

# Collective Decision-making: Arguing & Voting

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# Purpose of this talk

- ☐ **Provide a light-weighted introduction to Argumentation Theory in the context of Collective Decision-Making**
- ☐ **Put forth a number of research topics at the interface of the Theory of Aggregation, Argumentation theory, and Rational Dynamics**
- ☐ **... put together techniques from Social Choice, Artificial Intelligence, Logic (and Game theory)**
- ☐ **Watch a good movie!**

# Voting

*Preference & Judgment Aggregation*

# Majority voting on two issues

K. May (1952) "A set of Independent Necessary and Sufficient Conditions for Simple Majority Decisions",  
Econometrica, 20:680-684

**Theorem (May, 1952)** If the number of voters is odd, and the number of issues is two, then pairwise majority voting is the only rule which is anonymous, neutral, monotonic and resolute.



- The theorem is a PA theorem, but has an obvious counterpart in JA when voting on any number of *logically unrelated* propositions.

# Majority voting on more issues (i)

Marie. J. A. N. de Caritat (1785) "Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité de voix"



| $\{x, y\}$  | $\{y, z\}$  | $\{x, z\}$  |
|-------------|-------------|-------------|
| $y \prec x$ | $z \prec y$ | $z \prec x$ |
| $y \prec x$ | $y \prec z$ | $x \prec z$ |
| $x \prec y$ | $z \prec y$ | $x \prec z$ |
| $y \prec x$ | $z \prec y$ | $x \prec z$ |

- Intransitivity of collective judgment in pairwise majority voting

# Majority voting on more issues (ii)

Kornhauser, L.A., and L. G. Sager (1986) "Unpacking the Court" Yale Law Journal 96: 82-117



| $p$           | $p \rightarrow q$ | $q$           |
|---------------|-------------------|---------------|
| $\models$     | $\models$         | $\models$     |
| $\models$     | $\not\models$     | $\not\models$ |
| $\not\models$ | $\models$         | $\not\models$ |
| $\models$     | $\models$         | $\not\models$ |

- The collective judgment is impossible (under Propositional Logic)

# Majority voting on more issues (ii)

Kornhauser, L.A., and L. G. Sager (1986) "Unpacking the Court" Yale Law Journal 96: 82-117



| $p$           | $q$           | $p \wedge q$  |
|---------------|---------------|---------------|
| $\models$     | $\models$     | $\models$     |
| $\models$     | $\not\models$ | $\not\models$ |
| $\not\models$ | $\models$     | $\not\models$ |
| $\models$     | $\models$     | $\not\models$ |

- The collective judgment is impossible (under Propositional Logic)

# Majority voting on more issues (ii)

Kornhauser, L.A., and L. G. Sager (1986) "Unpacking the Court" Yale Law Journal 96: 82-117



| $p$    | $q$    | $p \vee q$ |
|--------|--------|------------|
| $\neq$ | $\neq$ | $\neq$     |
| $=$    | $\neq$ | $=$        |
| $\neq$ | $=$    | $=$        |
| $\neq$ | $\neq$ | $=$        |

- The collective judgment is impossible (under Propositional Logic)



# Majority voting on more issues (iii)

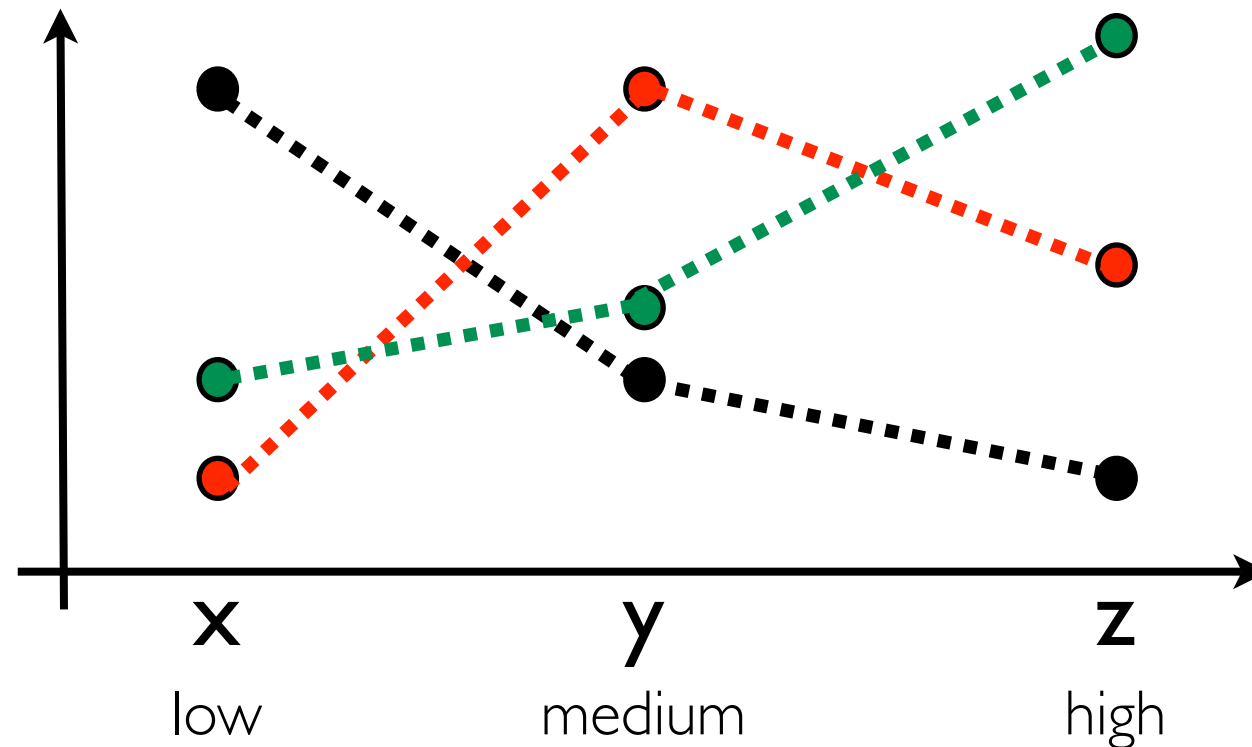
K.Arrow (1950) "A Difficulty in the Concept of Social Welfare", Journal of Political Economy, 58(4): 328-346

**Theorem (Arrow, 1950)** There exists no preference aggregation function which satisfies unanimity, independent of irrelevant alternatives and non-dictatorship for more than two issues.

C. List and P. Pettit (2002) "Aggregating Sets of Judgments: An Impossibility Result", Economics and Philosophy 18: 89-110

**Theorem (List and Pettit, 2002)** There exists no judgment aggregation function which satisfies systematicity and anonymity if the set of issues includes  $p, q$  and one of their Boolean compounds.

# “Well-behaved” preferences (i)



A preference profile  $\mathbf{p} = (\preceq_1, \dots, \preceq_n)$  of total preorders on  $\mathbf{Iss}$  is *single-peaked* if there exists a total order  $\preceq^*$  on  $\mathbf{Iss}$  s.t.  $\forall i \in \mathbf{Agn}$ :

$$y \preceq_i x \ \& \ B(x, y, z) \quad \Rightarrow \quad z \prec_i y$$

where  $B$  is the betweenness relation induced by  $\preceq^*$ .

# “Well-behaved” preferences (ii)

Black D. (1948) "On the Rationale of Group Decision Making", The Journal of Political Economy, 56: 23-34

**Theorem (Black, 1948)** If the number of voters is odd, and the domain of the aggregation function is restricted to single-peaked profiles, then there always exists a Condorcet winner.

- ☐ Since we restrict the domain of aggregation, majority becomes “non-resolute” in some problematic cases.
- ☐ Is such “non-resoluteness” the rationale of pre-voting deliberation?

# Unanimity

$$v(C) = \begin{cases} 1 & \text{if } |N| = |C| \\ 0 & \text{otherwise} \end{cases}$$

- ☐ Unanimity on more than two issues is a special case of majority under single-peakedness
- ☐ Unanimity is undertetermined also when voting on two issues.
- ☐ Is requesting a vote by unanimity a way to foster deliberation?

# Deliberating/Arguing

*Argumentation Theory*



# Arguing



- ☐ *The Economist*: “Berlusconi is unfit to lead Italy because he is involved in several trials”
- ☐ *Mr. Berlusconi*: “Berlusconi is actually the fittest to lead Italy because he is the most successful Italian entrepreneur”

# The Uses of Argument (1958)

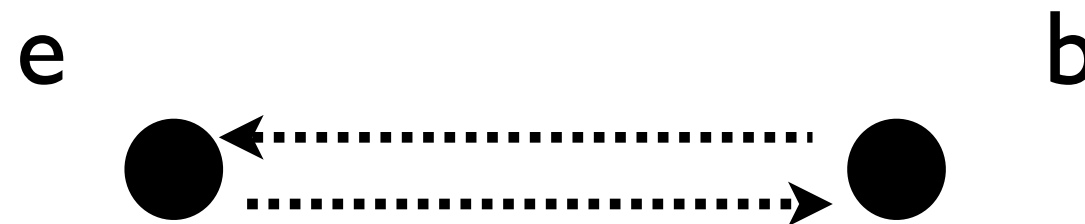


“What things about the form and merits of our arguments are *field-invariant* and what things about them are field-dependent? [...] The various phases [of assessment of an argument] may be encountered equally whether our argument is concerned with a question of physics or mathematics, ethics or law, or an everyday matter of fact. [...] The *force* of the conclusion [...] is the same regardless of fields: the criteria or sorts of grounds required to justify such a conclusion vary from field to field” [Toulmin, 1958]

- Abstract argumentation theory studies the field-invariant aspects of argumentation, i.e., what is common to arguing in any context

# Abstract argumentation (i)

P. M. Dung (1995) "On the Acceptability of Arguments and Its Fundamental Role in Argumentation, n-Persons games, and Logic Programming", Artificial Intelligence

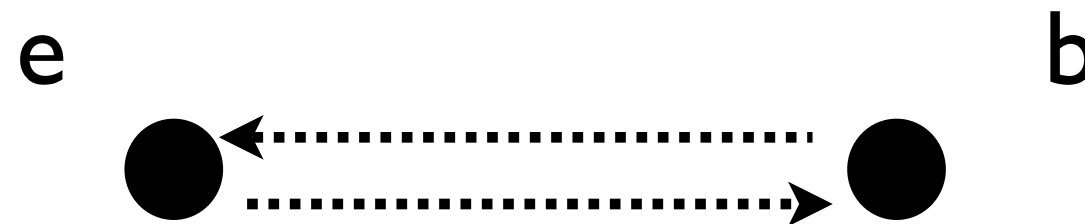


- ☐ Abstract argumentation is about arguments (points) and attacks (relations):  $(A, \rightarrow)$
- ☐ The question is, given an argumentation framework, which (sets of) arguments should be considered “justified”, or “acceptable” (justification-based ‘semantics’)?



# Abstract argumentation (ii)

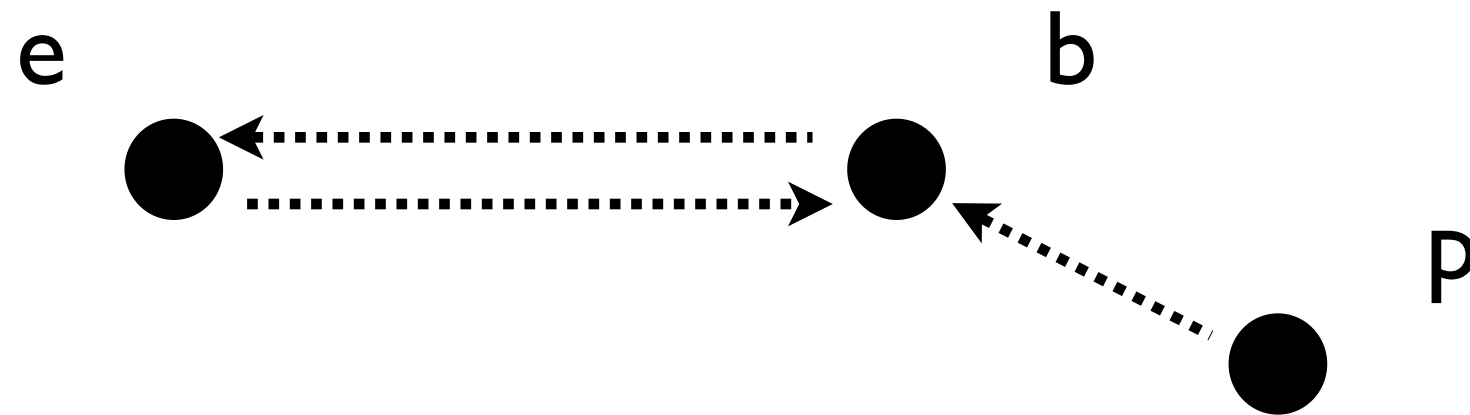
P. M. Dung (1995) "On the Acceptability of Arguments and Its Fundamental Role in Argumentation, n-Persons games, and Logic Programming", Artificial Intelligence



|               |                |  |
|---------------|----------------|--|
| GroundedSet   | $\emptyset$    | Least fixpoint of “attacking all attackers”                      |
| PreferredSets | $\{e\}, \{b\}$ | Maximal conflict-free post-fixpoint of “attacking all attackers” |
| StableSets    | $\{e\}, \{b\}$ | Fixpoint of “all attackers are out”                              |

# Abstract argumentation (iii)

P. M. Dung (1995) "On the Acceptability of Arguments and Its Fundamental Role in Argumentation, n-Persons games, and Logic Programming", Artificial Intelligence



|               |            |  |
|---------------|------------|--|
| GroundedSet   | $\{e, p\}$ | Least fixpoint of “attacking all attackers”                      |
| PreferredSets | $\{e, p\}$ | Maximal conflict-free post-fixpoint of “attacking all attackers” |
| StableSets    | $\{e, p\}$ | Fixpoint of “all attackers are out”                              |

# Conclusions

*Research questions*

# Deliberating & Arguing

- In the simplest case of deliberation (e.g. before casting a vote under Unanimity rule), the position of the group on a certain issue will be the set of “justified” or “acceptable” arguments put forth by the agents.
- So, each agents contributes to building an argumentation framework, then by applying a justification-based semantics, a collective conclusion is reached.
- This works for unanimity in PA or JA. How would such argument-based deliberation work in changing a profile of preferences to a single-peaked one (thus enabling Black’s theorem)?

# Deliberation as profile transformation (Umberto)

- ☐ Take “deliberation” to be a function from profiles to single-peaked profiles.
- ☐ What are the intuitive axioms they should satisfy?
- ☐ What are the properties of existing argument-based deliberation procedures (e.g. formal verification of parliamentary procedures)?

# Form vs. Formality (Johan)



“It now appears that arguments must not just have a particular shape, but must be set out and presented in a sequence of steps conforming to certain basic rules of procedure. In a word, rational assessment is an activity necessarily involving *formalities* [...] we must ask how far the formal character of sound arguments can be thought of more geometrico, as a matter of their having the right sort of shape, and how far it needs to be thought of, rather, in procedural terms, as a matter of their conforming to the formalities which must be observed if any rational assessment of arguments is to be possible” [Toulmin, 1958]

- If arguing happens according to some “rules of the game”, can we deploy game-theory to study the strategic aspects of deliberation via arguing (e.g. in trials)?

# Deliberation in DEL (Fernando)

- ☐ Can we use the DEL-machinery to model the type of changes in the epistemic attitudes of agents engaged in a deliberation and influencing each other's opinions? Does (van Benthem, 2008) suffice?
- ☐ Can argumentation frameworks be used to “update” DEL models so to capture the epistemic changes of agents engaged in a deliberation process?

# “12 Angry Men”

*Sydney Lumet, 1957*