Coursework #1

Deadline: Tuesday, 4 March 2008, 11:00am

Question 1 (10 marks)

A social welfare function is said to satisfy the axiom of *non-imposition* (NI) if any social preference ordering is achievable by *some* profile of individual preference orderings:

$$(\forall P \in \mathcal{P})(\exists \boldsymbol{P}' \in \mathcal{P}^n)(\forall x, y \in A)[xPy \leftrightarrow xP'y]$$

In other words, a social welfare function satisfying (NI) does not impose any restrictions that would *a priori* exclude a particular social preference ordering.

(a) Show that the Pareto condition (P) implies (NI).

(b) Show that Arrow's Theorem breaks down if we replace (P) by (NI).

(Adapted from A.D. Taylor, *Social Choice and the Mathematics of Manipulation*, Cambridge University Press, 2005.)

Question 2 (10 marks)

Some of the problems in social choice theory (impossibility results, paradoxes) can be circumvented if we make special assumptions regarding the preferences of individuals. An example is the condition of single-peakedness, which allows us to get around the Condorcet paradox. Let $<$ be a fixed linear ordering over the set of candidates. You may think of $<$ as ordering the candidates from left to right according to their political views. Then the preference ordering $\prec$ of a voter is called *single-peaked* iff (1) that voter has a most preferred candidate c^* and (2) $c \prec c'$ whenever $c < c' < c^*$ or $c^* < c' < c$.

Now suppose that all voters have single-peaked preferences and that the number of voters is odd. Prove that there must be a Condorcet winner.

Question 3 (10 marks)

Prove that the Copeland rule is easy to manipulate. This is in fact a corollary to a more general result by Bartholdi, Tovey and Trick (1989). Do not refer to their general result in your answer, but rather give a direct proof for the Copeland rule only.

(See J.J. Bartholdi III, C.A. Tovey, and M.A. Trick. The Computational Difficulty of Manipulating an Election. *Social Choice and Welfare*, 6(3):227–241, 1989.)