TiLPS

Expressive Voting: Modelling a Voter's Decision to Vote

Workshop on Logical Models of Group Decision Making

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Content of the talk

- Relationship between Voting Theory and Rational Choice Theory
- Two explanatory schemes for voting: expressive vs. instrumental.
- Expressive voting-based analysis of voting systems
- Discuss a current approach by Gilboa et al. and present an alternative

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Voting and Rational Choice

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- Voting as an instrument to influence outcome Instrumental Account of Voting
- Strategic Considerations prominently studied in voting theory: Gibbard Sattertwaithe

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Why do people vote?

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- See Brennan/Lomasky (1993) for a deeper discussion

G.Brennan and L.Lomasky. *Democracy & Decision*. CUP 1993.

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- Study both kinds of motivations seperately to understand voting behaviour

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Discuss voting systems in an expressive framework

- Majority voting: Voter votes for a single candidate
- Approval voting: Voter picks an arbitrary subset of candidates
- Majority Judgment/Graded voting: Voter gives grades to candidates (1-10)

We

- present a formal Framework of Gilboa, Aragones and Weiss (2011) to compare approval and majority voting under expressive voting
- discuss this approach
- present an alternative framework

E.Aragones, I.Gilboa and A. Weiss. *Making statements and Approval Voting*. Voting Theory and Decision, 71:461-472, 2011.

The framework

- Political debate consists of n-topics $T_1 \dots T_n$.
- Stance on a topic is a number in $[-1:1]$
- every party $\vec{p}$ is a vector in $\{-1;1\}^n$
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Majority Vote

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- *dist* is the euclidean distance
- The party with the most votes gets elected.

Approval voting

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Approval voter: The position of a subset $I \subseteq \mathcal{P}$ is taken to be the straight average of its components:

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in **approval voting** each voter $\vec{v}$ approves of the coalition whose position is closest to his own:

$$\min_{I \subseteq \mathcal{P}} dist(pos(I), \vec{v})$$

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General Question: How much is required to motivate all voters to participate

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- In majority voting, the number of parties required to guarantee that everybody votes is exponential in the number of issues
- In approval voting 4 parties are enough to guarantee that everyone votes
- some stochastic results for number parties $=$ number issues

Our critique

- Implicit coalition making highly improbable
  - Equal weight assumption
  - Discourse is shaped by single winner intuitions

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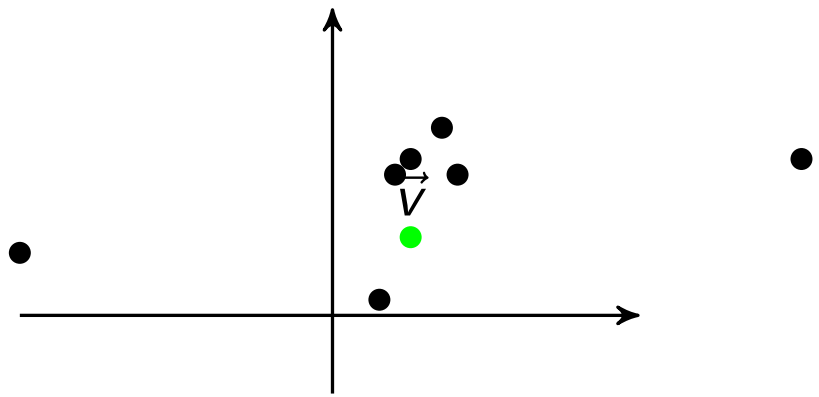

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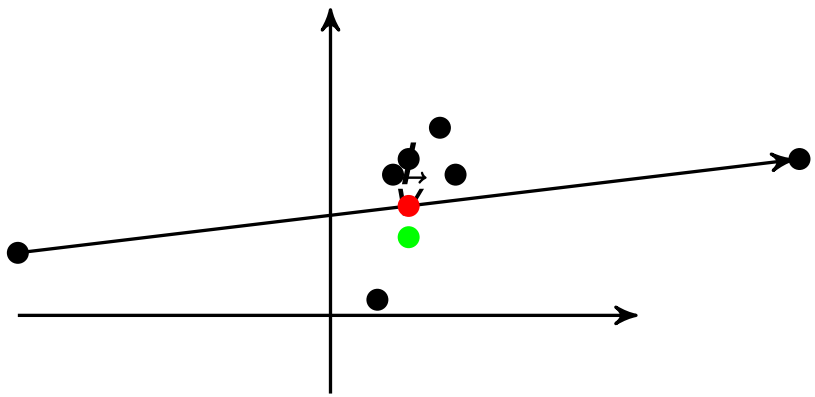

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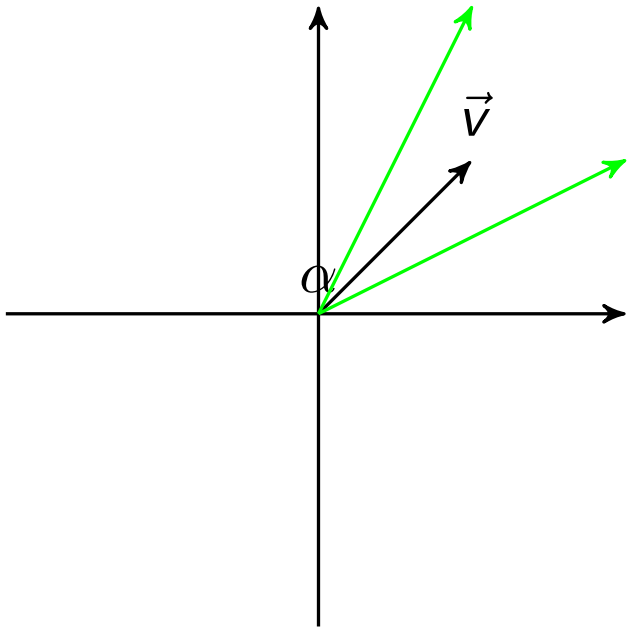
$\vec{v}$ approves of p if

$$\sum p_i v_i \geq k \cdot \sum |v_i|$$

For some threshold $k \in (-1; 1]$. (Typically $k \geq 0$)

Geometric Interpretation

The algebraic definition is equivalent to: Accept a party p if it is within an α-cone round $\vec{v}$



α depends upon n, k and $\vec{p}$.
Holds $\arccos(k) \leq \alpha \leq \arccos(\frac{k}{\sqrt{n}})$

Justification of cone

- Reasoning about individual alternatives: Individual Criterion
- v gives the relative weights of the different positions
- Cone represents level of satisfaction

Remark

Approval and Majority vote are compatible in the following sense:
For any voter $\vec{v}$ and every party $\vec{p}$ holds:

$$p \text{ minimizes } dist(p, v) \text{ iff } p \text{ maximizes } \frac{\vec{p}\vec{v}}{\sum |v_i|}$$

Results

For $k = 0$, i.e. $\alpha = 90°$ we have exactly the same results as in Gilboa et al:

- 4 (resp $2n$) parties are enough to make everyone vote
- For fixed $\vec{v}$ and randomly chosen n parties: $\lim_{n\to\infty} P(\exists \vec{p} | \vec{v}$ approves of $\vec{p}) = 1$
- For $k > 0$ exponentially many parties needed.

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- Plausible/in line with reasoning ✓
- Does not facilitate the election of unfavourable parties ✓
- Easily extendible to grade voting

Extension/Outlook: Focus Dynamics

- Focus of public attention changes over time
- Focus change has bigger impact on electoral outcome than opinion change
- Parties attempt to guide public focus to their areas of expertise
- Relative weights are not intrinsic
- Focus modelled by relative weights

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General Question: Which focus change should a party induce to maximize their electoral outcome?

Wrap up

- Interplay between rational choiche theory and voting theory: Algebraic models as input
- Expressive voting changes discussion of voting systems
- Semantics for approval voting in line with natural intuitions
- Dynamic Aspects: Focus Change

Thank You