($\Box q$), then not walking a dog implies that a violation does occur ($\neg q \rightarrow v$). Such accounts have been proposed as a solution to the *free choice* puzzle, most recently by Asher and Bonevac [17] and Barker [18], as standardly when disjunction is embedded in the antecedent of an implication then the consequent holds for each disjunct. This means that the following equivalences hold in standard propositional logic.

(1) $$\Diamond(p \vee q) \equiv (p \vee q) \rightarrow \neg v \equiv (p \rightarrow \neg v) \wedge (q \rightarrow \neg v)$$

The equivalences in (1) illustrate how an Andersonian deontic semantics predicts that both disjuncts in a *free choice* example are permitted. We adopted such an Andersonian approach to deontic modals to account for the *free choice puzzle*, and this turned out to be especially well suited for a framework intended to be applied to the interpretation problems of lawyers as our investigation of WTO legal texts revealed that WTO adjudicators explicitly discuss violations. Incorporating violations as propositions directly into the semantics allows us to mimic discussion of permissions, obligations and prohibitions in the courtroom.

In brief, our proposal is to adopt a semantics for deontic modals where we quantify universally over alternatives and treat modals as implications to violations. For permission, $\Diamond\varphi$, to hold, none of the alternatives for φ can include a violation. For obligation, $\Box\varphi$, to hold, all the alternatives for $\neg\varphi$ must include a violation.

MadRis provides a three-fold deontic characterization of statements. Consider the simple case p.

Permitted When all p worlds are also $\neg v$ worlds, we say that p is *permitted.*

Prohibited When all p worlds are v worlds, we say that p is *prohibited.*[2]

Neutral When some p worlds are v worlds and others $\neg v$ worlds, we say that p is *neutral.*

The deontic modals are realized within the framework of radical inquisitive semantics developed by Groenendijk and Roelofsen [44] and Sano [89]. The framework was chosen because it provides several advantages. For example, radical inquisitive semantics provides a treatment where *or* brings about alternatives, although, strictly speaking, it is not a standard alternative-based semantics as the alternatives are generated by the semantics rather than the syntax. Furthermore, radical inquisitive semantics provides a treatment of *conditional sentences* the predictions of which are very close to the widely accepted Kratzer account of conditionals and it also avoids the puzzles of material implication. Crucially, as radical inquisitive semantics specifies both support and rejection conditions for its connectives, we can easily define deontic modals as universally quantifying over alternatives both on the support and reject side. This accounts for the intuition that prohibitions, the contraries of permission, also resist *ignorance readings.*

These components give MadRis its name as Ris refers to Radical Inquisitive Semantics which is extended to incorporate a Modified account of Andersonian Deontic modals or Mad in abbreviated form.

The modification of the Andersonian proposal by introducing quantification over alternatives also brings about the crucial difference with regard to Anderson's original intuition. Despite the similarity of MadRis modals to implication, the deontic modals are not defined via implication as they behave differently in certain cases. Consider the prohibition in (2) where the prejacent gives rise to alternatives.

(2) A country may not establish a research center or a laboratory.

The salient reading of (2) says that it is against the rules for a country to establish a research center and it is against the rules for a country to establish a laboratory. According to Anderson, the prohibition in (2) could be expressed as the implication in (3).

[2] Obligations such as $\Box p$ are thus understood as prohibitions against doing p.

(3) It is not the case that if a country establishes a research center or a laboratory then no violation occurs.

The salient reading of (3) is different from (2) as it says that a country will avoid a violation either when it establishes a research center or a laboratory, but it is not known which. The conditional in (3) is generally represented in propositional logic as (4-a), which is equivalent with (4-b).

(4) a. $\neg((p \vee q) \rightarrow \neg v)$
b. $\neg(p \rightarrow \neg v) \vee \neg(q \rightarrow \neg v)$

If deontic permission were defined through implication, as it is in Anderson's account, then we would predict that a country will not avoid a violation either when it establishes a research center or a laboratory, but it is not know which as was the case with (3). But the salient reading of (2) is that a country is prohibited from establishing either, rather than only one.

In MadRis this difference between negated permission sentences and conditionals is reflected in the quantification over alternatives. While deontic modals resist *ignorance readings* and thus have universal quantification over alternatives in both the positive and negative case, negated conditionals have ignorance readings, which is reflected in MadRis by existential quantification over alternatives. So, even though there is indeed considerable overlap between implication and modals, they are distinct in the semantics.

With these components, MadRis makes intuitive predictions concerning the *free choice* phenomenon. As the deontic modals are Andersonian, the standard behaviour of disjunction in the antecedent of an implication accounts for the intuition that permission is granted to both disjuncts. Furthermore, the modification to add universal quantification over alternatives to Andersonian violation-semantics provides a semantic solution to the behaviour of *free choice* examples under negation, which has proven challenging for most accounts in the literature.

Recall that we said that introducing violations directly into the semantics allows us to make conceptual choices in our semantics. At the end of chapter 5, we consider alternative conceptualizations for dealing with *deontic conflicts*, i.e., situations where one cannot avoid violating all rules. For example, when there is a *conflict of obligations*, and one cannot avoid violating at least one of the obligations, a semantics ought to represent this situation as a choice between unfortunate consequences. The violations can be differentiated either by associating them with different authorities or different rules. A closer look at WTO panel reports demonstrated that WTO adjudicators make reference to different rules, even when the authority remains the same, so we standardly assign to each different rule a different violation.

Consider the example of a teenager who is given conflicting orders.

(5) a. You must leave your room. $\Box p$
b. You may not leave your room. $\neg\Diamond p$

According to the standard account (5-a) is the case when all best worlds are p worlds and (5-b) is the case when there are no p worlds among the best worlds. As the best worlds cannot both be p worlds and not p worlds, the standard account predicts that (5-a) and (5-b) are contradictory. This is not intuitively correct as such deontic conflicts are commonplace. The same puzzle also resurfaced as part of the *dr. Procrastinate puzzle* discussed earlier in sections 4.2.3.1 and 6.3.1.2.

MadRis solves the problem of deontic conflicts by making the different violations explicit. This can be easily done in an Andersonian semantics as violations are introduced as a designated atom. The literature has assumed that there is only one such atom, but we follow the way in which WTO adjudicators reason and allow each permission, obligation and prohibition sentence to generate a different violation atom. This means that examples such as (5-a) and (5-b) are no longer contradictory as they refer to different violations. In chapter 6 we demonstrated that such an explication provides intuitive predictions regarding the standard examples of deontic conflicts in the literature.

Another class of puzzles for the standard account of modals concerns *upward monotonicity*. When an operator O is upward monotone, then all entailments that hold between sentences in the propositional case also hold when embedded under the operator. The standard account of deontic modals satisfies the property of upward monotonicity which led to problematic examples such as *Ross's puzzle*[86].

(6) a. A country must establish a research center. $\Box p$

b. A county must establish a research center or invade its neighbour. $\Box(p \vee q)$

In the propositional case, p entails $p \vee q$, so any upward monotone semantics for deontic modals predicts that whenever (6-a) holds, so does (6-b). But this is highly counter-intuitive. Deontic operators are not upward monotonic in MadRis and we demonstrated in chapter 6 that such counter-intuitive predictions do not go through. This also applies to the *dr. Procrastinate* puzzle where the combination of introducing multiple violations and non-monotonicity of modals provides an intuitive prediction concerning the situation. Some authors have argued that upward monotonicity is a required property of deontic modals, but we demonstrate in chapter 6, section 6.3.3 that MadRis makes intuitive predictions in the cited cases.

MadRis is non-monotonic but it is not a defeasible semantics. In a defeasible semantics, inferences do not go through in all contexts, particularly in those contexts which are deemed abnormal. The main idea behind rejecting a defeasible semantics is laid out in chapter 4, section 4.5.2.3. According to the legal principle *legal certainty*, a subject of laws wondering whether doing something is illegal should be able to read the relevant law and decide the matter on his own. To achieve this, legal interpretation ought to hew closely to the ordinary meaning of words and judges should be constrained in their discretion to reinterpret the meaning of law. To make deontic modals themselves defeasible erodes this principle as the meaning of a deontic statements will always depend on interpretation.

Kratzer semantics utilizes the standard account of modality to propose an approach to conditional sentences. In Kratzer semantics the antecedent of a conditional restricts the domain for a (generally covert) operator that quantifies over the consequent. For example, when there is an overt deontic modal, the antecedent restricts this permission, obligation or prohibition modal. Such an approach generates additional dubious entailments. We consider the new *all or nothing* puzzle which concerns eliminating conditions for granting permission.

(7) a. If the car passed its technical inspection and you have your license, you may drive. $(p \wedge q) \rightarrow \Diamond r$

b. If the car passed its technical inspection, you may drive. $p \rightarrow \Diamond r$

The problem is that in Kratzer semantics, for (7-a) to hold, the best worlds must include an r world among the pq worlds. But any such world also satisfies (7-b) as well. Intuitively, it is not plausible, however, that one can discard a condition for granting permission.

MadRis utilizes the account of conditional sentences from radical inquisitive semantics [44] which does not depend on the antecedent restricting the domain for a covert or overt modal in the consequent, so it straightforwardly avoids making such deviant predictions.

MadRis also provides a solution to the *conditional oughts* puzzle by Jackson [50]. This was originally considered an upward monotonicity puzzle, but the permission and obligation versions behave slightly differently. The permission version receives a straightforward solution in MadRis and we discuss the obligation version in the outlook section below.

Furthermore, Lojko [67] has shown that radical inquisitive semantics avoids the puzzles of material implication. We discuss and illustrate the solution to some of the more prominent puzzles in chapter 6 to demonstrate this new approach to conditionals. This prepares the ground to consider *modus tollens*, the inference pattern which says that if both $\varphi \rightarrow \psi$ and $\neg\psi$ are the case, then so is $\neg\varphi$. While it appears intuitive for simple cases, this inference pattern has been criticized in conjunction to conditional sentences at least as early as the 19th century, and recently by Yalcin [103] who noted that problematic examples include both conditionals and deontic modals embedded in the consequent. In MadRis, *modus tollens* is not a valid inference pattern so neither the puzzling entailments with conditionals nor deontic modals go through.

Furthermore, MadRis introduces a notion of entailment which is a combination of support-entailment and rejection-entailment, which explains why in simple cases *modus tollens* appears to be a valid inference pattern. In these simple examples, the premise support-entails the consequent but it does not rejection-entail it.

7.1.1 Outlook

The treatment of deontic modals in this dissertation also opens up several avenues for future work We already mentioned that we excluded gradability and some puzzles of deontic modals. But also recall from chapter 3, section

3.4.1 that modal auxiliaries can receive several interpretations. Consider the example sentences in (8).

(8) a. John *must* pay his taxes.
    b. John *may* drive a car.

In this dissertation, we mostly discussed the deontic reading according to which (8-a) obligates John to pay his taxes and (8-b) gives John permission to drive a car. The modal auxiliaries can also be interpreted epistemically, according to which (3-a) says that, as far as we know, John pays his taxes. (3-b) says that as far as we know, it is possible that John drives a car.

In chapter 3, section 3.4.1 we observed that one of the advantages of the standard account is that the different interpretations of modals can be accounted for by merely changing the accessibility relation. For a deontic reading, the worlds are accessible according to rules and regulations. For an epistemic reading, the worlds are accessible according to what is known.

A treatment of deontic modals in terms of implications to violations does not lend itself to a similar explanation of the interpretations. This is because there is no accessibility relation to vary. The deontic interpretation arises due to the violation in the consequent. Instead, the different modal interpretations will have to be accounted for by some other structural similarity between the treatment of deontic modals in MadRis and a treatment of epistemic modals to be developed.

As there is no widely accepted treatment of epistemic modals as implication, we need to say a bit more about what such an account might look like. The first aspect to vary to attain the different modal interpretations is the content of the consequent, so that, for example, the epistemic treatment of (8-a) would say that the state of affairs where John does not pay his taxes implies something absurd. Whether such an account is viable cannot be determined before its details are fleshed out, so discussion must necessarily remain speculative, but such a treatment of epistemic modals is currently in development by the inquisitive semantics group in connection to *suppositional inquisitive semantics*.[3]

Some puzzles with *deontic conflicts* and *conditional oughts* also suggest that the behaviour of the proposed deontic modals should be further investigated in suppositional inquisitive semantics or some other formal treatment of suppositional content. For example, the conjunction of a permission and prohibition in (9) results in a special kind of *deontic conflict*. Previously, in (5-a) and (5-b), a violation occurred irrespective of whether p or $\neg p$ is the case.

(9) $\Diamond p \wedge \neg\Diamond p$

According to (9), assuming that both the permission and prohibition are assigned the same violation, then when p is the case, a violation both occurs and does not occur, which is absurd. Yet, the conditional as a whole is not absurd as it merely eliminates all worlds in which p is the case. This result

[3]Unfortunately there is no draft paper to refer to at this point. Please see https://sites.google.com/site/inquisitivesemantics/ site for updates.

signals that something is wrong with (9) because permission and prohibitions sentences should not say whether a fact of the world is the case or not. As (9) is only supported by states in which $\neg p$ holds, it lends itself to an analysis where one supposes that the antecedent or, in this case, the prejacent, holds, and when such a supposition is impossible, the analysis would predict that the sentence is infelicitous.

We also considered the *conditional oughts* puzzle by Jackson [50]. Recall that MadRis accounts for the intuition that the permission and obligation sentence versions behave differently. In relation to suppositional content, we are interested in the version with obligation, repeated in (10).

(10) $p \to \Box p$

As we discussed in chapters 4 and 6, (10) is a tautology in both the standard treatment of deontic modals and in MadRis. But the result comes about in a very different manner. Part of the meaning of (10) in MadRis can be paraphrased as: supposing that p is the case, suppose also that p is not the case. This is also an example of supposition failure as we cannot suppose both the antecedent p and the prejacent $\neg p$ at the same time. This means that we need a suppositional extension to add to the MadRis explanation of these phenomena.

7.2 Summary for lawyers

In chapter 2, we introduced a number of example sentences which have created interpretation problems in courtrooms. Most of the dissertation was dedicated to searching for a suitable semantic framework to apply to the examples in chapter 2. We formulated a semantics called MadRis which behaves better in legal contexts because, unlike previous semantic accounts, it adheres to well established intuitions regarding the behaviour of natural language connectives *or* and *if-then* in permission, prohibition and obligation sentences. This means that we can use MadRis to bring clarity to the discussions happening in courts of law. The following application of these new formal tools to the problematic examples is intended to be comprehensible without knowledge of chapters 5 and 6 where we introduced MadRis.

In chapter 2 we discussed the problematic examples by referring to the linguistics literature on the same issues. The discussion essentially summarized the consensus view in linguistics at the moment, which gave us the following three findings.

***Or* is not exclusive** Exclusivity inferences fail in all linguistic tests, both in everyday and legal language. So, without additional information, when *A or B* is the case, *A and B* might be the case as well.

If A, then B* does not imply *if B, then A The semantics literature includes a large number of examples where conditional sentences cannot be reversed. So, *if-then* sentences are not biconditional.

If A, then B* does not imply *if not A, then not B The semantics literature also includes a large number of examples where B is brought about without A being the case. So, *if* clauses provide a sufficient but not a necessary cause for B to be the case.

In the following sections, we will demonstrate how one can apply formal tools from semantics to these examples, especially when the problematic expressions are embedded under permission or obligation. We will see that the findings remain the same also in embedded cases. What a lawyer can expect from a linguist is an expert analysis that combines discussion of the linguistics literature with a formal application of semantics to present the results. We will demonstrate in the following sections how the framework MadRis makes predictions in each of these cases so as to make the interpretation issues explicit for discussion in the courtroom. We will also introduce notation and illustrative graphics to help present the results of the analysis.

7.2.1 Notation

In this section we will re-introduce notation needed to present the results of the analysis. This is meant for the benefit of those readers that did not read chapter 5. If you did read that chapter, feel free to skip to the next subsection.

It is sufficient for the analysis of the problematic examples to look at whole sentences as our basic units. Consider the following simple examples.

(11) a. There is a cup on the table.
  b. There is no cup on the table.

We capture the meaning of (11-a) by referring to contexts where the sentence can be truthfully used. To represent the sentence (11-a) in logic, we take a letter of the alphabet and ascribe to it the contexts in which (11-a) can be used, and to represent (11-b), we add the negation operator $\neg$.

(12) a. p
  b. $\neg p$

(12-b) is the case in all contexts where (12-a) is not used. To discuss different contexts, we will use the technical device of referring to possible worlds. With respect to (11-a) and (11-b), we can capture all relevant contexts with just two possible worlds as shown in figure 7.1. All contexts where p, or (11-a), is the case are represented by the p world and contexts where $\neg p$, or (11-b), is the case are represented by the $\overline{p}$ world.

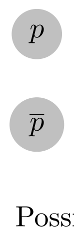


Figure 7.1: Possible worlds

When someone sincerely says that p is the case, their mental state excludes all worlds where p is not the case. We will represent such states as sets of possible worlds, which are drawn on the figures as a shape with the possible worlds inside it. This is illustrated in figure 7.2 for (12-a). Anything outside of the box is out of the realm of possibility for the speaker, so, figure 7.2 represents the fact that there is a cup on the table. We will see later that there can be several different (but potentially overlapping) states in which a sentence is felicitous.

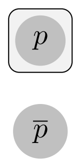


Figure 7.2: p

The semantics we outlined in chapter 5 specifies the meaning of a number of connectives - *or*, *and* and *if-then*. These are expressed by $\wedge, \vee, \rightarrow$, respectively.

Besides connectives and negation, for legal language we need formal tools to represent obligation ($\Box p$), permission ($\Diamond p$) and prohibition ($\neg\Diamond p$). The *must* in the following sentence obligates.

(13) There must be a cup on the table.

We will represent the obligation sentences in (13) with $\Box p$. The square marks the obligation, and the arbitrary letter p represents that which is obligatory, in this case that there is a cup on the table. Permission sentences will be represented by $\Diamond p$ and prohibition is the contrary of permission, represented by $\neg\Diamond p$.

Finally, we also need to introduce semantic inferences. For example, whenever (14-a) is the case, so is (14-b).

(14) a. There is a cup and a plate on the table. $p \wedge q$
b. There is a cup on the table. p

We say that a sentence entails ($\models$) another if in every situation where the premise sentence is the case the concluded sentence cannot be false. We can use the figures to check whether every world that supports the premise supports the conclusion. In this case, figures 7.3 and 7.4 illustrate that the world that supports (14-a) also supports (14-b).

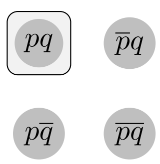


Figure 7.3: $p \wedge q$

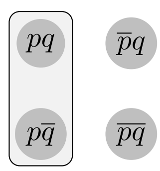


Figure 7.4: p

7.2.2 Is *or* exclusive?

The first puzzle concerns the question whether the connective *or* is inclusive or exclusive. As we previously stated, in this summary we will limit ourselves to only presenting the results and avoid discussing the motivation for the semantic framework MadRis. We will also not reiterate the discussion of the literature in chapter 2.

It is worth noting, however, that the following analysis is concerned with the meaning of *or* itself and its interaction with modal operators, and does not consider the role of pragmatic inferences stemming from the context of the utterance. Such inferences would necessarily figure in a fuller treatment of *or*. For the following basic examples, however, we will constrain ourselves to semantic inferences. To elaborate the difference between semantic and pragmatic inferences, consider (15)

(15) I will invite John or Mary to the party.

The salient reading of (15) says that either John or Mary will be invited but not both. Yet it is commonly accepted in the linguistics literature (see, for example, Huddleston [48, p. 1294]) that *or* does have an *inclusive* reading in examples such as (15). The salient reading is considered to be the result of a pragmatic inference in the Gricean [40] sense, i.e., we guess from the speaker's choice of words what his intent is. The person who uttered (15) should be aware of the possibility of uttering the stronger alternative with *and* provided in (16).

(16) I will invite John and Mary to the party.

If the speaker intended to invite both, (16) would capture that idea much more precisely than (15). From the fact that the speaker chose to use (15) instead, we can thus infer that the speaker intends not to invite both. But such pragmatic inferences are about what the speaker meant to say, rather than the meaning of *or* in what the speaker did say.

If *or* is analyzed exclusively, we can semantically infer that both disjuncts are not the case. This means that the question whether *or* is exclusive can be paraphrased in our notation as follows. If $p \vee q$ is the case, is it valid to semantically infer that $\neg(p \wedge q)$ also has to be the case?

We will discuss this in three steps, looking at *or* in *everyday sentences*, *permission sentences* and *obligation sentences*.

7.2.2.1 Everyday sentences

An everyday example of *or* is reproduced here with its representation in MadRis.

(17) a. There is a cup or a plate on the table.
b. $p \vee q$

For everyday sentences, when *or* is exclusive, whenever the sentence in (17-a) is the case, one could infer that the following negated *and* sentence would also be the case.

(18) a. There isn't a cup and a plate on the table.
b. $\neg(p \wedge q)$

For (17-a) to be exclusive there would need to be an entailment relation from (17-b) to (18-b). This relation would say that whenever (17-a) is the case, (18-a) would also hold. But the entailment does not hold in MadRis.

(19) $p \vee q \not\models \neg(p \wedge q)$

The reason why there is no entailment is illustrated in the following figures. Figure 7.5 shows the maximal supporting states for (17-b) and figure 7.6 shows them for (18-b). Note that there are two such states for both sentences; for (17-a) there is one which includes worlds where there is a cup on the table and a second with worlds where there is a plate on the table. As there can be both a cup and plate on the table, the sets of worlds overlap. For the inference to be valid, every state of affairs that supports (17-b) needs to support (18-b). However, the maximal supporting states for (17-b) include the world where both p and q are the case, and this world obviously does not support (18-b). Thus, entailment does not hold in MadRis and, in *everyday sentences*, *or* is predicted to be inclusive.

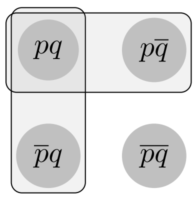


Figure 7.5: $p \vee q$

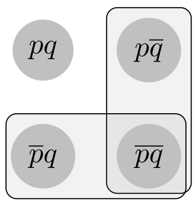


Figure 7.6: $\neg(p \wedge q)$

MadRis helps illustrate that it is possible for someone to say that (17-a) is the case, and yet believe that there is both a cup and a plate on the table. As we said earlier, there could also exist contextual clues that the speaker meant only one of the two things, but this would be additional to this standard meaning of *or*. Next, we also need to investigate whether *or* remains inclusive in legal language.

7.2.2.2 Permission sentences

As the first instance of a sentence in legal language, we will take a look at permission sentences. What this means is that we take the modal auxiliary *may* which grants permission and embed *or* sentences under *may* such that permission is granted to bring about the disjuncts.

As already noted, an advantage of using MadRis is that it provides a plausible account of the behaviour of disjunction under permission modals. Here we shall demonstrate how MadRis captures inclusive readings of *or* under permission, illustrated by the example from chapter 2 repeated below as (20-a).

(20) a. You may take a starter or a dessert.
 b. $\Diamond(p \vee q)$

If (20-a) were exclusive, each situation in which (20-a) is the case, would also be a situation in which it is prohibited to take both a starter and a dessert i.e., (21-a) would hold.

(21) a. You may take neither a starter nor a dessert.
 b. $\neg\Diamond(p \wedge q)$

Formally, were (20-a) exclusive, each situation where (20-b) holds would be a situation where (21-b) also holds. But this is not the case.

(22) $\Diamond(p \vee q) \not\models \neg\Diamond(p \wedge q)$

To see this, compare figures 7.7 and 7.8, with special attention to the world pq. Recall that rules can be violated and we mark possible worlds where laws are violated. Any possible world with a v is a world where a violation occurs, and a $\overline{v}$ world is one in which no violation occurs. Furthermore, to make the figures easier to grasp, we have coloured worlds where no violation of rules occurs green and worlds where a violation does occur red.[4]

We are comparing pairs of worlds where the same sentences, for example p and q, hold to see whether v, $\neg v$ or both are inside the supporting state.

Prohibition For taking both a starter and a dessert ($p \wedge q$) to be prohibited, it must be that pq is a violation world and all non-violation pq worlds are excluded i.e., $pq\overline{v}$ is outside of the drawn shapes.

Neutrality If both pqv and $pq\overline{v}$ worlds remain within the drawn shapes, we say that pq is neutral, i.e., it is neither prohibited nor explicitly permitted.

Permission If only non-violation pq worlds remain within the drawn shapes and all violation pq worlds have been excluded, we say that the sentence grants permission to bring about pq, and so one is permitted to take both a starter and a dessert.

The supporting states for *or* under the permission modal *may* are shown below.

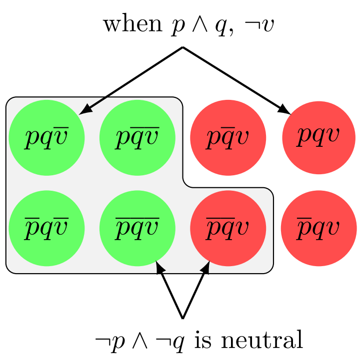


Figure 7.7: $\Diamond(p \vee q)$

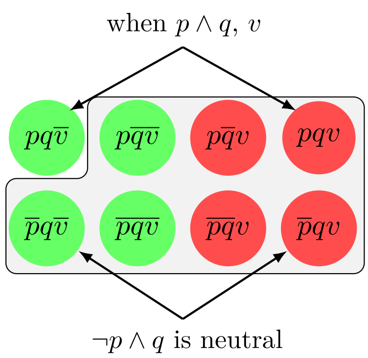


Figure 7.8: $\neg\Diamond(p \wedge q)$

[4] If you are using a greyscale printer, then the violation worlds are darker, and non-violation worlds are lighter.

To see whether taking both is permitted or prohibited, we compare the pq worlds, i.e., worlds where one takes both a starter and a dessert. As one can see from the figures, the world in which one takes both a starter and a dessert and no violation occurs ($pq\overline{v}$) remains among the worlds that support $\Diamond(p \vee q)$, but it is excluded from those that support $\neg\Diamond(p \wedge q)$. So one cannot infer from (20-a) that you may not take both a starter and a dessert as required for the exclusive reading of *or*.

After we saw how MadRis captures the intuition that *or* is inclusive also in permission sentences, we also need to check whether *or* remains inclusive in obligation sentences.

7.2.2.3 Obligation sentences

Consider (23-a) which is based on the relevant example from chapter 2. We have shortened the example for easier analysis.

(23) a. It is obligatory for such parent or other person legally chargeable to contribute.
b. $\Box(p \vee q)$

For (23-a) to be exclusive, it should follow that it is not obligatory that both contribute.

(24) a. It is not obligatory for both such parent and other person legally chargeable to contribute.
b. $\neg\Box(p \wedge q)$

Under an exclusive treatment of *or* embedded under obligation, whenever (23-a) is the case, (24-a) should be the case as well.

A comparison of maximal supporting states for (23-b) in figure 7.9 and those for (24-b) in figure 7.10 shows that the inference does not hold since the situations that licence each sentence are very different.

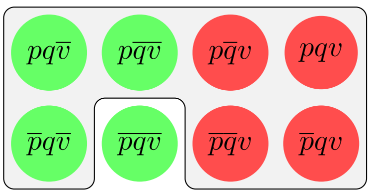


Figure 7.9: $\Box(p \vee q)$

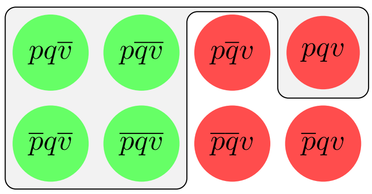


Figure 7.10: $\neg\Box(p \wedge q)$

Recall that Mrs. Siebel made the argument that, assuming an exclusive reading of *or*, since one parent is already contributing, no other person needs to contribute. What was not recognized, however, is how obligation interacts with *or* sentences such as (23-a) or (24-a).

When (23-a) is the case, one knows that the relevant obligation is being violated when neither a parent nor another person contributes. So an obligation sentence with *or* does not actually say much about situations where one or both contribute. These situations are neutral according to (23-a).

(24-a), one the other hand, grants permission for either one parent, another person or both of them not to contribute. The situation when both do contribute is neutral.

(23-a) and (24-a) are contraries with respect to what they say about the situation when neither the parent nor another person contribute. According to (23-a), when neither contributes, a violation occurs, and (24-a) says that no violation occurs in this situation.

Neither sentence says that the situation in which both parents contribute is a violation or not a violation, so they are in fact neutral with respect to the situation that Mrs. Siebel was referring to.

From figures 7.9 and 7.10 one can determine that the exclusivity inference in (25) does not hold in MadRis.

(25) $\Box(p \vee q) \not\models \neg\Box(p \wedge q)$

For (25) to hold, each situation that supports (23-b) should also support (24-b) but the worlds $p\bar{q}v$, $\overline{pq}v$ and $\bar{p}qv$ support (23-b) but not (24-b). Thus, the law in (23-a) only prohibited the situation where neither such parent nor other person legally chargeable contributes. It is a misunderstanding to claim that a semantic inference would help Mrs. Siebel as (24-a) grants permission for such parent, other person legally chargeable or both not to contribute, which goes against the idea behind (23-a) as it grants permission for neither to contribute.

Thus, with the help of MadRis we can see both that *or* receives an inclusive reading also when embedded under obligation and also that the behaviour of *or* under modals can lead to misunderstandings.

7.2.2.4 Conclusions regarding the *inclusive/exclusive or* puzzle

There are two lessons to be learned from the preceding application of MadRis to the examples from chapter 2. Firstly, MadRis captures and illustrates the consensus view among linguists that *or* is not exclusive. In each of the sentences, whether with or without permission and obligation, the sentences do not permit the exclusivity inferences. This means that in any situation where *or* receives an exclusive interpretation, that interpretation needs to follow from reasoning about what the speaker meant to say, rather than what the speaker literally said. Yet the meaning of *or* alone is inclusive.

Secondly, the interaction of connectives in a sentence can lead to entirely different effects compared to the same connectives taken alone. As we saw with example (23-a), the salient reading says that the situation when neither p nor q is the case leads to a violation. In a sense, the sentence says very little about the situations when p or q is the case.

Such differences in embedded interpretations can be demonstrated even more clearly with negated sentences.

(26) There is no cup or plate on the table.

The salient reading of (26) says that there is neither a cup nor a plate. But *or* usually has an *one is sufficient* reading, which disappeared entirely under *no*. So, merely by the interaction of components of a sentence, the readings can be entirely different.

7.2.3 Puzzles with conditionals

The second set of problematic examples concern conditional sentences. We will use the simplified version of the first puzzle sentence for illustration.

(27) a. If Odysseus' plan works, then he will be a hero.
b. $p \to q$

The salient reading of (27-a) says that all those situations where people would say that Odysseus' plan works, they would also be willing to say that he is a hero.

But the question in chapter 2 was what happens when we know that the plan failed. Does it always hold that from (27-a) it follows that (28-a)?

(28) a. If Odysseus' plan does not work, then he will not be a hero.
b. $\neg p \to \neg q$

Recall that in chapter 2 we saw an example which shows that intuitively it does not have to be that when Odysseus' plan fails, he will not be a hero. The basic idea is that there could be other ways in which he became a hero.

What we will do here is discuss the predictions of MadRis for both examples of conditional sentences and the show the results of the analysis guide us to the counter-example from chapter 2.

For conditionals to have the associated negative reading explicated in (28-a), then whenever someone believes that (27-a) is the case, they would also believe that (28-a) is the case. Yet, in MadRis, this is not the case.

(29) $p \to q \not\models \neg p \to \neg q$

We see that this inference does not hold by looking at situations which would allow one to utter each sentence. Consider figures 7.11 and 7.12.

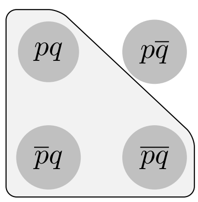


Figure 7.11: $p \to q$

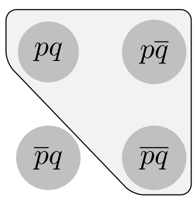


Figure 7.12: $\neg p \to \neg q$

The contexts for uttering (27-a) differ from those for (28-a), in that the possible world where the plan fails but Odysseus still becomes a hero, $\overline{p}q$, supports (27-a) but does not support (28-a). Thus it cannot be that whenever (27-a) holds, so does (28-a). Barring additional information, even though one of Odysseus' plans could fail, it is possible that there is another reason why he will be considered a hero.

Before we look at the counter-example, also consider the second conditional.

(30) a. If delivery is before June 30, 2004, then purchaser will tender cash.
b. $p \rightarrow q$

The discussion centered around whether it would follow from (30-a) that whenever cash was tendered, the delivery will be before the 30th of June 2004.

(31) a. If purchaser will tender cash, then delivery is before June 30, 2004.
b. $q \rightarrow p$

We have reversed the order of the antecedent and consequent in the conditional. The question is, does someone who believes (30-a) also believe (31-a)? Intuitively, the answer is no, because a purchaser is free to tender cash regardless of the delivery date. MadRis also predicts this intuitive result because the inference from (30-b) to (31-b) is not valid.

(32) $p \rightarrow q \not\models q \rightarrow p$

The reason for the inference failing is demonstrated in figures 7.13 and 7.14.

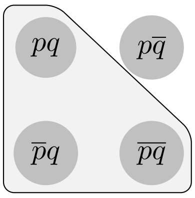


Figure 7.13: $p \rightarrow q$

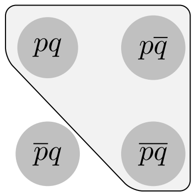


Figure 7.14: $q \rightarrow p$

The figures show that every context in which (30-a) is the case is not a context in which (31-a) is the case. The situation in which the delivery is not made before the respective date, but the purchaser still tenders cash ($\overline{p}q$) supports (30-a) but does not support (31-a). So, even though purchaser elects to tender cash, because this choice is made independently of the rule in (30-a), the delivery may also occur after June 30, 2004.

Note that according to MadRis, both example sentences have the same structure ($p \rightarrow q$) and thus the same meaning. More remarkably, despite (28-a) and (31-a) seeming to differ in their logical form ($\neg p \rightarrow \neg q$ and $q \rightarrow p$, respectively), they are also supported by the same worlds as both only exclude the $\overline{p}q$ world. This means that we can construct a single counter-example for both problematic examples by focusing on the single world that supports (27-b)/(30-b) but is excluded by (28-b) and (31-b).

The story we will construct about a young chess player, John, was already presented in chapter 2. We introduced the Sicilian opening which can be played both with the white and the black pieces. We say that p represents "John had white" and q represents "John played a Sicilian opening." Chess also has the useful feature of only having white and black pieces so that not playing with white pieces, means that the person was playing with black pieces, $\neg p$. Thus, $\neg p$ represents playing with black pieces and $\neg q$ represents "John did not play a Sicilian opening."

From the semantic analysis we know that the world we need to focus on is $\overline{p}q$, i.e., situations where John plays black and a Sicilian opening. To find counter-examples, we need to construct such a situation where it is possible to utter $p \to q$ but neither $\neg p \to \neg q$ nor $q \to p$.

The story goes as follows, John is practising the Sicilian whether he gets black or white. He has played a total of 20 games, 10 with white and 10 with black and in all of them he has played the Sicilian opening. In this case, the following is a reasonable sentence to utter.

(33) a. If John had white, then John played a Sicilian opening.
b. $p \to q$

If the entailment in (29) were valid, then it ought to be the case that when (33-a) is the case then (34) is the case as well.

(34) If John had black, then John did not play a Sicilian opening.

Yet, we know that John was practising the Sicilian opening in all his games, both when he had white and black pieces. So, based on this information, even though we agree with (33-a) we reject (34). Thus, the semantics guides us to an example that contradicts the above inference.

As predicted, we can use the same context to investigate the other example, (30-a) and its potential semantic inference in (32). In terms of (33-a) this means that whenever (33-a) is the case, (35) is as well.

(35) If John played a Sicilian opening, then John played black.

But from the story we know that John played 10 Sicilian opening games with white. So we accept (33-a) but reject (35).

We have demonstrated how to illustrate the meaning of connectives with the aid of MadRis. This can be applied to examples of natural language that have been proven to be problematic in courts of law. So semantics can contribute by clarifying the meaning of laws so that lawyers and judges can make their respective judgements.

7.2.4 Future steps

This last subsection is meant to share my own experience with working with lawyers so it may help future researchers or lawyers in bringing the two fields together.

When talking to lawyers, I have often been asked which is the right interpretation. And the answer is that there is no catch-all. MadRis or some other semantic framework cannot solve all language-oriented problems for lawyers. This dissertation was written with the purpose of taking a single insufficient step towards active and fruitful collaboration between lawyers and linguists and there are still a number of crucial components missing.

Firstly, we did not discuss syntax but the literature includes a wealth of information about the syntax of natural language which might be pertinent when discussing other problematic examples in courts of law.

Secondly, the primary goal was to provide an analysis of the behaviour of *or* and *if-then* under permission, obligation and prohibition, so we said very little about a wealth of other puzzling constructions. Yet, the linguistics literature might already include helpful analysis on other problems that lawyers face in courtrooms.

Thirdly, we were concerned only with inferences that stem from the meaning of connectives. As we discussed in 7.2.2, there is a different kind of inference which stems from the context of the utterance regarding the intent of the speaker. Such pragmatic inferences were not explored in this dissertation but they play a role in natural language interpretation.

Fortunately, in legal language, pragmatic inferences are not as likely to play a role. When a legal text states prohibitions or obligations, one is first concerned with what the law says and rarely does one need to know the author's intention. In cases where the intent of the author is important to establish the spirit of the law, there are already tools available to lawyers and judges. For example, many legal texts maintain records of the process of drafting, including explanatory notes by authors. The most famous of these might be the Commentaries on the Constitution of the United States.[97] A semanticist like me has very little to add to such an analysis.

The main message of this summary is that linguists have been working on unravelling the meaning of language for decades, and have amassed a wealth of clarifying examples and useful formal tools that can provide the means to assist lawyers and judges in interpreting difficult language.

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