The Game of Chaos

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STACS'99 @ TRIER

Noam Nisan
Algorithms for Selfish Agents
Mechanism Design for Distributed Computing
Proc. STACS'99, Springer LNCS 1563, pp. 1--15
Invited paper.

Elias Koutsoupias & Christos Papadimitriou
Worst-Case Equilibria, Proc. STACS'99, Springer LNCS 1563, pp. 404--413

Game Theory for Controlling the Internet !

The Game of Chaos

Sorry: it is a French Card
Game of Chaos
Sorcery

Play head or tails against a target opponent. The looser of the game looses one life. The winner of the game gains one life, and may choose to repeat the procedure. For every repetition the ante in life is doubled.

Magic; the Gathering

Customizable card game: build a deck using a very large collection of available cards.
Both players start out with **20 lives**.
Number of lives ≤ 0 means you have **lost** the duel.
Move = playing land, casting a spell, combat, ...
Attack: **summoning** creatures, damaging spells, damaging effects
Defense: **Blocking** attacking creatures, protecting spells and effects
Spells require **Mana** obtained by **tapping** lands or activating other Mana sources. Mana exists in **5 colors** and a **generic** variant.
Spells exist in the same 5 colors or a generic variant (**artifacts**)

For almost every rule in the game there exists a card creating an exception against it when successfully cast.....

The Players

URGAT

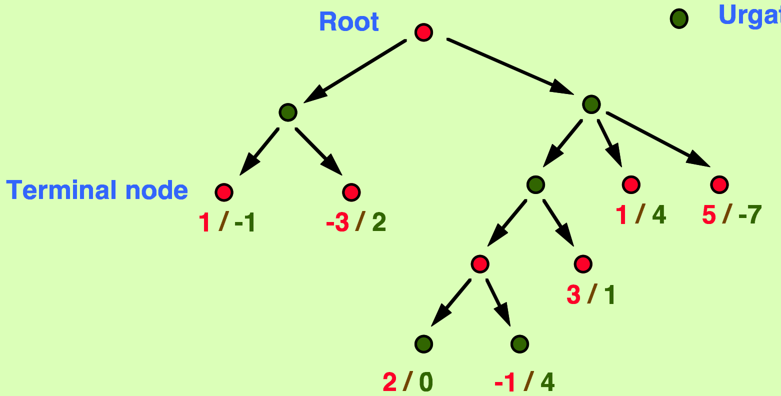
THORGRIM

Game Trees

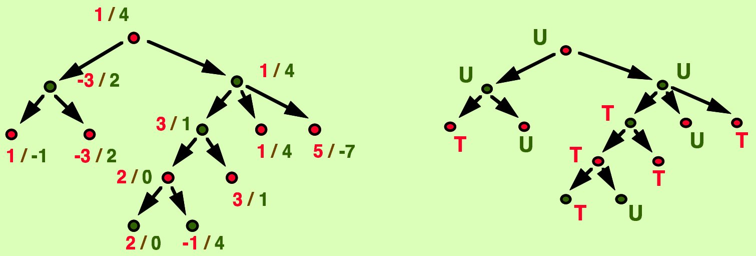
Non Zero-Sum Game:
Payoffs explicitly
designated at terminal node

● Thorgrim's turn

● Urgat's turn



Backward Induction



At terminal nodes: Pay-off as explicitly given

At Thorgrim's nodes: Pay-off inherited from Thorgrim's optimal choice

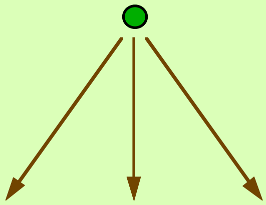
At Urgat's nodes: Pay-off inherited from Urgat's optimal choice

For strictly competitive games this is the Max-Min rule

CHANCE MOVES

- Chance moves controlled by another player (Nature) who is not interested in the result
- Nature is bound to choose his moves fairly with respect to commonly known probabilities
- Resulting outcomes for active players become lotteries

Lotteries

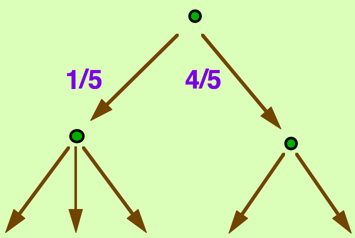


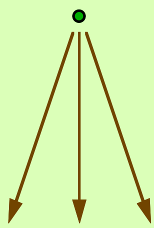
price	-$2	$12	$3
prob.	1/2	1/6	1/3

Expectation:

$$1/2 \cdot -2 + 1/6 \cdot 12 + 1/3 \cdot 3 = 2$$

Compound Lottery



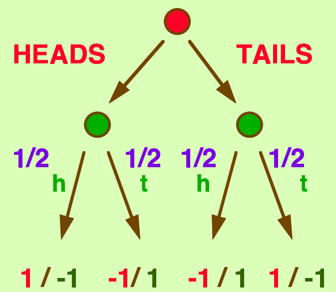


price	-$2	$12	$3	-$2	$3
prob.	1/2	1/6	1/3	1/2	1/2

price	-$2	$12	$3
prob.	1/2	1/30	7/15

In compound lotteries all drawings are assumed to be independent

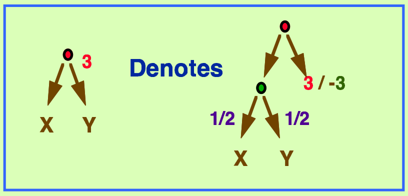
Flipping a coin



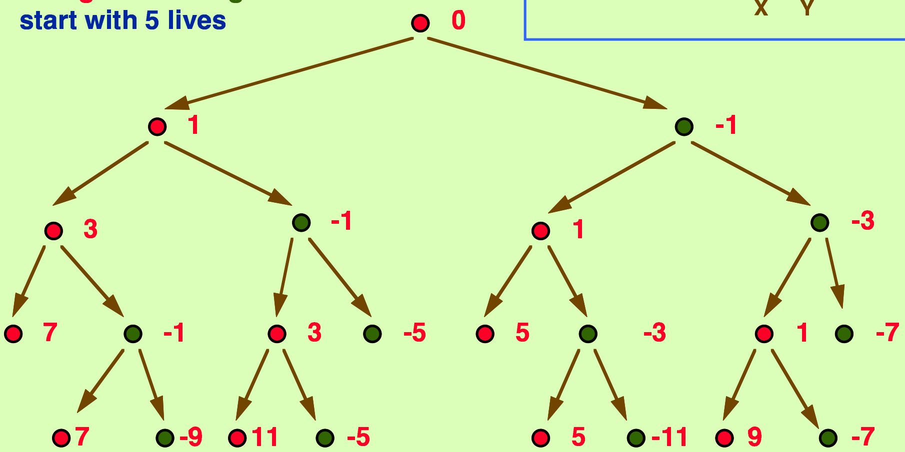
Expectation 0 / 0 0 / 0

Thorgrim calls head or tails and Urgat flips the coin.
Urgat's move is irrelevant. Nature determines the outcome.

The Game Tree



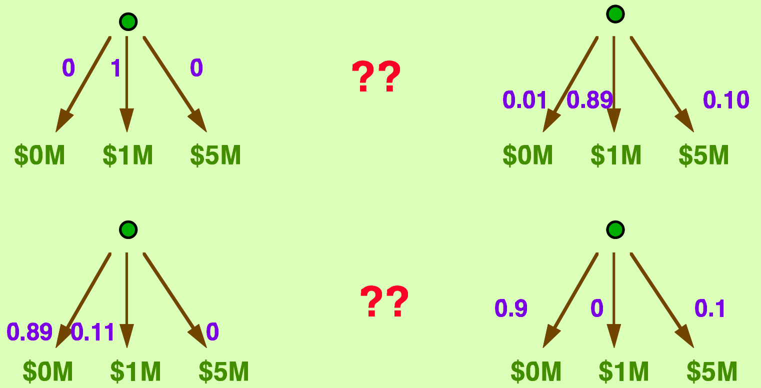
Thorgrim and Urgat both start with 5 lives



WHY UTILITY FUNCTIONS?

- Backward Induction is based on preferences rather than numbers
- Numbers as a tool for expressing preferences works OK when chance moves are absent
- We like to compute expected pay-off at chance nodes.
- Expected pay-off is sensitive to scaling
- Comparing complex lotteries is non-trivial

Comparing Complex Lotteries Allais Example



Von Neumann-Morgenstern Utility

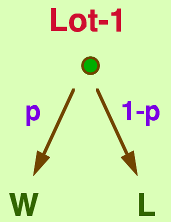
Rational Players may be assumed to maximize the expectation of Something.
Let's call this Something Utility.

Works nice for 2-outcome Lotteries:
Something = chance of winning.

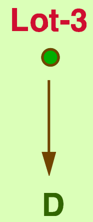
So let's reduce the n-outcome Lotteries to 2-outcome Compound Lotteries:
Each intermediate outcome is "equivalent" to a suitable 2-outcome Lottery. The involved chance determines the Utility.

Utility Intermediate Outcome

p :=



u(L) = a
u(W) = b
u(D) = x
a < b



$E_{Lot\text{-}1}(u) = p.b + (1-p).a$

$E_{Lot\text{-}3}(u) = x$

If p is large (almost 1) : Lot-1 > Lot-3
For p small (almost 0) : Lot-1 < Lot-3
So for some intermediate p, say q: Lot-1 ≈ Lot-3

q ≈ Lot-3 whence u(D) = q.b + (1-q).a !

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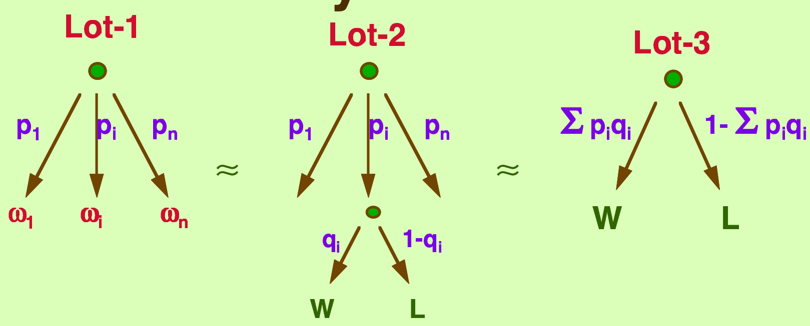
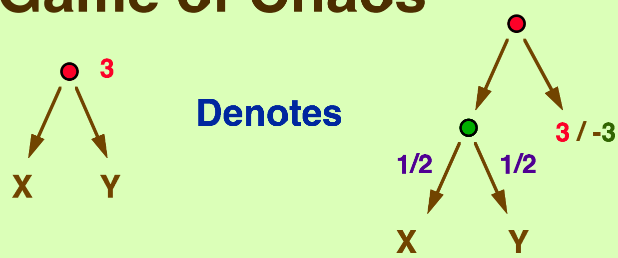
Utility Lottery = Expected Utility Outcomes



$$u(W) = 1 \ , \ u(L) = 0 \ , \ u(\omega_i) = q_i$$

$$\Sigma p_i q_i = u(\text{Lot-3}) = \Sigma p_i u(\omega_i) = \mathbf{E}_{\text{Lot-1}}\, u(\text{outcome})$$

Game of Chaos



Structure of the game tree independent of the choice of the utilities.

$u_{T,1}$: $u_{T,1}(n) = n$
$u_{T,2}$: $u_{T,2}(n) =$ <u>if</u> $n \geq v_{opp}$ <u>then</u> 1 <u>elif</u> $n \leq -v_{self}$ <u>then</u> -1 <u>else</u> 0 <u>fi</u>

$u_{U,1}$: $u_{U,1}(n) = -n$
$u_{U,2}$: $u_{U,2}(n) =$ <u>if</u> $n \geq v_{self}$ <u>then</u> - 1 <u>elif</u> $n \leq -v_{opp}$ <u>then</u> 1 <u>else</u> 0 <u>fi</u>

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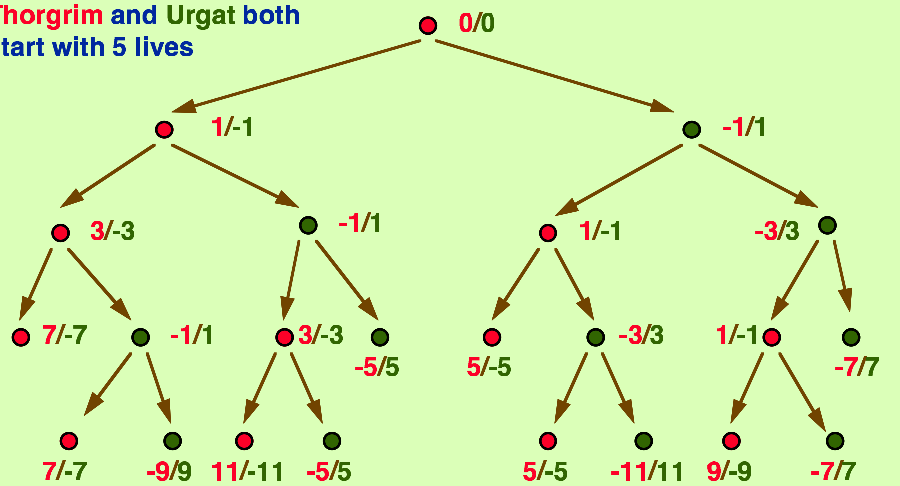
Linear Utilities

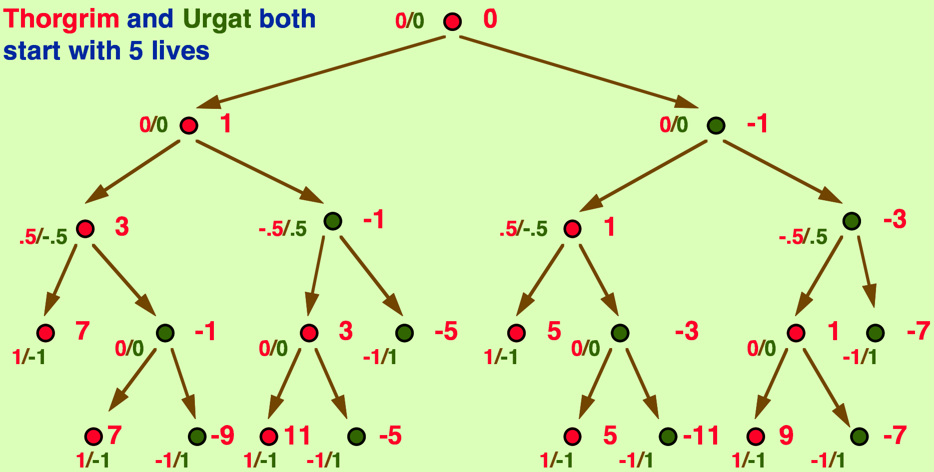
Thorgrim and Urgat both start with 5 lives



Both Thorgrim and Urgat use utility u_1

Go for the Kill!

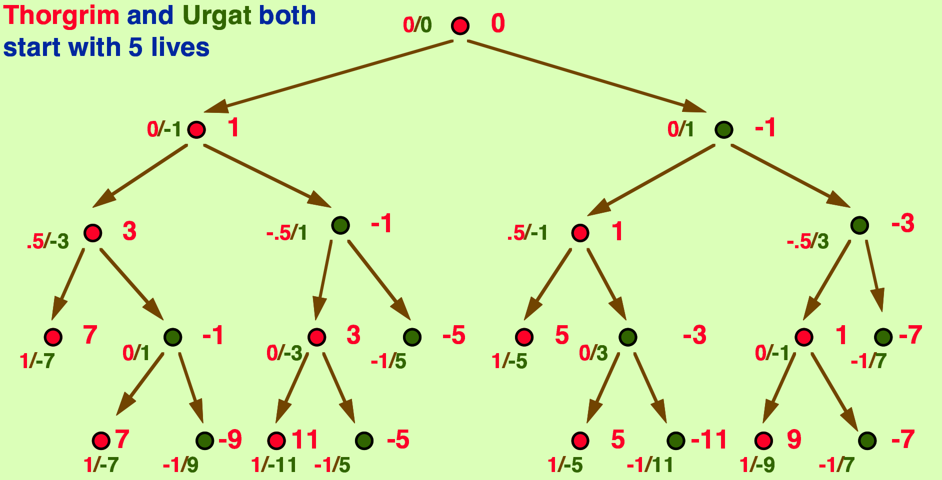
Thorgrim and Urgat both start with 5 lives



Both Thorgrim and Urgat use utility u_2

Mixed Utilities

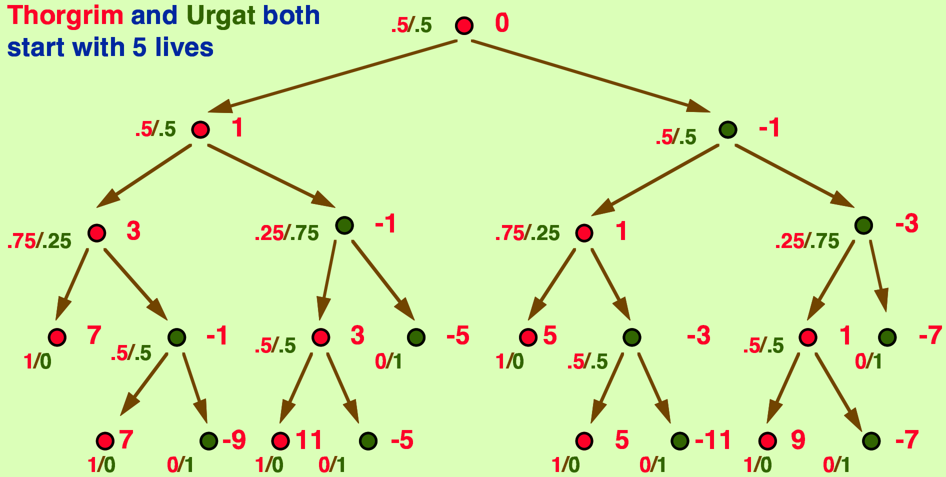
Thorgrim and Urgat both start with 5 lives



Thorgrim uses u_2 ; Urgat uses u_1

Winning is all

Thorgrim and Urgat both start with 5 lives



Utilities: Thorgrim uses $u_{3,T}$: $u_{3.T}(n)$ = if $n \geq v_{opp}$ then 1 else 0 fi

Urgat uses $u_{3,U}$: $u_{3.U}(n)$ = if $-n \geq v_{opp}$ then 1 else 0 fi

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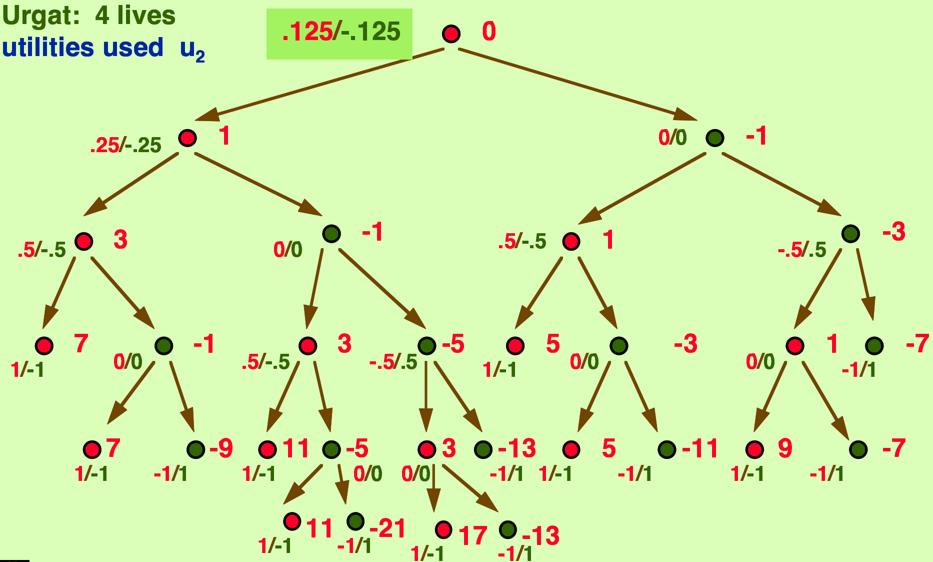
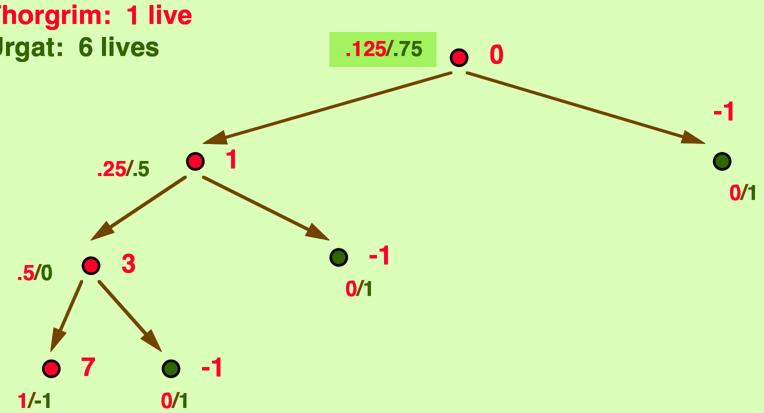
Unequal Start

Thorgrim: 6 lives
Urgat: 4 lives
utilities used u_2



Thorgrim's last stand

Thorgrim: 1 live
Urgat: 6 lives



Utilities: Thorgrim uses u_3: $u_3(n) =$ if $n \geq v_{opp}$ then 1 else 0 fi
Urgat uses u_2