IMC 2010 – Dialogical Aspects of *Obligationes*
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Relations between Medieval and Modern Logical Dialogue Games

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Warning: I presuppose some familiarity with *obligationes*.

Disclaimer:
No attempt at a direct interpretation of *Obligationes* will be made.

Motivation:
A fruitful interpretation of *Obligationes* from a contemporary logical point of view presupposes an understanding of (dialogue) games in modern logic.

We aim at providing the corresponding background.

Frequently one only refers to Lorenzen style dialogue games and to Hintikka style evaluation games.

We shall provide a birds eyes view on the varied landscape of logical games (of dialogical form) in mathematical logic.

The profit might be go in two directions:

- better understanding of *Obligationes* as logical games
- new topics arising for modern logic games

Are *obligationes* logical dialogue games?

It seems obvious that (all variants of) *Obligationes* can be classified as logical dialogue games in some sense.
But what is implied by this classification?

- logical?
  !: *obligationes* involve truth, conistency, entailment ...
  ?: no concern about logical connectives or validity
- dialogue?
  !: involves two talking agents, referring to each other
  ?: only the *Opponent* utters 'propositions'
- games?
  !: we can discern 'players', 'states', 'moves', etc.
  ?: winning conditions remain somewhat unclear

Lorenzen style dialogue games

Proponent defends a statement against attacks by **O**pponent.
The role of defender/attacker may change during a dialogue.

Logical rules: (**X**/**Y** stand for **P**/**O** or for **P**/**O**)

X asserts	challenge by **Y**	answer by **X**
$A \to B$	A	B
$A \vee B$	'?'	A or B (**X** chooses)
$A \wedge B$	'l?' or 'r?' (**Y** chooses)	A or B (accordingly)
$\forall x\, A(x)$	't?' (**Y** chooses)	$A(t)$
$\exists x\, A(x)$	'?'	$A(t)$ (**X** chooses)

Note: $\neg A$ abbreviates $A \to \bot$, where '$\bot$' is not defensible.
To obtain a game we still need:

1. regulations defining admissible runs of a dialogue
2. winning conditions

Winning conditions and regulations

Lorenzen's original game for intuitionistic logic:

1. **P** wins if **O** attacks an already granted formula ('ipse dixisti!')
2. quite special regulations, like, e.g.,:
   – **P**, in contrast to **O**, can never assert an atomic statement
   – **O** can choose which statement she wants to defend next, but **P** has always to defend the last attacked statement

A formula is valid if **P** has a winning strategy.

More liberal regulations characterize classical logic.

Note: 'truth' plays no role!

For classical logic, Lorenzen also defines material dialogues, where Proponent **P** is free to assert true atomic statements.

Beyond Lorenzen:
Variants for different types of logics: modal, linear, relevant, intermediary, paraconsistent, free logic, many-valued, ...
(Rahman, Rückert, Keiff, Carnielli, Blackburn, Blass, Giles, F, ...)

Hintikka's evaluation games (GTS)

Hintikka based on ideas by Henkin and others:
A game about truth of a formula F in a model $\mathcal{M}$ between 'initial verifier' **E**loise and 'initial falsifier' **A**belard.

Moves depend on the current subformula of F:

current	move	next subformula
$A \vee B$	**E** picks 'l' or 'r'	A or B (accordingly)
$A \wedge B$	**A** picks 'l' or 'r'	A or B (accordingly)
$\neg A$	role switch	A
$\forall x\, A(x)$	**A** picks t	$A(t)$
$\exists x\, A(x)$	**E** picks t	$A(t)$

Atoms are verified/falsified by direct inspection of $\mathcal{M}$.

F is true in $\mathcal{M}$ if **E** has a winning strategy.

A hybrid form: Giles's game for Łukasiewicz logic

Originally introduced to model reasoning in physical theories.
Later used to interpret fuzzy logic ('approximative reasoning')

Two independent phases:

1. logical dialogue rules for reduction to atomic states
2. betting on experiments associated with atomic statements

[Ad 1]

Dialogue states: symmetric pairs of multisets of formulas

$$\overset{\text{you}}{[A_1, \ldots, A_m} \,\|\, \overset{\text{me}}{B_1, \ldots, B_n]}$$

Complex assertions are stepwise reduced into their components.

Lorenzen's rules can be augmented, e.g., by 'strong conjunction'.

No regulations needed: the resulting atomic states are equivalent!

[Ad 2] The betting part of Giles's game

Giles's idea:

Let the players bet on the truth of their (atomic) claims!

(Yes/no-)experiments decide — but they may be dispersive!

- I pay 1£ to you for each of my false atomic assertions, if you agree to do the same for your atomic assertions

My expected payoff at atomic (final) state $[p_1, \ldots, p_m \| q_1, \ldots, q_n]$:

$$\Big(\sum_{i=1}^{m} \langle p_i \rangle - \sum_{j=1}^{n} \langle q_j \rangle\Big) £$$

$\langle p \rangle$ ... risk value = (subjective) probability of "no" as result for p

<u>Giles's Theorem:</u>

I have a strategy for avoiding expected loss in dialogues starting with my assertion of F iff F is valid in Łukasiewicz logic.
(Has been generalized to other logics and game variants.)

NB: risk assignment $\langle \cdot \rangle$ amounts to evaluation in a fixed model!

Other types of games in modern logic

Many more types of games are used in contemporary logic.
(Even an exhaustive classification is almost impossible.)

Some important forms of games in logic:

- games for model construction (Hodges et.al.)
- model comparison games ('back-and-forth games' – Ehrenfeucht-Fraïssé)
- forcing games (*descriptive set theory* – Banach-Mazur)
- Japaridze's games for 'computability logic'
- 'trees, automata, and games' (Gurevich, Harrington)
- proof as games (*complexity theory* – Pudlak)
- (Mundici's logical analysis of) the Ulam-Renyi game
- ...

A space for *Obligationes*?

Note: none of the mentioned games fits *Obligationes*

However:
van Benthem's *Logic in Games* presents a simplified, modern version of *Obligationes* as a game of consistency maintenance:

> *A certain finite number of rounds is chosen, the 'severity' of the exam. The teacher gives the student successive abstract assertions $P_1, \ldots P_n$ which the student has to 'accept' or 'reject' as they are put forward. In the former case, the statement P_i is added to the student's current stock of commitments – in the latter, its negation $\neg P_i$ is so added. The student passes if she can maintain consistency throughout.* v.Benthem (2000), p. 2

Comments on van Benthem's '*Obligatio Game*'

- van Benthem: *Old records show that, in one such exam, a student was exposed to the statements* $B \vee \neg(A \vee C)$, $A \rightarrow B$, $\neg B \vee C$
  Where does this come from?
- van Benthem analyzes this example using a game tree and observes that student has a winning strategy.

  A particularly simple strategy: construct a model $\mathcal{M}$ of teacher's first statement and reply to consecutive statements according to evaluation in $\mathcal{M}$.

  NB: an (unexpected?) connection to Hintikka games but also to model construction games arises!
- Complexity issues: checking consistency in NP-complete, but model evaluation is in PTIME
- A connection with Mundici's take on Ulam-Renyi games?
  Allow student to make a fixed amoun of mistakes.
  Can Łukasiewicz logic be characterized in analogy with Mundici's Pinocchio?

Historical Obligationes and modern logic

NB: While van Benthem's analysis is certainly interesting for contemporary logicians, it hardly does justice to subtleties of medieval logic.

Do we have to choose between 'a modern logical point of view' and an 'historic-philological analysis'?

No! As the following formal models of *Obligationes* show:

- Catarina Dutilh Novaes: $Ob = \langle K_C, \Phi, \Gamma, R(\phi_n) \rangle$
  – 'additive framework'
- Sara Uckelman: $O = \langle \Theta, R, \Gamma \rangle$
  – employing dynamic epistemic logic

Some neglected(?) features of *Obligationes*

Interpreted language? Common knowledge?

Respondent has to respect presumed background knowledge K.

Formalizing K as a set of sentences seems not fully adequate.

In conrete *Obligationes* the language is certainly 'interpreted'.

But is it an 'interpreted language' in the modern sense?

Logical inference and (in)compatibility

How to judge '*incompatible with the positum*' or '*follows from the positum*'? What types of inference are available?

Reflective (modal) statements

(From *Burley*, cf. Ueckelman)

Opponent's statements:

– ϕ or ϕ must be granted

– ϕ must be granted

– ϕ follows from the positum and the opposite of something denied

– ϕ must be granted

Conclusions

- Whether *Obligationes* can be classified as logical dialogue games depends on how wide or how narrow these terms are interpreted. Modern logical games might help to provide a logical context, however the differences are essential.
- *Obligationes*, understood as consistency maintenance games, connect in interesting ways to other types of logic games.
- Looking into distinctly non-modern features might lead not only to a better understanding of *Obligationes*, but may also serve as a source of inspiration for as yet unexplored topics in contemporary logic.