Optimal Outcomes of Distributed Negotiation in Utilitarian and Egalitarian Settings

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Talk Overview

1. Distributed Resource Allocation
   - MARA...the setting
   - Our Framework : Main definitions
2. Experiments
3. Egalitarian Social Welfare
   - Theorical and experimental results
   - Condition under which egalitarian SW=0
   - Limit

Outline

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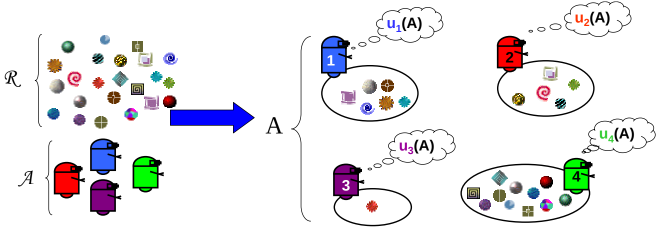
MARA...the setting (1)

- Resource Allocation Framework
  - Finite set of agents A and finite set of discrete resources R

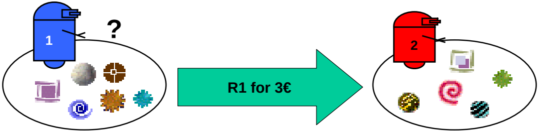


Definition (Allocation)

An allocation of resources for the system$(\mathcal{A}, \mathcal{R})$ is a function A from agents in $\mathcal{A}$ to subsets of $\mathcal{R}$ such that $A(i) \cap A(j) = \{\}$ for $i \neq j$ and $\cup_{i \in \mathcal{A}} A(i) = \mathcal{R}$

MARA...the setting (2)

- Restriction on deal type : Bilateral deals
  $\Rightarrow$ One-resource-at-a-time (from an agent to another)



- Each agent i $\in \mathcal{A}$ has a utility function u_i to express his personal welfare

Example (k-additive functions : multinomial of degree k)

$u_1(R) = 1r_1 + 3r_2 + 7r_3 \quad \Rightarrow \quad$ 1_additive

$u_2(R) = 2r_1 + 3r_2 + 7r_1 r_3 \quad \Rightarrow \quad$ k_additive : 2_additive

$u_2(r_1, r_3) = 9$

How to measure social welfare?

- well-being of the society
- 2 classical measures of Social Welfare

Definition (Utilitarian Social Welfare)

$$sw_u(A) = \sum_{i \in \mathcal{A}} u_i(A)$$

Definition (Egalitarian Social Welfare)

$$sw_e(A) = \min\{u_i(A)\}$$

Agents and rationality

- individual rationality

Lemma (Rational deals)

A deal δ = (A,A') is individualy rational iff $sw_u(A) < sw_u(A')$

- payment function

Definition (Payment function)

$$\sum p(i) = 0$$

- We assume that money is unlimited

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MultiAgent Resource Allocation Modelling

- We have some theorical results
  - AAMAS 03 : In rational negotiation, with one-deals we know that we rise to the utilitarian optimum.
  - What about the others optimum?
- Why experiments?
  - for better understanding theorical results
  - to induce new results
    $\Rightarrow$ We know that we reach some results like optimal Utilitarian Social Welfare but we want to know in other cases
- Test all the kinds of Social Welfare
  - egalitarian
  - utilitarian

Experimental protocol

- During experimentation, variation of some parameters
  - number of resources
  - number of agents
  - Complexity of the utility function
- **4 steps**
  1. System creation
  2. Exhaustive search of the optimal allocation
  3. Negotiation until no more deal is possible
  4. Change parameters and Go to 1

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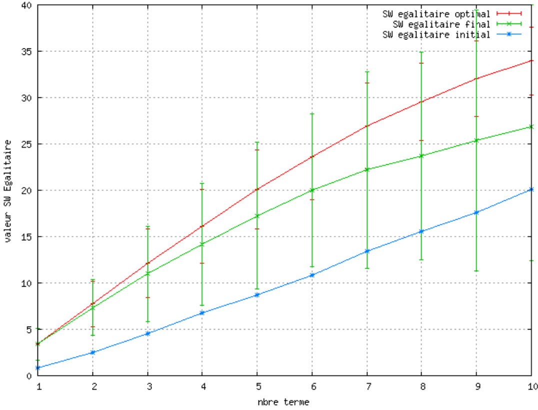
Egalitarian Social Welfare : theorical results

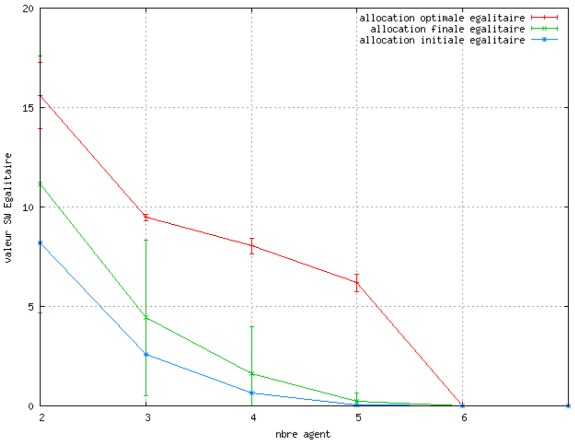
Theorem (Bouveret & al, AAMAS 05)

Even with additive utility, to find the egalitarian optimum is a NP-hard problem.

- $\sharp Resource = 10$
- $\sharp Term = 1..10$
- $\sharp Agent = 2$
- k=1



Egalitarian Social Welfare : experimental results



- $\sharp Resource = 5$
- $\sharp Term = 5$
- $\sharp Agent = 1..7$
- k=1

- When the number of agent rise, the poorest is poor and poor.
- When will Egalitarian Social Welfare = 0?

Condition under which egalitarian SW=0 (1)

Lemma (shortage of resource)

if the number of agents exceeds the number of resources, then one of the agents will be necessarily deprived of resource, in which case $sw_e = 0$

Lemma (generalization to k-additive function)

if an agent requires for at least k resources to have a non-null utility, the egalitarian SW will be null if $k_{min} > \frac{\sharp Resource}{\sharp Agent}$

Example (k=3)

3 agents, 2 resources

$u_1 = r_1$

$u_2 = 3r_1$

$u_3 = 4r_2$

the agents couldn't be satisfied in the same time, so one agent had a utility =0

Condition under which egalitarian SW=0 (2)

Example

$u_1 = r_1 r_2$

$u_2 = r_2 r_3 + r_4$

$u_3 = r_3 r_1 + r_4$

When an agent have a utility > 0, the other has to be 0.

We can see that there exist a blocking situation which is different than shortage of resource.

Limit for Egalitarian Social Welfare

Example (α_{max})

$$u_1 = 2r_1 + 4r_2 \text{ and } u_2 = 1r_1$$
$$\Rightarrow \text{then } \alpha_{max} = 4$$

Lemma (limit with k=1)

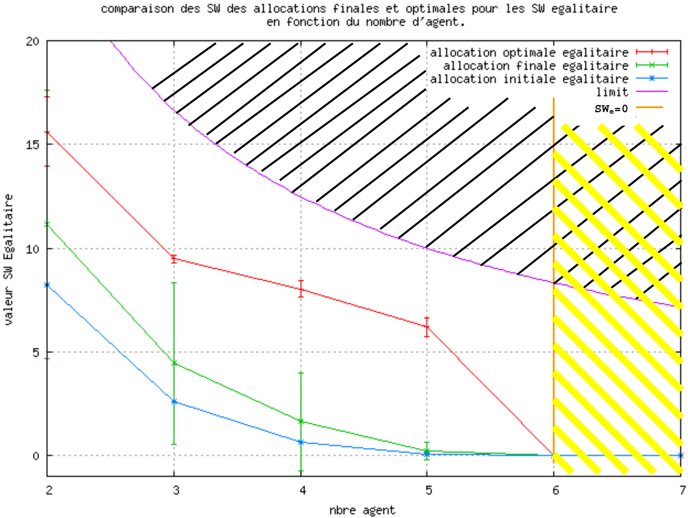
$$sw_e \leq \frac{\sharp Resource}{\sharp Agent} * \alpha_{max}$$
$$sw_e \leq \sharp terms * \alpha_{max}$$

with α_{max} maximal coefficient of all the utility functions

Lemma (limit for all k)

$$sw_e \leq \frac{\sharp Resource.\sharp Term}{k.\sharp Agent} * \alpha_{max}$$

condition under which egalitarian SW=0 and limit



Methodological question

- To what extant does it make sense to assess the egalitarian welfare when payment are allowed?
  - When we are interested only by the quality of the allocation and when payments are only virtual.
  - Run the experiments with a payment function

Example (Uniform payment function)

$$p(i) = u_i(A') - u_i(A) - \frac{sw_u(A') - sw_u(A)}{|\delta^{\mathcal{A}}|} \text{ for i} \in \delta^{\mathcal{A}}$$

Conclusion and Future work

- Conclusion
  - We found a limit and a condition where Egalitarian Social Welfare $= 0$
- Future work :
  - More tests with various parameters
  - Studying Utilitarian social welfare
  - Studying swap deals and other deal types