Logic and Group Decision Making

Eric Pacuit

Department of Philosophy
University of Maryland, College Park
pacuit.org
epacuit@umd.edu

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An Email

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"Interesting

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"Interesting...but what does logic have to do with group decision making??? I've never seen logic prevail at any of our faculty meetings."

Logic and Group Decision Making

Group decision making from a logicians perspective...

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Group decision making from a logicians perspective...

1. Logical (and algebraic) methods can be used to prove various results (Eckert & Herzberg, Nehring & Pivato)
2. Two non-standard logics for reasoning about social choice
3. A challenge: probabilities in group decision making (Goranko & Bulling)
4. Logics for *social epistemology* (Rendsvig)

Arrow's Theorem

Theorem Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.

K. Arrow. *Social Choice & Individual Values*. 1951.

Broader Applications

- Is it possible to choose rationally among rival scientific theories on the basis of the accuracy, simplicity, scope and other relevant criteria? No

S. Okasha. *Theory choice and social choice: Kuhn versus Arrow*. Mind, 120, 477, pgs. 83 - 115, 2011.

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- Is it possible to choose rationally among rival scientific theories on the basis of the accuracy, simplicity, scope and other relevant criteria? ~~No~~ Yes

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- Is it possible to rationally merge evidence from multiple methods?

J. Stegenga. *An impossibility theorem for amalgamating evidence*. Synthese, 2011.

Broader Applications

- Is it possible to merge classic AGM belief revision with the Ramsey test?

P. Gärdenfors. *Belief revisions and the Ramsey Test for conditionals*. The Philosophical Review, 95, pp. 81 - 93, 1986.

H. Leitgeb and K. Segerberg. *Dynamic doxastic logic: why, how and where to?*. Synthese, 2011.

H. Leitgeb. *A Dictator Theorem on Belief Revision Derived From Arrow's Theorem*. Manuscript, 2011.

Two non-standard logics for reasoning about social choice

D. Osherson and S. Weinstein. *Preference based on reasons*. Review of Symbolic Logic, 2012.

$\varphi \succeq_X \psi$ "The agent considers φ at least as good as ψ for reason X"

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The agent envisions a situation in which φ is true and that otherwise differs little from his actual situation. Likewise she envisions a world where ψ is true and otherwise differs little from his actual situation. Finally, the utility according to u_X of the first imagined situation exceeds that of the second.

p: "i purchases a fire alarm"

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$p \succ_1 \neg p$: u_1 measures safety

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What is the status of $p \succ_{1,2} \neg p$? $p \prec_{1,2} \neg p$?

At a set of atomic proposition, $\mathbb{S}$ a set of **reasons**.

$$\langle W, s, u, V \rangle$$

- W is a set of states
- $s : W \times \wp_{\neq\emptyset}(W) \rightarrow W$ is a selection function ($s(w, A) \in A$)
- $u : W \times \mathbb{S} \rightarrow \mathfrak{R}$ is a utility function
- $V : \mathrm{At} \rightarrow \wp(W)$ is a valuation function

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$\mathcal{M}, w \models \theta \succeq_X \psi$ iff $u_X(s(w, [\![\theta]\!]_{\mathcal{M}})) \geq u_X(s(w, [\![\psi]\!]_{\mathcal{M}}))$ *provided* $[\![\theta]\!]_{\mathcal{M}} \neq \emptyset$ *and* $[\![\psi]\!]_{\mathcal{M}} \neq \emptyset$

Universal Modality is Definable

$$\Diamond\varphi =_{\text{def}} \quad \varphi \succeq_X \varphi$$

$$\Box\varphi =_{\text{def}} \quad \neg(\neg\varphi \succeq_X \neg\varphi)$$

Reflexive: for all w if $w \in A$ then $s(w, A) = w$.

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$\mathcal{M}$ is reflexive implies $(p \succeq_X \top) \vee (\neg p \succeq_X \top)$ is valid.

Regular: Suppose that $A \subseteq B$ and $w_1 \in A$ then, if $s(w, B) = w_1$ then $s(w, A) = w_1$.

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$\mathcal{M}$ is regular implies

$$((p \vee q) \succ_X r) \rightarrow ((p \succ_X r) \vee (q \succ_X r))$$

is valid.

Modeling Social Choice Problems

The set of reasons: $\{1\}, \ldots, \{k\}$, $\{1, 2, \ldots, k\}$.

The signature contains a monadic predicate P.

$Px_1 \succ_i Px_2$: "agent i strictly prefers the object assigned to x_1 over the object assigned to x_2"

$Px_1 \succ_{1,\ldots,k} Px_2$: "society strictly prefers the object assigned to x_1 over the object assigned to x_2"

Universal Domain

Fix a set of variables $x^1, x^2, \ldots, x^m$ (with $m \geq 3$). Let $\chi(x_1, x_2, \ldots, x_m)$ be the formula saying that each of $x_1, \ldots, x_m$ is equal to exactly one of the $x^1, \ldots, x^m$.

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$$\chi(x_1, \ldots, x_m) \wedge (Px_1 \succ_1 Px_2) \wedge \cdots \wedge (Px_{m-1} \succ_1 Px_m)$$

$$\vdots$$

$$\chi(x_1, \ldots, x_m) \wedge (Px_1 \succ_k Px_2) \wedge \cdots \wedge (Px_{m-1} \succ_k Px_m)$$

Let φ_{univ} be the universal closure of $\Diamond\psi$

Pareto

Let φ_{pareto} be the universal closure of the above formula

$$\Box((Px_1 \succ_1 Px_2 \wedge \cdots Px_k \succ_1 Px_k) \rightarrow Px \succ_{1,\ldots,k} Py)$$

IIA

Fix two variables x, y. Let $\psi(x', y')$ be the formula that says each of x', y' is equal to exactly one of x, y. The formula φ_{iia} is the universal closure of:

$$\begin{aligned} (\psi(x_1, y_1) \wedge \cdots \wedge \psi(x_k, y_k)) \to \\ (\Diamond((Px_1 \succ_1 Py_1 \wedge \cdots \wedge (Px_k \succ_k Py_k) \wedge Px \succ_{1,\ldots,k} Py)) \to \\ \Box(((Px_1 \succ_1 Py_1) \wedge \cdots \wedge (Px_k \succ_k Py_k)) \to Px \succ_{1,\ldots,k} Py)) \end{aligned}$$

Dictator

Let $\varphi_{dictator}$ be the disjunction of:

$$\forall x_1 \cdots x_m \Box(((Px_1 \succ_1 Px_2) \wedge (Px_2 \succ_1 Px_3) \cdots (Px_{m-1} \succ_1 Px_m)) \leftrightarrow ((Px_1 \succ_{1,\ldots k} Px_2) \wedge (Px_2 \succ_{1,\ldots k} Px_3) \wedge \cdots (Px_{m-1} \succ_{1,\ldots k} Px_m)))$$

$$\vdots$$

$$\forall x_1 \cdots x_m \Box(((Px_1 \succ_k Px_2) \wedge (Px_2 \succ_k Px_3) \cdots (Px_{m-1} \succ_k Px_m)) \leftrightarrow ((Px_1 \succ_{1,\ldots k} Px_2) \wedge (Px_2 \succ_{1,\ldots k} Px_3) \wedge \cdots (Px_{m-1} \succ_{1,\ldots k} Px_m)))$$

Arrow's Theorem

$$\{\varphi_{univ}, \varphi_{pareto}, \varphi_{iia}\} \models \varphi_{dictator}$$

Dependence Logic

J. Väänänen. *Dependence Logic*. Cambridge University Press, 2007.

E. Grädel and J. Väänänen. *Dependence and Independence*. Studia Logica, vol. 101(2), pp. 399-410, 2013.

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A **substitution** is a function $s : \mathcal{V} \to D$.

A **team** X is a set of substitutions.

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$X \models {=}(x_1, \ldots, x_n, y)$ iff for all $s, s' \in X$,
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$X \models (x_1, \ldots x_n) \perp y$ iff for all $s, s' \in X$, there exists $s'' \in X$ such that $s''(x_1, \ldots, x_n) = s(x_1, \ldots, x_n)$ and $s''(y) = s'(y)$

- $\mathcal{M}, X \models x = y$ iff for all $s \in X$, $s(x) = s(y)$

- $\mathcal{M}, X \models \neg x = y$ iff for all $s \in X$, $s(x) \neq s(y)$

- $\mathcal{M}, X \models R(x_1, \ldots, x_n)$ iff for all $s \in X$, $(s(x_1), \ldots, s(x_n)) \in R^{\mathcal{M}}$

- $\mathcal{M}, X \models \neg R(x_1, \ldots, x_n)$ iff for all $s \in X$, $(s(x_1), \ldots, s(x_n)) \notin R^{\mathcal{M}}$

- $\mathcal{M}, X \models \varphi \wedge \psi$ iff $\mathcal{M}, X \models \varphi$ and $\mathcal{M}, X \models \psi$

- $\mathcal{M}, X \models \varphi \vee \psi$ iff there are X_1, X_2 such that $X = X_1 \cup X_2$ and $\mathcal{M}, X_1 \models \varphi$ and $\mathcal{M}, X_2 \models \psi$.

- $\mathcal{M}, X \models \exists x \varphi$ iff $\mathcal{M}, X' \models \varphi$ for some X' such that for all $s \in X$, there is a $d \in D$ such that $s[x/d] \in X'$.

- $\mathcal{M}, X \models \forall x \varphi$ iff $\mathcal{M}, X' \models \varphi$ for some X' such that for all $s \in X$, for all $d \in D$, $s[x/d] \in X'$

Dependence Logic Formalization

Voters are variables $x_1, x_2, \ldots, x_n$

Society's Ranking is the variable y

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Profiles are assignments ($s : \{x_1, \ldots, x_n, y\} \to \mathcal{P}$), where $\mathcal{P}$ is the set of preferences over a set.

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A team is a set of profiles (the "constitution")

$P_{ab}(s(x))$ is true if $s(x)$ ranks a strictly above b (similarly for weak preference R and indifference I).

J. Väänänen. *Introduction to Dependence Logic*. Dagstuhl Workshop on Dependence and Independence, 2013.

To state Arrow's Theorem (and other social choice results), we only need *propositional dependence*:

$=\!(\varphi_1, \ldots, \varphi_n, \psi)$ (the truth of ψ *depends* on the truth of $\varphi_1, \ldots, \varphi_n$).

Unanimity

If each agent ranks a above b, then so does the social welfare function

DL formula φ_{unam}: $\bigwedge_i P_{ab}(x_i) \to P_{ab}(y)$

Universal Domain

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$X \models \forall x_i$ iff for all $R \in \mathcal{P}$, there is an $s \in X$, such that $s(x_i) = R$

DL formula φ_{univ1}: $\forall x_1 \wedge \cdots \wedge \forall x_n$

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DL formula φ_{univ1}: $\forall x_1 \wedge \cdots \wedge \forall x_n$

DL formula φ_{univ2}: $\{x_j \mid j \neq i\} \perp x_i$

Independence of Irrelevant Alternatives

The social relative ranking (higher, lower, or indifferent) of two alternatives a and b depends only the relative rankings of a and b for each individual.

DL formula φ_{iia}: $=(R_{ab}(x_1), \ldots, R_{ab}(x_n), R_{ab}(y))$

Dictatorship

There is an individual $d \in \mathcal{A}$ such that the society strictly prefers a over b whenever d strictly prefers a over b.

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DL formula $\varphi_{dictator}$: $=\!(P_{ab}(x_d), P_{ab}(y))$

$$\{\varphi_{univ1}, \varphi_{univ2}, \varphi_{pareto}, \varphi_{iia}\} \models \varphi_{dictator}$$

Independence?

If for each $i \in \mathcal{A}$, aR_ib iff aR'_ib, then $aF(\vec{R})b$ iff $aF(\vec{R}')b$.

Two profiles p and q agree on a set B provided $p_i = q_i$ on B (i.e., the preferences are restricted to candidates in B) for each voter i.

(full) IIA: every set B is independent,

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Theorem (Blau) If there are at least $m+1$ candidates, then m-ary implies $m-1$-ary

Theorem. Arrow's Theorem can be provided under these weaker conditions: If $|X| > m > 1$, then Universal Domain, Unanimity, and m-ary implies that the social welfare function is a dictatorship.

A challenge: probabilities in group decision making

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Probabilities in group decision making:

1. Linear pooling
2. Stochastic choice

K. McConway. *Marginalization and Linear Opinion Pools*. Journal of the American Statistical Association, 76:374, pgs. 410 - 414, 1981.

Suppose there are n agents who have assessed distributions $\pi_1, \ldots, \pi_n$ over a space Ω.

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Let S be a σ-algebra over Ω, then $\pi_i : S \rightarrow [0, 1]$ (satisfying the usual Kolmogrov axioms). Let $\Delta(S)$ be the set of all probability measures on S. Let Σ be the set of all σ-algebras over Ω.

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For a σ-algebra S, a **consensus function** is a map $C_S : \Delta(S)^n \rightarrow \Delta(S)$.

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For a σ-algebra S, a **consensus function** is a map $C_S : \Delta(S)^n \to \Delta(S)$.

Linear Pooling: $C_S(A) = \sum_1^n \alpha_i \pi_i(A)$ for each $A \in S$, where the weights α_i are non-negative and sum to 1.

Pareto: For all $S \in \Sigma$, for all $\pi_1, \ldots, \pi_n \in \Delta(S)$ and for all $A \in S$,
If $\pi_1(A) = \pi_2(A) = \cdots = \pi_n(A) = 0$, then $C_S(\pi_1, \ldots, \pi_n)(A) = 0$

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Weak setwise function property (Independence): Suppose that Q is $\wp(\Omega) - \{\emptyset, \Omega\} \times [0,1]^n \cup \{(\emptyset, 0, \ldots, 0), (\Omega, 1, \ldots, 1)\}$. There exists a function $F : Q \to [0,1]$ such that for all $S \in \Sigma$,

$$C_S(\pi_1 \ldots, \pi_n)(A) = F(A, \pi_1(A), \ldots, \pi_n(A))$$

for all $A \in S$ and $\pi_1, \ldots, \pi_n \in \Delta(S)$.

Strong setwise function property (Systematicity): There exists a function $G : [0,1]^n \rightarrow [0,1]$ such that for all $S \in \Sigma$,

$$C_S(\pi_1 \ldots, \pi_n)(A) = G(\pi_1(A), \ldots, \pi_n(A))$$

for all $A \in S$ and $\pi_1, \ldots, \pi_n \in \Delta(S)$.

Theorem. The following are equivalent: (a) The consensus function satisfies Pareto and independence and (b) The consensus function satisfies systematicity.

Theorem. If there are at least three distinct points in Ω, then for a class of consensus functions the following are equivalent

a. The class satisfies systematicity
b. There exists real numbers $\alpha_1, \ldots, \alpha_n$ that are non-negative and sum to 1 such that for all $S \in \Sigma$, all $A \in S$ and $\pi_1, \ldots, \pi_n \in \Delta(S)$,

$$C_S(\pi_1, \ldots, \pi_n)(A) = \sum_{i=1}^{n} \alpha_i \pi_i(A)$$

General Aggregation Theory

F. Dietrich and C. List. *The aggregation of propositional attitudes: Towards a general theory*. Oxford Studies in Epistemology, Vol. 3, pgs. 215 - 234, 2010.

F. Herzberg. *Universal algebra for general aggregation theory: Many-valued propositional-attitude aggregators as MV-homomorphisms*. Journal of Logic and Computation, 2013.

T. Daniëls and EP. *A general approach to aggregation problems*. Journal of Logic and Computation, 19, pgs. 517 - 536, 2009.

M. Intriligator. *A Probabilistic Model of Social Choice*. The Review of Economic Studies, 40:4, pgs. 553 - 560, 1973.

Stochastic Choice

$\mathbf{q}_i = (q_{i1}, \ldots, q_{in})$ such that for all i, j $q_{ij} \geq 0$ and for all i, $\sum_{j=1}^{n} q_{ij} = 1$

q_{ij} is the probability that agent i would choose alternative A_j if he could act alone in deciding among the alternatives.

D. Luce. *A Probabilistic Theory of Utility*. Econometrica, 26, pgs. 193 - 224, 1958.

$$\mathbf{p} = (p_1, \ldots, p_n) \text{ such that for all } j \ p_j \geq 0 \text{ and } \sum_{ij=1}^{n} p_j = 1$$

p_i is the probability that society will choose alternative A_i

Universal Domain: Given any set of individual probabilities, the rule specifies a unique set of social probabilities.

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Unanimity of Loser: If all individuals reject an alternative then so does society.

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Unanimity of Loser: If all individuals reject an alternative then so does society. If $q_{ij_0} = 0$ for all i, then $p_{j_0} = 0$.

Strict Sensitivity to Individual Probabilities: Social probabilities are strictly sensitive to the changes in individual probabilities and all agents are treated equally.

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$$p_j = f_j(q_{11}, \ldots q_{m1}, \ldots, q_{1j}, \ldots, q_{mj}, \ldots q_{1n}, \ldots, q_{mn})$$

$$\frac{\partial f_j}{\partial q_{ik}} = \begin{cases} \mu_j \neq 0 & \text{if } k = j \\ 0 & \text{if } k \neq j \end{cases}$$

Average Rule: For all j,

$$p_j = \frac{1}{m}\sum_{i=1}^{m} q_{ij}$$

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Theorem. The average rule is the only rule satisfying universal domain, unanimity of a loser and strict sensitivity to individual probabilities.

Logics for *social epistemology*

“Wisdom” of the Crowd

A. Lyon and EP. *The Wisdom of Crowds: Methods of Human Judgement Aggregation*. The Handbook of Human Computation, 2013.

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- The power of averaging (Diversity Theorem)
- Dynamics of group deliberation (information cascades, anchoring effect, "common knowledge" effect)
- Prediction markets (Combinatorial markets: bets are made on events of the form "horse A will win" rather than "horse A will beat horse B which will beat horse C", "horse A will win and horse B will come in third" or "horse A will win if horse B comes in second")

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Group decision making from a logicians perspective...

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2. Two non-standard logics for reasoning about social choice
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Thank you!