Interfacing Probabilistic and Epistemic Update

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Plan

- (apologize for changing the topic)
- (apologize for not being a probability theorist)
- Try to see if we can import tools from Dynamic Epistemic Logic to Probabily Theory.
- Specifically: use Baltag-Moss-Solecki style action models to represent probabilistic evidence.

Epistemic and Probabilistic update

$$\textbf{information state} \xrightarrow{\textbf{update}} \textbf{new state}$$

epistemic update with ϕ

$$\text{Set of worlds} \xrightarrow{\text{eliminate non-}\phi\text{ worlds}} \text{Set of worlds}$$

epistemic update generalizes to: *Bayesian conditioning*

$$\text{Probability measure} \xrightarrow{\text{Condition on }\phi} \text{Probability measure}$$

Action models

$$\textbf{information state} \xRightarrow{\textbf{update}} \textbf{new state}$$

epistemic update à la Baltag-Moss-Solecki:

$$\text{Set of worlds} \xRightarrow{\text{product with action model}} \text{Set of worlds}$$

epistemic update generalizes to: ??

$$\text{Probability measure} \xRightarrow{\text{?? probabilistic action model}} \text{Probability measure}$$

Running Example

Johnny has stolen a cookie.

You ask him: ‘Did you steal the cookie?’

Johnny mumbles something — you heard ‘no’; but you are not sure.

Anyway, he might be lying about it.

Simplified Action models

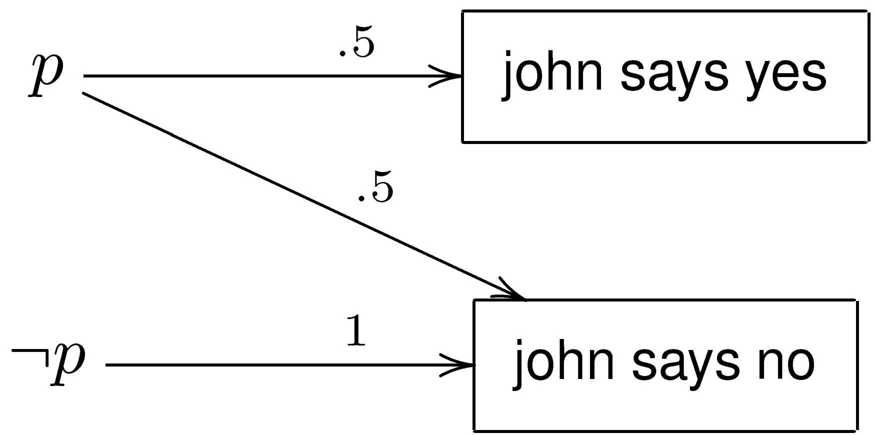
(for the probability people: *Event* Models)

- An action model is a set of *actions*
- The agent is not sure which action from the set has happened
- Each action has a precondition
- An update of an information state with an action models consists of all pairs (w, a) such that the preconditions of a are fulfilled in w

information state	action model	result
p	johnny says yes^{p}	$(p$, says yes$)$
$\neg p$	johnny says no$^{\top}$	$(p$, says no$)$
		$(\neg p$, says no$)$

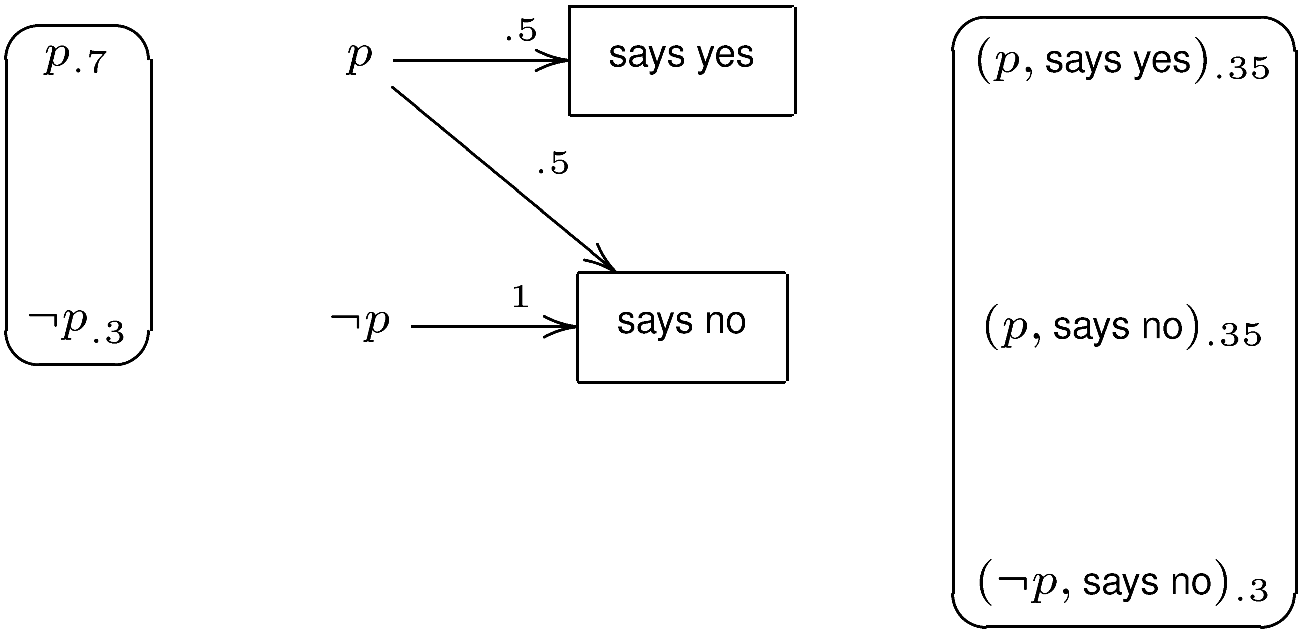
Probabilistic action models

- Add (conditional) probabilities to the actions (van Benthem, JoLLI, 2003)



Updates with Probalistic Action Models

- Update as if probabilities represent Bayes' factors

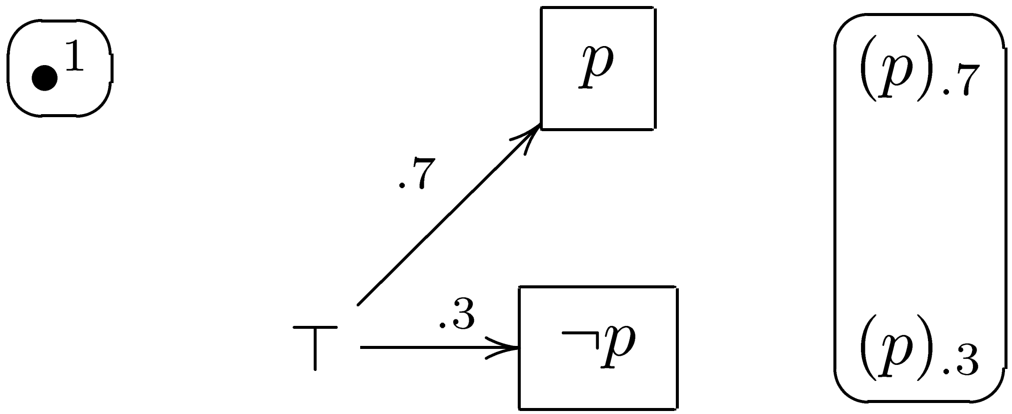


What Probabilistic Action models might give

- Explicit representation of ‘internal structure’ of probabilistic evidence
- A logic of probabilistic update
- ‘Building rich models in a structured way’
- (Probabilistic updates for models with more agents)

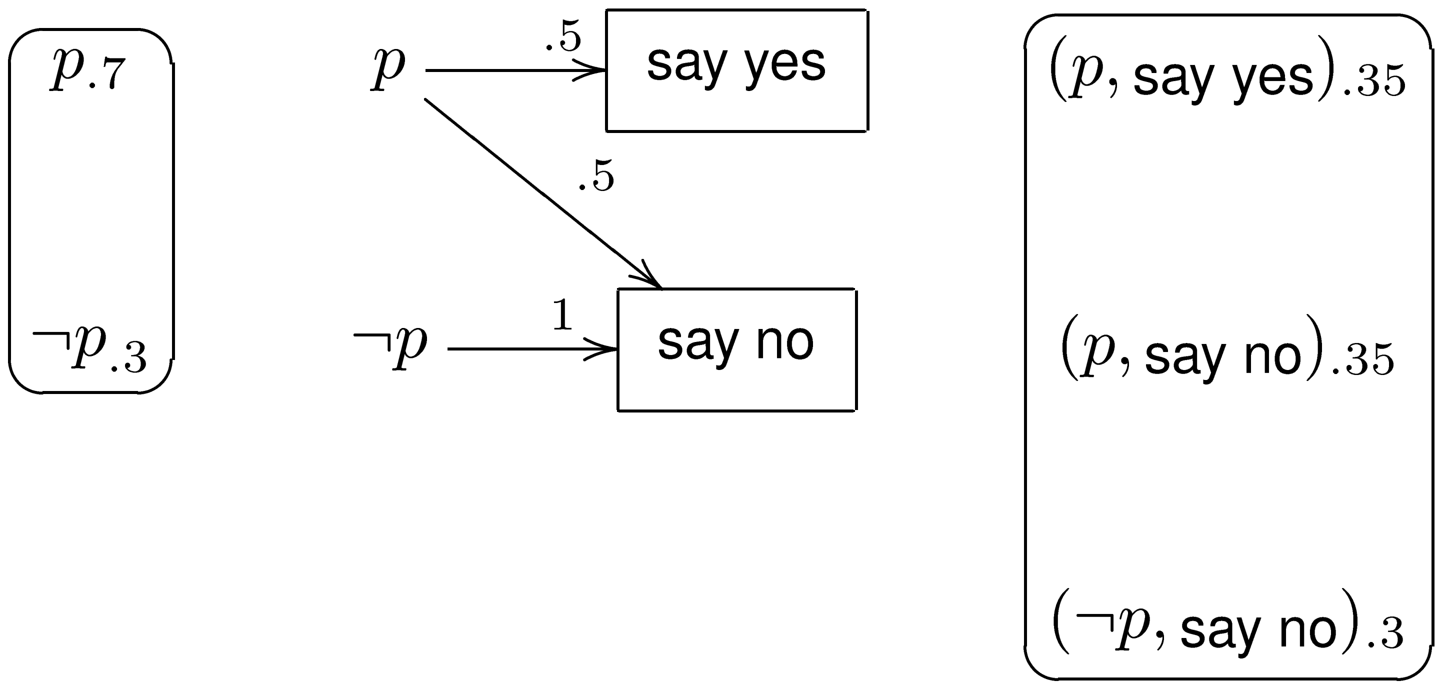
Johnny and the cookie

You learn that Johnny might have stolen a cookie (probability .7)



Johnny and the cookie

You learn that Johnny is a liar



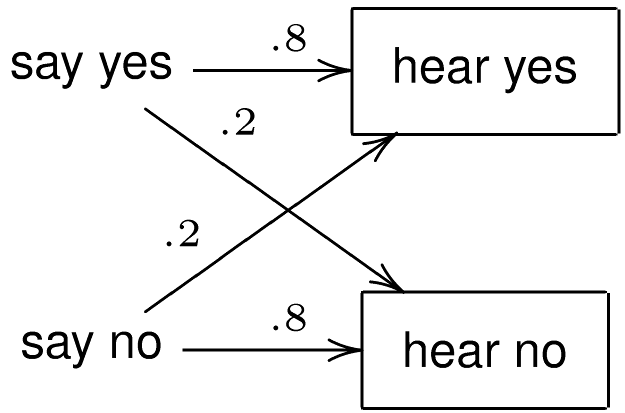
Johnny and the cookie

You learn that Johnny mumbles

$(p, \text{say yes})_{.35}$

$(p, \text{say no})_{.35}$

$(\neg p, \text{say no})_{.3}$



$(p, \text{say yes,hear yes})_{.28}$

$(p, \text{say yes, hear no})_{.07}$

$(p, \text{say no, hear yes})_{.07}$

$(p, \text{say no, hear no})_{.28}$

$(\neg p, \text{say no, hear yes})_{.06}$

$(\neg p, \text{say no, hear no})_{.24}$

Johnny and the cookie

You hear Johnny say ‘yes’

$(p, \text{say yes,hear yes})\ .28$
$(p, \text{say yes, hear no})\ .07$
$(p, \text{say no, hear yes})\ .07$
$(p, \text{say no, hear no})\ .28$
$(\neg p, \text{say no, hear yes})\ .06$
$(\neg p, \text{say no, hear no})\ .24$

$\top \xrightarrow{1}$ hear yes

$(p, \text{say yes, hear yes})\ \frac{28}{41}$
$(p, \text{say no, hear yes})\ \frac{7}{41}$
$(\neg p, \text{say no, hear yes})\ \frac{6}{41}$

What we like about this

- Modular
  - Scenarios can be decomposed into more or less independent action models
  - Action models can be composed to make more complex ones (and the order in which we do this does not matter)
- Structured
  - Create a *logic of probabilistic evidence*
  - $P(p \mid \phi) = \alpha \cdot P(q \mid \phi) = \beta \rightarrow P(p \wedge q \mid \phi) = \alpha \cdot \beta$

What we would like to know

- Does this make sense to a probability theorist?
- Does this seem useful to a probability theorist?